Year 12 Mathematics Extension 2

Topic: Complex Numbers (2 of 5)
Review Questions: De Moivre, Powers & Roots
Term 3, 2026

How to use this review set

These are real past-paper questions, grouped by question pattern. The grey label on each question shows where it is from and its marks.

- The questions are grouped by the pattern that keeps coming up in exams, a few of each in a row, running from the plainer patterns into the harder ones. Work through them and you will meet every type an exam can throw at you.
- This set **is your diagnostic**. Working through it shows you exactly which patterns are solid and which need work. Attempt every question properly in your own book, and go back to your notes to help you work through tough questions if needed.
- Every question you get wrong, need help with or struggle with goes in your **Mistakes Log**, which you should be working on daily to get you exam-ready.

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De Moivre's theorem

1. Powers via De Moivre's theorem

Q1 2025 Killara Trial Q5 · **1 mark**

Evaluate the complex number $\left(-2+2i\sqrt{3}\right)^3$.

(A) $-2+2i\sqrt{3}$

(B) $8+6i\sqrt{3}$

(C) 8

(D) 64

Q2 2025 St George Girls Trial Q3 · **1 mark**

Evaluate $\left(\sqrt{3}+i\right)^8$.

(A) $128-128i\sqrt{3}$

(B) $128+128i\sqrt{3}$

(C) $256\left(\cos 120^\circ + i\sin 120^\circ\right)$

(D) $256\left(\cos\left(-\frac{2\pi}{3}\right)+i\sin\left(-\frac{2\pi}{3}\right)\right)$

Q3 2022 HSC Q11(c) · **4 marks**

(i) Write the complex number $-\sqrt{3}+i$ in exponential form.

(ii) Hence find the exact value of $\left(-\sqrt{3}+i\right)^{10}$, giving your answer in the form $x+iy$.

Q4 2025 Ascham Trial Q12(b) · **3 marks**

Given that x is a real number and $(x+i)^4$ is purely imaginary, find all possible values of x in exact form.

Q5 2025 Kinross Wolaroi Trial Q11(a) · **4 marks**

(i) Write $1 - i\sqrt{3}$ in modulus-argument form.

(ii) Hence write $\dfrac{\left(1 - i\sqrt{3}\right)^4}{1 + i\sqrt{3}}$ in exact Cartesian form.

Q6 2025 Killara Trial Q12(b) · **2 marks**

Prove or disprove that there exists an integer k for which $\dfrac{(1+i)^k}{(1-i)^{k-1}}$ is purely real or purely imaginary.

2. Deriving trig identities with De Moivre's theorem

Q7 2025 Ascham Trial Q14(c) · **3 marks**

(i) Use De Moivre's theorem to show that $\cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta$ and $\sin 3\theta = 3\cos^2\theta\sin\theta - \sin^3\theta$.

(ii) Deduce that $\tan 3\theta = \dfrac{\tan^3\theta - 3\tan\theta}{3\tan^2\theta - 1}$.

Q8 2025 Hurlstone Trial Q12(c) · **5 marks**

Let $z = \cos\theta + i\sin\theta$.

(i) Show that $z^n + z^{-n} = 2\cos n\theta$.

(ii) Expand $\left(z + z^{-1}\right)^4$.

(iii) Use parts (i) and (ii) to show that $8\cos^4\theta = \cos 4\theta + 4\cos 2\theta + 3$.

Q9 2025 Caringbah Trial Q13(c) · **4 marks**

(i) Show that $\left(z - \dfrac{1}{z}\right)^5 = \left(z^5 - z^{-5}\right) - 5\left(z^3 - z^{-3}\right) + 10\left(z - z^{-1}\right)$.

(ii) By letting $z = \cos\theta + i\sin\theta$, show that $32\sin^5\theta = 2\sin 5\theta - 10\sin 3\theta + 20\sin\theta$.

