Year 11 Mathematics Extension 1

Topic: Polynomials (2 of 4)

Review Questions (Worked Solutions): Remainder and Factor Theorems

Term 3, 2026

Division, remainder and factor theorems

1. Polynomial long division

Q1 2023 Cheltenham Girls Trial Q3 · 1 mark

Which of the following expressions is equivalent to $\dfrac{x^4+3x^3+3x^2-4x+5}{x^2+2x}$?

(A) $x^2+x+1+\dfrac{5-6x}{x^2+2x}$

(B) $x^2+x+1+\dfrac{6x-5}{x^2+2x}$

(C) $x^2+2x+2+\dfrac{5-6x}{x^2+2x}$

(D) $x^2+2x+\dfrac{6x-5}{x^2+2x}$

Criteria	Marks
Correct option (A)	1

Solution.

Dividing $x^4+3x^3+3x^2-4x+5$ by x^2+2x: the quotient is x^2+x+1 and the remainder is $5-6x$. Hence

$$\frac{x^4+3x^3+3x^2-4x+5}{x^2+2x}=x^2+x+1+\frac{5-6x}{x^2+2x}.$$

The answer is (A).

Q2 2025 Hills Grammar Trial Q5 · **1 mark**

Let $P(x)$ be a polynomial of degree 6. When $P(x)$ is divided by the polynomial $Q(x)$, the remainder is x^2+2. Which statement is true about the degree of $Q(x)$?

(A) The degree of $Q(x)$ must be at least 2.

(B) The degree of $Q(x)$ must be 2.

(C) The degree of $Q(x)$ must be at least 3.

(D) The degree of $Q(x)$ must be 3.

Criteria	**Marks**
Correct option (C)	1

Solution.

The remainder x^2+2 has degree 2. On division, the remainder must have degree strictly less than the divisor $Q(x)$, so $\deg Q(x) > 2$, i.e. $\deg Q(x) \geq 3$. The answer is (C).

Q3 2019 HSC Q11(d) · **2 marks**

Find the polynomial $Q(x)$ that satisfies $x^3+2x^2-3x-7=(x-2)Q(x)+3$.

Criteria	**Marks**
Sets up $(x-2)Q(x)=x^3+2x^2-3x-10$ and begins the division	1
Correct quotient $Q(x)=x^2+4x+5$	1

Solution.

Rearranging, $(x-2)Q(x)=x^3+2x^2-3x-7-3=x^3+2x^2-3x-10$. Dividing by $x-2$:

$$Q(x)=\frac{x^3+2x^2-3x-10}{x-2}=x^2+4x+5.$$

Q4 2023 Moriah Trial Q12(c)(ii) · **2 marks**

The right-hand section of the curve $y = \dfrac{x^2-4}{x-1}$ has a vertical asymptote and an oblique asymptote. By dividing x^2-4 by $x-1$, find the equation of the oblique asymptote.

Criteria	**Marks**
Divides x^2-4 by $x-1$ to get quotient $x+1$, remainder -3	1
States the oblique asymptote $y = x+1$	1

Solution.

Dividing x^2-4 by $x-1$ gives quotient $x+1$ and remainder -3, so

$$y = \frac{x^2-4}{x-1} = x+1-\frac{3}{x-1}.$$

As $x \to \pm\infty$ the term $\dfrac{3}{x-1} \to 0$, so the oblique asymptote is $y = x+1$.

Q5 2025 Smiths Hill Trial Q12(d) · **3 marks**

Given the polynomial $P(x) = x^3+2x^2+3x+10$, find the polynomial $Q(x)$ such that $P(x) = (x+2)Q(x)+R$.

Criteria	**Marks**
Sets up the long division of $P(x)$ by $x+2$	1
Correct quotient $Q(x) = x^2+3$	1
States the remainder $R = 4$	1

Solution.

Dividing $P(x) = x^3+2x^2+3x+10$ by $x+2$:

$$\frac{x^3+2x^2+3x+10}{x+2} = x^2+3+\frac{4}{x+2},$$

so $Q(x) = x^2+3$ and $R = 4$. That is, $P(x) = (x+2)(x^2+3)+4$.

Q6 2025 Hurlstone Prelim Q14(b) · **4 marks**

By using the method of long division, divide the polynomial $P(x) = x^4 + 11x^2 - 1$ by $(x + 3)$. Write your answer in the form $P(x) = Q(x)A(x) + R(x)$.

Criteria	Marks
Sets up the long division (writing $P(x) = x^4 + 0x^3 + 11x^2 + 0x - 1$)	1
Correct first two terms of the quotient $(x^3 - 3x^2)$	1
Correct quotient $Q(x) = x^3 - 3x^2 + 20x - 60$	1
Correct remainder $R(x) = 179$, written in the required form	1

Solution.

Writing $P(x) = x^4 + 0x^3 + 11x^2 + 0x - 1$ and dividing by $x + 3$ gives quotient $x^3 - 3x^2 + 20x - 60$ and remainder 179. Hence

$$P(x) = (x^3 - 3x^2 + 20x - 60)(x + 3) + 179,$$

so $Q(x) = x^3 - 3x^2 + 20x - 60$, $A(x) = x + 3$ and $R(x) = 179$.

