Year 12 Mathematics Extension 2

Topic: The Nature of Proof (2 of 5)
Review Questions: Illustrations of proofs
Term 3, 2026

How to use this review set

These are real past-paper questions, grouped by question pattern. The grey label on each question shows where it is from and its marks.

- The questions are grouped by the pattern that keeps coming up in exams, a few of each in a row, running from the plainer patterns into the harder ones near the end. Work through them and you will meet every type an exam can throw at you.
- This set **is your diagnostic**. Working through it shows you exactly which patterns are solid and which need work. Attempt every question properly in your own book, and go back to your notes to help you work through tough questions if needed.
- Every question you get wrong, need help with or struggle with goes in your **Mistakes Log**, which you should be working on daily to get you exam-ready.

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Direct proofs about integers

1. Proving number and parity properties

Q1 2025 Pymble Trial Q11(e) · **2 marks**

Prove algebraically that the difference between two consecutive square numbers is odd.

Q2 2025 Penrith Trial Q11b · **2 marks**

Suppose that $a, b \in \mathbb{Z}$. If ab and $a - b$ are both even, prove that a and b are both even.

Q3 2025 Newington Trial Q12a · **2 marks**

Prove that $8^n - 1$ is a composite number for all positive integers $n > 1$.

Q4 2025 Hills Grammar Trial Q12c · **3 marks**

Prove that $(n + 1)^n - n^{n+1}$ is odd for all positive integers n.

Q5 2025 Riverview Trial Q12e · **2 marks**

Show that for $x \in \mathbb{Z}$, if $x^2 - 6x + 5$ is even, then x is odd.

Q6 2025 Caringbah Trial Q13d · **3 marks**

Prove that the product of any three consecutive numbers is even.

Q7 2025 James Ruse Trial Q12a · **3 marks**

Prove for all integers n, that n^2 is odd if and only if n is odd.

Q8 2025 Sydney Girls Trial Q14c · **3 marks**

Let a and b be consecutive positive integers with $a < b$.

By considering the expansion of $(a^2 + a + 1)^2$, prove that if $c = ab$, then $\sqrt{a^2 + b^2 + c^2}$ is an odd integer.

2. Proving divisibility results

Q9 2025 Baulkham Hills Trial Q13b · **2 marks**

Prove, without the use of mathematical induction, that $5^{2n} - 1$ is divisible by 8, for all positive integers n.

Q10 2024 HSC Q14a · **2 marks**

Prove that if a is any odd integer, then $a^2 - 1$ is divisible by 8.

Q11 2020 HSC Q15(a(i)) · **2 marks**

In the set of integers, let P be the proposition:

$$\text{“If } k + 1 \text{ is divisible by 3, then } k^3 + 1 \text{ is divisible by 3.”}$$

Prove that the proposition P is true.

Q12 2025 Merewether Trial Q12a · **2 marks**

Prove that, if n is an even integer, then $n^3 - n$ is divisible by 6.

Q13 2025 Killara Trial Q11c · **2 marks**

Prove that if $n \in \mathbb{Z}$, then $n^2 + 2$ is not divisible by 4.

Q14 2025 Riverview Trial Q14a · **2 marks**

Suppose that n and $n + 1$ are positive integers, neither of which is divisible by 3. Prove that $n^3 + (n + 1)^3$ is divisible by 9.

Q15 2025 Penrith Trial Q14a · **3 marks**

Prove that a three-digit number is divisible by 3 if and only if the sum of its digits is divisible by 3.

Disproof, contradiction and irrationality

3. Disproof by counterexample

Q16 2025 Sydney Girls Trial Q11a · 1 mark

Give a counterexample to disprove the following statement:

$$\text{For all } x, y \in \mathbb{R}, \text{ if } x^2 \geq y^2, \text{ then } x \geq y.$$

Q17 2025 Pymble Trial Q1 · 1 mark

Consider the proposition:

"Only the diagonals of rectangles are equal".

Which of the following statements is a counterexample to this proposition?

(A) The diagonals of a rhombus are equal.

(B) The diagonals of a kite are equal.

(C) The diagonals of a hexagon are equal.

(D) The diagonals of an isosceles trapezium are equal.

Q18 2021 HSC Q5 · 1 mark

Which of the following statements is FALSE?

(A) $\forall a, b \in \mathbb{R}, \quad a < b \Rightarrow a^3 < b^3$

(B) $\forall a, b \in \mathbb{R}, \quad a < b \Rightarrow e^{-a} > e^{-b}$

(C) $\forall a, b \in (0, +\infty), \quad a < b \Rightarrow \ln a < \ln b$

(D) $\forall a, b \in \mathbb{R}, \text{ with } a, b \neq 0, \quad a < b \Rightarrow \dfrac{1}{a} > \dfrac{1}{b}$

Q19 2025 Gosford Trial Q11b · 2 marks

Prove or disprove the statement $3^n + 2$ is prime for all integers $n \geq 0$.

Q20 2020 HSC Q7 · **1 mark**

Consider the proposition:

'If $2^n - 1$ is not prime, then n is not prime'.

Given that each of the following statements is true, which statement disproves the proposition?

(A) $2^5 - 1$ is prime

(B) $2^6 - 1$ is divisible by 9

(C) $2^7 - 1$ is prime

(D) $2^{11} - 1$ is divisible by 23

Q21 2025 St George Girls Trial Q13e · **2 marks**

Prove or disprove the following statement:

$$\forall x : x = 2k + 1 \ (k \in \mathbb{Z}), \quad \exists a, b \in \mathbb{Z} : a^2 - b^2 = x.$$

4. Proof by contradiction

Q22 2025 Gosford Trial Q11g · **2 marks**

Prove by contradiction that there are no integers a, b such that $25a + 15b = 1$.

Q23 2025 Hornsby Girls Trial Q12d · **2 marks**

Prove by contradiction that if a and b are both positive integers then

$$\frac{a}{b} + \frac{b}{a} \geq 2.$$

Q24 2025 HSC Q11e · **2 marks**

Prove by contradiction that $\sqrt{3} + \sqrt{5} > \sqrt{11}$.

Q25 2025 Manly Trial Q13b · **2 marks**

Use a proof by contradiction to prove that there are no positive integers p and q such that $p^2 - q^2 = 2$.

Q26 2025 Baulkham Hills Trial Q12a · **3 marks**

Prove, by the method of contradiction, that there are no integers a and b which satisfy $a^2 - 8b = 7$.

Q27 2025 Sydney Tech Trial Q13a · **3 marks**

Prove by contradiction that there are an infinite number of prime numbers.

Q28 2025 Newington Trial Q16b · **4 marks**

Use proof by contradiction to prove that if $a^2 + b^2 = c^2$ for positive integers a, b and c, then a and b cannot both be odd.

5. Proving irrationality

Q29 2025 Abbotsleigh Trial Q12d · **2 marks**

Prove that $\log_2 9$ is irrational.

Q30 2025 Kinross Wolaroi Trial Q11d · **3 marks**

Prove that $\sqrt{2}$ is irrational by contradiction.

Q31 2023 HSC Q12a · **3 marks**

Prove that $\sqrt[3]{2}$ is irrational.

Q32 2025 Cheltenham Girls Trial Q11b · **3 marks**

Prove by contradiction that $\sqrt{5}$ is irrational.

Q33 2025 Penrith Trial Q12a · **3 marks**

Prove by contradiction that $\log_4 9$ is an irrational number.

Q34 2020 HSC Q14(d) · **3 marks**

Prove that for any integer $n > 1$, $\log_n(n+1)$ is irrational.