Year 11 Mathematics Extension 1

Topic: Polynomials (1 of 4)

Review Questions (Worked Solutions): Language and Graphs of Polynomials

Term 3, 2026

Language and properties of polynomials

1. Polynomial language: degree, leading and constant term, monic

Q1 2025 Hurlstone Prelim Q8 · 1 mark

What is the degree and leading coefficient of $f(x) = -x + 5x^2 - 6x^3 + 10$?

(A) degree 1 with a leading coefficient of -1

(B) degree 3 with a leading coefficient of 6

(C) degree 3 with a leading coefficient of -6

(D) degree 6 with a leading coefficient of -1

Criteria	Marks
Correct option (C)	1

Solution.

Write f in descending order: $f(x) = -6x^3 + 5x^2 - x + 10$. The highest power is x^3, so the degree is 3 and the leading coefficient is the coefficient of that term, -6. The answer is (C).

Q2 2022 St George Girls Prelim Q1 · 1 mark

Which of the following is not a polynomial?

(A) $2x^3 + x^{-1}$

(B) -17π

(C) $3x^2 - 2x + 5^{-\frac{1}{3}}$

(D) $\sqrt{3}\,x^4 + x - 14$

Criteria	Marks
Correct option (A)	1

Solution.

A polynomial has only non-negative integer powers of x. Option (A) contains x^{-1} (a negative power of x), so it is not a polynomial. The other options are polynomials: (B) is the constant -17π (degree 0), (C) has constant term $5^{-1/3}$ (just a real number) with the rest in non-negative integer powers, and (D) has the real coefficient $\sqrt{3}$ on x^4. The answer is (A).

Q3 2023 Sydney Grammar Prelim Q2 · **1 mark**

In which polynomial is the constant term greater than the degree?

(A) $3x^2 + x + 2$

(B) $(2x - 3)^3$

(C) $x(5x^2 + 4)$

(D) $(x - 2)^4$

Criteria	Marks
Correct option (D)	1

Solution.

Compare each polynomial's constant term with its degree.

$$(A):\ \deg = 2,\ \text{const} = 2\ (2 \not> 2); \qquad (B):\ (2x-3)^3,\ \deg = 3,\ \text{const} = (-3)^3 = -27\ (-27 \not> 3).$$

$$(C):\ x(5x^2+4) = 5x^3+4x,\ \deg = 3,\ \text{const} = 0\ (0 \not> 3); \qquad (D):\ (x-2)^4,\ \deg = 4,\ \text{const} = (-2)^4 = 16.$$

Only in (D) is the constant term greater than the degree $(16 > 4)$. The answer is (D).

Q4 2022 Baulkham Hills Prelim Q8 · **1 mark**

Consider the polynomial $P(x) = (k^3 - k)x^4 + (k^2 - k)x^3 + x^2$. What value(s) of k will make $P(x)$ a non-zero polynomial of degree 3?

(A) $k = -1$

(B) $k = -1,\ k = 0$

(C) $k = -1,\ k = 1$

(D) $k = -1,\ k = 0,\ k = 1$

Criteria	Marks
Correct option (A)	1

Solution.

For $P(x)$ to have degree 3, the x^4 coefficient must vanish while the x^3 coefficient must be non-zero.

$$k^3 - k = 0: \quad k(k-1)(k+1) = 0 \implies k = -1, 0, 1.$$

$$k^2 - k = 0: \quad k(k-1) = 0 \implies k = 0, 1 \text{ (these must be excluded)}.$$

Removing $k = 0$ and $k = 1$ from $\{-1, 0, 1\}$ leaves $k = -1$. The answer is (A).

Q5 2023 Knox Prelim Q8 · **1 mark**

The polynomial $H(x)$ has degree 5. When $H(x)$ is divided by another polynomial $G(x)$ the remainder is $x^2 - x + 2$. Which of the following is true about the degree of the polynomial $K(x) = H(x)\big(G(x)\big)^2$?

(A) The degree could be $7, 8, 9$ or 10.

(B) The degree could be $11, 13$ or 15.

(C) The degree could be $9, 11, 13$ or 15.

(D) The degree could be $13, 15$ or 17.

Criteria	Marks
Correct option (B)	1

Solution.

The remainder must have degree less than $\deg G$, and the remainder here has degree 2, so $\deg G > 2$; also $\deg G \leq \deg H = 5$. Hence $\deg G \in \{3, 4, 5\}$. Then

$$\deg K = \deg H + 2\deg G = 5 + 2\deg G.$$

For $\deg G = 3, 4, 5$ this gives $\deg K = 11, 13, 15$ respectively. The answer is (B).

2. Zeroes, roots and multiplicity

Q6 2020 Caringbah Prelim Q11(d)(i) · **1 mark**

If $P(x) = x\left(x^2 - 1\right)^2$, find all the roots of $P(x)$.

Criteria	Marks
All roots stated: $x = -1,\ 0,\ 1$ (with ± 1 as double roots)	1

Solution.

Factor the difference of squares: $x^2 - 1 = (x - 1)(x + 1)$, so

$$P(x) = x\,(x - 1)^2(x + 1)^2.$$

Setting $P(x) = 0$, the roots are $x = 0$, $x = 1$ (a double root) and $x = -1$ (a double root). Hence the roots are $x = -1,\ 0,\ 1$.

Q7 2020 Cherrybrook Tech Trial Q3 · **1 mark**

The polynomial $P(x) = x^3 - 12x^2 + 21x + k$ has a double root. What are the possible values of k?

(A) $k = -1$ or $k = -7$

(B) $k = 1$ or $k = 7$

(C) $k = 10$ or $k = -98$

(D) $k = -10$ or $k = 98$

Criteria	**Marks**
Correct option (D)	1

Solution.

A double root of P is also a root of P'. Now

$$P'(x) = 3x^2 - 24x + 21 = 3(x^2 - 8x + 7) = 3(x - 1)(x - 7),$$

so the repeated root is $x = 1$ or $x = 7$. Substituting into $P(x) = 0$:

$$P(1) = 1 - 12 + 21 + k = 0 \implies k = -10, \qquad P(7) = 343 - 588 + 147 + k = 0 \implies k = 98.$$

Hence $k = -10$ or $k = 98$, which is option **(D)**.