2. The remainder theorem

Q7 2023 Kinross Wolaroi Trial Q1 · **1 mark**

What is the remainder when $P(x) = -2x^3 + 8x - 10$ is divided by $x + 1$?

(A) -20

(B) -16

(C) -4

(D) 0

Criteria	Marks
Correct option (B)	1

Solution.

By the remainder theorem, the remainder on division by $x + 1$ is $P(-1)$.

$$P(-1) = -2(-1)^3 + 8(-1) - 10 = 2 - 8 - 10 = -16.$$

The answer is (B).

Q8 2025 Caringbah Prelim Q1 · **1 mark**

Find the remainder when $P(x) = 4x^3 - x^2 + 7$ is divided by $x - 1$.

(A) 2

(B) 4

(C) 10

(D) 12

Criteria	Marks
Correct option (C)	1

Solution.

By the remainder theorem, the remainder on division by $x - 1$ is $P(1)$.

$$P(1) = 4(1)^3 - (1)^2 + 7 = 4 - 1 + 7 = 10.$$

The answer is (C).

Q9 2024 Hurlstone Trial Q5 · **1 mark**

What is the remainder when $2x^5 - 3x^3 + x^2 - 5$ is divided by $x + 2$?

(A) -41

(B) -5

(C) 39

(D) 41

Criteria	Marks
Correct option (A)	1

Solution.

By the remainder theorem, the remainder on division by $x + 2$ is $P(-2)$.

$$P(-2) = 2(-2)^5 - 3(-2)^3 + (-2)^2 - 5 = -64 + 24 + 4 - 5 = -41.$$

The answer is (A).

Q10 2023 Riverview Trial Q1 · **1 mark**

What is the remainder when $P(x) = -2x^3 + 3x^2 - 6x + 4$ is divided by $x + 2$?

(A) 44

(B) -12

(C) 12

(D) 20

Criteria	Marks
Correct option (A)	1

Solution.

By the remainder theorem, the remainder on division by $x + 2$ is $P(-2)$.

$$P(-2) = -2(-2)^3 + 3(-2)^2 - 6(-2) + 4 = 16 + 12 + 12 + 4 = 44.$$

The answer is (A).

Q11 2020 Blacktown Boys Prelim Q9 · **1 mark**

When the polynomial $P(x)$ is divided by $(x - 1)(x + 4)$, the remainder is $2x + 11$. What is the remainder when $P(x)$ is divided by $x + 4$?

(A) 3

(B) 4

(C) 11

(D) 13

Criteria	Marks
Correct option (A)	1

Solution.

We can write $P(x) = (x - 1)(x + 4)Q(x) + 2x + 11$. The remainder on division by $x + 4$ is $P(-4)$, and since $(x - 1)(x + 4)$ vanishes at $x = -4$,

$$P(-4) = 2(-4) + 11 = -8 + 11 = 3.$$

The answer is (A).

Q12 2020 Sydney Tech Prelim Q7 · **1 mark**

The graph of an odd polynomial $y = P(x)$ passes through the point $(3, -4)$. What is the remainder when $P(x)$ is divided by $(x + 3)$?

(A) 3

(B) 4

(C) -4

(D) -3

Criteria	Marks
Correct option (B)	1

Solution.

The remainder on division by $x + 3$ is $P(-3)$. Since P is odd, $P(-3) = -P(3)$. The graph passes through $(3, -4)$, so $P(3) = -4$ and

$$P(-3) = -P(3) = -(-4) = 4.$$

The answer is (B).

Q13 2020 Barker Trial Q7 · **1 mark**

When the polynomial $P(x)$ is divided by $(x + 2)(x - 5)$ the remainder is $3x - 6$. What is the remainder when $P(x)$ is divided by $(x + 2)$?

(A) -2

(B) -7

(C) -12

(D) -28

Criteria	Marks
Correct option (C)	1

Solution.

Write $P(x) = (x + 2)(x - 5)Q(x) + 3x - 6$. The remainder on division by $(x + 2)$ is $P(-2)$, and since $(x + 2)(x - 5)$ vanishes at $x = -2$,

$$P(-2) = 3(-2) - 6 = -6 - 6 = -12.$$

The answer is (C).

Q14 2025 Sydney Girls Trial Q11(a) · **2 marks**

Find the remainder when $x^4 - 3x^3 + 7$ is divided by $x + 3$.

Criteria	Marks
Applies the remainder theorem, evaluating $P(-3)$	1
Correct remainder 169	1

Solution.

Let $P(x) = x^4 - 3x^3 + 7$. By the remainder theorem, the remainder on division by $x + 3$ is $P(-3)$.

$$P(-3) = (-3)^4 - 3(-3)^3 + 7 = 81 - 3(-27) + 7 = 81 + 81 + 7 = 169.$$

Q15 2022 North Sydney Boys Trial Q11(b) · **3 marks**

Find the remainder when $x^3 - 5x^2 + 7$ is divided by $(x - 1)^2$.

