Year 11 Mathematics Extension 1

Topic: Polynomials (2 of 4)
Review Questions: Remainder and Factor Theorems
Term 3, 2026

How to use this review set

These are real past-paper questions, grouped by question pattern. The grey label on each question shows where it is from and its marks.

- The questions are grouped by the pattern that keeps coming up in exams, a few of each in a row, running from the plainer patterns into the harder ones near the end. Work through them and you will meet every type an exam can throw at you.
- This set **is your diagnostic**. Working through it shows you exactly which patterns are solid and which need work. Attempt every question properly in your own book, and go back to your notes to help you work through tough questions if needed.
- Every question you get wrong, need help with or struggle with goes in your **Mistakes Log**, which you should be working on daily to get you exam-ready.

Contents

Division, remainder and factor theorems

1. Polynomial long division
2. The remainder theorem
3. The factor theorem: factorising and solving

Finding unknown coefficients

4. Finding unknown coefficients from factor and remainder conditions

Division, remainder and factor theorems

1. Polynomial long division

Q1 2023 Cheltenham Girls Trial Q3 · 1 mark

Which of the following expressions is equivalent to $\dfrac{x^4+3x^3+3x^2-4x+5}{x^2+2x}$?

(A) $x^2+x+1+\dfrac{5-6x}{x^2+2x}$

(B) $x^2+x+1+\dfrac{6x-5}{x^2+2x}$

(C) $x^2+2x+2+\dfrac{5-6x}{x^2+2x}$

(D) $x^2+2x+\dfrac{6x-5}{x^2+2x}$

Q2 2025 Hills Grammar Trial Q5 · 1 mark

Let $P(x)$ be a polynomial of degree 6. When $P(x)$ is divided by the polynomial $Q(x)$, the remainder is x^2+2. Which statement is true about the degree of $Q(x)$?

(A) The degree of $Q(x)$ must be at least 2.

(B) The degree of $Q(x)$ must be 2.

(C) The degree of $Q(x)$ must be at least 3.

(D) The degree of $Q(x)$ must be 3.

Q3 2019 HSC Q11(d) · 2 marks

Find the polynomial $Q(x)$ that satisfies $x^3+2x^2-3x-7=(x-2)Q(x)+3$.

Q4 2023 Moriah Trial Q12(c)(ii) · 2 marks

The right-hand section of the curve $y=\dfrac{x^2-4}{x-1}$ has a vertical asymptote and an oblique asymptote. By dividing x^2-4 by $x-1$, find the equation of the oblique asymptote.

Q5 2025 Smiths Hill Trial Q12(d) · 3 marks

Given the polynomial $P(x)=x^3+2x^2+3x+10$, find the polynomial $Q(x)$ such that $P(x)=(x+2)Q(x)+R$.

Q6 2025 Hurlstone Prelim Q14(b) · **4 marks**

By using the method of long division, divide the polynomial $P(x) = x^4 + 11x^2 - 1$ by $(x + 3)$. Write your answer in the form $P(x) = Q(x)A(x) + R(x)$.

2. The remainder theorem

Q7 2023 Kinross Wolaroi Trial Q1 · **1 mark**

What is the remainder when $P(x) = -2x^3 + 8x - 10$ is divided by $x + 1$?

(A) -20

(B) -16

(C) -4

(D) 0

Q8 2025 Caringbah Prelim Q1 · **1 mark**

Find the remainder when $P(x) = 4x^3 - x^2 + 7$ is divided by $x - 1$.

(A) 2

(B) 4

(C) 10

(D) 12

Q9 2024 Hurlstone Trial Q5 · **1 mark**

What is the remainder when $2x^5 - 3x^3 + x^2 - 5$ is divided by $x + 2$?

(A) -41

(B) -5

(C) 39

(D) 41

Q10 2023 Riverview Trial Q1 · **1 mark**

What is the remainder when $P(x) = -2x^3 + 3x^2 - 6x + 4$ is divided by $x + 2$?

(A) 44

(B) -12

(C) 12

(D) 20

Q11 2020 Blacktown Boys Prelim Q9 · **1 mark**

When the polynomial $P(x)$ is divided by $(x - 1)(x + 4)$, the remainder is $2x + 11$. What is the remainder when $P(x)$ is divided by $x + 4$?

(A) 3

(B) 4

(C) 11

(D) 13

Q12 2020 Sydney Tech Prelim Q7 · **1 mark**

The graph of an odd polynomial $y = P(x)$ passes through the point $(3, -4)$. What is the remainder when $P(x)$ is divided by $(x + 3)$?

