Year 12 Mathematics Extension 2

Topic: Complex Numbers (4 of 5)

Review Questions: The Argand Plane, Loci, Regions & Geometry

Term 3, 2026

How to use this review set

These are real past-paper questions, grouped by question pattern. The grey label on each question shows where it is from and its marks. For the sketching questions, draw the diagrams in your own book.

- The questions are grouped by the pattern that keeps coming up in exams, a few of each in a row, running from the plainer patterns into the harder ones. Work through them and you will meet every type an exam can throw at you.
- This set **is your diagnostic**. Working through it shows you exactly which patterns are solid and which need work. Attempt every question properly in your own book, and go back to your notes to help you work through tough questions if needed.
- Every question you get wrong, need help with or struggle with goes in your **Mistakes Log**, which you should be working on daily to get you exam-ready.

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Loci and regions

1. Loci in the complex plane
2. Regions in the complex plane
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4. Maximum & minimum modulus on a locus

Geometry with complex numbers

5. Rotations, vectors & geometric proofs

Loci and regions

1. Loci in the complex plane

Q1 2025 Trinity Grammar Trial Q13(c)(i) · **2 marks**

Sketch the locus $|z-1|=4$ on an Argand diagram.

Q2 2025 Riverview Trial Q11(f)(i) · **2 marks**

The point P moves in the complex plane so that $|z-3-4i|=5$. Sketch the locus of P, noting the intercepts.

Q3 2025 Sydney Tech Trial Q15(a) · **3 marks**

Sketch on the Argand diagram the values of z for which $\dfrac{z}{z-2}$ is purely imaginary.

Q4 2025 North Sydney Boys Trial Q6 · **1 mark**

Let a and b be real numbers with $a \neq b$, and let $z = x+iy$ satisfy $|z-a|^2 - |z-b|^2 = 1$. Which of the following best describes the locus of z?

(A) a circle with centre $(0,0)$

(B) a circle with centre (a,b)

(C) a ray

(D) a vertical line

Q5 2025 Caringbah Trial Q12(e) · **3 marks**

Let $z = x+iy$, where a and b are real. Prove that $\operatorname{Re}\left(\dfrac{z-ib}{z-a}\right) = 0$ is the equation of a circle, and state its centre and radius in terms of a and b.

Q6 2025 Penrith Trial Q13(b)(i) · **3 marks**

Let $A = 1+i$ and $B = -1+i$. Find the Cartesian equation of the locus of z satisfying $|z-A| = 2|z-B|$.

2. Regions in the complex plane

Q7 2025 HSC Q12(c) · **3 marks**

Sketch the region of the complex plane defined by $|z + 5 - i| > |z - 3 + 3i|$.

Q8 2024 HSC Q11(f) · **3 marks**

Sketch the region defined by $|z| < 3$ and $0 \leq \text{Arg}(z - i) \leq \dfrac{\pi}{2}$.

Q9 2025 Normanhurst Boys Trial Q13(a) · **6 marks**

Consider the region defined by $|z - 2 + 2i| \leq 4$ and $-\dfrac{\pi}{3} \leq \text{Arg}(z - 1 + 3i) \leq \dfrac{\pi}{4}$.

(i) On an Argand diagram, clearly sketch the region above.

(ii) Let w represent any complex number that lies in the region. Find the possible range of values of $\text{Arg}(w)$, correct to two decimal places.

Q10 2025 Manly Trial Q13(d) · **3 marks**

Sketch the region defined by $|z - i| < |z + i|$ and $-\dfrac{\pi}{6} \leq \text{Arg}(z - i) \leq \dfrac{\pi}{6}$.

Q11 2025 St George Girls Trial Q11(d) · **3 marks**

Let $z = x + iy$. Sketch the region in the complex plane defined by $|z - 2| \leq 3$ and $z + \bar{z} \geq 0$.

Q12 2025 Killara Trial Q12(e) · **4 marks**

(i) Sketch on an Argand diagram the locus $\arg\left(\dfrac{z-2}{z+2}\right) = \dfrac{\pi}{4}$, showing all important features.

(ii) On the same diagram, sketch the locus $\arg\left(\dfrac{z-2}{z+2}\right) = \dfrac{\pi}{2}$.