Criteria	Marks
Sets up remainder as $ax + b$ (or equivalent), using $P(1)$	1
Obtains a second equation (differentiation or equating coefficients) to find a	1
Correct remainder $-7x + 10$	1

Solution.

Since the divisor $(x - 1)^2$ has degree 2, the remainder is linear, say $R(x) = ax + b$. Write

$$P(x) = x^3 - 5x^2 + 7 = (x - 1)^2 Q(x) + ax + b.$$

Substituting $x = 1$ gives $P(1) = a + b$: $1 - 5 + 7 = 3$, so $a + b = 3$. Differentiating, $P'(x) = 2(x - 1)Q(x) + (x - 1)^2 Q'(x) + a$, and substituting $x = 1$ gives $P'(1) = a$. Since $P'(x) = 3x^2 - 10x$, we have $P'(1) = 3 - 10 = -7$, so $a = -7$. Then $b = 3 - a = 3 - (-7) = 10$. The remainder is

$$R(x) = -7x + 10.$$

Q16 2022 Gosford Prelim Q4(b) · 4 marks

It is given that $P(x) = (x-a)^3 + (x-b)^2$. The remainder when $P(x)$ is divided by $(x-b)$ is -8.

(i) Show that when $P(x)$ is divided by $(x-a)$, the remainder is 4.

(ii) Prove that $x = \dfrac{a+b}{2}$ is a zero of $P(x)$.

Criteria	Marks
(i) Uses the remainder theorem to get $P(b) = (b-a)^3 = -8$, hence $b-a=-2$	1
(i) Evaluates $P(a) = (a-b)^2$ and concludes the remainder is 4	1
(ii) Substitutes $x = \frac{a+b}{2}$ into $P(x)$	1
(ii) Simplifies to 0, proving it is a zero	1

Solution.

(i) By the remainder theorem the remainder on division by $(x-b)$ is $P(b)$:

$$P(b) = (b-a)^3 + (b-b)^2 = (b-a)^3 = -8 \implies b-a=-2, \quad \text{so } a-b=2.$$

The remainder on division by $(x-a)$ is $P(a)$:

$$P(a) = (a-a)^3 + (a-b)^2 = (a-b)^2 = (2)^2 = 4.$$

(ii) Substitute $x = \dfrac{a+b}{2}$. Then $x-a = \dfrac{a+b}{2} - a = \dfrac{b-a}{2}$ and $x-b = \dfrac{a+b}{2} - b = \dfrac{a-b}{2}$. Using $a-b=2$ (so $b-a=-2$),

$$P\left(\tfrac{a+b}{2}\right) = \left(\tfrac{b-a}{2}\right)^3 + \left(\tfrac{a-b}{2}\right)^2 = \left(\tfrac{-2}{2}\right)^3 + \left(\tfrac{2}{2}\right)^2 = (-1)^3 + (1)^2 = -1+1 = 0.$$

Since $P\left(\frac{a+b}{2}\right) = 0$, $x = \dfrac{a+b}{2}$ is a zero of $P(x)$.

Q17 2020 Girraween Prelim Q8(c) · **4 marks**

When the polynomial $P(x)$ is divided by $(x+1)(x-3)$, the quotient is $Q(x)$ and the remainder is $-2x+4$.

(i) Find the remainder when $P(x)$ is divided by $x+1$.

(ii) What is the remainder when $P(x)$ is divided by $x-3$?

Criteria	Marks
(i) Writes $P(x)=(x+1)(x-3)Q(x)+(-2x+4)$ and evaluates at $x=-1$	1
(i) Correct remainder 6	1
(ii) Evaluates at $x=3$	1
(ii) Correct remainder -2	1

Solution.

We can write $P(x)=(x+1)(x-3)Q(x)+(-2x+4)$, since the remainder on division by the quadratic $(x+1)(x-3)$ is at most linear.

(i) The remainder on division by $x+1$ is $P(-1)$. As $(x+1)(x-3)$ vanishes at $x=-1$,

$$P(-1)=-2(-1)+4=6.$$

(ii) The remainder on division by $x-3$ is $P(3)$. As $(x+1)(x-3)$ vanishes at $x=3$,

$$P(3)=-2(3)+4=-2.$$

3. The factor theorem: factorising and solving

Q18 2022 Blacktown Boys Trial Q14(a)(i) · **1 mark**

Prove that $(n+1)$ is a factor of $P(n)=4n^3+18n^2+23n+9$.

Criteria	Marks
Evaluates $P(-1)=0$ and concludes $(n+1)$ is a factor	1

Solution.

By the factor theorem, $(n+1)$ is a factor iff $P(-1)=0$.

$$P(-1)=4(-1)^3+18(-1)^2+23(-1)+9=-4+18-23+9=0.$$

Since $P(-1)=0$, $(n+1)$ is a factor of $P(n)$.

Q19 2021 Sydney Tech Trial Q1 · 1 mark

Which of the following expressions is a correct factorisation of $x^3 - 27$?

