Year 11 Mathematics Extension 1

Topic: Permutations & Combinations (1 of 2)
Review Questions: Permutations & Combinations
Term 3, 2026

How to use this review set

These are real past-paper questions, grouped by question pattern. The grey label on each question shows where it is from and its marks.

- The questions are grouped by the pattern that keeps coming up in exams, a few of each in a row, running from the plainer patterns into the harder, combined ones near the end. Work through them and you will meet every type an exam can throw at you.
- This set **is your diagnostic**. Working through it shows you exactly which patterns are solid and which need work. Attempt every question properly in your own book, and go back to your notes to help you work through tough questions if needed.
- Every question you get wrong, need help with or struggle with goes in your **Mistakes Log**, which you should be working on daily to get you exam-ready.

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Counting arrangements

1. Multiplication principle & basic counting

Q1 2020 Cherrybrook Tech Trial Q1 · **1 mark**

A PIN is to be chosen using four of the digits 0 through 9. Digits cannot be repeated. How many possible PINs are there?

(A) 256

(B) 210

(C) 10 000

(D) 5040

Q2 2025 Hornsby Girls Trial Q3 · **1 mark**

A four-digit number is formed by choosing four counters from the set $\{9, 8, 7, 6, 5, 4\}$ and placing them in order. How many numbers greater than 6 000 can be formed?

(A) 240

(B) 360

(C) 840

(D) 3024

Q3 2021 Sydney Girls Prelim Q1(a) · **1 mark**

The number plates for scooters appear as 3 letters followed by 2 numbers such as MAX21. How many different number plates are possible?

Q4 2024 North Sydney Boys Trial Q11(d) · **2 marks**

How many numbers greater than 8000 can be formed from the digits 1, 2, 4, 6, 9 if no digit is repeated?

Q5 2023 Trinity Grammar Trial Q11(e) · **2 marks**

A 6-character password is to be chosen from the following characters.

Digits	4	5	7
Letters	T	G	S
Symbols	#	&	%

No character may be used more than once in any password. Find the number of different passwords that may be chosen if the password starts with two letters and ends with two digits.

2. Basic arrangements (nPr, distinct)

Q6 2023 Penrith Trial Q14(a) · **1 mark**

In how many ways can an Academic Award, Citizenship Award and Integrity Award be awarded in a class of 24 students?

Q7 2023 Gosford Trial Q5 · **1 mark**

In how many ways can 5 people be selected from a group of 6 and then arranged in a line so that the two oldest people in the selected group are at either end of the line? (Note, no two people are the same age.)

(A) 720

(B) 144

(C) 72

(D) 36

Q8 2020 Hunters Hill Trial Q10 · **1 mark**

The letters A, B, C and D are used to form a four letter word. How many words can be written such that D comes before A?

(A) 3

(B) 6

(C) 12

(D) 24

Q9 2020 Baulkham Hills Prelim Q11(d)(iii) · **1 mark**

A group of 12 students is to be divided into discussion groups. In how many ways can a president and vice president be chosen for the group of 12?

Q10 2022 Killara Trial Q12(f) · **2 marks**

A set of encyclopedias numbered 1 to 13 is placed randomly on a bookshelf. In how many ways can they be placed if encyclopedias 1 and 2 are in the correct order, but not necessarily next to each other?

3. Arrangements with identical objects

Q11 2022 Abbotsleigh Trial Q1 · **1 mark**

How many distinct arrangements of the letters of the word ARRANGEMENT are possible?

(A) 2 494 800

(B) 4 989 600

(C) 9 979 200

(D) 39 916 800

Q12 2023 HSC Q11(b) · **2 marks**

In how many different ways can all the letters of the word CONDOBOLIN be arranged in a line?

Q13 2025 Abbotsleigh Trial Q11(c) · **2 marks**

In how many different ways can all the letters of the word TENNANT be arranged in a line?

Q14 2023 Sydney Grammar Trial Q11(c) · **2 marks**

Which boy's name has more unique arrangements of letters: MILO or SAVVAS? Show your working.

Q15 2024 Baulkham Hills Prelim Q14(b)(i) · **2 marks**

Consider the word PHOTOGRAPHS. How many arrangements of the letters of the word are possible?

4. Basic circular arrangements

Q16 2022 Girraween Trial Q13(c)(i) · **1 mark**

A class of fifteen Mathematics students are seated around a circular table to discuss Mathematical problems. In how many different ways can the students be arranged?

