Year 12 Mathematics Extension 2

Topic: Complex Numbers (5 of 5)

Review Questions: Mixed one-off exam questions

Term 3, 2026

How to use this set

These are real past-paper questions on complex numbers that do not fit any one recurring pattern: genuine one-offs. They are here so you also meet the rarer, less predictable exam questions. Work through them once you are solid on the patterns in the main review docs. Every question you get wrong or struggle with goes in your **Mistakes Log**.

Mixed one-off exam questions

Q1 2020 HSC Q9 · **1 mark**

What is the maximum value of $|e^{i\theta} - 2| + |e^{i\theta} + 2|$ for $0 \leq \theta \leq 2\pi$?

(A) $\sqrt{5}$

(B) 4

(C) $2\sqrt{5}$

(D) 10

Q2 2025 HSC Q9 · **1 mark**

The points U, V, W and Z represent the complex numbers u, v, w and z respectively. It is given that $v + z = u + w$ and $u + kiz = w + kiv$, where k is real and $k > 1$. Which quadrilateral best describes $UVWZ$?

(A) Parallelogram

(B) Rectangle

(C) Rhombus

(D) Square

Q3 2025 Fort St Trial Q13(d) · **2 marks**

Let z be a complex number and λ a real constant. Find the values of $|z|$ that satisfy $\lambda(z + z^n) = i(z - z^n)$, where $n \geq 2$.

Q4 2022 HSC Q16(d) · **3 marks**

Find all the complex numbers z_1, z_2, z_3 that satisfy the following three conditions simultaneously:

$$|z_1| = |z_2| = |z_3|, \qquad z_1 + z_2 + z_3 = 1, \qquad z_1 z_2 z_3 = 1.$$

Q5 2025 North Sydney Boys Trial Q16(b) · **3 marks**

The complex number z satisfies $|z| = 1$ and $z^{n+1} - z^n - 1 = 0$. Prove that $n + 2$ is a multiple of 6.

Q6 2025 Sydney Girls Trial Q16(d) · **3 marks**

Suppose z_1, z_2, z_3 are complex numbers satisfying $|z_1| = |z_2| = |z_3| = 1$, $\dfrac{z_1}{z_2} + \dfrac{z_2}{z_3} + \dfrac{z_3}{z_1} = 1$, $z_2 \neq z_3$ and $z_3 \neq z_1$. Prove that for real a, b, c,

$$|az_1 + bz_2 + cz_3| = \sqrt{a^2 + b^2 + c^2 + 2ab}.$$

Q7 2025 Hurlstone Trial Q11(c) · **4 marks**

Let z and w be two complex numbers.

(i) Sketch the complex numbers z, w and $z + w$ on an Argand diagram, representing each as a vector.

(ii) Justify why $|z + w| \leq |z| + |w|$.

(iii) If $|z| = 10$, prove that $|13z^2 - 2z + 5| \leq 1325$.

Q8 2025 Fort St Trial Q14(b) · **4 marks**

Consider a square $ABCD$ of side length a in the complex plane. The point P lies on the circle of radius $\dfrac{a}{2}$, centred at the origin, inscribed in the square $ABCD$. Find, in terms of a, the value of $PA^2 + PB^2 + PC^2 + PD^2$.

Diagram to be inserted: `mx2-cn-oneoff-q8.png`

Q9 2025 Sydney Grammar Trial Q16(b)(ii) · **4 marks**

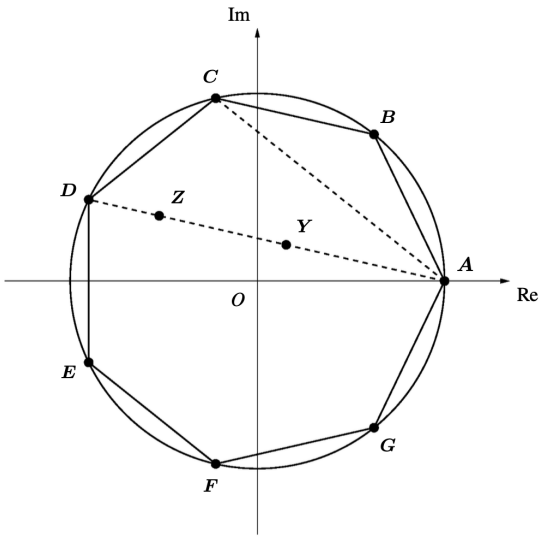
Let n be a positive integer and $z_k = e^{\frac{2k\pi i}{n}}$. Prove that three distinct complex numbers z_a, z_b, z_c satisfy $\dfrac{z_b}{z_a} = \dfrac{z_c}{z_b}$ if and only if the points representing 0, z_b and $z_a + z_b + z_c$ are collinear.

Q10 2025 Fort St Trial Q16(c) · **5 marks**

Let z_1, z_2, z_3 be complex numbers with $|z_1| = |z_2| = |z_3| = 1$, $z_1 + z_2 + z_3 \neq 0$ and $z_1^2 + z_2^2 + z_3^2 = 0$. Show that for all integers $n \geq 1$, $|z_1^n + z_2^n + z_3^n|$ can only take certain integer values, and state those values.

Q11 2025 James Ruse Trial Q15(b) · **7 marks**

The points A, B, C, D, E, F and G, corresponding to the complex roots of $z^7 = 1$, form the vertices of a regular heptagon in the Argand diagram, as shown. Let $\omega = \cos\dfrac{\pi}{7} + i\sin\dfrac{\pi}{7}$.



(i) By considering appropriate arguments, show that $\angle CAB = \dfrac{\pi}{7}$ and $\angle DAB = \dfrac{2\pi}{7}$.

(ii) The points Y and Z are chosen on the line segment AD such that $AY = AB$ and $AZ = AC$. Find the complex numbers representing Y and Z in terms of ω.

(iii) Hence, or otherwise, prove that $\dfrac{1}{AB} = \dfrac{1}{AC} + \dfrac{1}{AD}$.