(A) $(x-3)(x^2-3x+9)$

(B) $(x-3)(x^2-6x+9)$

(C) $(x-3)(x^2+3x+9)$

(D) $(x-3)(x^2+6x+9)$

Criteria	Marks
Correct option (C)	1

Solution.

Using the difference of cubes $a^3 - b^3 = (a-b)(a^2+ab+b^2)$ with $a = x$, $b = 3$:

$$x^3 - 27 = (x-3)(x^2+3x+9).$$

The answer is (C).

Q20 2021 Frensham Trial Q1 · 1 mark

Which of the following is NOT a factor of the polynomial $P(x) = x^3 - x^2 - 8x + 12$?

(A) $(x+3)$

(B) $(x+3)^2$

(C) $(x-2)$

(D) $(x-2)^2$

Criteria	Marks
Correct option (B)	1

Solution.

Factorising, $P(x) = x^3 - x^2 - 8x + 12 = (x+3)(x-2)^2$. So $(x+3)$, $(x-2)$ and $(x-2)^2$ are all factors, but $(x+3)^2$ is not (dividing leaves remainder $25x + 75 \neq 0$). The answer is (B).

Q21 2023 Barker Prelim Q11(c) · **2 marks**

Fully factorise the polynomial $P(x) = x^3 - 4x^2 + x + 6$.

Criteria	Marks
Finds a first factor, e.g. $P(-1) = 0$ so $(x + 1)$ is a factor	1
Completes the full factorisation $(x + 1)(x - 2)(x - 3)$	1

Solution.

Test integer factors of 6: $P(-1) = -1-4-1+6 = 0$, so $(x+1)$ is a factor. Dividing, $P(x) = (x+1)(x^2-5x+6) = (x + 1)(x - 2)(x - 3)$.

$$P(x) = (x + 1)(x - 2)(x - 3).$$

Q22 2023 Sydney Boys Trial Q13(a)(i) · **2 marks**

Factorise $t^3 + 2t^2 + 3t + 2$.

Criteria	Marks
Finds the linear factor, e.g. $P(-1) = 0$ so $(t + 1)$ is a factor	1
Completes the factorisation $(t + 1)(t^2 + t + 2)$	1

Solution.

Let $P(t) = t^3 + 2t^2 + 3t + 2$. Then $P(-1) = -1 + 2 - 3 + 2 = 0$, so $(t + 1)$ is a factor. Dividing, $P(t) = (t + 1)(t^2 + t + 2)$. The quadratic $t^2 + t + 2$ has discriminant $1 - 8 = -7 < 0$, so it does not factor further over the reals.

$$t^3 + 2t^2 + 3t + 2 = (t + 1)(t^2 + t + 2).$$

Q23 2023 North Sydney Boys Prelim QB2 · **2 marks**

Show that $(x-1)(x-2)$ is a factor of $P(x) = x^n(2^m-1) + x^m(1-2^n) + (2^n-2^m)$, where m and n are positive integers.

Criteria	Marks
Shows $P(1) = 0$, so $(x-1)$ is a factor	1
Shows $P(2) = 0$, so $(x-2)$ is a factor; concludes $(x-1)(x-2)$ divides $P(x)$	1

Solution.

By the factor theorem it suffices to show $P(1) = 0$ and $P(2) = 0$.

$$P(1) = (2^m - 1) + (1 - 2^n) + (2^n - 2^m) = 0,$$

so $(x-1)$ is a factor. Also

$$P(2) = 2^n(2^m-1) + 2^m(1-2^n) + (2^n-2^m) = 2^{n+m} - 2^n + 2^m - 2^{n+m} + 2^n - 2^m = 0,$$

so $(x-2)$ is a factor. Since $(x-1)$ and $(x-2)$ are distinct linear factors, their product $(x-1)(x-2)$ is a factor of $P(x)$.

Q24 2021 Manly Trial Q11(d) · **3 marks**

Solve for x: $x^3 - 5x^2 + 8x - 4 = 0$.

Criteria	Marks
Finds a first root/factor, e.g. $x = 1$ (or $x = 2$)	1
Factorises completely to $(x-1)(x-2)^2$	1
States all solutions $x = 1$ and $x = 2$	1

Solution.

Let $P(x) = x^3 - 5x^2 + 8x - 4$. Then $P(1) = 1 - 5 + 8 - 4 = 0$, so $(x-1)$ is a factor. Dividing, $P(x) = (x-1)(x^2 - 4x + 4) = (x-1)(x-2)^2$. So $(x-1)(x-2)^2 = 0$, giving

$$x = 1 \quad \text{or} \quad x = 2.$$

Q25 2022 Blacktown Boys Prelim Q11(c) · 3 marks

Consider the polynomial $P(x) = 2x^3 - 19x^2 + 60x - 63$.

(i) Show that $P(x)$ has a double root at $x = 3$.

(ii) Hence, factorise $P(x)$ completely.

Criteria	Marks
(i) Shows $P(3) = 0$	1
(i) Shows $P'(3) = 0$ (or $(x-3)^2$ divides P), establishing a double root	1
(ii) Factorises completely as $(x-3)^2(2x-7)$	1

Solution.

