Year 11 Mathematics Extension 1

Topic: Polynomials (4 of 4)
Review Questions: Mixed One-off Exam Questions
Term 3, 2026

How to use this set

These are real past-paper questions on polynomials that do not fit any one recurring pattern: genuine one-offs. They are here so you also meet the rarer, less predictable exam questions. Work through them once you are solid on the patterns in the main review docs. Every question you get wrong or struggle with goes in your **Mistakes Log**.

Contents

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Q1 2023 Blacktown Boys Trial Q10 · **1 mark**

Given that the roots of $2x^2 - \sqrt{3}x + 1 = 0$ are $\tan\alpha$ and $\tan\beta$, what is the value of $\alpha + \beta$?

(A) $\dfrac{\pi}{6}$

(B) $\dfrac{\pi}{4}$

(C) $\dfrac{\pi}{3}$

(D) $\dfrac{2\pi}{3}$

Q2 2022 Normanhurst Boys Trial Q12(b)(i) · **1 mark**

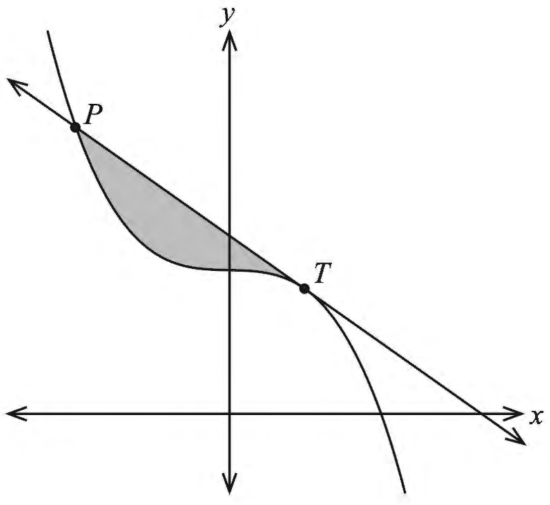
Show that

$$\frac{x^2}{x+1} = x - 1 + \frac{1}{x+1}.$$

Q3 2020 Killara Trial Q13(d)(i) · **1 mark**

Part of the graph of $x^3 + 8y = 64$ is shown. A tangent is drawn to the curve at the point $P(2, 7)$, intersecting the curve again at the point K. The equation of the line KP is

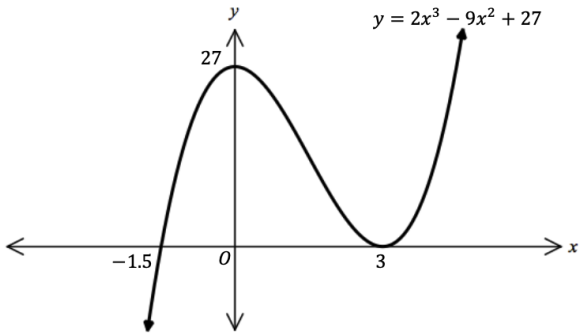
$$y = -\frac{3}{2}x + 10.$$



Find the coordinates of K.

Q4 2021 Scots Trial Q10 · **1 mark**

The diagram shows the graph of $y = 2x^3 - 9x^2 + 27$ with x-intercepts at $x = 3$ and $x = -1.5$ and turning points at $(0, 27)$ and $(3, 0)$.



For what values of k does the function $y = 2x^3 - 9x^2 + k$ have one x-intercept?

(A) $0 < k < 27$

(B) $k < -1.5$ or $k > 27$

(C) $k < -1.5$ or $k > 3$

(D) $k < 0$ or $k > 27$

Q5 2020 Sydney Boys Trial Q12(f)(i) · **2 marks**

Suppose $Q(x) = Ax^3 + Bx^2 + Cx + D$ with $A, B, C, D \in \mathbb{R}$, $A \neq 0$.

(i) What can be deduced about the zeros of $Q(x)$ if $B^2 - 3AC < 0$?

Q6 2025 Pymble Trial Q14(c)(ii) · **2 marks**

(i) Solve $\tan 4\alpha = 1$ for $0 \leq \alpha < \pi$.

(ii) Given

$$\tan 4\alpha = \frac{4\tan\alpha - 4\tan^3\alpha}{1 - 6\tan^2\alpha + \tan^4\alpha}$$

and by letting $x = \tan\alpha$, solve the equation $x^4 + 4x^3 - 6x^2 - 4x + 1 = 0$.

Q7 2023 Glenwood Trial Q13(b) · **3 marks**

(i) Consider the identity $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$. By letting $x = \cos\theta$ in the cubic equation $8x^3 - 6x - 1 = 0$, show that $\cos 3\theta = \dfrac{1}{2}$.