Q8 2023 Presbyterian Prelim Q10 · **1 mark**

The polynomial $P(x)$ has degree 5 and $P(x)$ is an odd function. Which of the following could be true?

I: The sum of the real roots of $P(x)$ is positive.

II: $P(x) = 0$ has one distinct solution.

III: $P(x)$ has a double root.

(A) I and II

(B) II only

(C) II and III

(D) I and III

Criteria	**Marks**
Correct option (C)	1

Solution.

An odd degree-5 polynomial has only odd powers, $P(x) = ax^5 + bx^3 + cx$, so there is no x^4 term and by the sum-of-roots relation the sum of all roots is 0. As the coefficients are real, the roots (real and complex) still sum to 0, and the non-real roots occur in conjugate pairs that themselves sum to a real total, forcing the real roots to sum to 0 as well. So statement I *cannot* be true.

Statement II is achievable: $P(x) = x^5$ has the single distinct solution $x = 0$.

Statement III is achievable: $P(x) = x\left(x^2 - r^2\right)^2$ is odd of degree 5 and has double roots at $x = \pm r$.

Hence II and III could be true, giving option **(C)**.

Q9 2025 Caringbah Prelim Q6(b) · 3 marks

Consider the polynomial $P(x) = x^3 + 5x^2 + 3x - 9$.

(i) Show that $P(x)$ has a double root at $x = -3$.

(ii) Hence, or otherwise, factorise $P(x)$ completely.

Criteria	Marks
(i) Shows $P(-3) = 0$ (i.e. $x = -3$ is a root)	1
(i) Shows $P'(-3) = 0$, confirming a double root	1
(ii) Complete factorisation $P(x) = (x-1)(x+3)^2$	1

Solution.

(i) $P(-3) = (-3)^3 + 5(-3)^2 + 3(-3) - 9 = -27 + 45 - 9 - 9 = 0$, so $x = -3$ is a root.

$$P'(x) = 3x^2 + 10x + 3, \qquad P'(-3) = 3(9) + 10(-3) + 3 = 27 - 30 + 3 = 0.$$

Since $P(-3) = P'(-3) = 0$, $x = -3$ is a double root.

(ii) As $(x+3)^2$ is a factor, dividing gives the remaining linear factor $(x-1)$:

$$P(x) = (x-1)(x+3)^2.$$

Q10 2024 Girraween Prelim Q7(b) · **3 marks**

Consider the polynomial $p(x) = x^3 - px^2 + qx - pq$, where $q > 0$.

(i) Show that $(x - p)$ is a factor of $p(x)$.

(ii) Show that the equation $p(x) = 0$ has only one real root.

Criteria	**Marks**
(i) Evaluates $p(p) = 0$ and concludes $(x - p)$ is a factor by the factor theorem	1
(ii) Factorises $p(x) = (x - p)(x^2 + q)$	1
(ii) Uses $q > 0$ to argue $x^2 + q > 0$ has no real zeros, so $x = p$ is the only real root	1

Solution.

(i) By the factor theorem, $(x - p)$ is a factor iff $p(p) = 0$:

$$p(p) = p^3 - p \cdot p^2 + q \cdot p - pq = p^3 - p^3 + pq - pq = 0.$$

Hence $(x - p)$ is a factor.

(ii) Grouping the terms,

$$p(x) = x^2(x - p) + q(x - p) = (x - p)(x^2 + q).$$

Since $q > 0$, we have $x^2 + q > 0$ for every real x, so $x^2 + q = 0$ has no real solutions. Therefore the only real root of $p(x) = 0$ is $x = p$.

Q11 2023 HSC Q14(b) · **4 marks**

Consider the hyperbola $y = \dfrac{1}{x}$ and the circle $(x-c)^2 + y^2 = c^2$, where c is a constant.

(i) Show that the x-coordinates of any points of intersection of the hyperbola and circle are zeros of the polynomial $P(x) = x^4 - 2cx^3 + 1$.

(ii) The graphs of $y = x^4 - 2cx^3 + 1$ for $c = 0.8$ and $c = 1$ are shown. By considering the given graphs, or otherwise, find the exact value of $c > 0$ such that the hyperbola $y = \dfrac{1}{x}$ and the circle $(x-c)^2 + y^2 = c^2$ intersect at only one point.

Criteria	**Marks**
(i) Substitutes $y = \frac{1}{x}$ into the circle and clears x^2 to obtain $x^4 - 2cx^3 + 1 = 0$	1
(ii) Recognises a single intersection means P has a repeated (double) root, so $P(x) = P'(x) = 0$	1
(ii) Solves for the tangency point $x = \sqrt[4]{3}$	1
(ii) Correct exact value $c = \dfrac{2\sqrt[4]{3}}{3}$	1

Solution.

(i) At a point of intersection, $y = \dfrac{1}{x}$ satisfies the circle:

$$(x-c)^2 + \frac{1}{x^2} = c^2 \;\Rightarrow\; x^2 - 2cx + c^2 + \frac{1}{x^2} = c^2 \;\Rightarrow\; x^2 - 2cx + \frac{1}{x^2} = 0.$$

Multiplying through by x^2 (note $x \neq 0$ on the hyperbola):

$$x^4 - 2cx^3 + 1 = 0,$$

so each such x-coordinate is a zero of $P(x) = x^4 - 2cx^3 + 1$.

(ii) From the graphs, as c increases through the value between 0.8 and 1 the curve becomes tangent to the x-axis: the single intersection occurs when $P(x) = 0$ has a repeated root, i.e. $P(x) = P'(x) = 0$ simultaneously. Now

$$P'(x) = 4x^3 - 6cx^2 = 2x^2(2x - 3c) = 0 \;\Rightarrow\; x = \tfrac{3c}{2} \quad (x \neq 0).$$

Substituting the repeated root also satisfies $P(x) = 0$. Setting $P(x) = 0$ and $P'(x) = 0$ together gives the tangency at

$$x = \sqrt[4]{3}, \qquad c = \frac{2\sqrt[4]{3}}{3} \approx 0.877.$$

This value lies between 0.8 and 1, consistent with the graphs, so $c = \dfrac{2\sqrt[4]{3}}{3}$.