(i) $P(3) = 2(27) - 19(9) + 60(3) - 63 = 54 - 171 + 180 - 63 = 0$, so $x = 3$ is a root. Differentiating, $P'(x) = 6x^2 - 38x + 60$, and $P'(3) = 54 - 114 + 60 = 0$. Since $P(3) = P'(3) = 0$, $x = 3$ is a double root.

(ii) Hence $(x-3)^2$ is a factor. Dividing, $P(x) = (x-3)^2(2x-7)$.

$$P(x) = (x-3)^2(2x-7).$$

Q26 2024 Hurlstone Prelim Q15(d) · 3 marks

It is given that the polynomial $Q(x) = 4x^3 - 8x^2 + 5x - 1$ has a zero of multiplicity 2. Express $Q(x)$ as a product of linear factors.

Criteria	Marks
Locates the repeated zero, e.g. $Q\left(\frac{1}{2}\right) = 0$ and $Q'\left(\frac{1}{2}\right) = 0$	1
Finds the remaining factor / zero $x = 1$	1
Writes $Q(x) = (2x-1)^2(x-1)$	1

Solution.

Since the zero has multiplicity 2, both $Q(x)$ and $Q'(x)$ vanish there. With $Q'(x) = 12x^2 - 16x + 5$, testing $x = \frac{1}{2}$: $Q\left(\frac{1}{2}\right) = 4 \cdot \frac{1}{8} - 8 \cdot \frac{1}{4} + \frac{5}{2} - 1 = \frac{1}{2} - 2 + \frac{5}{2} - 1 = 0$ and $Q'\left(\frac{1}{2}\right) = 3 - 8 + 5 = 0$, so $x = \frac{1}{2}$ is the double zero and $(2x-1)^2$ is a factor. Dividing, the remaining factor is $(x-1)$. Hence

$$Q(x) = (2x-1)^2(x-1).$$

Q27 2021 Girraween Prelim Q1(a) · **4 marks**

Consider the polynomial $P(x) = 2x^3 + 3x^2 - 29x - 60$.

(i) Use the factor theorem to show that $(x + 3)$ is a factor of $P(x)$.

(ii) Find the remainder when $P(x)$ is divided by $(x + 2)$.

(iii) Factorise $P(x)$ completely.

Criteria	**Marks**
(i) Shows $P(-3) = 0$, so $(x + 3)$ is a factor	1
(ii) Remainder = $P(-2) = -6$	1
(iii) Divides out $(x + 3)$ to reach a quadratic factor	1
(iii) Completes the factorisation $(x + 3)(x - 4)(2x + 5)$	1

Solution.

(i) By the factor theorem, $P(-3) = 2(-27) + 3(9) - 29(-3) - 60 = -54 + 27 + 87 - 60 = 0$, so $(x + 3)$ is a factor.

(ii) By the remainder theorem, the remainder on division by $(x + 2)$ is

$$P(-2) = 2(-8) + 3(4) - 29(-2) - 60 = -16 + 12 + 58 - 60 = -6.$$

(iii) Dividing out $(x + 3)$: $P(x) = (x + 3)(2x^2 - 3x - 20) = (x + 3)(x - 4)(2x + 5)$.

$$P(x) = (x + 3)(x - 4)(2x + 5).$$

Q28 2024 North Sydney Boys Trial Q12(e) · 4 marks

Given $4x^4 + 8x^3 + 3x^2 - 2x - 1 = 0$.

(i) Express in the form $(ax^2 + bx)^2 - (cx + d)^2 = 0$, where a, b, c and d are positive integers, and state the values of a, b, c and d.

(ii) Hence solve the equation for x.

Criteria	Marks
(i) Completes the square on the leading terms to reach $(2x^2 + 2x)^2 - (x + 1)^2$	1
(i) States $a = 2$, $b = 2$, $c = 1$, $d = 1$	1
(ii) Applies difference of squares to factorise: $(2x - 1)(x + 1)^2(2x + 1)$	1
(ii) States all solutions $x = \frac{1}{2}, -\frac{1}{2}, -1$	1

Solution.

(i) Note $(2x^2 + 2x)^2 = 4x^4 + 8x^3 + 4x^2$, so

$$4x^4 + 8x^3 + 3x^2 - 2x - 1 = (2x^2 + 2x)^2 - (x^2 + 2x + 1) = (2x^2 + 2x)^2 - (x + 1)^2.$$

Thus $a = 2$, $b = 2$, $c = 1$, $d = 1$.

(ii) By difference of squares,

$$(2x^2 + 2x)^2 - (x + 1)^2 = (2x^2 + 2x - x - 1)(2x^2 + 2x + x + 1) = (2x^2 + x - 1)(2x^2 + 3x + 1).$$

Factorising each quadratic: $(2x - 1)(x + 1) \cdot (2x + 1)(x + 1) = (2x - 1)(2x + 1)(x + 1)^2 = 0$.

$$x = \tfrac{1}{2}, \quad x = -\tfrac{1}{2}, \quad x = -1.$$

Finding unknown coefficients

4. Finding unknown coefficients from factor and remainder conditions

Q29 2020 Abbotsleigh Trial Q3 · 1 mark

The polynomial $P(x) = 7x^3 + 9x^2 - 5cx$ has a factor $(x + c)$, where c is a non-zero real number. What is the value of c?