(ii) Hence prove that $\cos\dfrac{\pi}{9}\cos\dfrac{5\pi}{9}\cos\dfrac{7\pi}{9} = \dfrac{1}{8}$.

Q8 2025 HSC Q14(e) · **3 marks**

It is given that $\tan\alpha$, $\tan\beta$ and $\tan\gamma$ are the three real roots of the polynomial equation

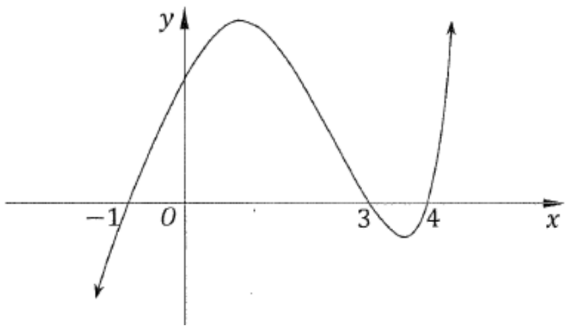
$$x^3 + bx^2 + cx - 1 + b + c = 0,$$

where b and c are real numbers and $c \neq 1$. Find the smallest positive value of $\alpha + \beta + \gamma$.

Q9 ★ 2022 Moriah Trial Q13(d) · **4 marks**

★ *Uses a Year 12 topic (the second derivative): kept as genuine cross-topic exam practice.*

Consider the cubic polynomial $f(x) = ax^3 + bx^2 + cx + d$ where a, b, c and d are real numbers and $a \neq 0$. Let α, β and γ be the zeros of $f(x)$.



(i) Write down an expression for $\alpha + \beta + \gamma$.

(ii) All cubic polynomial functions have a single point of inflexion where the second derivative is equal to zero. Using part (i), or otherwise, show that the x-coordinate of the point of inflexion on the curve $y = f(x)$ is given by

$$x = \frac{\alpha + \beta + \gamma}{3}.$$

(iii) A certain cubic polynomial has x-intercepts at -1, 3 and 4. Find the x-coordinate of its point of inflexion.

Q10 2025 Baulkham Hills Prelim Q12(a) · **4 marks**

The polynomial $P(x)$ is given by $P(x) = x^3 - (k-1)x^2 + (1-k)x + 1$ for some real number k.

(i) Show that $x = -1$ is a root of the equation $P(x) = 0$.

(ii) Given that $P(x) = (x+1)(x^2 - kx + 1)$, find the set of values of k such that the equation $P(x) = 0$ has three distinct real roots.

Q11 2021 St George Girls Trial Q15(b) · 6 marks

(i) By using $\cos 4\theta = 2\cos^2 2\theta - 1$, show that

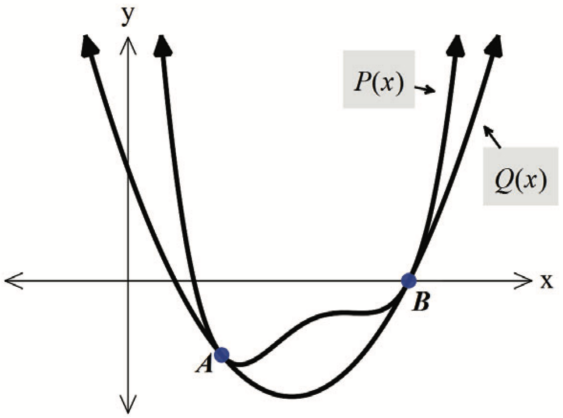
$$8\cos^4\theta - 8\cos^2\theta + 1 - \cos 4\theta = 0.$$

(ii) Using part (i) and by letting $x = \cos\theta$ into the equation $16x^4 - 16x^2 + 2 - \sqrt{2} = 0$, show that $\cos 4\theta = \dfrac{\sqrt{2}}{2}$.

(iii) Hence, prove that $\cos^2\dfrac{\pi}{16}\cos^2\dfrac{7\pi}{16} = \dfrac{2-\sqrt{2}}{16}$.

Q12 2023 Barker Prelim Q15(c) · 6 marks

The polynomial $P(x) = 2x^4 - 16x^3 + 46x^2 - 55x + 21$ is tangent to the quadratic function $Q(x) = 2x^2 - 7x + 3$ at two distinct points A and B. The point B lies on the x-axis.



(i) Find the coordinates of the points A and B.

(ii) Hence, show that the line passing through A and B is tangent to $P(x)$ at the midpoint of AB.