3. Repeated (double) roots: finding unknowns and factorising

Q12 2022 Shore Trial Q3 · 1 mark

The polynomial $P(x) = x^3 - x^2 - 5x - 3$ has a double root at $x = \alpha$. What is the value of α?

(A) $\alpha = -\dfrac{5}{3}$

(B) $\alpha = -1$

(C) $\alpha = 1$

(D) $\alpha = \dfrac{5}{3}$

Criteria	Marks
Correct option (B)	1

Solution.

A double root of P is also a root of $P'(x) = 3x^2 - 2x - 5 = (3x - 5)(x + 1)$, so the repeated root is $x = \frac{5}{3}$ or $x = -1$. Testing in P: $P(-1) = -1 - 1 + 5 - 3 = 0$, while $P\left(\frac{5}{3}\right) \neq 0$. Hence the double root is $\alpha = -1$, and indeed

$$P(x) = (x + 1)^2(x - 3).$$

The answer is option **(B)**.

Q13 2020 Gosford Prelim Q9(b) · 2 marks

The equation $P(x) = 0$, where $P(x) = x^3 + cx + d$, has a double root at $x = 1$. Find the values of c and d.

Criteria	Marks
Uses the double root: $P(1) = 0$ and $P'(1) = 0$	1
Solves to get $c = -3$ and $d = 2$	1

Solution.

A double root at $x = 1$ means $x = 1$ is a root of both P and P'. With $P(x) = x^3 + cx + d$ and $P'(x) = 3x^2 + c$:

$$P'(1) = 3 + c = 0 \implies c = -3.$$

$$P(1) = 1 + c + d = 0 \implies d = -1 - c = -1 - (-3) = 2.$$

Hence $c = -3$ and $d = 2$, and indeed $P(x) = x^3 - 3x + 2 = (x - 1)^2(x + 2)$, confirming the double root at $x = 1$.

Q14 2023 Abbotsleigh Trial Q11(a) · **3 marks**

The polynomial $P(x) = 8x^4 - 38x^3 + 9x^2 + ax + b$ has a double root at $x = 3$. Find the value of a and b, where a and b are real numbers.

Criteria	Marks
Recognises a double root at $x = 3$ means $P(3) = 0$ and $P'(3) = 0$	1
Sets up the two equations correctly	1
Solves to get $a = 108$ and $b = -27$	1

Solution.

A double root at $x = 3$ means both $P(3) = 0$ and $P'(3) = 0$.

$$P(3) = 8(81) - 38(27) + 9(9) + 3a + b = 648 - 1026 + 81 + 3a + b = 3a + b - 297 = 0.$$

Differentiating, $P'(x) = 32x^3 - 114x^2 + 18x + a$, so

$$P'(3) = 32(27) - 114(9) + 18(3) + a = 864 - 1026 + 54 + a = a - 108 = 0.$$

Hence $a = 108$, and substituting back $3(108) + b - 297 = 0$ gives $b = -27$. So $a = 108,\ b = -27$.

Q15 2022 Sydney Tech Trial Q12(b) · **3 marks**

The polynomial $P(x) = ax^3 + bx^2 + c$ has a double root at $x = 3$ and has remainder -36 when divided by $x + 3$. Find the values of a, b and c.

Criteria	Marks
Uses the double root to write $P(3) = 0$ and $P'(3) = 0$	1
Uses the remainder theorem: $P(-3) = -36$	1
Solves the system to get $a = \frac{2}{3},\ b = -3,\ c = 9$	1

Solution.

A double root at $x = 3$ gives $P(3) = 0$ and $P'(3) = 0$, where $P'(x) = 3ax^2 + 2bx$.

$$P(3) = 27a + 9b + c = 0, \qquad P'(3) = 27a + 6b = 0 \implies b = -\tfrac{9}{2}a.$$

By the remainder theorem the remainder on division by $x + 3$ is $P(-3) = -27a + 9b + c = -36$. Subtracting this from $P(3) = 0$:

$$(27a + 9b + c) - (-27a + 9b + c) = 54a = 36 \implies a = \tfrac{2}{3}.$$

Then $b = -\frac{9}{2} \cdot \frac{2}{3} = -3$, and $c = -27a - 9b = -27\left(\frac{2}{3}\right) - 9(-3) = -18 + 27 = 9$. Hence $a = \frac{2}{3},\ b = -3,\ c = 9$, i.e. $P(x) = \frac{2}{3}x^3 - 3x^2 + 9$.

Q16 2020 Caringbah Prelim Q13(b) · **3 marks**

A polynomial of degree 4 has a double zero at 2, with the other two zeros being k and $-k$. When $x = 1$ it takes the value 8, and when $x = 3$ it takes the value 80. Find the possible values of k.

Criteria	**Marks**
Writes $P(x) = a(x-2)^2(x-k)(x+k)$ using the given zeros	1
Sets up the two equations from $P(1) = 8$ and $P(3) = 80$	1
Eliminates a and solves to get $k = \pm\frac{1}{3}$	1

Solution.

From the zeros, write the polynomial with leading coefficient a:

$$P(x) = a(x-2)^2(x-k)(x+k) = a(x-2)^2(x^2-k^2).$$

Substituting the two given values:

$$P(1) = a(1-2)^2(1-k^2) = a(1-k^2) = 8,$$

$$P(3) = a(3-2)^2(9-k^2) = a(9-k^2) = 80.$$

Dividing the second equation by the first eliminates a:

$$\frac{9-k^2}{1-k^2} = 10 \implies 9-k^2 = 10(1-k^2) \implies 9k^2 = 1.$$

Therefore $k^2 = \frac{1}{9}$, giving

$$k = \pm\frac{1}{3}.$$

Q17 2020 St George Girls Prelim Q10(c) · **4 marks**

Factorise $P(x) = x^4 + x^3 - 15x^2 - 9x + 54$, given it has a double root. Express your answer in fully factored form.

