Year 11 Mathematics Extension 1

Topic: Polynomials (3 of 4)
Review Questions: Sums and Products of Zeroes
Term 3, 2026

How to use this review set

These are real past-paper questions, grouped by question pattern. The grey label on each question shows where it is from and its marks.

- The questions are grouped by the pattern that keeps coming up in exams, a few of each in a row, running from the plainer patterns into the harder ones near the end. Work through them and you will meet every type an exam can throw at you.
- This set **is your diagnostic**. Working through it shows you exactly which patterns are solid and which need work. Attempt every question properly in your own book, and go back to your notes to help you work through tough questions if needed.
- Every question you get wrong, need help with or struggle with goes in your **Mistakes Log**, which you should be working on daily to get you exam-ready.

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1. Sum, product and pairwise sums of the roots
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Sums and products of the roots

1. Sum, product and pairwise sums of the roots

Q1 2023 St George Girls Trial Q2 · 1 mark

Consider the equation $3x^3 + 2x^2 - x + 5 = 0$. What is the product of the roots of this polynomial equation?

(A) $\frac{5}{3}$

(B) $\frac{1}{3}$

(C) $-\frac{1}{3}$

(D) $-\frac{5}{3}$

Q2 2024 HSC Q1 · 1 mark

The polynomial $x^3 + 2x^2 - 5x - 6$ has zeros -1, -3 and a. What is the value of a?

(A) -2

(B) 2

(C) 3

(D) 6

Q3 2022 Sydney Boys Trial Q1 · 1 mark

What is the sum of the zeroes of $Q(x) = 4x^5 + 8x^3 - 2x^2 + 5x - 2$?

(A) -8

(B) -2

(C) 0

(D) 2

Q4 2022 Glenwood Trial Q6 · **1 mark**

The equation $x^3 - 8x^2 - 4x + 2 = 0$ has roots α, β and γ. What is the value of $\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha}$?

(A) -4

(B) -2

(C) 2

(D) 4

Q5 2023 Penrith Prelim Q7 · **1 mark**

The roots of $4x^4 - 2x^3 + 9x - 10$ are α, β, γ and δ. What is the value of $\alpha\beta\gamma\delta(\alpha + \beta + \gamma + \delta)$?

(A) $\frac{5}{4}$

(B) $-\frac{5}{4}$

(C) $\frac{9}{4}$

(D) $-\frac{9}{4}$

Q6 2020 Ascham Trial Q11(b) · **3 marks**

The polynomial $P(x) = x^3 - 4x^2 + tx + 10$ has roots α, $-\alpha$ and β. Find the three roots and hence find the value of t.

Q7 2020 Normanhurst Boys Prelim Q11(a) · **3 marks**

Solve $3x^3 - 17x^2 - 8x + 12 = 0$, given that the product of two of the roots is 4.

Q8 2021 Sydney Grammar Prelim Q4(b) · **4 marks**

The polynomial $P(x) = 2x^3 + 6x^2 + cx + d$ has a triple zero α.

(i) Find the value of α.

(ii) Find the values of c and d.

Q9 2020 Fort St Prelim Q9(c) · **5 marks**

The cubic equation $2x^3 - 3x + 2 = 0$ has three real roots α, β and γ. Evaluate:

(i) $\alpha^3\beta^3\gamma^2 + \alpha^3\beta^2\gamma^3 + \alpha^2\beta^3\gamma^3$

(ii) $\frac{\alpha}{\beta\gamma} + \frac{\beta}{\alpha\gamma} + \frac{\gamma}{\alpha\beta}$

2. Symmetric functions of the roots

Q10 2021 Caringbah Trial Q3 · 1 mark

The equation $x^3 - 2x^2 - 4x + 8 = 0$ has roots α, β and γ. What is the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$?

(A) $-\frac{1}{2}$

(B) $-\frac{1}{4}$

(C) $\frac{1}{4}$

(D) $\frac{1}{2}$

Q11 2025 Normanhurst Boys Trial Q3 · 1 mark

The equation $x^3 - 3x^2 - 18x + 40 = 0$ has roots α, β, γ. What is the value of $\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\alpha\gamma}$?

(A) $-\frac{1}{18}$

(B) $\frac{1}{18}$

(C) $-\frac{3}{40}$

(D) $\frac{3}{40}$

Q12 2022 Presbyterian Prelim Q5 · 1 mark

Let α and β be the roots of the polynomial $P(x) = 3x^2 - x - 2$. What is the value of $\frac{1}{\alpha} + \frac{1}{\beta}$?

(A) $-\frac{2}{3}$

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) $\frac{2}{3}$

Q13 2019 HSC Q7 · 1 mark

Let $P(x) = qx^3 + rx^2 + rx + q$ where q and r are constants, $q \neq 0$. One of the zeros of $P(x)$ is -1. Given that a is a zero of $P(x)$, $a \neq -1$, which of the following is also a zero?