(A) 3

(B) 4

(C) -4

(D) -3

Q13 2020 Barker Trial Q7 · **1 mark**

When the polynomial $P(x)$ is divided by $(x + 2)(x - 5)$ the remainder is $3x - 6$. What is the remainder when $P(x)$ is divided by $(x + 2)$?

(A) -2

(B) -7

(C) -12

(D) -28

Q14 2025 Sydney Girls Trial Q11(a) · **2 marks**

Find the remainder when $x^4 - 3x^3 + 7$ is divided by $x + 3$.

Q15 2022 North Sydney Boys Trial Q11(b) · **3 marks**

Find the remainder when $x^3 - 5x^2 + 7$ is divided by $(x - 1)^2$.

Q16 2022 Gosford Prelim Q4(b) · **4 marks**

It is given that $P(x) = (x - a)^3 + (x - b)^2$. The remainder when $P(x)$ is divided by $(x - b)$ is -8.

(i) Show that when $P(x)$ is divided by $(x - a)$, the remainder is 4.

(ii) Prove that $x = \dfrac{a + b}{2}$ is a zero of $P(x)$.

Q17 2020 Girraween Prelim Q8(c) · **4 marks**

When the polynomial $P(x)$ is divided by $(x + 1)(x - 3)$, the quotient is $Q(x)$ and the remainder is $-2x + 4$.

(i) Find the remainder when $P(x)$ is divided by $x + 1$.

(ii) What is the remainder when $P(x)$ is divided by $x - 3$?

3. The factor theorem: factorising and solving

Q18 2022 Blacktown Boys Trial Q14(a)(i) · **1 mark**

Prove that $(n + 1)$ is a factor of $P(n) = 4n^3 + 18n^2 + 23n + 9$.

Q19 2021 Sydney Tech Trial Q1 · **1 mark**

Which of the following expressions is a correct factorisation of $x^3 - 27$?

(A) $(x - 3)(x^2 - 3x + 9)$

(B) $(x - 3)(x^2 - 6x + 9)$

(C) $(x - 3)(x^2 + 3x + 9)$

(D) $(x - 3)(x^2 + 6x + 9)$

Q20 2021 Frensham Trial Q1 · **1 mark**

Which of the following is NOT a factor of the polynomial $P(x) = x^3 - x^2 - 8x + 12$?

(A) $(x + 3)$

(B) $(x + 3)^2$

(C) $(x - 2)$

(D) $(x - 2)^2$

Q21 2023 Barker Prelim Q11(c) · **2 marks**

Fully factorise the polynomial $P(x) = x^3 - 4x^2 + x + 6$.

Q22 2023 Sydney Boys Trial Q13(a)(i) · **2 marks**

Factorise $t^3 + 2t^2 + 3t + 2$.

Q23 2023 North Sydney Boys Prelim QB2 · **2 marks**

Show that $(x - 1)(x - 2)$ is a factor of $P(x) = x^n(2^m - 1) + x^m(1 - 2^n) + (2^n - 2^m)$, where m and n are positive integers.

Q24 2021 Manly Trial Q11(d) · **3 marks**

Solve for x: $x^3 - 5x^2 + 8x - 4 = 0$.

Q25 2022 Blacktown Boys Prelim Q11(c) · **3 marks**

Consider the polynomial $P(x) = 2x^3 - 19x^2 + 60x - 63$.

(i) Show that $P(x)$ has a double root at $x = 3$.

(ii) Hence, factorise $P(x)$ completely.

Q26 2024 Hurlstone Prelim Q15(d) · **3 marks**

It is given that the polynomial $Q(x) = 4x^3 - 8x^2 + 5x - 1$ has a zero of multiplicity 2. Express $Q(x)$ as a product of linear factors.

Q27 2021 Girraween Prelim Q1(a) · **4 marks**

Consider the polynomial $P(x) = 2x^3 + 3x^2 - 29x - 60$.

(i) Use the factor theorem to show that $(x + 3)$ is a factor of $P(x)$.

(ii) Find the remainder when $P(x)$ is divided by $(x + 2)$.

(iii) Factorise $P(x)$ completely.

Q28 2024 North Sydney Boys Trial Q12(e) · **4 marks**

Given $4x^4 + 8x^3 + 3x^2 - 2x - 1 = 0$.

(i) Express in the form $(ax^2 + bx)^2 - (cx + d)^2 = 0$, where a, b, c and d are positive integers, and state the values of a, b, c and d.

(ii) Hence solve the equation for x.

Finding unknown coefficients

4. Finding unknown coefficients from factor and remainder conditions

Q29 2020 Abbotsleigh Trial Q3 · 1 mark

The polynomial $P(x) = 7x^3 + 9x^2 - 5cx$ has a factor $(x + c)$, where c is a non-zero real number. What is the value of c?