Criteria	**Marks**
Uses $P(x) = P'(x) = 0$ (or trial of factors) to locate the double root $x = -3$	1
Extracts the repeated factor $(x + 3)^2$	1
Divides to obtain the remaining quadratic $x^2 - 5x + 6$	1
Complete factorisation $P(x) = (x + 3)^2(x - 2)(x - 3)$	1

Solution.

A double root is a common root of P and $P'(x) = 4x^3 + 3x^2 - 30x - 9$. Testing integer factors of 54, $P(-3) = 81 - 27 - 135 + 27 + 54 = 0$ and $P'(-3) = -108 + 27 + 90 - 9 = 0$, so $x = -3$ is the double root and $(x + 3)^2$ is a factor. Dividing,

$$P(x) = (x + 3)^2 \left(x^2 - 5x + 6\right) = (x + 3)^2(x - 2)(x - 3).$$

Graphs of polynomials

4. Sketching a polynomial from its factors

Q18 2021 Riverview Trial Q2(c)(i) · **1 mark**

Sketch the graph of $y = 3x^3 - 2x^2$.

Criteria	Marks
Correct cubic shape with the double root at $x = 0$ and single root at $x = \frac{2}{3}$ shown	1

Solution.

Factorise: $y = x^2(3x - 2)$. The zeros are $x = 0$ (double root, so the curve *touches* the axis there) and $x = \frac{2}{3}$ (single root, so the curve *crosses*). The leading coefficient is positive, so as $x \to -\infty$, $y \to -\infty$ and as $x \to +\infty$, $y \to +\infty$. The y-intercept is 0.

$$y = 3x^3 - 2x^2 = x^2(3x - 2)$$

Q19 2022 Sydney Girls Trial Q13(a)(i) · **2 marks**

Sketch the function $y = f(x)$ where $f(x) = 9x - x^3$.

Criteria	Marks
Correct factorisation / all three intercepts $x = -3, 0, 3$	1
Correct shape, i.e. inverted (negative) cubic crossing at each root	1

Solution.

Factorise: $y = 9x - x^3 = x(9 - x^2) = x(3 - x)(3 + x)$. The zeros are $x = -3,\ 0,\ 3$, each a single root, so the curve crosses the axis at all three. The leading term is $-x^3$ (negative leading coefficient), so as $x \to -\infty$, $y \to +\infty$ and as $x \to +\infty$, $y \to -\infty$. The y-intercept is 0.

$$y = 9x - x^3 = -x(x - 3)(x + 3)$$

Q20 2023 Knox Prelim Q11(d)(i) · **2 marks**

Sketch the polynomial $y = x(x+3)(x-2)^2$.

Criteria	Marks
All intercepts correct: $x = -3, 0, 2$ and y-intercept 0	1
Correct shape: crosses at $x = -3$ and $x = 0$, touches at $x = 2$	1

Solution.

The zeros are $x = -3$ (single, cross), $x = 0$ (single, cross) and $x = 2$ (double, touch). This is a quartic with positive leading coefficient, so both ends rise: $y \to +\infty$ as $x \to \pm\infty$. The y-intercept is 0.

$$y = x(x+3)(x-2)^2$$

Q21 2020 Sydney Girls Prelim Q1(b) · **3 marks**

(i) Sketch the graph of $y = (x+4)(x-3)^2$.

(ii) Hence, or otherwise, solve the inequality $(x+4)(x-3)^2 \geq 0$.

Criteria	Marks
(i) Correct cubic shape: crosses at $x = -4$, touches at $x = 3$	2
(ii) Correct solution $x \geq -4$	1

Solution.

(i) The zeros are $x = -4$ (single root, curve crosses) and $x = 3$ (double root, curve touches). Positive leading coefficient, so $y \to -\infty$ as $x \to -\infty$ and $y \to +\infty$ as $x \to +\infty$. The y-intercept is $(0+4)(0-3)^2 = 4 \cdot 9 = 36$.

$$y = (x+4)(x-3)^2$$

(ii) Since $(x-3)^2 \geq 0$ for all x, the sign of y matches the sign of $(x+4)$, except at $x = 3$ where $y = 0$. Thus $y \geq 0$ when $x + 4 \geq 0$, i.e.

$$x \geq -4.$$

(The point $x = 3$ is included since $y = 0$ there, and it already lies in $x \geq -4$.)

Q22 2020 Caringbah Prelim Q12(a) · **3 marks**

Sketch the graph of $y = (1-x)^3(2+x)^2(3-x)$, showing all the intercepts.

Criteria	Marks
All x-intercepts and the y-intercept identified: $x = 1, -2, 3$ and $y = 12$	1
Correct multiplicity behaviour: inflectional crossing at $x = 1$, touch at $x = -2$, cross at $x = 3$	1
Correct end behaviour (both ends up, degree 6 with positive leading coefficient)	1

Solution.

The zeros are $x = 1$ (multiplicity 3, so the curve crosses with a horizontal inflection), $x = -2$ (multiplicity 2, so the curve touches) and $x = 3$ (multiplicity 1, so the curve crosses). The polynomial has degree $3+2+1 = 6$. Expanding, the leading term is $(-x)^3(x)^2(-x) = x^6$, i.e. a positive leading coefficient, so both ends rise: $y \to +\infty$ as $x \to \pm\infty$. The y-intercept is $(1)^3(2)^2(3) = 1 \cdot 4 \cdot 3 = 12$.

$$y = (1-x)^3(2+x)^2(3-x)$$

Q23 2024 Penrith Prelim Q11(b)(iv) · **3 marks**

Consider the polynomial $P(x) = x^3 - x^2 - 33x - 63$. Fully factorise $P(x)$ and sketch the graph of this polynomial, showing all intercepts.

Criteria	Marks
Correct full factorisation $P(x) = (x-7)(x+3)^2$	1
All intercepts marked: $x = 7$, $x = -3$ and y-intercept -63	1
Correct shape: crosses at $x = 7$, touches at $x = -3$	1

Solution.