Q17 2025 North Sydney Boys Trial Q11(d)(i) · **1 mark**

The letters from the word VOLUME are placed at random on the circumference of a circle. Each letter is used only once. Find the number of different arrangements that can be formed.

Q18 2021 Knox Trial Q8 · **1 mark**

A team of 7 is to be selected from 10 people and then sat around a circular table. In how many ways can this be done?

(A) $\dfrac{10!}{7!\,3!} \times 7!$

(B) $\dfrac{10!}{7!\,3!} \times \dfrac{1}{5!}$

(C) $\dfrac{10!}{7 \times 3!}$

(D) $\dfrac{10!}{7!\,3!}$

Q19 2025 Blacktown Boys Trial Q8 · **1 mark**

In how many ways can 8 people from a group of 21 people be chosen and then arranged in a circle?

(A) $\dfrac{20!}{12!}$

(B) $\dfrac{20!}{12! \times 8}$

(C) $\dfrac{21!}{13!}$

(D) $\dfrac{21!}{13! \times 8}$

Q20 2020 International Grammar Prelim Q11(e)(i) · **1 mark**

Four Year 12 students, three Year 11 students, and two Year 10 students are seated around a circular table randomly. How many seating arrangements are possible?

Arrangements with a restriction

5. Keeping a group together (block method)

Q21 2019 HSC Q8 · **1 mark**

In how many ways can all the letters of the word PARALLEL be placed in a line with the three Ls together?

(A) $\frac{6!}{2!}$

(B) $\frac{6!}{2!\,3!}$

(C) $\frac{8!}{2!}$

(D) $\frac{8!}{2!\,3!}$

Q22 2020 Abbotsleigh Trial Q1 · **1 mark**

In how many different ways can 5 boys and 3 girls stand in a queue if the girls all stand next to each other?

(A) 40 320

(B) 720

(C) 4 320

(D) 3 136

Q23 2020 James Ruse Trial Q3 · **1 mark**

How many arrangements of the letters of the word OLYMPIC are possible if the C and the L are to be together?

(A) 120

(B) 720

(C) 240

(D) 1440

Q24 2023 Cranbrook Trial Q11(e) · **2 marks**

In how many ways can the letters of the word POSSIBILITY be arranged so that the two Ss are together?

Q25 2024 Fort St Trial Q12(a) · **2 marks**

There are four teams consisting of two tennis players in each team. They are seated around a circular table. In how many ways can the teams be arranged around the table if the players in each team are to stay together?

Q26 2024 Killara Trial Q11(c) · **3 marks**

Three men and three women are to sit at a round table. How many seating arrangements are possible:

(i) If there are no restrictions?

(ii) If two particular women must sit together?

Q27 2024 Baulkham Hills Trial Q12(c) · **4 marks**

In how many ways can the letters of the word DIFFERENCE be arranged if:

(i) there are no restrictions?

(ii) all three Es are next to each other with no letters separating them, and this string of three Es are somewhere between the two Fs?

6. Keeping objects apart (gap method)

Q28 2021 Newington Trial Q4 · **1 mark**

In how many ways can the letters of the word OLYMPICS be arranged if the 2 vowels are not next to each other?

(A) 40320

(B) 30240

(C) 10080

(D) 2520

Q29 2020 North Sydney Girls Trial Q11 · **2 marks**

Donald, Melania and five other people get on a bus one at a time. In how many ways can the seven people board the bus if Melania refuses to get on immediately following Donald?

Q30 2024 James Ruse Trial Q12(a) · **2 marks**

Three adults and five children go to the cinema and are seated next to each other in a row of eight seats. How many ways can these eight people sit so that at least two of the adults sit next to each other?

Q31 2025 Barker Trial Q12 · **3 marks**

The letters of the word SUCCESS are to be arranged.

(i) In how many ways can the letters of the word SUCCESS be arranged in a line if there are no restrictions?

(ii) In how many ways can the letters of the word SUCCESS be arranged in a line if the two Cs are NOT adjacent?

Q32 2022 Cherrybrook Tech Trial Q11(b) · **3 marks**

The letters of the word EXAMINATION are to be arranged in a row.

(i) How many eleven letter words can be made?

(ii) How many eleven letter words will NOT contain adjacent N's?

Q33 2023 Baulkham Hills Trial Q11 · **3 marks**

At a dinner party, the host, hostess and their 6 guests sit at a round table. In how many ways can they be arranged if:

(i) the host and hostess are separated?

(ii) the host and hostess are sitting directly opposite each other?

