Year 12 Mathematics Extension 2

Topic: The Nature of Proof (1 of 5)
Review Questions: The language and notation of proof
Term 3, 2026

How to use this review set

These are real past-paper questions, grouped by question pattern. The grey label on each question shows where it is from and its marks.

- The questions are grouped by the pattern that keeps coming up in exams, a few of each in a row, running from the plainer patterns into the harder ones near the end. Work through them and you will meet every type an exam can throw at you.
- This set **is your diagnostic**. Working through it shows you exactly which patterns are solid and which need work. Attempt every question properly in your own book, and go back to your notes to help you work through tough questions if needed.
- Every question you get wrong, need help with or struggle with goes in your **Mistakes Log**, which you should be working on daily to get you exam-ready.

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Statements, negation and quantifiers

1. Negating statements

Q1 2025 HSC Q2 · **1 mark**

Consider the statement:

$$\exists x \in \mathbb{Z}, \text{ such that } x^2 \text{ is odd.}$$

Which of the following is the negation of the statement?

(A) $\forall x \in \mathbb{Z},\ x^2$ is odd

(B) $\forall x \in \mathbb{Z},\ x^2$ is even

(C) x^2 is even $\quad x \in \mathbb{Z}$

(D) $\exists x \in \mathbb{Z}, \text{ such that } x^2$ is even

Q2 2025 North Sydney Boys Trial Q2 · **1 mark**

Which of the following is the negation of the statement

If I know how to negate this statement, then I will select the correct answer.

(A) If I don't know how to negate this statement, then I will not select the correct answer

(B) I know how to negate this statement and I will not select the correct answer

(C) If I do not select the correct answer, then I do not know how to negate this statement

(D) I do not know how to negate this statement and I will select the correct answer

Q3 2025 Cheltenham Girls Trial Q2 · **1 mark**

Which of the following is the negation of "$y > 6$ or $y \leq -2$"?

(A) $-2 < y < 6$

(B) $-2 \leq y < 6$

(C) $-2 < y \leq 6$

(D) $-2 \leq y \leq 6$

Q4 2025 Sydney Girls Trial Q11b · **1 mark**

Write down the negation of the following statement:

$$\text{There exists } k \in \mathbb{R} \text{ such that } k^2 < 0.$$

Q5 2025 Trinity Grammar Trial Q9 · **1 mark**

Which of the following is the negation of $\forall\, x \in \mathbb{R}^+,\ \exists\, y \in \mathbb{R}^+ : xy = 1$?

(A) $\exists\, x, y \in \mathbb{R}^+ : xy \neq 1$

(B) $\forall\, x, y \in \mathbb{R}^+,\ xy \neq 1$

(C) $\exists\, x \in \mathbb{R}^+ : \forall\, y \in \mathbb{R}^+,\ xy \neq 1$

(D) $\forall\, x \in \mathbb{R}^+,\ \exists\, y \in \mathbb{R}^+ : xy \neq 1$

Q6 2025 Manly Trial Q3 · **1 mark**

Consider the following statement: $\forall\, x, y \in \mathbb{R},\ xy \leq 9 \Rightarrow x \leq 3$ or $y \leq 3$. Which of the following is the negation of this statement?

(A) $\forall\, x, y \in \mathbb{R},\ x \leq 3$ or $y \leq 3 \Rightarrow xy \leq 9$

(B) $\forall\, x, y \in \mathbb{R},\ x > 3$ and $y > 3 \Rightarrow xy > 9$

(C) $\exists\, x, y \in \mathbb{R},$ with $x > 3$ or $y > 3$ such that $xy \leq 9$

(D) $\exists\, x, y \in \mathbb{R},$ with $x > 3$ and $y > 3$ such that $xy \leq 9$

Q7 2025 Killara Trial Q3 · **1 mark**

What is the negation of the statement $\forall\, x \in \mathbb{Z},\ \exists\, y \in \mathbb{Z},\ y \geq x$?