Test integer factors of 63. $P(-3) = (-3)^3 - (-3)^2 - 33(-3) - 63 = -27 - 9 + 99 - 63 = 0$, so $(x+3)$ is a factor; $P(7) = 343 - 49 - 231 - 63 = 0$, so $(x-7)$ is a factor. Since P is a cubic with these two roots and deg $= 3$, checking the remaining factor gives

$$P(x) = (x-7)(x+3)^2.$$

The zeros are $x = 7$ (single, cross) and $x = -3$ (double, touch). Positive leading coefficient, so $y \to -\infty$ as $x \to -\infty$ and $y \to +\infty$ as $x \to +\infty$. The y-intercept is $P(0) = -63$.

Q24 2023 Sydney Grammar Prelim Q9(d) · **3 marks**

Sketch the polynomial $y = (x+1)^2(x-1)^3(x-3)$, labelling all intercepts with the coordinate axes.

Criteria	Marks
All intercepts labelled: $x = -1, 1, 3$ and y-intercept 3	1
Correct multiplicity behaviour: touch at $x = -1$, inflectional crossing at $x = 1$, cross at $x = 3$	1
Correct end behaviour (both ends up, degree 6 with positive leading coefficient)	1

Solution.

The zeros are $x = -1$ (multiplicity 2, touch), $x = 1$ (multiplicity 3, crosses with a horizontal inflection) and $x = 3$ (multiplicity 1, crosses). The degree is $2 + 3 + 1 = 6$ with positive leading coefficient, so both ends rise: $y \to +\infty$ as $x \to \pm\infty$. The y-intercept is $(1)^2(-1)^3(-3) = 1 \cdot (-1) \cdot (-3) = 3$.

$$y = (x+1)^2(x-1)^3(x-3)$$

Q25 2023 Penrith Prelim Q12(c) · **5 marks**

Sketch the graph of $y = x^4 - 5x^3 - 3x^2 + 17x - 10$, given that it has a root at $x = 1$ which has multiplicity 2. Label all intercepts on your graph. (Do not use calculus and do not calculate the coordinates of the turning points; sketch them roughly, according to your scale and intercepts.)

Criteria	Marks
Uses the double root at $x = 1$ to extract $(x-1)^2$ as a factor	1
Correct full factorisation $y = (x+2)(x-1)^2(x-5)$	2
All intercepts labelled: $x = -2, 1, 5$ and y-intercept -10	1
Correct shape: crosses at $x = -2$ and $x = 5$, touches at $x = 1$; both ends up	1

Solution.

Since $x = 1$ is a root of multiplicity 2, $(x-1)^2 = x^2 - 2x + 1$ is a factor. Dividing,

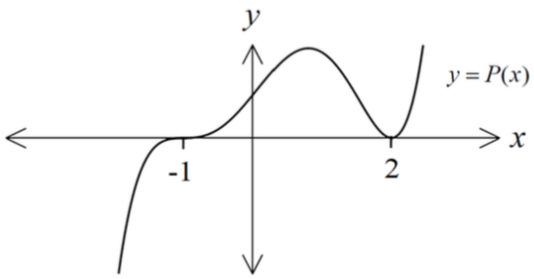
$$x^4 - 5x^3 - 3x^2 + 17x - 10 = (x-1)^2(x^2 - 3x - 10) = (x-1)^2(x+2)(x-5).$$

The zeros are $x = -2$ (single, cross), $x = 1$ (double, touch) and $x = 5$ (single, cross). This is a quartic with positive leading coefficient, so both ends rise: $y \to +\infty$ as $x \to \pm\infty$. The y-intercept is -10.

5. Identifying a polynomial from its graph

Q26 2022 Abbotsleigh Trial Q2 · 1 mark

Which of the following could be the equation of the graphed polynomial, $P(x)$?



(A) $P(x) = -(x+1)^3(x-2)^2$

(B) $P(x) = (x-1)^3(x+2)^2$

(C) $P(x) = (x+1)^3(x-2)^2$

(D) $P(x) = (x-1)^3(2-x)^2$

Criteria	Marks
Correct option (C)	1

Solution.

The graph has a root at $x = -1$ where it flattens and crosses (an odd multiplicity, i.e. the cube $(x+1)^3$) and a root at $x = 2$ where it touches the axis (an even multiplicity, i.e. the square $(x-2)^2$). This rules out (B) and (D), whose roots are at $x = 1$ and $x = \pm 2$. Between $x = -1$ and $x = 2$ the curve lies *above* the axis, so the leading coefficient is positive: checking $x = \frac{1}{2}$ gives $(x+1)^3(x-2)^2 = \dfrac{243}{32} > 0$, whereas option (A) is negative there. Hence

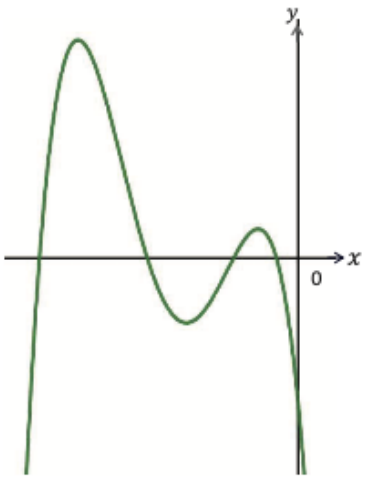
$$P(x) = (x+1)^3(x-2)^2,$$

which is option **(C)**.

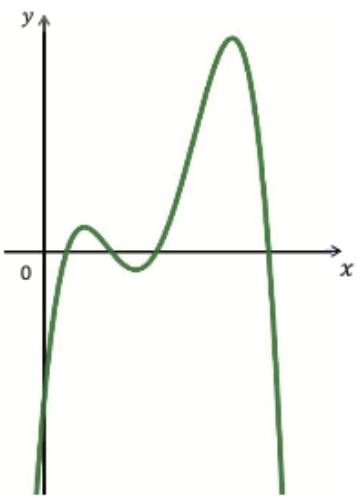
Q27 2024 Knox Trial Q7 · **1 mark**

Consider the polynomial $f(x) = ax^4 + bx^3 + cx + d$, where the constant a is negative and the constant b is positive. Which graph could represent $f(x)$?