Q34 2025 Roseville Trial Q13 · **3 marks**

There are seven distinct people who are to be seated around a round table. Of these seven people, Alice and Bob must sit next to each other, while Charlie and Dean must not sit next to each other. In how many ways could these seven people be seated with the above conditions?

7. Alternating arrangements

Q35 2020 Riverview Trial Q5 · **1 mark**

Four males and four females are to sit around a table. In how many ways can this be done if the males and females alternate?

(A) 144

(B) 2880

(C) 5040

(D) 40 320

Q36 2020 Newcastle Grammar Trial Q7 · **1 mark**

Four female and four male students are to be seated around a circular table. In how many ways can this be done if the males and females must alternate?

(A) $4! \times 4!$

(B) $3! \times 4!$

(C) $3! \times 3!$

(D) $2 \times 3! \times 3!$

Q37 2022 Blacktown Boys Trial Q1 · **1 mark**

Three women and three men are to be seated around a circular table. In how many ways can this be done if the men and women must alternate?

(A) $3! \times 2!$

(B) $2! \times 2!$

(C) $5!$

(D) $6!$

Q38 2021 Hurlstone Trial Q11 · **1 mark**

Four politicians and four activists are seated alternately at a round table (i.e. alternating so that no two politicians or two activists sit next to each other). In how many ways can this be done?

Q39 2025 Normanhurst Boys Trial Q11(d) · **5 marks**

A group of six males and five females, including Aden and Ina, are to be placed into two separate lines. Line A consists of six people and line B consists of five people. How many ways can they be arranged across the two lines if:

(i) Aden and Ina are to be placed in separate lines?

(ii) The members in line A are selected first and must alternate between males and females within A?

8. Fixed positions / relative order / numbers

Q40 2022 Sydney Tech Trial Q1 · **1 mark**

The number of different arrangements of the letters of the word NORTHERN which begin and end with the letter N is:

(A) $\dfrac{8!}{2!}$

(B) $\dfrac{6!}{2!}$

(C) $\dfrac{8!}{2!\,2!}$

(D) $\dfrac{6!}{2!\,2!}$

Q41 2022 Glenwood Trial Q13(b)(ii) · **1 mark**

Eight students, including Ari, Bobbi, and Cali, form a single file line and walk into their classroom. How many ways can the students walk into the classroom if Ari enters before Bobbi?

Q42 2024 Trinity Grammar Trial Q5 · **1 mark**

Using the digits 0, 1, 2, 3, 4 and 5, how many odd numbers between 2000 and 5000 can be formed if no digits are repeated?

(A) 48

(B) 96

(C) 120

(D) 360

Q43 2022 St George Girls Trial Q11(c) · **2 marks**

In how many ways can the word GEOMETRY be arranged in a straight line if the vowels must occupy the 2nd, 4th and 6th position?

Q44 2021 Girraween Prelim Q2(b) · **2 marks**

How many numbers greater than 3 000 can be formed from the digits 1, 3, 5, 7, 9 if no repetition is permitted?

Choosing (selections)

9. Basic selections (nCr)

Q45 2022 Moriah Trial Q11(c)(i) · 1 mark

A school has ten prefects. John and Alex are two of those prefects. A committee of six prefects is to be formed to organise a function. How many different committees can be formed?

Q46 2025 Caringbah Trial Q13(a)(i) · 1 mark

A four-person team is to be chosen at random from nine women and seven men. In how many ways can the team be chosen?

Q47 2021 HSC Q11(d) · 1 mark

A committee containing 5 men and 3 women is to be formed from a group of 10 men and 8 women. In how many different ways can the committee be formed?

Q48 2024 Penrith Trial Q11(b) · 2 marks

A committee of 7 is to be chosen from 6 men and 8 women. How many different committees can be formed if the committee must have exactly 4 men?

Q49 2022 Penrith Trial Q12(c) · 3 marks

In how many ways can 5 people be selected from a group of 12 if:

(i) one particular person must be included.

(ii) two particular people cannot be selected.

(iii) one particular person must be included, and two particular people are excluded from the selection.

Q50 2024 Girraween Trial Q16(c) · 3 marks

A committee of 5 is to be chosen from six men and seven women.

(i) How many committees are possible if there are no restrictions?

(ii) How many committees are possible if there are more women than men?

10. Selections: at least / at most / cases

Q51 2020 Trinity Grammar Trial Q7 · 1 mark

From 8 green and 8 white cards, 4 are chosen at random. If each outcome is equally likely, how many combinations are possible that have at most 1 green card?