Q10 2021 HSC Q14(c)(i) · **3 marks**

Using de Moivre's theorem and the binomial expansion of $(\cos\theta + i\sin\theta)^5$, show that

$$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta.$$

Q11 2025 Sydney Boys Trial Q12(c)(i) · **3 marks**

Using De Moivre's theorem, prove that

$$\cos 6\theta = 32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1.$$

Q12 2025 Cheltenham Girls Trial Q12(b)(i) · **2 marks**

Let $z = \cos\theta + i\sin\theta$, so that $z^n + z^{-n} = 2\cos n\theta$. By expanding $\left(z + z^{-1}\right)^4$, prove that $\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$.

Q13 2025 Sydney Girls Trial Q12(c) · **7 marks**

(i) By expanding $(\cos\theta + i\sin\theta)^3$ and using de Moivre's theorem, show that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$.

(ii) Hence solve $8x^3 - 6x + 1 = 0$, where $x = \cos\theta$.

(iii) Deduce that $\cos\dfrac{\pi}{9} - \cos\dfrac{2\pi}{9} - \cos\dfrac{4\pi}{9} = 0$.

Roots of complex numbers

3. nth roots of a complex number

Q14 2025 Ascham Trial Q12(c) · **3 marks**

Find all the solutions of $z^5 = 4 + 4i$, giving your answers in modulus-argument form.

Q15 2025 Girraween Trial Q11(d) · **2 marks**

Find the six sixth roots of $64i$. You may leave your answer in modulus-argument form.

Q16 2025 St George Girls Trial Q12(a) · **3 marks**

Find the fourth roots of $8 + 8\sqrt{3}\,i$.

Q17 2024 HSC Q9 · **1 mark**

Consider the solutions of the equation $z^4 = -9$. What is the product of all of the solutions that have a positive principal argument?

(A) 3

(B) -3

(C) $3i$

(D) $-3i$

Q18 2025 Hornsby Girls Trial Q5 · **1 mark**

One of the complex solutions to $z^5 = -a$, where a is a positive real constant, is $a^{\frac{1}{5}}\,\text{cis}\,\dfrac{\pi}{5}$. One of the other solutions is a real number and is equal to

(A) $a^{\frac{1}{5}}\,\text{cis}\,\dfrac{3\pi}{5}$

(B) $-a^{\frac{1}{5}}$

(C) $a^{\frac{1}{5}}$

(D) $a^{\frac{1}{5}}\,\text{cis}\,\dfrac{9\pi}{5}$

Q19 2025 Baulkham Hills Trial Q14(a) · **3 marks**

If $z = r(\cos\theta + i\sin\theta)$, use De Moivre's theorem to solve the equation $z^5 = \overline{z}$. Express your answers in the form $a + ib$.

Q20 2025 North Sydney Girls Trial Q16(b)(i) · **4 marks**

Consider the equation $(z+2)^8 = z^8$. Show that the six non-real roots of the equation can be expressed in the form

$$-1 - i\cot\left(\frac{k\pi}{8}\right), \qquad k = \pm 1,\ \pm 2,\ \pm 3.$$

4. Roots of unity

Q21 2025 Girraween Trial Q16(b)(i) · **1 mark**

State the solutions to $z^9 - 1 = 0$, leaving your solutions in modulus-argument form.

Q22 2025 Sydney Tech Trial Q13(c)(i) · **1 mark**

Given that w is a non-real fifth root of unity, show that

$$1 + w + w^2 + w^3 + w^4 = 0.$$

Q23 2022 HSC Q13(c)(i) · **2 marks**

Solve the equation $z^5 + 1 = 0$ by finding the fifth roots of -1.

Q24 2025 Normanhurst Boys Trial Q3 · **1 mark**

Consider $z^6 = 1$. Which of the following are the roots of the equation?