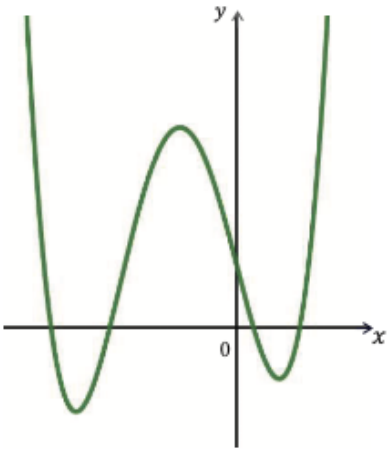
A)



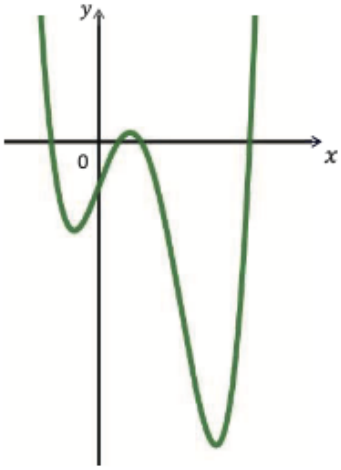
B)



C)



D)



Criteria	Marks
Correct option (B)	1

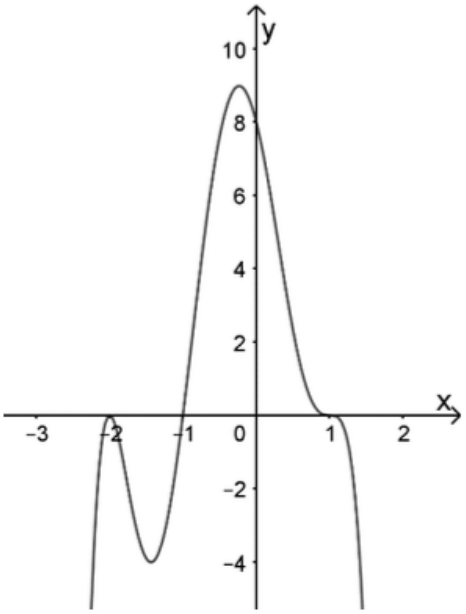
Solution.

Since $a < 0$, the quartic opens downward: as $x \to \pm\infty$, $f(x) \to -\infty$, so both ends point down. This eliminates the graph whose ends point upward. To choose between the remaining downward graphs, compare the two sides: $f(t) - f(-t) = 2bt^3 + 2ct$, and with $b > 0$ dominating for larger t this is positive for $t > 0$, so

the graph sits *higher on the right*. The turning behaviour is therefore skewed to the right, with the principal maximum occurring for $x > 0$. The graph matching a downward quartic with its main hump displaced to the right is option **(B)**.

Q28 2020 Blacktown Boys Prelim Q7 · **1 mark**

The diagram below shows the graph of $y = a(x+b)(x+c)^2(x+d)^3$. What are possible values of a, b, c and d?



(A) $a = -2,\ b = 1,\ c = 2,\ d = -1$

(B) $a = -2,\ b = -1,\ c = -2,\ d = 1$

(C) $a = 8,\ b = 1,\ c = 2,\ d = -1$

(D) $a = 8,\ b = -1,\ c = -2,\ d = 1$

Criteria	Marks
Correct option (A)	1

Solution.

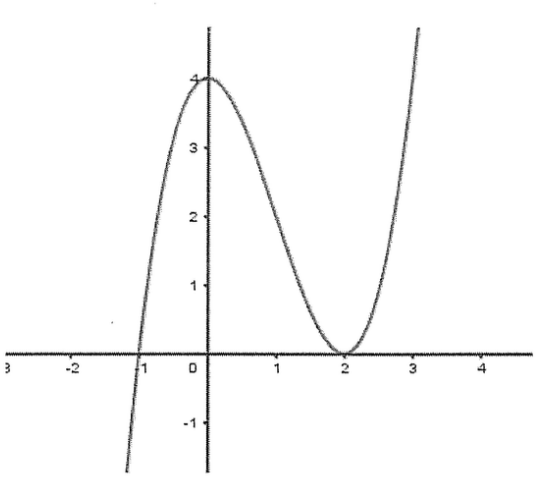
From the graph there is a single root at $x = -1$, a double root at $x = -2$ and a triple root at $x = 1$, so $(x+b) = (x+1)$, $(x+c)^2 = (x+2)^2$ and $(x+d)^3 = (x-1)^3$, giving $b = 1$, $c = 2$, $d = -1$. The y-intercept is 8:

$$a(1)(2)^2(-1)^3 = -4a = 8 \Rightarrow a = -2.$$

Hence $a = -2,\ b = 1,\ c = 2,\ d = -1$, which is option **(A)**.

Q29 2022 Girraween Prelim Q4 · **1 mark**

The diagram below shows the graph of a monic cubic polynomial $P(x)$.



The values of k such that the curve $y = P(x-k)$ passes through the origin are

(A) $k = 1$ or $k = -2$

(B) $k = -1$ or $k = 2$

(C) $k = 1$ or $k = 2$

(D) $k = -1$ or $k = -2$

Criteria	**Marks**
Correct option (A)	1

Solution.

The graph crosses the axis at $x = -1$ (single root) and touches it at $x = 2$ (double root), so as a monic cubic

$$P(x) = (x+1)(x-2)^2.$$

The curve $y = P(x-k)$ passes through the origin when $P(0-k) = P(-k) = 0$, i.e. when $-k$ is a root of P. The roots are $x = -1$ and $x = 2$, so

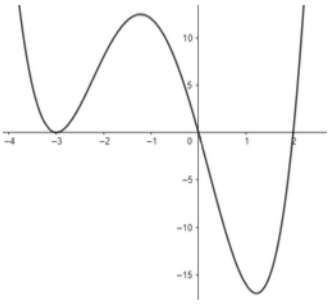
$$-k = -1 \implies k = 1, \qquad -k = 2 \implies k = -2.$$

Hence $k = 1$ or $k = -2$, which is option **(A)**.