(A) 518

(B) 930

(C) 1820

(D) 3920

Q52 2022 Barker Trial Q11(d) · 2 marks

A state senate sub-committee is to be formed by choosing 3 senators from among 6 rural senators and 6 urban senators. How many different sub-committees can be formed if at least 1 rural and 1 urban senator must be chosen?

Q53 2022 Shore Trial Q12(d) · 2 marks

A mixed volleyball team of eight players is selected from ten males and nine females. In how many ways can this be done if the team must have at least 2 female players?

Q54 2023 Fort St Trial Q12(b) · 2 marks

In a 16 member soccer squad 12 are right-handed while 4 are left-handed. If 11 members are to be selected as the starting line up, in how many ways will there be at least three left-handed players in the starting line up?

Q55 2025 Manly Trial Q14(a)(ii) · 3 marks

A standard pack of 52 cards consists of 13 cards from each of the four suits: Spades, Hearts, Clubs and Diamonds. In how many ways can six cards be selected without replacement if at least five cards must be of the same suit?

11. Grouping & splitting

Q56 2023 Penrith Trial Q9 · **1 mark**

In how many ways can twelve table tennis players be assigned into three groups each containing four players for the elimination round?

(A) 5 775

(B) 11 550

(C) 15 400

(D) 34 650

Q57 2024 Girraween Trial Q12(d) · **2 marks**

In how many ways can 8 people be placed in groups of 2?

Q58 2023 Fort St Trial Q13(d)(i) · **2 marks**

Ten friends are going to be divided into groups of 5, 3 and 2 members as part of a competition. In how many ways can the groups be formed?

Q59 2023 Normanhurst Boys Trial Q11(e) · **2 marks**

There are twelve people that need to be divided into groups.

(i) In how many ways can they be divided into two groups of six people?

(ii) In how many ways can they be divided into two groups of six people if two particular people must be in the same group?

Q60 2024 Ascham Trial Q14(e) · **2 marks**

Two teams of four players and an umpire are to be formed from nine people. If Amelia and Chloe cannot be on the same team, although they can be an umpire, find how many different ways the nine people can be grouped into two teams and an umpire.

Q61 2024 Sydney Tech Trial Q14(d) · **3 marks**

(i) In how many ways can n students be placed in two distinct rooms so that neither room is empty? Give your answer in simplest form.

(ii) In how many ways can five students be placed in three distinct rooms so that no room is empty?

12. Inclusion-exclusion

Q62 2020 Sydney Grammar Trial Q6 · **1 mark**

A school mathematics department consists of 5 male and 5 female teachers. How many different exam-writing committees comprising three teachers could be formed if there must be at least one teacher of each sex on the committee?

(A) 50

(B) 100

(C) 120

(D) 600

Q63 2021 Caringbah Trial Q7 · **1 mark**

A committee of 5 people is to be chosen from a group of 6 girls and 4 boys. How many different committees could be formed that have at least one boy?

(A) ${}^{10}C_5 - 1$

(B) ${}^{4}C_1 \times {}^{6}C_4$

(C) ${}^{4}C_1 + {}^{6}C_4$

(D) ${}^{10}C_5 - 6$

Q64 2023 Sydney Boys Trial Q11(d) · **2 marks**

A school principal calls a meeting with two student representatives from each year group from Year 7 to Year 12. The 12 students and the principal sit at a round table. In how many ways can the 13 participants sit around the table if at least one pair of students from a year group sits apart?

Q65 2022 James Ruse Trial Q11(d) · **4 marks**

You are asked to provide an 8-letter password which may only be comprised of the letters A, B and C.

(i) How many 8-letter passwords are there to choose from?

(ii) How many of the 8-letter passwords contain only B's and C's?

(iii) If you decide that your 8-letter password must use each of the letters A, B and C at least once, how many possible passwords are there to choose from?

Q66 2020 Baulkham Hills Trial Q14(c) · **4 marks**

Suppose there are n different books, and there are p identical copies of each of these n books. (You may assume n and p are non-zero integers by definition.)

(i) If all the books are to be placed in a line on one shelf, in how many ways can this be done?

(ii) Consider now a library with n different books, where there are p identical copies of each of these n books on one shelf. The library barcodes each book differently. Suppose you want to borrow at least one book from the shelf, but you do not want to borrow multiple copies of the same book. In how many ways can this be done?

Specialised techniques & proofs

13. Geometric counting

Q67 2024 North Sydney Girls Trial Q8 · **1 mark**

A set of five distinct parallel lines intersects a different set of seven distinct parallel lines. How many possible parallelograms can be made from these lines?

