Year 12 Mathematics Extension 2

Topic: Complex Numbers (1 of 5)
Review Questions: Arithmetic, Conjugates & Forms
Term 3, 2026

How to use this review set

These are real past-paper questions, grouped by question pattern. The grey label on each question shows where it is from and its marks.

- The questions are grouped by the pattern that keeps coming up in exams, a few of each in a row, running from the plainer patterns into the harder ones. Work through them and you will meet every type an exam can throw at you.
- This set **is your diagnostic**. Working through it shows you exactly which patterns are solid and which need work. Attempt every question properly in your own book, and go back to your notes to help you work through tough questions if needed.
- Every question you get wrong, need help with or struggle with goes in your **Mistakes Log**, which you should be working on daily to get you exam-ready.

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Arithmetic in the complex plane

1. Adding, subtracting, multiplying & powers of i

Q1 2025 Abbotsleigh Trial Q1 · 1 mark

What is the value of i^{2025}?

(A) 1

(B) -1

(C) i

(D) $-i$

Q2 2025 Sydney Tech Trial Q1 · 1 mark

Let $z = 3 + 2i$. What is the value of $i\overline{z}$, expressed in the form $x + iy$?

(A) $2 + 3i$

(B) $-2 + 3i$

(C) $-2 - 3i$

(D) $2 - 3i$

Q3 2023 HSC Q1 · 1 mark

Which of the following is equal to $(a + ib)^3$?

(A) $(a^3 - 3ab^2) + i(3a^2b + b^3)$

(B) $(a^3 + 3ab^2) + i(3a^2b + b^3)$

(C) $(a^3 - 3ab^2) + i(3a^2b - b^3)$

(D) $(a^3 + 3ab^2) + i(3a^2b - b^3)$

Q4 2024 HSC Q11(b) · 2 marks

Let $z = 2 + 3i$ and $w = 1 - 5i$.

(i) Find $\overline{z + w}$.

(ii) Find z^2.

Q5 2025 HSC Q11(c) · **3 marks**

The complex number z is given by $x + iy$. Find, in Cartesian form:

(i) z^2

(ii) $\dfrac{1}{z}$.

Q6 2025 Hills Grammar Trial Q12(a) · **2 marks**

Consider the complex number $z = a + ib$.

- The product of z and $\overline{z}$ is 25.
- The sum of z and $\overline{z}$ is 8.

Find z.

Q7 2025 Penrith Trial Q11(a) · **3 marks**

Given $z = 1 + i$, find $z^2 + \dfrac{1}{z^2}$ in the form $a + ib$, where a and b are real numbers.

2. Division & realising denominators

Q8 2022 HSC Q11(a) · **2 marks**

Express $\dfrac{3 - i}{2 + i}$ in the form $x + iy$, where x and y are real numbers.

Q9 2021 HSC Q11(e) · **2 marks**

The complex numbers $z = 5 + i$ and $w = 2 - 4i$ are given. Find $\dfrac{\overline{z}}{w}$, giving your answer in Cartesian form.

Q10 2025 Normanhurst Boys Trial Q11(a) · **2 marks**

Find $\dfrac{-3 + i}{1 - 2i}$, showing all working out.

Q11 2025 Caringbah Trial Q2 · **1 mark**

If $z = \dfrac{3+4i}{1+2i}$, the imaginary component of z is:

(A) -2

(B) $\dfrac{2}{5}$

(C) $-\dfrac{2}{5}$

(D) 2

Q12 2025 Pymble Trial Q12(a)(i) · **2 marks**

The complex number z is such that $z = \dfrac{1+i}{1-ki}$, where k is a real number. Find the real part of z and the imaginary part of z, giving your answers in terms of k.

Q13 2025 Gosford Trial Q12(c) · **2 marks**

If $\dfrac{3+2i\sin\theta}{1-2i\sin\theta}$ is purely imaginary, find θ where $-\pi < \theta \leq \pi$.

3. Square roots of a complex number

Q14 2025 HSC Q3 · **1 mark**

What are the square roots of $3-4i$?

(A) $1-2i$ and $-1+2i$

(B) $1+2i$ and $-1-2i$

(C) $2-i$ and $-2+i$

(D) $-2-i$ and $2+i$

Q15 2025 James Ruse Trial Q11(b)(i) · **2 marks**

Find the square roots of $2i$, giving the answers in the form $x+iy$, where x and y are real numbers.

Q16 2025 Caringbah Trial Q11(e)(i) · **2 marks**

Find the square roots of $3+4i$.

Q17 2025 Gosford Trial Q11(d) · **2 marks**

Determine the square roots of $-15 + 8i$. Show all relevant working.