(A) 2

(B) -2

(C) -1

(D) 1

Q30 2021 Killara Trial Q4 · 1 mark

Find the values of a and b given that $x + 1$ and $x - 2$ are factors of $ax^3 - 8x^2 + bx - 24$.

(A) $a = -20,\ b = 52$

(B) $a = -\dfrac{88}{7},\ b = \dfrac{312}{7}$

(C) $a = \dfrac{88}{7},\ b = -\dfrac{312}{7}$

(D) $a = 20,\ b = -52$

Q31 2023 Penrith Trial Q2 · 1 mark

The polynomial $P(x) = 6x^3 - 2x + bx - 4$ has a factor of $2x - 1$. What is the value of b?

(A) 8

(B) -7.5

(C) 8.5

(D) -7

Q32 2021 Hornsby Girls Prelim Q1 · 1 mark

If $(x+m)$ is a factor of $8x^3 - 14x^2 - m^2x$ where $m \in \mathbb{R}$, $m \neq 0$, then the value of m is

(A) 7

(B) 4

(C) 1

(D) -2

Q33 2020 Sydney Girls Trial Q10 · 1 mark

The polynomial $x^2 + x + 2$ is a factor of the polynomial $x^4 + 4x^2 + x + a$. Determine the value of the constant a.

(A) $a = -6$

(B) $a = -3$

(C) $a = 3$

(D) $a = 6$

Q34 2022 Ascham Trial Q11(a) · 2 marks

Let $P(x) = 2x^3 + rx^2 + x - 3$ and $D(x) = x^2 + 6x + 4$ have the same remainder when divided by $x - 1$. Find r.

Q35 2021 Presbyterian Prelim Q7(a) · 2 marks

Let $P(x) = x^3 - 3x^2 + kx - 5$, where k is a rational number. When $P(x)$ is divided by $(x+2)$, the remainder is 7. Find k.

Q36 2025 Merewether Trial Q12(e) · 2 marks

When the polynomial $P(x)$ is divided by $(x+1)(x-4)$ the quotient is $Q(x)$ and the remainder is $R(x)$.

(i) Given that $P(4) = -5$, show that $R(4) = -5$.

(ii) When $P(x)$ is divided by $(x+1)$, the remainder is 5. Find $R(x)$.

Q37 2021 Fort St Prelim Q2(b) · 3 marks

$P(x) = 2x^3 - ax^2 + bx + 3$, where a and b are constants. When $P(x)$ is divided by $x^2 - 1$ the remainder is $-2x - 2$. Find a and b.

Q38 2025 Hurlstone Trial Q15(c) · 3 marks

Let $P(x) = (x+1)(x-3)Q(x) + ax + b$, where $Q(x)$ is a polynomial and a and b are real numbers. The polynomial $P(x)$ has a factor of $x-3$. When $P(x)$ is divided by $x+1$ the remainder is 8.

(i) Find the values of a and b.

(ii) Find the remainder when $P(x)$ is divided by $(x+1)(x-3)$.

Q39 2022 Sydney Girls Prelim Q1(g) · 3 marks

Let $P(x) = x^3 + ax^2 + bx + 8$ where a and b are real numbers. Find the values of a and b given that $(x-2)^2$ is a factor of $P(x)$.

Q40 2021 Sydney Girls Prelim Q1(c) · 4 marks

Consider the polynomial $P(x) = x^3 + ax + b$ where a and b are real numbers. $P(x)$ has a factor of $(x-2)$ and when $P(x)$ is divided by $(x+1)$ the remainder is 6.

(i) Show that $2a + b = -8$ and $-a + b = 7$.

(ii) Find the values of a and b.

Q41 2022 Sydney Boys Trial Q11(d) · 5 marks

When a polynomial $p(x)$, with rational coefficients, is divided by $(x-\alpha)^2$ the remainder is $R_2(x-\alpha) + R_1$, where $R_1, R_2 \in \mathbb{R}$, i.e.

$$p(x) = (x-\alpha)^2 Q(x) + R_2(x-\alpha) + R_1,$$

where $Q(x)$ is a polynomial.

(i) Show that $R_1 = p(\alpha)$ and $R_2 = p'(\alpha)$.

(ii) For $n > 1$, $n \in \mathbb{N}$, show that $x^n - n(x-1) - 1$ is divisible by $(x-1)^2$.

(iii) For $n > 1$, $n \in \mathbb{N}$, deduce that $2^{4n} - 15n - 1$ is divisible by 225.