(A) $-\dfrac{1}{a}$

(B) $-\dfrac{q}{a}$

(C) $\dfrac{1}{a}$

(D) $\dfrac{q}{a}$

Q14 2025 Hills Grammar Trial Q11(f) · 2 marks

The cubic polynomial $P(x) = x^3 + x^2 - 8x - 12$ has roots α, β and γ. Find $\alpha^2 + \beta^2 + \gamma^2$.

Q15 2020 St George Girls Trial Q11(a) · 2 marks

The polynomial $2x^3 - 4x^2 + 3x - 6 = 0$ has roots α, β and γ. Calculate the value of $\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma}$.

Q16 2024 Hurlstone Prelim Q15(c) · 2 marks

If α, β, γ are the roots of the equation $2x^3 - 5x^2 - 3x + 1 = 0$, find the value of $\dfrac{1}{\alpha\beta} + \dfrac{1}{\beta\gamma} + \dfrac{1}{\alpha\gamma}$.

Q17 2023 Girraween Prelim Q9(b) · 3 marks

The polynomial $P(x) = x^4 + 4x^3 - 3x^2 + 5x - 7$ has roots α, β, γ and δ. Find the value of $\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma} + \dfrac{1}{\delta}$.

Q18 2022 SCEGGS Prelim Q8(f) · 3 marks

Given that α, β, γ are the roots of the equation $2x^3 - 6x^2 + 10x - 5 = 0$, find:

(i) $\alpha + \beta + \gamma$

(ii) $\alpha^2 + \beta^2 + \gamma^2$

Q19 2024 Glenwood Prelim Q9(e) · 3 marks

If α, β, γ are roots of $3x^3 + 8x^2 - 1 = 0$, find the value of $\left(\beta + \dfrac{1}{\gamma}\right)\left(\gamma + \dfrac{1}{\alpha}\right)\left(\alpha + \dfrac{1}{\beta}\right)$. (Do not find the values of α, β and γ.)

Q20 2023 Moriah Trial Q9(a) · **4 marks**

The quadratic equation $x^2 + 3x + 1 = 0$ has roots α and β.

(i) Find the value of $\alpha + \beta$.

(ii) Find the value of $\alpha\beta$.

(iii) Hence, or otherwise, find the value of $\alpha^2 + \beta^2$.

Q21 2021 Sydney Grammar Prelim Q2(b) · **4 marks**

The polynomial equation $x^3 + 9x^2 - 2x - 24 = 0$ has roots α, β and γ. Find the value of:

(i) $\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma}$

(ii) $\alpha^2 + \beta^2 + \gamma^2$

Q22 2022 James Ruse Trial Q12(a) · **6 marks**

The quartic polynomial $P(x) = x^4 + px^3 + qx^2 + px + 1$ has $x = \alpha$ as a double zero.

(i) Show that α^{-1} is also a zero of $P(x)$.

(ii) Find the fourth root in terms of α.

(iii) Hence show that $p^2 = 4(q - 2)$.

Q23 2020 St George Girls Prelim Q9(b) · **7 marks**

Let α, β and γ be the roots of $2x^3 - 4x + 5 = 0$. Find, in simplest form:

(i) $\alpha + \beta + \gamma$

(ii) $\alpha\beta + \beta\gamma + \gamma\alpha$

(iii) $\alpha\beta\gamma$

(iv) $\dfrac{1}{\alpha\beta} + \dfrac{1}{\beta\gamma} + \dfrac{1}{\gamma\alpha}$

(v) $\alpha^3 + \beta^3 + \gamma^3$

Relationships between roots

3. Finding a coefficient from a root condition

Q24 2020 Glenwood Prelim Q4 · **1 mark**

It is known that two of the roots of the equation $3x^3 + x^2 - kx + 6 = 0$ are reciprocals of each other. What is the value of k?

(A) -2

(B) 6

(C) 7

(D) 17

Q25 2020 Normanhurst Boys Prelim Q3 · **1 mark**

Two of the roots of $4x^3 + 8x^2 + kx - 18 = 0$ are equal in magnitude but have opposite sign. What is the value of k?

(A) -9

(B) 9

(C) 2

(D) -2

Q26 2024 Penrith Prelim Q9 · **1 mark**

Two of the roots of the equation $x^3 - 3x^2 + kx + 15 = 0$ are reciprocals of each other. What is the value of k?

(A) -69

(B) -169

(C) -269

(D) -369

Q27 2025 Pymble Trial Q11(e) · **2 marks**

The sum of two of the roots of the equation $2x^3 + 6x^2 + kx - 150 = 0$ is zero. Find the value of k.

Q28 2023 Baulkham Hills Prelim Q13(d) · **3 marks**

Two of the roots of the equation $4x^3 + px^2 - 20x + 12 = 0$ are reciprocals. Find the value of p.