(iii) Hence shade the region where $\dfrac{\pi}{4} \leq \arg\left(\dfrac{z-2}{z+2}\right) \leq \dfrac{\pi}{2}$.

Q13 2021 HSC Q16(c) · **3 marks**

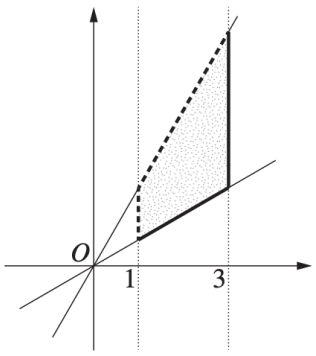
Sketch the region of the complex plane defined by $\text{Re}(z) \geq \text{Arg}(z)$, where $\text{Arg}(z)$ is the principal argument of z.

3. Reading loci & regions from a diagram

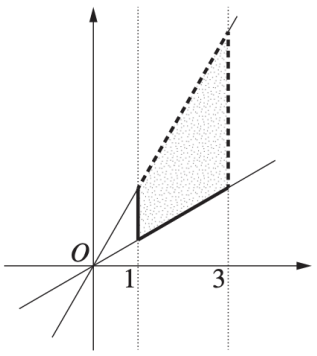
Q14 2022 HSC Q1 · **1 mark**

Let R be the region in the complex plane defined by $1 < \text{Re}(z) \leq 3$ and $\frac{\pi}{6} \leq \text{Arg}(z) < \frac{\pi}{3}$. Which diagram best represents the region R?

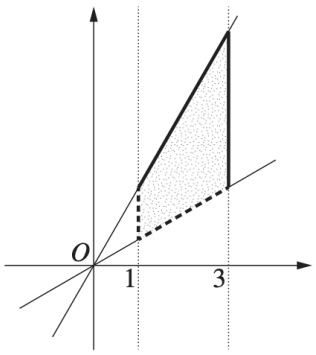
A.



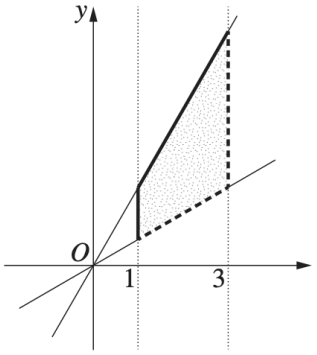
B.



C.

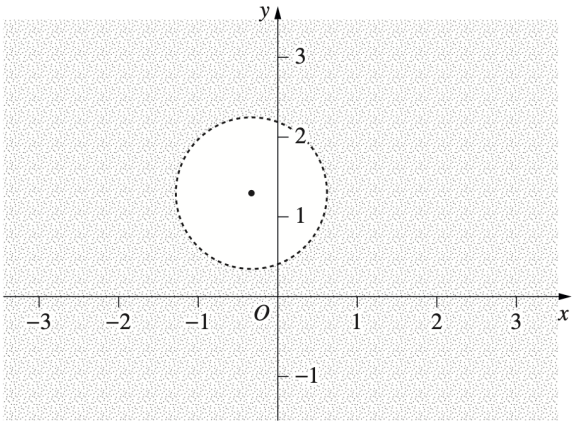


D.



Q15 2023 HSC Q8 · **1 mark**

Which relation best describes the region shaded on the complex plane?