(A) $e^{\frac{k\pi}{3}i}$ for $k = \pm 1, \pm 2, \pm 3$

(B) $e^{\frac{k\pi}{6}i}$ for $k = \pm 1, \pm 2, \pm 3$

(C) $e^{\frac{k\pi}{3}i}$ for $k = 0, 1, 2, \ldots, 5$

(D) $e^{\frac{k\pi}{6}i}$ for $k = 0, 1, 2, \ldots, 5$

Q25 2025 Cheltenham Girls Trial Q11(d) · **4 marks**

(i) Solve $z^5 - 1 = 0$ over the complex field, giving your answers in modulus-argument form.

(ii) Hence express $z^5 - 1$ as a product of linear and quadratic factors.

Q26 2025 Sydney Grammar Trial Q13(c)(i) · **3 marks**

Use De Moivre's theorem to find the solutions to $z^5 + 1 = 0$, and hence show that

$$\cos\frac{\pi}{5} + \cos\frac{3\pi}{5} = \frac{1}{2}.$$

Q27 2024 HSC Q16(b)(i) · **3 marks**

The number $w = e^{\frac{2\pi i}{3}}$ is a complex cube root of unity, and γ is a cube root of w. Show that $\gamma + \overline{\gamma}$ is a real root of $z^3 - 3z + 1 = 0$.

5. Evaluating and simplifying expressions in a root of unity

Q28 2025 Kinross Wolaroi Trial Q1 · **1 mark**

ω is a complex fifth root of unity. What is the value of ω^{2025}?

(A) 1

(B) -1

(C) i

(D) $-i$

Q29 2025 Hornsby Girls Trial Q10 · **1 mark**

Suppose that $x + \dfrac{1}{x} = -1$. What is the value of $x^{2025} + \dfrac{1}{x^{2025}}$?

(A) 1

(B) 2

(C) $\dfrac{2}{3}$

(D) $\dfrac{4}{3}$

Q30 2025 Sydney Girls Trial Q6 · 1 mark

Let ω be a non-real cube root of unity. What is the value of $(1 + 2\omega + 3\omega^2)(1 + 3\omega + 2\omega^2)$?

(A) 3

(B) 6

(C) 9

(D) 12

Q31 2025 Hornsby Girls Trial Q7 · 1 mark

If ω is a complex cube root of unity, the value of $(1 - 3\omega + \omega^2)(1 + \omega - 8\omega^2)$ is:

(A) $36\sqrt{3}$

(B) -36

(C) $-36\sqrt{3}$

(D) 36

Q32 2025 Sydney Grammar Trial Q8 · 1 mark

Let w be a complex cube root of unity. Which of these is equivalent to $\dfrac{1}{1-w} + \dfrac{1}{1-w^2}$?

(A) 0

(B) 1

(C) w

(D) w^2

Q33 2025 Normanhurst Boys Trial Q6 · 1 mark

Let ω be a non-real cube root of unity. Which of the following is equivalent to $(1 + 2\omega - \omega^2 + \omega^3)^5$?

(A) -243ω

(B) $-243\omega^2$

(C) 243ω

(D) $243\omega^2$

Q34 2025 Ascham Trial Q10 · **1 mark**

Let $w = e^{\frac{2\pi i}{5}}$. What is the value of $(1-w)(1-w^2)(1-w^3)(1-w^4)$?

(A) -5

(B) -4

(C) 4

(D) 5

Q35 2025 Sydney Boys Trial Q9 · **1 mark**

ω is a non-real cube root of unity. Evaluate $(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)\cdots(1+\omega^{2^n})$, where n is an even positive integer.

(A) 1

(B) ω

(C) $\dfrac{1}{\omega}$

(D) $-\dfrac{1}{\omega}$

Q36 2025 Merewether Trial Q13(a) · **2 marks**

Given that w is a complex cube root of unity, find the exact value of $1 + w + w^2 + w^3 + \cdots + w^{1001}$.

Q37 2025 Abbotsleigh Trial Q12(e) · **2 marks**

If ω is a complex (non-real) solution of $z^3 = 1$, simplify $(1+\omega)^2 + (1+\omega^2)^2$.