(A) ${}^{12}C_4$

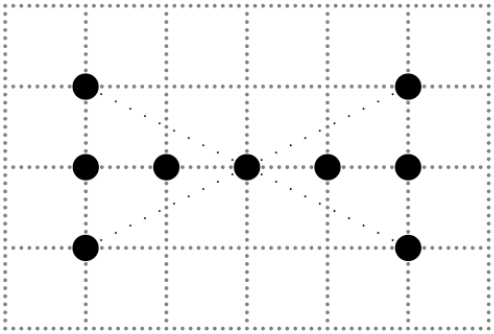
(B) ${}^{5}C_2 \times {}^{7}C_2$

(C) ${}^{5}C_2 + {}^{7}C_2$

(D) ${}^{5}P_2 \times {}^{7}P_2$

Q68 2020 Normanhurst Boys Trial Q10 · **1 mark**

The diagram below shows a shape made by 9 points.



How many combinations of 3 points are collinear?

(A) 10

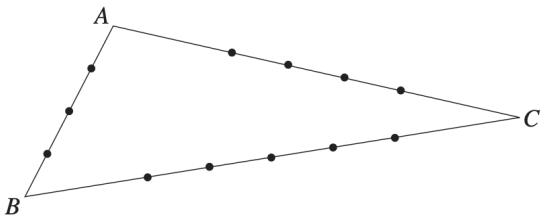
(B) 14

(C) 12

(D) 8

Q69 2022 HSC Q7 · **1 mark**

The diagram shows triangle ABC with points chosen on each of the sides. On side AB, 3 points are chosen. On side AC, 4 points are chosen. On side BC, 5 points are chosen.



How many triangles can be formed using the chosen points as vertices?

(A) 60

(B) 145

(C) 205

(D) 220

Q70 2024 James Ruse Trial Q9 · **1 mark**

There are 11 points in a plane such that 4 of the points are collinear. Which of the following gives the number of lines that may be formed such that those lines pass through at least two of the 11 points?

(A) 55

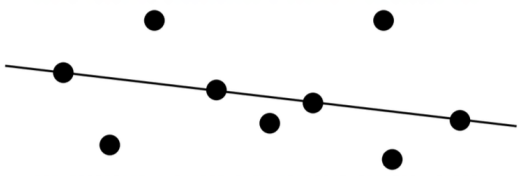
(B) 49

(C) 50

(D) 52

Q71 2020 Abbotsleigh Trial Q13(a) · **2 marks**

Nine points are drawn on a page as shown, with four of them lying on one line. Other than these four points, no other set of three points is collinear.



How many different triangles can be drawn using the nine points as vertices?

14. Stars & bars / lattice paths

Q72 2022 Killara Trial Q8 · **1 mark**

How many solutions are there to the equation $x + y + z = 4$ given that x, y and z are non-negative integers?

(A) 4

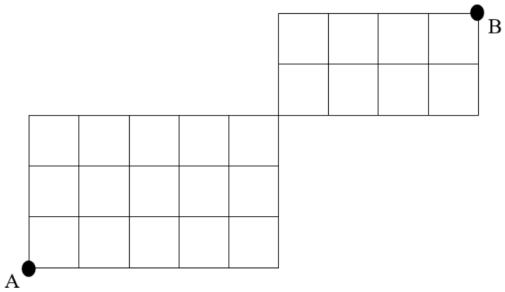
(B) 15

(C) 17

(D) 24

Q73 2024 Manly Trial Q10 · **1 mark**

Moving only right or up, how many different paths exist to get from Point A to Point B?



(A) 45

(B) 840

(C) 480480

(D) 16!

Q74 2022 James Ruse Prelim Q12(e)(i) · **1 mark**

A customer wishes to purchase a dozen donuts from a bakery that offers four different types of donut. The customer wishes to know how many ways there are of buying a dozen donuts. To do this, they represent each donut as a 'D' and place those twelve Ds in a line:

$$DDDDDDDDDDDD$$

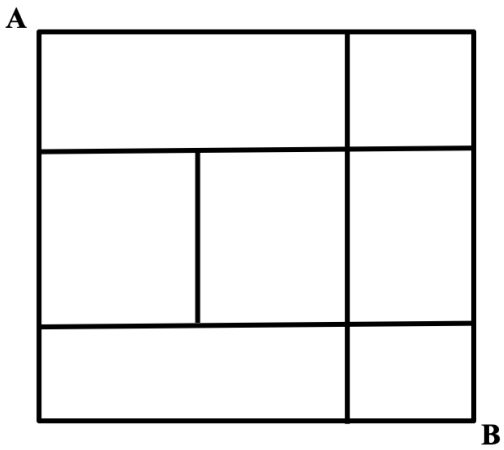
They then insert three $*$s as place markers to indicate how many of each type of donut they buy. For example:

$$DD * DDD * DDDDDD * DD$$

means they buy 2 donuts of the first type, 3 donuts of the second type, 6 donuts of the third type and 2 donuts of the fourth type. In how many ways can the customer buy 12 donuts if there are 4 available types? Assume there are at least 12 donuts available for each type.

Q75 2023 Baulkham Hills Prelim Q10 · **1 mark**

Moving only to the right or down, how many different routes are there from A to B?



(A) 10

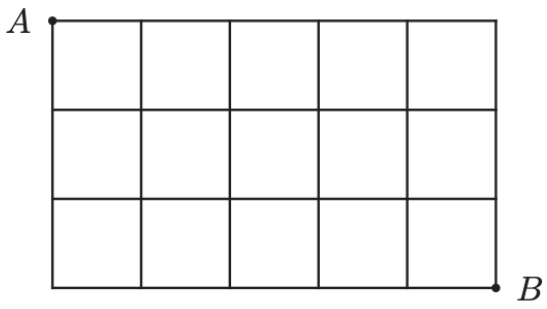
(B) 12

(C) 14

(D) 18

Q76 2022 Normanhurst Boys Trial Q11(e) · **2 marks**

The diagram shows a grid consisting of unit squares. Chris needs to travel from point A to point B but can only do so by moving right or down along the grid lines.



How many paths are there for Chris?

15. Solving nPr / nCr equations

Q77 2020 Sydney Boys Trial Q12(c) · **2 marks**

Solve the equation $(n+2)! = 72\,n!$.

Q78 2025 Barker Trial Q12(d) · **2 marks**

Solve for n: ${}^nC_3 = 3\,({}^nP_2)$.

Q79 2025 Blacktown Boys Trial Q12(a) · **3 marks**

(i) Show that ${}^nC_2 = \dfrac{n(n-1)}{2}$.

(ii) Hence, or otherwise, solve for n where $2 \times {}^nC_4 = 51 \times {}^nC_2$.

Q80 2023 Knox Prelim Q13(b) · **3 marks**

There are n swimmers in a swimming club. The trainer is to select a group of four or five of these swimmers to swim in a competition. The number of different groups of five swimmers he can select is twice the number of different groups of four swimmers. Find n, the number of swimmers in the swimming club.

Q81 2024 Hurlstone Prelim Q11(c) · **3 marks**

Solve the equation $16(n-1)! = 5\,n! + (n+1)!$ where n is any positive integer.

16. Factorial manipulation: simplify & prove

Q82 2025 Roseville Trial Q14(c) · **1 mark**

Show that ${}^{n}C_{r} = {}^{n}C_{n-r}$.

Q83 2023 Abbotsleigh Trial Q12(c)(i) · **1 mark**

Show that $\dfrac{1}{(n+1)!} - \dfrac{n+1}{(n+2)!} = \dfrac{1}{(n+2)!}$.

Q84 2020 Girraween Prelim Q2 · **1 mark**

Simplify $\dfrac{(n+1)!}{(n-1)!}$.

Q85 2022 Girraween Prelim Q2 · **1 mark**

Simplify $\dfrac{1}{n!} - \dfrac{1}{(n+1)!}$.

Q86 2024 Hurlstone Trial Q15(c) · **2 marks**

Show that ${}^{n-1}C_{k-1} + {}^{n-1}C_{k} = \dfrac{n!}{(n-k)!\,k!}$.

Q87 2024 Blacktown Boys Trial Q12(d) · **2 marks**

It is known that ${}^{n}C_{r} = {}^{n-1}C_{r-1} + {}^{n-1}C_{r}$ for all integers such that $1 \leq r \leq n-1$. (Do NOT prove this.) Find ONE possible set of values for p and q such that

$$ {}^{2026}C_{1924} - {}^{2024}C_{100} - {}^{2024}C_{101} = {}^{p}C_{q}. $$

Q88 2024 Girraween Prelim Q8(a) · **2 marks**

Write $\dfrac{1}{(n+1)!} - \dfrac{1}{(n-1)!}$ as a single fraction.