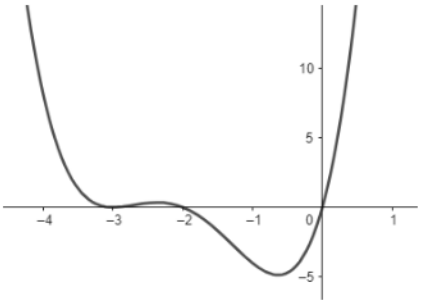
Q30 2024 Hurlstone Prelim Q9 · **1 mark**

Which of the following is the graph of $y = x(x+3)^2(2-x)$?

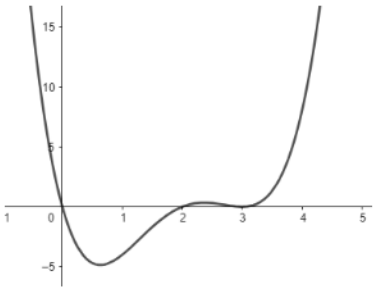
A.



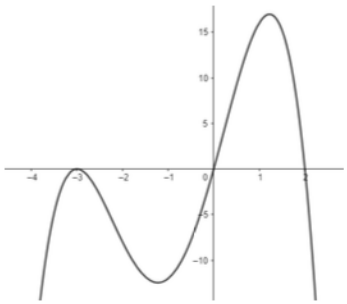
B.



C.



D.



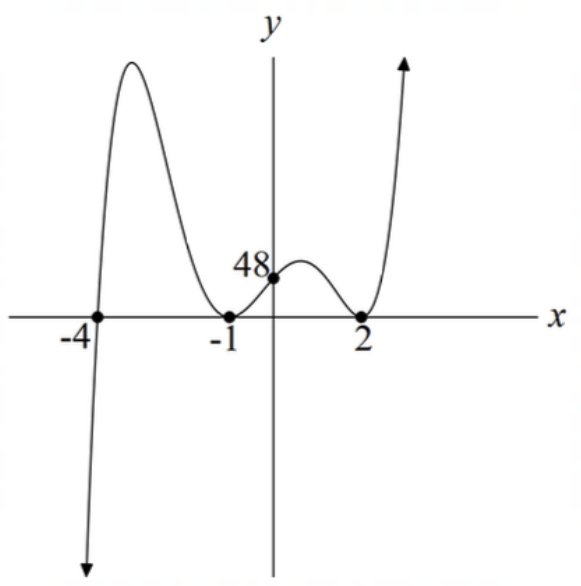
Criteria	Marks
Correct option (D)	1

Solution.

The zeros are $x = 0$ and $x = 2$ (single roots, where the curve crosses) and $x = -3$ (a double root, where it touches). Writing $y = -x(x-2)(x+3)^2$, the leading term is $-x^4$, so the quartic opens downward and both ends point down. The correct graph has a double root (turning point on the axis) at $x = -3$ and simple crossings at $x = 0$ and $x = 2$, with both ends downward; this is option **(D)**.

Q31 2020 International Grammar Prelim Q8 · **1 mark**

The diagram shows the graph of $y = a(x + b)^2(x + c)^2(x + d)$. What are possible values of a, b, c and d?



(A) $a = 3,\ b = 4,\ c = 1,\ d = 2$

(B) $a = 6,\ b = 4,\ c = 1,\ d = 2$

(C) $a = 3,\ b = -2,\ c = 1,\ d = 4$

(D) $a = 6,\ b = -2,\ c = 1,\ d = 4$

Criteria	Marks
Correct option (C)	1

Solution.

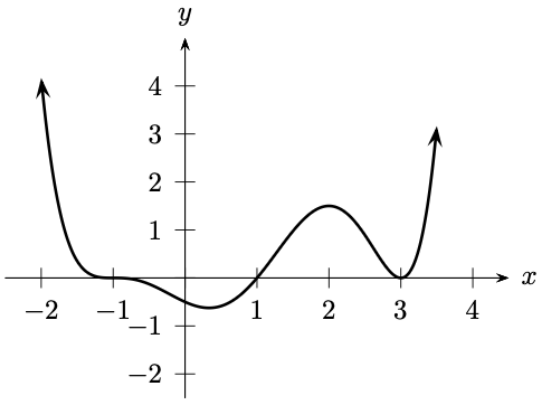
The curve crosses the axis at $x = -4$ (single root), so $(x + d) = (x + 4)$ and $d = 4$. It has double roots (turning points on the axis) at $x = -1$ and $x = 2$, so the squared factors are $(x + 1)^2$ and $(x - 2)^2$, i.e. b and c are 1 and -2. Using the y-intercept 48:

$$a(-2)^2(1)^2(4) = 16a = 48 \Rightarrow a = 3.$$

Hence $a = 3,\ b = -2,\ c = 1,\ d = 4$, which is option **(C)**.

Q32 2023 Normanhurst Boys Trial Q11(c)(i) · **1 mark**

The graph of the polynomial function $y = P(x)$ is given below.



Without using calculus, write down a possible equation of $y = P(x)$.

Criteria	**Marks**
A correct possible equation, e.g. $P(x) = (x+1)^3(x-1)(x-3)^2$	1

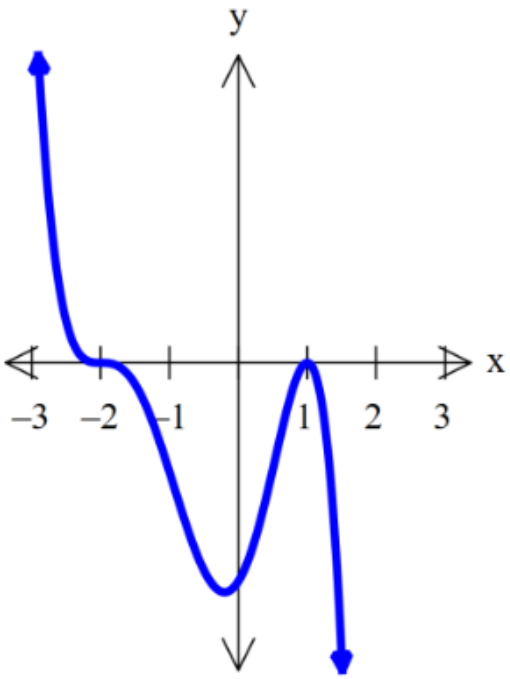
Solution.