Q38 2025 Killara Trial Q14(a) · **3 marks**

Let z be a solution of $z^7 - 1 = 0$ with $z \neq 1$. Find the value of $z^{10} + \dfrac{1}{z^{10}} + z^{30} + \dfrac{1}{z^{30}} + z^{50} + \dfrac{1}{z^{50}}$.

6. Series and sums via De Moivre

Q39 2021 HSC Q11(b) · **2 marks**

Find $\displaystyle\sum_{n=1}^{5} i^n$.

Q40 2025 Caringbah Trial Q5 · **1 mark**

Find the value of $i + i^2 + i^3 + \cdots + i^{2025}$.

(A) i

(B) -1

(C) 1

(D) $-i$

Q41 2025 North Sydney Boys Trial Q3 · **1 mark**

For the complex number $z = a + ib$, which of the following equals $\sum_{k=0}^{2025} i^k z$?

(A) $(a + b) + (a + b)i$

(B) $(a - b) + (a + b)i$

(C) $(a + b) + (a - b)i$

(D) $(a - b) + (b - a)i$

Q42 2025 Normanhurst Boys Trial Q15(b) · **4 marks**

Consider the series

$$C = 1 + \cos\theta + \cos 2\theta + \cdots + \cos(n-1)\theta, \qquad S = \sin\theta + \sin 2\theta + \cdots + \sin(n-1)\theta.$$

Given that $z = \cos\theta + i\sin\theta$:

(i) Show that $C + iS = \dfrac{1 - z^n}{1 - z}$, where $z \neq 1$.

(ii) Hence show that $\sin\theta + \sin 2\theta + \cdots + \sin(n-1)\theta = \dfrac{(\sin\theta - \sin n\theta) + \sin(n-1)\theta}{2(1 - \cos\theta)}$.

7. The $\frac{1+z}{1-z} = i\cot\frac{\theta}{2}$ identity and $\left(\frac{z-1}{z+1}\right)^n$ equations

Q43 2025 HSC Q15(c) · 7 marks

(i) Show that $\dfrac{1+\cos\theta+i\sin\theta}{1-\cos\theta-i\sin\theta} = i\cot\dfrac{\theta}{2}$.

(ii) Use De Moivre's theorem to show that the sixth roots of -1 are $\cos\dfrac{(2k+1)\pi}{6} + i\sin\dfrac{(2k+1)\pi}{6}$ for $k = 0, 1, 2, 3, 4, 5$.

(iii) Hence, or otherwise, show the solutions of $\left(\dfrac{z-1}{z+1}\right)^6 = -1$ are $z = i\cot\dfrac{(2k+1)\pi}{12}$ for $k = 0, 1, 2, 3, 4, 5$.

Q44 2025 Trinity Grammar Trial Q14(d) · 5 marks

The complex number z lies on the unit circle with $\text{Arg}(z) = \theta$.

(i) Show that $\dfrac{1+z}{1-z} = i\cot\dfrac{\theta}{2}$.

(ii) Hence, or otherwise, solve $\left(\dfrac{w-1}{w+1}\right)^8 = 1$, where w is a complex number.

Q45 2025 Hills Grammar Trial Q16(b) · 6 marks

The complex number w lies on the unit circle with $\text{Arg}(w) = \theta$.

(i) Show that $\dfrac{1+w}{1-w} = i\cot\dfrac{\theta}{2}$.

(ii) Hence solve $\left(\dfrac{z-1}{z+1}\right)^8 = -1$.

Q46 2025 Fort St Trial Q15(b) · 5 marks

The complex number w lies on the unit circle with $\text{Arg}(w) = \theta$.

(i) Show that $\dfrac{1+w}{1-w} = i\cot\dfrac{\theta}{2}$.

(ii) Hence find all solutions of $(z-1)^5 + (z+1)^5 = 0$, where z is a complex number.