Q89 2020 Girraween Prelim Q9(a) · **2 marks**

Write $\dfrac{8}{(n+2)!} - \dfrac{1}{n!}$ as a single fraction.

Q90 2024 Sydney Girls Trial Q13(c) · **4 marks**

For positive integers n and k, where $k \leq n$:

(i) Show algebraically that $\frac{k}{n}\binom{n}{k} = \binom{n-1}{k-1}$, where $\binom{n}{k} = {}^nC_k$.

(ii) Prove the above identity by providing a combinatorial argument (i.e. by providing a counting argument).

17. Combinatorial-argument and explain-why proofs

Q91 2021 Sydney Grammar Prelim Q5(b) · **2 marks**

Twelve pupils are taking a class that requires them to complete the same group project. They are to be split into three groups, with each group consisting of four pupils. In calculating the number of ways this can be done, one of the pupils suggests:

> "The first group can be selected in ${}^{12}C_4$ ways. The second group can be selected in ${}^8C_4$ ways. The third group can then be selected in ${}^4C_4$ ways. Therefore there are ${}^{12}C_4 \times {}^8C_4 \times {}^4C_4 = 34\,650$ ways we can be split into three groups of four."

Explain why this reasoning is incorrect, and calculate the correct number of ways that the twelve pupils can be split into three groups of four.

Q92 2023 Pymble Trial Q14(b) · **2 marks**

A Maths club has $2n$ members, half of which are male and half female. A Pi Day committee is made up of 3 members of this club. By considering all different combinations for this committee, show that

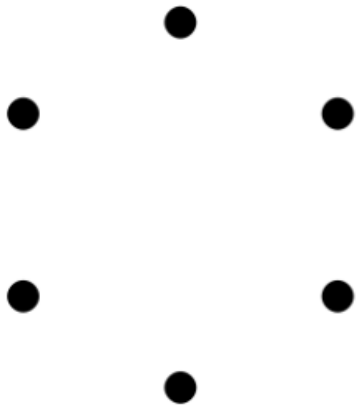
$$n\,{}^nC_2 + {}^nC_3 = \frac{1}{2}\,{}^{2n}C_3, \qquad \text{where } n \geq 3.$$

Q93 2022 James Ruse Prelim Q14(e) · **2 marks**

Using a combinatorial argument, show that $\dfrac{(n^2)!}{(n!)^n} \in \mathbb{N}$ for all natural numbers n.

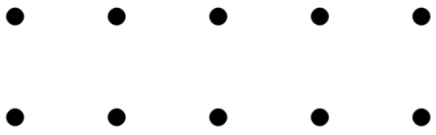
Q94 2023 Presbyterian (PLC Sydney) Prelim Q14(a) · 3 marks

Consider the dots below.



(i) Explain why ${}^6C_3$ distinct triangles can be formed using the dots as vertices.

(ii) How many distinct triangles can be formed using the dots below as vertices?



Q95 2021 Cherrybrook Tech Trial Q14(b) · 3 marks

Consider a set with n objects without regard to order. The set is to be split into r groups. The first group is of size n_1, the second is of size n_2, and so forth, with the last group of size n_r. Note $n_1 + n_2 + \cdots + n_r = n$.

(i) Explain why the number of ways to split the set into r groups is

$$\binom{n}{n_1}\binom{n-n_1}{n_2}\binom{n-n_1-n_2}{n_3}\cdots\binom{n-n_1-n_2-\cdots-n_{r-1}}{n_r}.$$

(ii) Show that the expression in part (i) is equal to

$$\frac{n!}{n_1!\,n_2!\,\ldots\,n_r!}.$$

Q96 2021 Knox Prelim Q12(c) · 3 marks

12 people need to be split up into teams for a quiz.

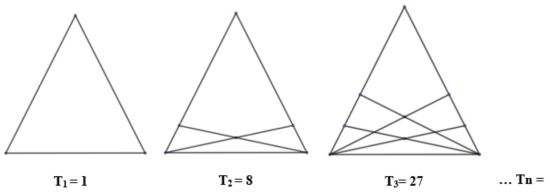
(i) Explain why the number of ways of splitting them into two groups of the same size is

$$\frac{1}{2}\binom{12}{6}.$$

(ii) How many ways are there of splitting them into two groups of any size (but there must be at least one person in each group)?