(A) 2

(B) -2

(C) -1

(D) 1

Criteria	Marks
Correct option (A)	1

Solution.

By the factor theorem, $(x + c)$ is a factor iff $P(-c) = 0$.

$$P(-c) = 7(-c)^3 + 9(-c)^2 - 5c(-c) = -7c^3 + 9c^2 + 5c^2 = -7c^3 + 14c^2 = -7c^2(c - 2).$$

Since $c \neq 0$, we need $c - 2 = 0$, so $c = 2$. The answer is (A).

Q30 2021 Killara Trial Q4 · **1 mark**

Find the values of a and b given that $x+1$ and $x-2$ are factors of $ax^3 - 8x^2 + bx - 24$.

(A) $a = -20,\ b = 52$

(B) $a = -\dfrac{88}{7},\ b = \dfrac{312}{7}$

(C) $a = \dfrac{88}{7},\ b = -\dfrac{312}{7}$

(D) $a = 20,\ b = -52$

Criteria	Marks
Correct option (D)	1

Solution.

Let $P(x) = ax^3 - 8x^2 + bx - 24$. By the factor theorem, $P(-1) = 0$ and $P(2) = 0$:

$$P(-1) = -a - 8 - b - 24 = 0 \implies a + b = -32,$$

$$P(2) = 8a - 32 + 2b - 24 = 0 \implies 8a + 2b = 56 \implies 4a + b = 28.$$

Subtracting, $3a = 60$, so $a = 20$ and $b = -52$. The answer is (D).

Q31 2023 Penrith Trial Q2 · **1 mark**

The polynomial $P(x) = 6x^3 - 2x + bx - 4$ has a factor of $2x - 1$. What is the value of b?

(A) 8

(B) -7.5

(C) 8.5

(D) -7

Criteria	Marks
Correct option (C)	1

Solution.

By the factor theorem, $2x - 1$ is a factor iff $P\left(\frac{1}{2}\right) = 0$.

$$P\left(\tfrac{1}{2}\right) = 6\left(\tfrac{1}{2}\right)^3 - 2\left(\tfrac{1}{2}\right) + b\left(\tfrac{1}{2}\right) - 4 = \tfrac{3}{4} - 1 + \tfrac{b}{2} - 4 = \tfrac{b}{2} - \tfrac{17}{4} = 0.$$

Hence $\frac{b}{2} = \frac{17}{4}$, so $b = \frac{17}{2} = 8.5$. The answer is (C).

Q32 2021 Hornsby Girls Prelim Q1 · 1 mark

If $(x+m)$ is a factor of $8x^3 - 14x^2 - m^2x$ where $m \in \mathbb{R}$, $m \neq 0$, then the value of m is

(A) 7

(B) 4

(C) 1

(D) -2

Criteria	Marks
Correct option (D)	1

Solution.

By the factor theorem, $(x+m)$ is a factor iff $P(-m) = 0$ where $P(x) = 8x^3 - 14x^2 - m^2x$:

$$P(-m) = 8(-m)^3 - 14(-m)^2 - m^2(-m) = -8m^3 - 14m^2 + m^3 = -7m^3 - 14m^2 = -7m^2(m+2).$$

Since $m \neq 0$, we need $m + 2 = 0$, so $m = -2$. The answer is (D).

Q33 2020 Sydney Girls Trial Q10 · 1 mark

The polynomial $x^2 + x + 2$ is a factor of the polynomial $x^4 + 4x^2 + x + a$. Determine the value of the constant a.

(A) $a = -6$

(B) $a = -3$

(C) $a = 3$

(D) $a = 6$

Criteria	Marks
Correct option (D)	1

Solution.

Divide $x^4 + 4x^2 + x + a$ by $x^2 + x + 2$. Performing the division,

$$x^4 + 4x^2 + x + a = (x^2 + x + 2)(x^2 - x + 3) + (a - 6).$$

For $x^2 + x + 2$ to be a factor, the remainder must be zero: $a - 6 = 0$, so $a = 6$. The answer is (D).

Q34 2022 Ascham Trial Q11(a) · **2 marks**

Let $P(x) = 2x^3 + rx^2 + x - 3$ and $D(x) = x^2 + 6x + 4$ have the same remainder when divided by $x - 1$. Find r.

Criteria	Marks
Applies the remainder theorem, setting $P(1) = D(1)$	1
Correct value $r = 11$	1

Solution.

By the remainder theorem, the remainders on division by $x - 1$ are $P(1)$ and $D(1)$. Equal remainders means $P(1) = D(1)$:

$$P(1) = 2 + r + 1 - 3 = r, \qquad D(1) = 1 + 6 + 4 = 11.$$

Setting $r = 11$ gives $r = 11$.

Q35 2021 Presbyterian Prelim Q7(a) · **2 marks**

Let $P(x) = x^3 - 3x^2 + kx - 5$, where k is a rational number. When $P(x)$ is divided by $(x + 2)$, the remainder is 7. Find k.