Q18 2025 Manly Trial Q11(c) · **2 marks**

Find the square roots of $12 - 16i$, giving your answers in the form $a + bi$, where a and b are real numbers.

Q19 2025 Sydney Grammar Trial Q12(a)(i) · **2 marks**

Find the two square roots of $5 - 12i$.

Q20 2021 HSC Q11(d) · **4 marks**

(i) Find the two square roots of $-i$, giving the answers in the form $x + iy$, where x and y are real numbers.

(ii) Hence, or otherwise, solve $z^2 + 2z + 1 + i = 0$, giving your solutions in the form $a + ib$ where a and b are real numbers.

Modulus, argument & polar form

4. Modulus & argument

Q21 2025 Fort St Trial Q11(a)(i) · 2 marks

Let $z = 3 - i$ and $w = 2 + 5i$. Find $|z + w|$.

Q22 2025 Cheltenham Girls Trial Q11(a)(i) · 2 marks

Consider the complex numbers $z = 1 - i$ and $w = \sqrt{3} + i$. Evaluate $|z| + |w|$, leaving your answer in exact form.

Q23 2025 Barker Trial Q1 · 1 mark

The argument of a complex number z is $\dfrac{\pi}{7}$. What is $\text{Arg}(z^{10})$?

(A) $\dfrac{3\pi}{7}$

(B) $-\dfrac{3\pi}{7}$

(C) $\dfrac{4\pi}{7}$

(D) $-\dfrac{4\pi}{7}$

Q24 2025 Trinity Grammar Trial Q11(a) · 3 marks

Let $z_1 = -3 - 2i$, $z_2 = 2 - i$, and $z_3 = i(z_1 - \overline{z_2})$. Calculate $|z_3|$.

Q25 2025 Sydney Girls Trial Q11(f) · 2 marks

Evaluate $\text{Arg}(i^{2025})$, where $\text{Arg}(z)$ denotes the principal argument of z.

Q26 2025 Baulkham Hills Trial Q3 · **1 mark**

What is the principal argument of $\dfrac{-3\sqrt{2}-i\sqrt{6}}{2+2i}$?

(A) $-\dfrac{11\pi}{12}$

(B) $\dfrac{7\pi}{12}$

(C) $\dfrac{11\pi}{12}$

(D) $\dfrac{13\pi}{12}$

Q27 2025 Riverview Trial Q9 · **1 mark**

The complex number $z = a + ib$, where $0 < a < b$. Which of the following best describes z^4?

(A) $\text{Re}(z^4) < 0$

(B) $\text{Re}(z^4) \leq 0$

(C) $\text{Im}(z^4) < 0$

(D) $\text{Im}(z^4) \leq 0$

5. Modulus-argument (polar) form & converting

Q28 2025 Trinity Grammar Trial Q12(a)(i) · **1 mark**

Consider the complex number $z = 1 + \sqrt{3}\,i$. Write z in modulus-argument form.

Q29 2024 HSC Q11(e)(i) · **2 marks**

Write the number $\sqrt{3} + i$ in modulus-argument form.

Q30 2025 Hornsby Girls Trial Q11(b)(i) · **2 marks**

Express $-1 + \sqrt{3}\,i$ in modulus-argument form.

Q31 2025 Sydney Grammar Trial Q11(a)(i) · **2 marks**

Consider the complex numbers $z = 1 + \sqrt{3}\,i$ and $w = 1 - i$. Express both z and w in modulus-argument form.

Q32 2025 Killara Trial Q11(a) · **4 marks**

Let $z = 1 + 3i$ and $w = 2 - 6i$.

(i) Find $\overline{z} + w$ in modulus-argument form.

(ii) Write $\dfrac{z}{w}$ in the form $x + iy$, where x and y are real numbers.

Q33 2025 Normanhurst Boys Trial Q11(b) · **2 marks**

Given that $w = 2 - 2\sqrt{3}\,i$ and $\text{Arg}(zw) = \dfrac{\pi}{3}$, find the exact value of $\text{Arg}(z)$.

6. Modulus, argument & conjugate identities

Q34 2025 Sydney Grammar Trial Q14(b)(i) · **1 mark**

Consider the equation $z^2 = \dfrac{az + w}{\overline{z}}$, where a is real, and both w and z are complex. Show that $\text{Im}\left(\dfrac{z}{w}\right) = 0$.

Q35 2025 Manly Trial Q12(c) · **2 marks**

Show that $\text{Re}\left(\dfrac{1 - z}{1 + z}\right) = 0$ for any complex number z with $|z| = 1$.