Q97 2020 James Ruse Prelim Q9(b) · 4 marks

Parminder is generating triangles from a given triangle, T_1. He takes the triangle and draws a ray from each corner of the base angles, as shown, and notes the number of triangles this creates in T_2. He then creates a third triangle, T_3, by drawing another ray from each base angle, and counts the number of triangles now. He continues doing this, creating a series: $T_1, T_2, T_3, \ldots$



(i) Explain why $T_n = \binom{2n+1}{3} - 2\binom{n+1}{3}, \quad n \geq 2.$

(ii) Hence, prove that $T_n = n^3$.

Probability

18. Arrangements, then a probability

Q98 2021 Hurlstone Trial Q11(b) · 1 mark

The letters of the word CONSIDER are rearranged at random. What is the probability that an arrangement begins with SI?

Q99 2021 Ascham Trial Q6 · 1 mark

Six girls and three boys are to be seated randomly around a circular table. What is the probability that the three boys are seated together?

(A) $\frac{1}{12}$

(B) $\frac{1}{24}$

(C) $\frac{3}{28}$

(D) $\frac{1}{56}$

Q100 2020 Sydney Tech Trial Q11(a) · 2 marks

Nine people are arranged around a circular table.

(i) How many ways can they be arranged?

(ii) If they are seated randomly, what is the probability that two individuals, Lois and Clark, are sitting next to each other?

Q101 2025 Killara Trial Q11(c) · 3 marks

The letters of the name NARUTO are arranged in a row.

(i) In how many ways can this be done using all the letters?

(ii) What is the probability that the vowels are all together?

Q102 2025 Hurlstone Trial Q11(a) · **3 marks**

The letters of the word CALCULUS are arranged in a row.

(i) How many different arrangements are there?

(ii) If one of these arrangements is selected at random, find the probability that it begins with "U" and ends with "U".

Q103 2025 Gosford Trial Q11(c) · **3 marks**

A group of six girls and two boys are to be seated around a round table.

(i) In how many different ways can the eight people be seated?

(ii) What is the probability the two boys won't be seated next to each other?

Q104 2022 Manly Trial Q11(f) · **3 marks**

A group of 10 people arrive to eat at a restaurant. The only seating available for them is at two circular tables, one that seats six people and another that seats four. Assuming that the seating arrangement is random, what is the probability that a particular couple will be seated at the same table? (The couple do not have to sit next to each other.)

19. Selections, then a probability

Q105 2024 HSC Q9 · 1 mark

A bag contains n metal coins, $n \geq 3$, that are made from either silver or bronze. There are k silver coins in the bag and the rest are bronze. Two coins are to be drawn at random from the bag, with the first coin drawn not being replaced before the second coin is drawn. Which of the following expressions will give the probability that the two coins drawn are made of the same metal?

(A) $\dfrac{k(k-1)+(n-k)(n-k-1)}{n(n-1)}$

(B) $\binom{n}{2}\binom{n}{k}\left(1-\dfrac{k}{n}\right)^{n-2}$

(C) $\dfrac{\binom{k}{2}+\binom{n-k}{2}}{n(n-1)}$

(D) $\dfrac{k^2+(n-k)^2}{n^2}$

Q106 2021 Ascham Trial Q11(d) · 2 marks

A committee of 5 is to be chosen from 5 men and 4 women. What is the probability that the committee contains three women?

Q107 2024 Newington Trial Q11(a) · 2 marks

A student representative committee of 7 people is to be chosen from a group of 10 men and 11 women. Find the probability that the committee is made of 3 men and 4 women.

Q108 2022 Sydney Boys Trial Q13(a) · 2 marks

A fireworks technician is testing fireworks for an upcoming display. He has 20 fireworks, of which 13 are red and 7 are yellow. He launches 8 of them in a random order. What is the probability that 5 of the launched fireworks are red?

Q109 2023 Manly Trial Q11(b) · 2 marks

A Mathematics department consists of 12 teachers. Nine of the teachers wear glasses and three of the teachers do not wear glasses. Five of these teachers go out for dinner together. What is the probability there will be more teachers who wear glasses than teachers who do not wear glasses in the group who go out for dinner?

Q110 2020 Newcastle Grammar Trial Q14(b) · 2 marks

A bag contains n red marbles and one blue marble. Three marbles are drawn (without replacement). The probability that the three marbles are red is $\dfrac{5}{8}$. Find the value of n.

Q111 2022 Normanhurst Boys Trial Q11(c) · **3 marks**

A school has twenty prefects. Jun and Alex are two of those prefects. A committee of seven prefects is to be formed to organise a function.

(i) How many different committees can be formed?

(ii) What is the probability that the committee contains Jun but does not contain Alex?