Q29 2025 Baulkham Hills Prelim Q11(c) · **3 marks**

The polynomial $P(x) = x^3 - 4x^2 + kx + 10$ has two zeros that are the same in magnitude but opposite in sign. Find the value of k and all zeros of the polynomial.

Q30 2021 Newington Trial Q6(b) · **5 marks**

Consider the polynomial $P(x) = x^3 - 4x^2 + kx + 12$, where k is a constant. It is given that α, β and γ are the roots of the equation $P(x) = 0$.

(i) Find the values of $\alpha + \beta + \gamma$ and $\alpha\beta\gamma$.

(ii) If two of the roots are equal in magnitude but opposite in sign, find the value of the roots and the value of k.

4. Root relationships, proofs and forming polynomials

Q31 2022 Barker Trial Q1 · **1 mark**

The equation $x^3 + bx^2 + cx + d = 0$ has roots $1 + \sqrt{2}$, $1 - \sqrt{2}$ and 4. What is the value of b?

(A) -6

(B) -4

(C) 4

(D) 6

Q32 2023 SCEGGS Prelim Q1 · **1 mark**

The sum of two roots of $4x^4 - 56x^3 + 239x^2 - 316x + 105 = 0$ is 12. What is the sum of the other two roots?

(A) $-\dfrac{14}{3}$

(B) -2

(C) 2

(D) $\dfrac{14}{3}$

Q33 2020 Knox Prelim Q3 · **1 mark**

A polynomial equation has roots α, β and γ where

$$\alpha + \beta + \gamma = -2, \qquad \alpha\beta + \alpha\gamma + \beta\gamma = 3 \qquad \text{and} \qquad \alpha\beta\gamma = 1.$$

Which polynomial equation has the roots α, β and γ?

(A) $x^3 + 2x^2 + 3x + 1 = 0$

(B) $x^3 + 2x^2 + 3x - 1 = 0$

(C) $x^3 - 2x^2 + 3x + 1 = 0$

(D) $x^3 - 2x^2 + 3x - 1 = 0$

Q34 2025 Hornsby Girls Trial Q13(b) · **2 marks**

The three solutions of the equation $x^3 - 2x^2 + 3x + 1 = 0$ are denoted by α, β and γ. Consider the equation $x^3 - x^2 + 7x + C = 0$ whose solutions are

$$2\alpha - 1, \quad 2\beta - 1 \quad \text{and} \quad 2\gamma - 1.$$

Find the value of C, where C is an integer.

Q35 2020 Gosford Trial Q12(a) · **3 marks**

The roots of $x^3 - x^2 - 5x + 2 = 0$ are α, β and γ.

(i) Show that $\beta + \gamma = 1 - \alpha$.

(ii) Using part (i), and similar results, evaluate $\dfrac{\beta+\gamma}{\alpha} + \dfrac{\gamma+\alpha}{\beta} + \dfrac{\alpha+\beta}{\gamma}$.

Q36 2023 Sydney Grammar Prelim Q11(a) · **3 marks**

The polynomial $P(x) = 2x^3 - x^2 + 16x - 8$ has zeroes α, β and γ. Find the value of:

(i) $\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2$

(ii) $(\alpha + 1)(\beta + 1)(\gamma + 1)$

Q37 2022 Fort St Prelim Q10(b) · **3 marks**

The three roots of $x^3 + px^2 + qx + r = 0$ are integers with a common difference of β. That is, if the roots are x_1, x_2 and x_3 then $x_2 - x_1 = x_3 - x_2 = \beta$. Show that $9pq = 27r + 2p^3$.

Q38 2022 Sydney Girls Prelim Q4(d) · **3 marks**

Suppose that the roots of $Q(x) = Ax^3 + Bx^2 + Cx + D$ are α, α and γ, where A, B, C and $D \in \mathbb{R}$, and $A \neq 0$. Show that $B\alpha^2 + 2C\alpha + 3D = 0$.

Q39 2021 Caringbah Trial Q13(b) · **4 marks**

One of the roots of the equation $x^3 + ax^2 + 1 = 0$ is equal to the sum of the other two roots.

(i) Using your knowledge of the sum and products of the roots, show that $x = -\dfrac{a}{2}$ is a root of the equation.

(ii) Find the value of a.

Q40 2023 Sydney Grammar Prelim Q13(c) · **4 marks**

Given that the polynomial $x^3 + px^2 + qx + r$ has zeroes α, β and $\alpha + \beta$, show that

$$4pq = p^3 + 8r.$$

Q41 2023 St George Girls Trial Q11(c) · **5 marks**

The polynomial $P(x) = 2x^3 + mx^2 + nx - 20$ has a double root and $P(2) = P'(2) = 0$.

(i) Find the values of m and n.

(ii) Hence, find the other root of $P(x)$.