Q13 2020 St George Girls Trial Q16(a) · 7 marks

(i) Prove the trigonometric identity $\sin 3A = 3\sin A - 4\sin^3 A$.

(ii) Hence, show that the equation $6x - 8x^3 = 1$ has the roots

$$\sin\frac{\pi}{18}, \quad \sin\frac{5\pi}{18} \quad \text{and} \quad \sin\frac{25\pi}{18}. \qquad \text{Hint: let } x = \sin A.$$

(iii) Hence show that $\sin\dfrac{\pi}{18}\times\sin\dfrac{5\pi}{18}\times\sin\dfrac{25\pi}{18} = -\dfrac{1}{8}$.

Q14 ★ 2019 HSC Q14(b) · **7 marks**

★ *Uses a Year 12 topic (Newton's method): kept as genuine cross-topic exam practice.*

The diagram shows the graph of $y = \dfrac{1}{x-k}$, where k is a positive real number.

(i) By considering the graphs of $y = x^2$ and $y = \dfrac{1}{x-k}$, explain why the function $f(x) = x^3 - kx^2 - 1$ has exactly one real zero.

Let this zero be α and let $x_1 = k$ be a first approximation to α.

(ii) Show that using one application of Newton's method gives a second approximation, $x_2 = k + \dfrac{1}{k^2}$.

(iii) Show that $x_1 < \alpha < x_2$.

Q15 2022 Normanhurst Boys Trial Q13(c) · **8 marks**

(i) You are given that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$ and $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$. Use this to show that

$$\sin^4\theta + \cos^4\theta = \frac{1}{4}\left(\cos 4\theta + 3\right).$$

(ii) By letting $x = \cos\theta$, show that the equation $8x^4 + 8\left(1 - x^2\right)^2 = 7$ has roots $\pm\cos\dfrac{\pi}{12}$ and $\pm\cos\dfrac{5\pi}{12}$.

(iii) Deduce that $\cos\dfrac{\pi}{12}\cos\dfrac{5\pi}{12} = \dfrac{1}{4}$.

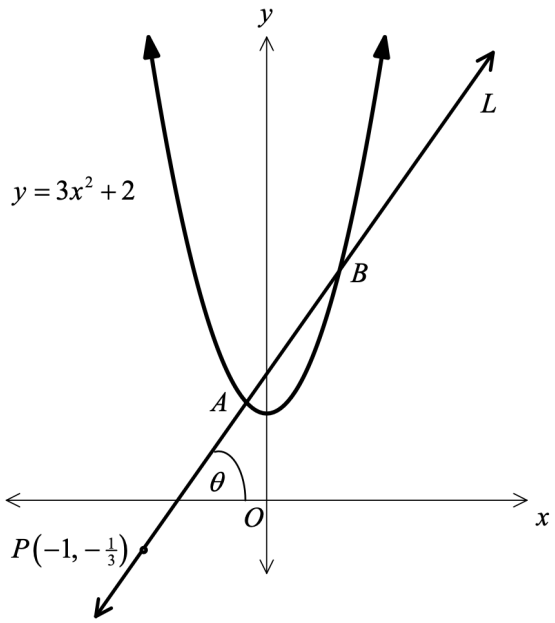
Q16 ★ 2022 Sydney Boys Trial Q14(b) · **11 marks**

★ *Uses Year 12 topics (parametric equations and angle of inclination): kept as genuine cross-topic exam practice.*

Let L be the straight line passing through the point $P(x_0, y_0)$ with an angle of inclination θ to the x-axis. The line L is defined parametrically by

$$x = x_0 + t\cos\theta, \qquad y = y_0 + t\sin\theta,$$

where $t \in \mathbb{R}$ and $\theta \in [0, \pi]$.



(i) Show that for any point T on the line L, $PT = |t|$.

(ii) The line L cuts the parabola $y = 3x^2 + 2$ at points A and B, where P is the point $\left(-1, -\frac{1}{3}\right)$. Let $PA = t_1$ and $PB = t_2$.

(α) Show that t_1 and t_2 are the roots of the equation

$$9t^2\cos^2\theta - 3t(\sin\theta + 6\cos\theta) + 16 = 0.$$

(β) Hence show that

$$AB^2 = \frac{\sin^2\theta + 12\sin\theta\cos\theta - 28\cos^2\theta}{9\cos^4\theta}.$$

(γ) If L is a tangent, show that the two possible gradients of L are 2 and -14.

(δ) Hence, if one of the tangents has a gradient of 2, show that $PZ = \frac{4\sqrt{5}}{3}$, where Z is the point of tangency of the line L to the parabola.