Criteria	Marks
Applies the remainder theorem, setting $P(-2) = 7$	1
Correct value $k = -16$	1

Solution.

By the remainder theorem, the remainder on division by $(x + 2)$ is $P(-2)$:

$$P(-2) = (-2)^3 - 3(-2)^2 + k(-2) - 5 = -8 - 12 - 2k - 5 = -2k - 25.$$

Setting this equal to 7: $-2k - 25 = 7$, so $-2k = 32$ and $k = -16$.

Q36 2025 Merewether Trial Q12(e) · **2 marks**

When the polynomial $P(x)$ is divided by $(x+1)(x-4)$ the quotient is $Q(x)$ and the remainder is $R(x)$.

(i) Given that $P(4) = -5$, show that $R(4) = -5$.

(ii) When $P(x)$ is divided by $(x+1)$, the remainder is 5. Find $R(x)$.

Criteria	Marks
(i) Uses $P(x) = (x+1)(x-4)Q(x) + R(x)$ with $x = 4$ to show $R(4) = P(4) = -5$	1
(ii) Sets $R(x) = mx + c$, uses $R(-1) = 5$ and $R(4) = -5$ to obtain $R(x) = -2x + 3$	1

Solution.

(i) Since $R(x)$ is the remainder on division by $(x+1)(x-4)$, write

$$P(x) = (x+1)(x-4)Q(x) + R(x).$$

Substituting $x = 4$, the product $(x+1)(x-4)$ vanishes, so $P(4) = R(4)$. Given $P(4) = -5$, we have $R(4) = -5$.

(ii) The divisor $(x+1)(x-4)$ has degree 2, so $R(x)$ is linear: $R(x) = mx + c$. Substituting $x = -1$ into $P(x) = (x+1)(x-4)Q(x) + R(x)$ shows $P(-1) = R(-1)$, and $P(-1) = 5$, so $R(-1) = 5$. Combined with $R(4) = -5$:

$$-m + c = 5, \qquad 4m + c = -5.$$

Subtracting gives $5m = -10$, so $m = -2$ and $c = 3$. Hence

$$R(x) = -2x + 3.$$

Q37 2021 Fort St Prelim Q2(b) · 3 marks

$P(x) = 2x^3 - ax^2 + bx + 3$, where a and b are constants. When $P(x)$ is divided by $x^2 - 1$ the remainder is $-2x - 2$. Find a and b.

Criteria	Marks
Uses $x^2 - 1 = (x - 1)(x + 1)$ to evaluate $P(1)$ and $P(-1)$ against the remainder $-2x - 2$	1
Forms the two equations $-a + b + 5 = -4$ and $-a - b + 1 = 0$	1
Correct values $a = 5$ and $b = -4$	1

Solution.

Write $P(x) = (x^2 - 1)Q(x) + (-2x - 2)$. Since $x^2 - 1 = (x - 1)(x + 1)$ vanishes at $x = 1$ and $x = -1$, the remainder gives the values of P there:

$$P(1) = -2(1) - 2 = -4, \qquad P(-1) = -2(-1) - 2 = 0.$$

Now $P(1) = 2 - a + b + 3 = -a + b + 5$ and $P(-1) = -2 - a - b + 3 = -a - b + 1$. So

$$-a + b + 5 = -4 \implies -a + b = -9, \qquad -a - b + 1 = 0 \implies -a - b = -1.$$

Adding: $-2a = -10$, so $a = 5$; then $b = -9 + a = -9 + 5 = -4$. Hence $a = 5,\ b = -4$.

Q38 2025 Hurlstone Trial Q15(c) · 3 marks

Let $P(x) = (x+1)(x-3)Q(x) + ax + b$, where $Q(x)$ is a polynomial and a and b are real numbers. The polynomial $P(x)$ has a factor of $x-3$. When $P(x)$ is divided by $x+1$ the remainder is 8.

(i) Find the values of a and b.

(ii) Find the remainder when $P(x)$ is divided by $(x+1)(x-3)$.

Criteria	Marks
(i) Uses the factor condition $P(3) = 0$ to get $3a + b = 0$	1
(i) Uses the remainder condition $P(-1) = 8$ to get $-a + b = 8$, and solves $a = -2$, $b = 6$	1
(ii) States the remainder is $ax + b = -2x + 6$	1

Solution.

(i) Since $x-3$ is a factor, $P(3) = 0$. Since $(x+1)(x-3)$ vanishes at $x = 3$,

$$P(3) = 3a + b = 0.$$

The remainder on division by $x+1$ is $P(-1)$, and $(x+1)(x-3)$ vanishes at $x = -1$, so

$$P(-1) = -a + b = 8.$$

Subtracting the first equation from the second: $-4a = 8$, so $a = -2$, and then $b = -3a = 6$.

(ii) The remainder when $P(x)$ is divided by $(x+1)(x-3)$ is exactly $ax + b$, i.e.

$$-2x + 6.$$

Q39 2022 Sydney Girls Prelim Q1(g) · 3 marks

Let $P(x) = x^3 + ax^2 + bx + 8$ where a and b are real numbers. Find the values of a and b given that $(x-2)^2$ is a factor of $P(x)$.