(A) $\forall\, x \in \mathbb{Z},\ \nexists\, y \in \mathbb{Z},\ y > x$

(B) $\forall\, x \in \mathbb{Z},\ \nexists\, y \in \mathbb{Z},\ y < x$

(C) $\exists\, x \in \mathbb{Z},\ \exists\, y \in \mathbb{Z},\ y < x$

(D) $\exists\, x \in \mathbb{Z},\ \forall\, y \in \mathbb{Z},\ y < x$

Q8 2020 HSC Q8 · **1 mark**

Consider the statement:

'If n is even, then if n is a multiple of 3, then n is a multiple of 6'.

Which of the following is the negation of this statement?

(A) n is odd and n is not a multiple of 3 or 6.

(B) n is even and n is a multiple of 3 but not a multiple of 6.

(C) If n is even, then n is not a multiple of 3 and n is not a multiple of 6.

(D) If n is odd, then if n is not a multiple of 3 then n is not a multiple of 6.

2. Quantifiers and logical notation

Q9 2025 Gosford Trial Q1 · **1 mark**

Which of the following statements is a correct interpretation of

$$\forall a \in \mathbb{Z},\ \exists b \in \mathbb{Z} \text{ such that } \frac{a}{b} \in \mathbb{Z}\ ?$$

(A) Every integer is also a rational number.

(B) There exists a rational number that corresponds to each pair of integers.

(C) Every rational number is also an integer.

(D) Every integer has an integer factor.

Q10 2025 Penrith Trial Q3 · **1 mark**

Consider the statement:

"Whichever positive number r you pick, it is possible to find a number x greater than zero such that $\dfrac{1}{x^2} < r$."

When this statement is written in a formal language of proof, which of the following is obtained?

(A) $\forall r > 0,\ \exists x > 0$ such that $\dfrac{1}{x^2} < r$

(B) $\exists r > 0,\ \exists x > 0$ such that $\dfrac{1}{x^2} < r$

(C) $\forall r > 0,\ \forall x > 0$ such that $\dfrac{1}{x^2} < r$

(D) $\exists r > 0,\ \forall x > 0$ such that $\dfrac{1}{x^2} < r$

Q11 2024 HSC Q2 · **1 mark**

Consider the following statement written in the formal language of proof.

$$\forall \theta \in \left(\tfrac{\pi}{2},\ \pi\right),\ \exists \varphi \in \left(\pi,\ \tfrac{3\pi}{2}\right),\ \sin\theta = -\cos\varphi.$$

Which of the following best represents this statement?

(A) There exists a θ in the second quadrant such that for all φ in the third quadrant $\sin\theta = -\cos\varphi$.

(B) There exists a φ in the third quadrant such that for all θ in the second quadrant $\sin\theta = -\cos\varphi$.

(C) For all θ in the second quadrant there exists a φ in the third quadrant such that $\sin\theta = -\cos\varphi$.

(D) For all φ in the third quadrant there exists a θ in the second quadrant such that $\sin\theta = -\cos\varphi$.

Q12 2023 HSC Q4 · **1 mark**

Consider the following statement about real numbers.

"Whichever positive number r you pick, it is possible to find a number x greater than 1 such that $\dfrac{\ln x}{x^3} < r$."

When this statement is written in the formal language of proof, which of the following is obtained?

(A) $\forall x > 1,\ \exists r > 0,\ \dfrac{\ln x}{x^3} < r$

(B) $\exists x > 1,\ \forall r > 0,\ \dfrac{\ln x}{x^3} < r$

(C) $\forall r > 0,\ \exists x > 1,\ \dfrac{\ln x}{x^3} < r$

(D) $\exists r > 0,\ \forall x > 1,\ \dfrac{\ln x}{x^3} < r$

Q13 2025 Hurlstone Trial Q6 · **1 mark**

Which of the following statements is true?