Q36 2025 St George Girls Trial Q13(c)(i) · **3 marks**

For any complex numbers $w_1 = a + bi$ and $w_2 = c + di$, where a, b, c, d are real, prove that

$$|w_1 + w_2|^2 + |w_1 - w_2|^2 = 2\left(|w_1|^2 + |w_2|^2\right).$$

Q37 2024 HSC Q14(c) · **2 marks**

For the complex numbers z and w, it is known that $\arg\left(\dfrac{z}{w}\right) = -\dfrac{\pi}{2}$. Find $\left|\dfrac{z + w}{z - w}\right|$.

Q38 2025 Ascham Trial Q16(c) · **4 marks**

Let z_1 and z_2 be two non-zero complex numbers. It is known that $z_1 + z_2 + 1 = 0$.

(i) Show that if $|z_1| = |z_2| = 1$, then $\dfrac{1}{z_1} + \dfrac{1}{z_2} + 1 = 0$.

(ii) Show that if $\dfrac{1}{z_1} + \dfrac{1}{z_2} + 1 = 0$, then $|z_1| = |z_2| = 1$.

(iii) Hence, or otherwise, show that if $|z_1| = |z_2| = 1$, then $z_1^2 + z_2^2 + 1 = 0$.

7. Real and imaginary conditions on z and $\overline{z}$

Q39 2025 St George Girls Trial Q5 · **1 mark**

Consider the complex number $z = a + ib$ (where $a, b \in \mathbb{R}$, $a \neq 0$, $b \neq 0$) and its complex conjugate $\overline{z}$. Which row of the table correctly classifies the sum, difference, product and quotient of z and $\overline{z}$?

	$z + \overline{z}$	$z - \overline{z}$	$z\overline{z}$	$z \div \overline{z}$
A.	Real	Imaginary	Real	Imaginary
B.	Complex	Real	Complex	Real
C.	Real	Imaginary	Real	Complex
D.	Real	Real	Imaginary	Complex

Q40 2025 Sydney Grammar Trial Q7 · **1 mark**

Let z be a complex number. Which of the following will always be real?

(A) $(z + \overline{z})(z - \overline{z})$

(B) $(z + i\overline{z})(z - i\overline{z})$

(C) $(z + i\overline{z})(\overline{z} + iz)$

(D) $(z - i\overline{z})(\overline{z} - iz)$

Q41 2022 HSC Q6 · 1 mark

It is known that a particular complex number z is NOT a real number. Which of the following could be true for this number z?

(A) $\overline{z} = iz$

(B) $\overline{z} = |z^2|$

(C) $\text{Re}(iz) = \text{Im}(z)$

(D) $\text{Arg}(z^3) = \text{Arg}(z)$

Q42 2025 Hurlstone Trial Q1 · 1 mark

Let $z = a + ib$, where a and b are real non-zero numbers. If $z + \dfrac{1}{z}$ is real, which of the following must be true?

(A) $\arg z = \dfrac{\pi}{4}$

(B) $a = -b$

(C) $a = b$

(D) $|z| = 1$

Q43 2025 Caringbah Trial Q3 · 1 mark

Given the complex number $z = a + ib$, where $a, b \in \mathbb{R}$, $a \neq 0$, the expression $\dfrac{4\overline{z}z}{(z + \overline{z})^2}$ is equivalent to:

(A) $4[\text{Re}(z) \times \text{Im}(z)]$

(B) $1 + \left(\dfrac{\text{Im}(z)}{\text{Re}(z)}\right)^2$

(C) $4\left([\text{Re}(z)]^2 + [\text{Im}(z)]^2\right)$

(D) $\dfrac{2 \times \text{Im}(z)}{[\text{Re}(z)]^2}$

Q44 2025 Normanhurst Boys Trial Q7 · **1 mark**

Let z be a complex number such that $\text{Im}(z) \neq 0$. Which of the following is equivalent to $\dfrac{8z\overline{z}}{(\overline{z}-z)^2}$?

(A) $2+2\left(\dfrac{\text{Re}(z)}{\text{Im}(z)}\right)^2$

(B) $2+2\left(\dfrac{\text{Im}(z)}{\text{Re}(z)}\right)^2$

(C) $-2-2\left(\dfrac{\text{Re}(z)}{\text{Im}(z)}\right)^2$

(D) $-2-2\left(\dfrac{\text{Im}(z)}{\text{Re}(z)}\right)^2$

Q45 2025 Pymble Trial Q4 · **1 mark**

The sum of the non-real complex numbers z and w is zero. Which of the following statements is FALSE?

(A) $\dfrac{w}{z}=-1$

(B) $z-w=2z$

(C) $w-z=2w$

(D) $z+\overline{w}=0$