Read off the zeros and their multiplicities: at $x = -1$ the curve flattens and crosses (a triple root, $(x+1)^3$); at $x = 1$ it crosses transversally (a single root, $(x-1)$); at $x = 3$ it touches the axis (a double root, $(x-3)^2$). Both ends point upward, so the leading coefficient is positive. A possible equation is

$$P(x) = (x+1)^3(x-1)(x-3)^2.$$

Q33 2021 Presbyterian Prelim Q3 · 1 mark

Which equation best represents the following graph?



(A) $y = (x-1)^2(x+2)^3$

(B) $y = -(x-1)^2(x+2)^3$

(C) $y = (x+1)^2(x-2)^3$

(D) $y = -(x+1)^2(x-2)^3$

Criteria	Marks
Correct option (B)	1

Solution.

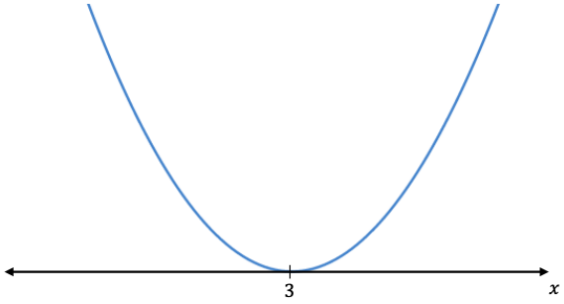
The graph has a root at $x = -2$ where it flattens and crosses (a triple root, $(x+2)^3$) and a root at $x = 1$ where it touches the axis (a double root, $(x-1)^2$). This eliminates (C) and (D), whose roots are at $x = -1$ and $x = 2$. The left end points up and the right end points down, so the degree is odd with a negative leading coefficient; the expansion of $-(x-1)^2(x+2)^3$ has leading term $-x^5$, matching. Hence

$$y = -(x-1)^2(x+2)^3,$$

which is option **(B)**.

Q34 2022 St George Girls Prelim Q4 · **1 mark**

Part of the graph of $y = P(x)$ is shown below.



Which of the following could be the equation of $P(x)$?

(A) $P(x) = (x-4)(x+3)^2(x-3)^2$

(B) $P(x) = (x-4)(x+3)^3(x-3)^3$

(C) $P(x) = -(x-4)(x+3)^2(x-3)^2$

(D) $P(x) = -(x-4)(x+3)^3(x-3)^3$

Criteria	**Marks**
Correct option (C)	1

Solution.

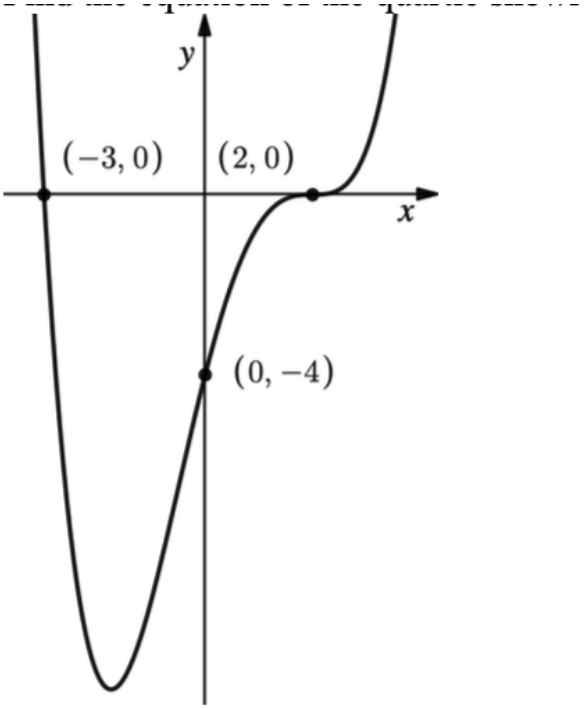
At $x = 3$ the visible part of the curve *touches* the axis (a turning point on the axis), so $x = 3$ is a root of even multiplicity, i.e. the squared factor $(x-3)^2$; this rules out (B) and (D), which have the cube $(x-3)^3$ (a crossing). Near $x = 3$ the curve lies above the axis, so $P(x) > 0$ there. Testing $x = 3.1$ in $-(x-4)(x+3)^2(x-3)^2$ gives a small positive value (≈ 0.33), whereas $(x-4)(x+3)^2(x-3)^2$ is negative near $x = 3$ (since $x - 4 < 0$). Hence

$$P(x) = -(x-4)(x+3)^2(x-3)^2,$$

which is option **(C)**.

Q35 2023 Glenwood Trial Q11(a) · **2 marks**

Find the equation of the quartic shown below.



Criteria	Marks
Identifies the factor form from the roots: single at $x = -3$, triple at $x = 2$	1
Uses the y-intercept $(0, -4)$ to find $a = \frac{1}{6}$, giving $P(x) = \frac{1}{6}(x+3)(x-2)^3$	1

Solution.

The curve crosses the axis transversally at $x = -3$ (a single root) and passes through $x = 2$ with a horizontal point of inflection (a triple root). For a quartic this gives

$$P(x) = a(x+3)(x-2)^3.$$

Using the y-intercept $(0, -4)$:

$$a(3)(-2)^3 = -24a = -4 \Rightarrow a = \tfrac{1}{6}.$$

Hence

$$P(x) = \tfrac{1}{6}(x+3)(x-2)^3.$$