(A) $\forall a, b \in \mathbb{R},\ \sin a = \sin b \Rightarrow a = b$

(B) $\forall a, b \in \mathbb{R},\ |a + b| > |a - b|$

(C) $\exists a, b \in \mathbb{R}$ such that $\log_a(a + b) = \log_a(ab)$

(D) $\exists a, b \in \mathbb{C},\ |a + b| > |a| + |b|$

Q14 2025 James Ruse Trial Q7 · **1 mark**

Consider the following propositions:

$$P: \quad \forall x > 0,\ \exists n \in \mathbb{Z} \text{ such that } x > \frac{1}{n}$$

$$Q: \quad \exists n \in \mathbb{Z} \text{ such that } \forall x > 0,\ x > \frac{1}{n}$$

Which of the following is false?

(A) $Q \Rightarrow \neg P$

(B) $\neg P \Rightarrow Q$

(C) $\neg Q \Rightarrow P$

(D) $P \Rightarrow Q$

Converse, contrapositive and proof

3. Converse and contrapositive: forming and identifying

Q15 2024 HSC Q3 · 1 mark

Consider the statement:

'If a polygon is a square, then it is a rectangle.'

Which of the following is the converse of the statement above?

(A) If a polygon is a rectangle, then it is a square.

(B) If a polygon is a rectangle, then it is not a square.

(C) If a polygon is not a rectangle, then it is not a square.

(D) If a polygon is not a square, then it is not a rectangle.

Q16 2021 HSC Q4 · 1 mark

Consider the statement:

'For all integers n, if n is a multiple of 6, then n is a multiple of 2'.

Which of the following is the contrapositive of the statement?

(A) There exists an integer n such that n is a multiple of 6 and not a multiple of 2.

(B) There exists an integer n such that n is a multiple of 2 and not a multiple of 6.

(C) For all integers n, if n is not a multiple of 2, then n is not a multiple of 6.

(D) For all integers n, if n is not a multiple of 6, then n is not a multiple of 2.

Q17 2023 HSC Q2 · **1 mark**

Consider the statement:

'If an animal is a herbivore, then it does not eat meat.'

Which of the following is the converse of this statement?

(A) If an animal is a herbivore, then it eats meat.

(B) If an animal is not a herbivore, then it eats meat.

(C) If an animal eats meat, then it is not a herbivore.

(D) If an animal does not eat meat, then it is a herbivore.

Q18 2025 Girraween Trial Q4 · **1 mark**

The converse of "If the sum of the digits of an integer is a multiple of 3, the integer is a multiple of 3" is

(A) "If an integer is a multiple of 3 the sum of its digits will be a multiple of 3".

(B) "If an integer is not a multiple of 3, the sum of its digits will NOT be a multiple of 3".

(C) "If the sum of the digits of an integer is NOT a multiple of 3, the number will NOT be a multiple of 3".

(D) "If the sum of the digits of an integer is NOT a multiple of 3, the integer is a multiple of 3".

Q19 2025 Baulkham Hills Trial Q1 · **1 mark**

Consider the following statement:

"If my cricket team plays badly, they are not training enough".

Which one of the following statements is the contrapositive of the statement above?

(A) If they are not training enough, then my cricket team plays badly.

(B) If they are training enough, then my cricket team does not play badly.

(C) If my cricket team doesn't play badly, then they are training enough.

(D) If my cricket team plays badly, then they need more training.

Q20 2025 Ascham Trial Q6 · **1 mark**

For which of these statements, for positive integers x and y, is the converse necessarily true?

(A) If x and y are odd, then $x + y$ is even.

(B) If x and y are odd, then $x^2 + y^2$ is even.

(C) If x and y are odd, then xy is odd.

(D) If x and y are odd, then x^y is odd.

Q21 2022 HSC Q7 · **1 mark**

Consider the statement P.

$$P: \text{ For all integers } n \geq 1, \text{ if } n \text{ is a prime number then } \frac{n(n+1)}{2} \text{ is a prime number.}$$

Which of the following is true about this statement and its converse?

(A) The statement P and its converse are both true.

(B) The statement P and its converse are both false.

(C) The statement P is true and its converse is false.

(D) The statement P is false and its converse is true.

4. Truth, equivalence and logical puzzles

Q22 2025 Sydney Tech Trial Q5 · **1 mark**

Consider the statement "If a quadrilateral is a parallelogram, then it is a rhombus". Which of the following is true?