(A) $|z - i| > 2|z - 1|$

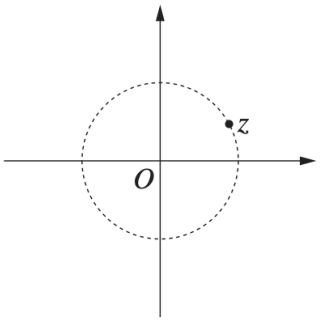
(B) $|z - i| < 2|z - 1|$

(C) $|z - 1| > 2|z - i|$

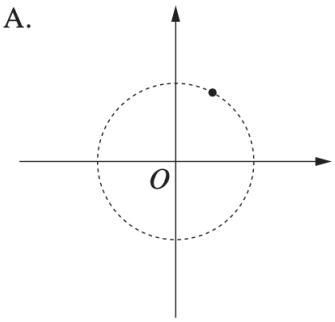
(D) $|z - 1| < 2|z - i|$

Q16 2020 HSC Q4 · **1 mark**

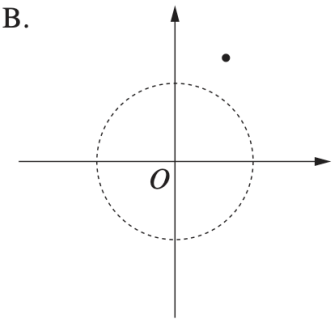
The diagram shows the complex number z on the Argand diagram. Which of the following diagrams best shows the position of $\dfrac{z^2}{|z|}$?



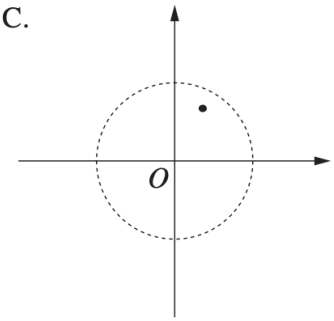
A.



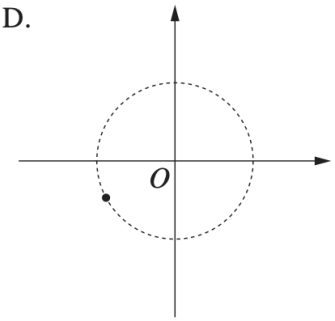
B.



C.

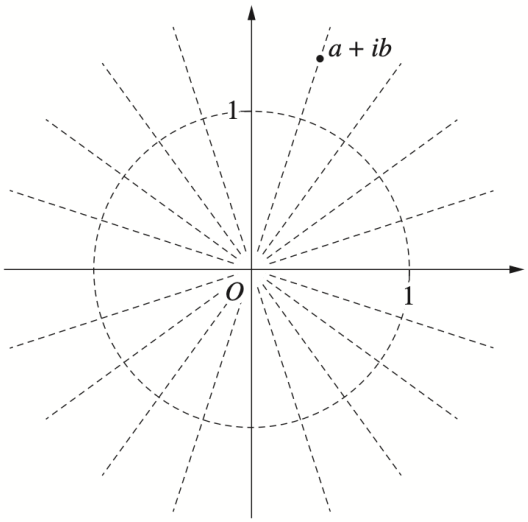


D.



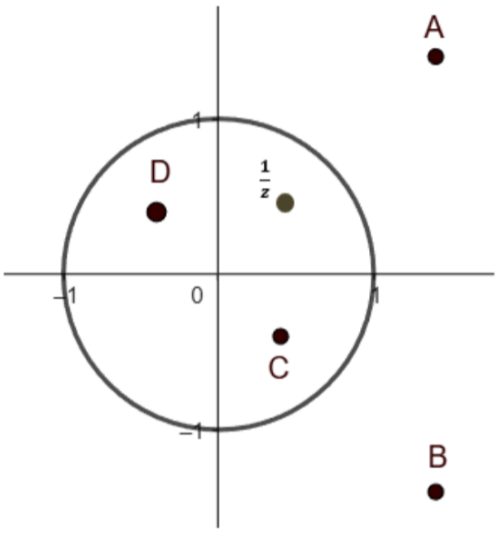
Q17 2021 HSC Q13(a) · **2 marks**

The location of the complex number $a+ib$ is shown on the Argand diagram below. On the diagram provided, indicate the locations of all four fourth roots of $a+ib$.



Q18 2025 Manly Trial Q6 · **1 mark**

The complex number $\frac{1}{z}$ is shown on the Argand diagram below. Which one of the points could correspond to the complex number z?



4. Maximum & minimum modulus on a locus

Q19 2024 HSC Q7 · **1 mark**

It is given that $|z - 1 + i| = 2$. What is the maximum possible value of $|z|$?

(A) $\sqrt{2}$

(B) $\sqrt{10}$

(C) $2 + \sqrt{2}$

(D) $2 - \sqrt{2}$

Q20 2025 Hills Grammar Trial Q2 · **1 mark**

It is given that $|z - 1 - i| = 1$. What is the maximum value of $\text{Arg}(z)$?