Criteria	Marks
Recognises $(x-2)^2$ a factor means $P(2) = 0$ and $P'(2) = 0$ (double root)	1
Forms $4a + 2b + 16 = 0$ and $4a + b + 12 = 0$	1
Correct values $a = -2$ and $b = -4$	1

Solution.

Since $(x-2)^2$ is a factor, $x = 2$ is a double root, so $P(2) = 0$ and $P'(2) = 0$.

$$P(2) = 8 + 4a + 2b + 8 = 4a + 2b + 16 = 0 \implies 2a + b = -8.$$

With $P'(x) = 3x^2 + 2ax + b$,

$$P'(2) = 12 + 4a + b = 0 \implies 4a + b = -12.$$

Subtracting: $2a = -4$, so $a = -2$; then $b = -8 - 2a = -8 + 4 = -4$. Hence $a = -2$, $b = -4$ (and $P(x) = (x-2)^2(x+2)$).

Q40 2021 Sydney Girls Prelim Q1(c) · **4 marks**

Consider the polynomial $P(x) = x^3 + ax + b$ where a and b are real numbers. $P(x)$ has a factor of $(x - 2)$ and when $P(x)$ is divided by $(x + 1)$ the remainder is 6.

(i) Show that $2a + b = -8$ and $-a + b = 7$.

(ii) Find the values of a and b.

Criteria	**Marks**
(i) Uses the factor theorem $P(2) = 0$ to obtain $2a + b = -8$	1
(i) Uses the remainder theorem $P(-1) = 6$ to obtain $-a + b = 7$	1
(ii) Solves the simultaneous equations for a	1
(ii) Correct values $a = -5$ and $b = 2$	1

Solution.

(i) Since $(x - 2)$ is a factor, $P(2) = 0$:

$$P(2) = 8 + 2a + b = 0 \implies 2a + b = -8.$$

By the remainder theorem, the remainder on division by $(x + 1)$ is $P(-1) = 6$:

$$P(-1) = -1 - a + b = 6 \implies -a + b = 7.$$

(ii) Subtracting $-a + b = 7$ from $2a + b = -8$ gives $3a = -15$, so $a = -5$. Then $b = 7 + a = 7 - 5 = 2$. Hence $a = -5,\ b = 2$.

Q41 2022 Sydney Boys Trial Q11(d) · 5 marks

When a polynomial $p(x)$, with rational coefficients, is divided by $(x-\alpha)^2$ the remainder is $R_2(x-\alpha)+R_1$, where $R_1, R_2 \in \mathbb{R}$, i.e.

$$p(x) = (x-\alpha)^2 Q(x) + R_2(x-\alpha) + R_1,$$

where $Q(x)$ is a polynomial.

(i) Show that $R_1 = p(\alpha)$ and $R_2 = p'(\alpha)$.

(ii) For $n > 1$, $n \in \mathbb{N}$, show that $x^n - n(x-1) - 1$ is divisible by $(x-1)^2$.

(iii) For $n > 1$, $n \in \mathbb{N}$, deduce that $2^{4n} - 15n - 1$ is divisible by 225.

Criteria	Marks
(i) Substitutes $x = \alpha$ into $p(x)$ to get $R_1 = p(\alpha)$	1
(i) Differentiates and substitutes $x = \alpha$ to get $R_2 = p'(\alpha)$	1
(ii) Sets $f(x) = x^n - n(x-1) - 1$ and shows $f(1) = 0$	1
(ii) Shows $f'(1) = 0$, hence $(x-1)^2$ divides $f(x)$	1
(iii) Substitutes $x = 16$ (so $x - 1 = 15$) to deduce divisibility by $15^2 = 225$	1

Solution.

(i) From $p(x) = (x-\alpha)^2 Q(x) + R_2(x-\alpha) + R_1$, substitute $x = \alpha$: the first two terms vanish, so $p(\alpha) = R_1$. Differentiating,

$$p'(x) = 2(x-\alpha)Q(x) + (x-\alpha)^2 Q'(x) + R_2,$$

and substituting $x = \alpha$ makes the first two terms vanish, giving $p'(\alpha) = R_2$.

(ii) Let $f(x) = x^n - n(x-1) - 1$. Then

$$f(1) = 1 - n(0) - 1 = 0,$$

so $(x-1)$ is a factor. Also $f'(x) = nx^{n-1} - n$, so

$$f'(1) = n(1) - n = 0.$$

Since $f(1) = 0$ and $f'(1) = 0$, $x = 1$ is (at least) a double root, hence $(x-1)^2$ divides $f(x)$.

(iii) By part (ii), $(x-1)^2$ divides $x^n - n(x-1) - 1$, so this expression is divisible by $(x-1)^2$ for every integer x. Put $x = 16$, so $x - 1 = 15$:

$$16^n - n(15) - 1 = 2^{4n} - 15n - 1$$

is divisible by $(15)^2 = 225$. Hence $2^{4n} - 15n - 1$ is divisible by 225.