Year 11 Mathematics Extension 1

Topic: Permutations & Combinations (2 of 2)

Review Questions (Worked Solutions): Mixed one-off exam questions

Term 3, 2026

Mixed one-off exam questions

Q1 2022 Hurlstone Prelim Q8 · **1 mark**

After a sports tournament every player shakes hands with every other player once. If there are 36 handshakes in total, how many players are at the tournament?

(A) 18

(B) 8

(C) 10

(D) 9

Criteria	Marks
Correct answer (D) 9	1

Solution.

Each handshake pairs two of the n players, so the number of handshakes is $\binom{n}{2}$. Setting this equal to 36:

$$\binom{n}{2} = \frac{n(n-1)}{2} = 36 \quad \Longrightarrow \quad n(n-1) = 72.$$

Since $9 \times 8 = 72$, we have $n = 9$. The answer is (D).

Q2 2023 North Sydney Boys Prelim Q2 · **1 mark**

Four digit numbers are formed from the digits 1, 2, 3 and 4. Each digit is used once only. The sum of all the numbers that can be formed is?

(A) 6666

(B) 44440

(C) 66660

(D) 266640

Criteria	**Marks**
Correct answer (C) 66660	1

Solution.

There are $4! = 24$ numbers in all. By symmetry, in each of the four place-value columns (units, tens, hundreds, thousands) each of the digits $1, 2, 3, 4$ appears the same number of times, namely $3! = 6$ times. The digit total in any one column is therefore

$$3!\,(1 + 2 + 3 + 4) = 6 \times 10 = 60.$$

Summing across the four columns, each weighted by its place value,

$$60 \times (1000 + 100 + 10 + 1) = 60 \times 1111 = 66660.$$

The answer is (C).

Q3 2024 Baulkham Hills Prelim Q9 · **1 mark**

Lawrence is about to walk up a set of stairs with 8 steps. He can take either one or two steps at a time. In how many different ways can he walk up the set of stairs?

(A) 5

(B) 20

(C) 34

(D) 163

Criteria	Marks
Correct answer (C) 34	1

Solution.

Let a_k be the number of ways to climb k steps. The final move is either a single step (from step $k-1$) or a double step (from step $k-2$), so

$$a_k = a_{k-1} + a_{k-2}, \qquad a_1 = 1, \quad a_2 = 2.$$

This generates the Fibonacci-type sequence

$$1,\ 2,\ 3,\ 5,\ 8,\ 13,\ 21,\ 34,$$

so $a_8 = 34$. The answer is (C).

Q4 2024 Girraween Prelim Q10(c) · **2 marks**

In how many ways can 8 different gifts be distributed to two children so that each child receives an odd number of gifts?

Criteria	Marks
Identifies the odd splits $(1,7), (3,5), (5,3), (7,1)$ (or equivalent case setup)	1
Correct total 128	1

Solution.

Label the children A and B. Since A and B together receive all 8 gifts, and 8 is even, giving A an odd number automatically gives B an odd number too. So we simply choose which gifts go to A, where A receives 1, 3, 5 or 7 gifts:

$$\binom{8}{1} + \binom{8}{3} + \binom{8}{5} + \binom{8}{7} = 8 + 56 + 56 + 8 = 128.$$

(Equivalently, exactly half of the 2^8 ways of assigning each gift to one of the two children give A an odd number, so the count is $2^8/2 = 2^7 = 128$.)

Q5 2023 Knox Prelim Q13(c) · **2 marks**

In a school athletic carnival 77 students are randomly divided into 3 groups, where each group has a different number of students. What is the maximum number of students that the smallest group can have?

Criteria	**Marks**
Recognises the smallest group is maximised by making the other two as small (and close) as possible	1
Correct answer 24	1

Solution.

Let the three group sizes be $a < b < c$ with $a + b + c = 77$. To make the smallest group a as large as possible, the other two groups should be as small as possible while still being distinct and larger than a; the tightest choice is $b = a + 1$ and $c = a + 2$.

Then $a + (a + 1) + (a + 2) = 3a + 3 = 77$ gives $a = 24\frac{2}{3}$, which is not a whole number. Taking $a = 25$ forces $b + c = 52$ with $b, c > 25$ distinct, and the smallest possibility $26 + 27 = 53 > 52$ is already too large, so $a = 25$ is impossible. Taking $a = 24$ works, for example

$$24 + 26 + 27 = 77.$$

Hence the maximum number of students in the smallest group is $\boxed{24}$.

Q6 2024 James Ruse Prelim Q9(c) · 3 marks

Shikung and Andrew are competing against each other in a competition in which the winner is the first to score five goals. The outcome is recorded by listing, in order, the initial of the person who scores each goal. For example, one possible outcome is recorded as $SAASSASS$.

(i) Explain why there are five different ways in which the outcome could be recorded if Andrew scores only one goal in the competition.

(ii) In how many different ways could the outcome of this competition be recorded?

Criteria	Marks
(i) Explains the record is an arrangement of $SSSSSA$ whose last letter must be S (the winner scores last)	1
(ii) Sets up the winning records for one player as $\sum_{k=0}^{4} \binom{4+k}{k}$ (or equivalent)	1
(ii) Correct answer 252	1

Solution.

(i) If Andrew (A) scores exactly one goal, the winner must be Shikung (S), who scores 5. The record is therefore an arrangement of the six letters S, S, S, S, S, A. The competition stops the moment S reaches five goals, so the last letter recorded must be S (the winning goal); the record cannot end in A. Fixing the final S, the remaining letters are S, S, S, S, A, and the single A can occupy any of the first five positions:

$$\frac{6!}{5!} - 1 = 6 - 1 = 5,$$

so there are 5 such records.

(ii) Count the records in which S wins, then double by symmetry. If S wins, the final letter is S (his fifth goal), and A scores k goals with $k \in \{0, 1, 2, 3, 4\}$. The record is then S appearing 5 times and A appearing k times, with the last letter fixed as S; the first $4 + k$ letters are an arrangement of four Ss and k As:

$$\binom{4+k}{k}.$$

Summing over $k = 0, 1, 2, 3, 4$ and using the hockey-stick identity,

$$\sum_{k=0}^{4} \binom{4+k}{k} = \binom{4}{0} + \binom{5}{1} + \binom{6}{2} + \binom{7}{3} + \binom{8}{4} = 1 + 5 + 15 + 35 + 70 = 126 = \binom{9}{4}.$$

By symmetry A can win in the same number of ways, so the total number of possible records is

$$2 \times 126 = 252.$$