(A) $-\dfrac{\pi}{2}$

(B) 0

(C) $\dfrac{\pi}{2}$

(D) π

Q21 2025 Girraween Trial Q7 · **1 mark**

It is given that $|z - 2 - i| = 1$. The maximum modulus of z is

(A) 1

(B) $\sqrt{5}$

(C) $\sqrt{5} - 1$

(D) $\sqrt{5} + 1$

Q22 2025 Gosford Trial Q10 · **1 mark**

Consider z such that $|z - z_1| = \sqrt{5}$, where $z_1 = \dfrac{7+4i}{3-2i}$. The largest possible value of $|z|$ is

(A) $2\sqrt{5}$

(B) $\sqrt{5}$

(C) $\sqrt{\dfrac{11}{5}}$

(D) $\sqrt{65}$

Q23 2025 Ascham Trial Q12(d) · **3 marks**

(i) On an Argand diagram sketch the locus of z such that $|z - (\sqrt{3} + i)| = 1$.

(ii) Find the maximum possible values of $|z|$ and $\text{Arg}(z)$.

Q24 2022 HSC Q15(d) · **3 marks**

The complex number z satisfies $\left|z - \dfrac{4}{z}\right| = 2$. Using the triangle inequality, or otherwise, show that $|z| \leq \sqrt{5}+1$.

Geometry with complex numbers

5. Rotations, vectors & geometric proofs

Q25 2025 Gosford Trial Q13(a) · **2 marks**

The point P represents the complex number $2+3i$. The segment OP is rotated anticlockwise about the origin O by $60°$ to OQ. Find the coordinates of Q.

Q26 2025 HSC Q6 · **1 mark**

The complex numbers z and w lie on the unit circle, and $|z+w| = \dfrac{3}{2}$. What is $|z-w|$?

(A) $\dfrac{1}{8}$

(B) $\dfrac{\sqrt{7}}{2}$

(C) $\dfrac{3}{2}$

(D) $\dfrac{7}{4}$

Q27 2021 HSC Q10 · **1 mark**

Consider two non-zero complex numbers z and w as vectors. Which of the following is the projection of z onto w?

(A) $\dfrac{\text{Re}(z\overline{w})}{|w|}\,w$

(B) $\dfrac{\overline{z}}{\overline{w}}\,w$

(C) $\text{Re}\left(\dfrac{z}{w}\right)w$

(D) $\dfrac{\text{Re}(\overline{z})}{|w|}\,w$

Q28 2025 Sydney Girls Trial Q15(a) · **3 marks**

Show that if $|z|^2 = |w|^2 = 2\,\text{Re}(\overline{z}w)$, then the triangle with vertices 0, z and w is equilateral.

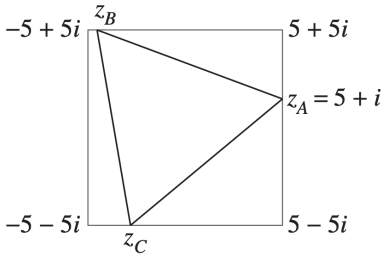
Q29 2020 HSC Q14(a)(i) · **2 marks**

Let z_1 be a complex number and let $z_2 = e^{\frac{i\pi}{3}} z_1$. In the Argand plane the point A represents z_1 and the point B represents z_2. Explain why triangle OAB is equilateral.

Diagram to be inserted: `argand-q29.png`

Q30 2022 HSC Q16(a) · **4 marks**

A square in the Argand plane has vertices $5+5i$, $5-5i$, $-5-5i$ and $-5+5i$. The complex numbers $z_A = 5+i$, z_B and z_C lie on the square and form the vertices of an equilateral triangle, as shown. Find the exact value of z_B.



NOT TO SCALE

Q31 2025 Baulkham Hills Trial Q15(c) · **4 marks**

The points A, B, C represent the complex numbers z_1, z_2, z_3. Prove that if $\dfrac{z_2 - z_3}{z_1 - z_3} = \dfrac{z_1 - z_3}{z_1 - z_2}$, then triangle ABC is equilateral.

Diagram to be inserted: `argand-q31.png`