(A) The statement and its converse are true.

(B) Neither the statement nor its converse is true.

(C) The statement is true and its converse is false.

(D) The statement is false and its converse is true.

Q23 2025 Newington Trial Q6 · **1 mark**

Consider the following statement concerning positive integers $n > 1$:

If n is odd, then n can be written as $x^2 - y^2$ for positive integers x and y.

Which of the following is correct?

(A) The contrapositive statement is false and the converse statement is false.

(B) The contrapositive statement is true and the converse statement is false.

(C) The contrapositive statement is false and the converse statement is true.

(D) The contrapositive statement is true and the converse statement is true.

Q24 2025 Normanhurst Boys Trial Q10 · **1 mark**

Consider the statement:

Period 0 is cancelled if and only if there is an excursion.

Which of the following statements is FALSE?

(A) The negation is an 'or' statement.

(B) The negation is 'Period 0 being cancelled does not imply that there is an excursion'.

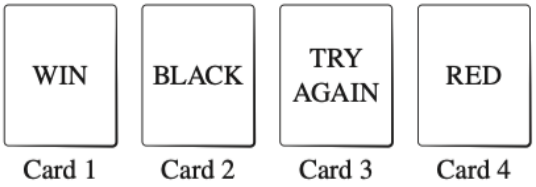
(C) A counterexample is 'There is no excursion and Period 0 is cancelled'.

(D) A counterexample is 'Period 0 is not cancelled and there is an excursion'.

Q25 2021 HSC Q9 · **1 mark**

Four cards have either RED or BLACK on one side and either WIN or TRY AGAIN on the other side.

Sam places the four cards on the table as shown below.



A statement is made: 'If a card is RED, then it has WIN written on the other side'.

Sam wants to check if the statement is true by turning over the minimum number of cards.

Which cards should Sam turn over?

(A) 1 and 4

(B) 3 and 4

(C) 1, 2 and 4

(D) 1, 3 and 4

Q26 2025 Killara Trial Q9 · **1 mark**

There are 4 trolls on the bridge.

- Troll 1: Every troll can see at least 1 Knave.
- Troll 2: I can see at least 1 troll that can only see Knaves.
- Troll 3: All trolls are afraid of goats.
- Troll 4: Some trolls are afraid of goats.

Knowing that all Trolls are either knights (always tell the truth) or Knaves (always Lie). What can be deduced?

(A) Three Trolls are Knaves

(B) No trolls are afraid of goats

(C) Some trolls are afraid of goats

(D) All trolls are afraid of goats

5. Proof using the contrapositive

Q27 2025 Abbotsleigh Trial Q11b · **2 marks**

Consider the statement about integers

$$\text{If } k^2 - 14k \text{ is odd, then } k \text{ is odd.}$$

By considering the contrapositive, prove that the statement is true.

Q28 2025 Cheltenham Girls Trial Q12d · **2 marks**

Prove that the statement "If n^2 is even, then n is even" is logically true, by considering its contrapositive.

Q29 2025 Sydney Tech Trial Q12b · **3 marks**

For $x \in \mathbb{Z}$, prove that if x^3 is even, then x is even.

Q30 2025 Killara Trial Q11f · **2 marks**

By using the contrapositive, prove that if a and b are integers and $a + b \geq 15$, then either $a \geq 7$ or $b \geq 8$.

Q31 2025 Normanhurst Boys Trial Q13c · **2 marks**

By considering the contrapositive, prove that for all $a, b \in \mathbb{Z}^+$ and where $n \in \mathbb{Z}$,

$$\text{If } a + b \geq 2n + 1, \text{ then } a \geq n + 1 \text{ or } b \geq n + 1.$$

Q32 2022 HSC Q13a · **3 marks**

Prove that for all integers n with $n \geq 3$, if $2^n - 1$ is prime, then n cannot be even.

Q33 2025 Ascham Trial Q15b · **3 marks**

Prove the following statement for all integers $n \geq 3$.

"If $3n - 4$ is a prime, then n is odd."