Year 11 Mathematics Extension 1

Topic: Further Trigonometry (4 of 4)

Review Questions (Worked Solutions): Mixed One-off Exam Questions

Term 3, 2026

Mixed one-off exam questions

Q1 2020 Fort St Prelim Q7 · 1 mark

What is the exact value of

$$\tan\left(\frac{\pi}{180}\right)\tan\left(\frac{2\pi}{180}\right)\tan\left(\frac{3\pi}{180}\right)\cdots\tan\left(\frac{88\pi}{180}\right)\tan\left(\frac{89\pi}{180}\right)?$$

(A) -1

(B) 0

(C) 1

(D) Not defined

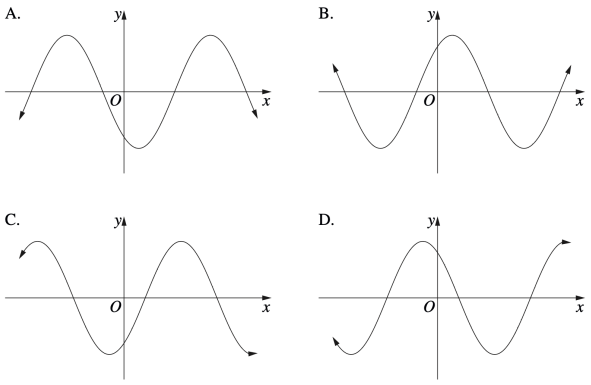
Criteria	Marks
Correct option (C)	1

Solution.

The angles are $1°, 2°, \ldots, 89°$. Pair each term with its complement: $\tan\theta\,\tan(90° - \theta) = \tan\theta\cot\theta = 1$. The pairs $(1°, 89°), (2°, 88°), \ldots, (44°, 46°)$ each multiply to 1, and the middle term $\tan 45° = 1$. Hence the whole product equals 1, so the answer is **(C)**.

Q2 2021 HSC Q7 · **1 mark**

Which curve best represents the graph of the function $f(x) = -a\sin x + b\cos x$ given that the constants a and b are both positive?



Criteria	Marks
Correct option (D)	1

Solution.

Write $f(x) = b\cos x - a\sin x = R\cos(x+\alpha)$ with $R = \sqrt{a^2+b^2}$ and $\alpha = \tan^{-1}\dfrac{a}{b}$ in the first quadrant, so the maximum sits at $x = -\alpha < 0$ (a shift to the left). Two features fix the curve: the y-intercept is $f(0) = b > 0$ (positive, and below the amplitude R), and the initial gradient is $f'(0) = -a < 0$, so the curve is already past its maximum and decreasing as it crosses the y-axis, heading down toward a minimum. The curve with a positive y-intercept that is decreasing at $x = 0$ is **(D)**.

Q3 2020 Sydney Girls Prelim Q3(e) · **3 marks**

Let A, B and C be the three angles of a triangle.

(i) Prove that $\tan(A+B) = -\tan C$.

(ii) Hence, or otherwise, prove that $\tan A + \tan B + \tan C = \tan A \tan B \tan C$.

Criteria	**Marks**
(i) Uses $A+B+C=\pi$, so $A+B=\pi-C$, and $\tan(\pi-C) = -\tan C$	1
(ii) Expands $\tan(A+B) = \dfrac{\tan A+\tan B}{1-\tan A\tan B}$ and sets it equal to $-\tan C$	1
(ii) Cross-multiplies and rearranges to the required identity	1

Solution.

(i) Since A, B, C are the angles of a triangle, $A+B+C=\pi$, so $A+B=\pi-C$. Hence

$$\tan(A+B) = \tan(\pi - C) = -\tan C.$$

(ii) Expanding the left-hand side of part (i),

$$\frac{\tan A+\tan B}{1-\tan A\tan B} = -\tan C.$$

Cross-multiplying, $\tan A+\tan B = -\tan C(1-\tan A\tan B) = -\tan C+\tan A\tan B\tan C$. Therefore

$$\tan A+\tan B+\tan C = \tan A\tan B\tan C.$$

Q4 2023 Blacktown Boys Prelim Q13(c) · **3 marks**

(i) Show that $\cos 3A = 4\cos^3 A - 3\cos A$.

(ii) Hence show that if $2\cos A = x + \dfrac{1}{x}$, then $2\cos 3A = x^3 + \dfrac{1}{x^3}$.

Criteria	**Marks**
(i) Expands $\cos(2A + A)$ and substitutes the double-angle results	1
(i) Simplifies to $4\cos^3 A - 3\cos A$	1
(ii) Uses $2\cos 3A = (2\cos A)^3 - 3(2\cos A)$ with $2\cos A = x + \frac{1}{x}$ to conclude	1

Solution.

(i) $\cos 3A = \cos(2A+A) = \cos 2A\cos A - \sin 2A\sin A$. With $\cos 2A = 2\cos^2 A - 1$ and $\sin 2A = 2\sin A\cos A$,

$$\cos 3A = (2\cos^2 A - 1)\cos A - 2\sin^2 A\cos A = 2\cos^3 A - \cos A - 2(1-\cos^2 A)\cos A = 4\cos^3 A - 3\cos A.$$

(ii) From part (i), $2\cos 3A = 8\cos^3 A - 6\cos A = (2\cos A)^3 - 3(2\cos A)$. Substituting $2\cos A = x + \dfrac{1}{x}$,

$$2\cos 3A = \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right) = x^3 + 3x + \frac{3}{x} + \frac{1}{x^3} - 3x - \frac{3}{x} = x^3 + \frac{1}{x^3}.$$

Q5 2023 Baulkham Hills Trial Q11(e) · **4 marks**

(i) Prove $\cot\theta - 2\cot 2\theta = \tan\theta$.

(ii) Hence, or otherwise, prove $\tan\theta + 2\tan 2\theta + 4\tan 4\theta = \cot\theta - 8\cot 8\theta$.

Criteria	**Marks**
(i) Writes $\cot\theta - 2\cot 2\theta = \dfrac{1}{\tan\theta} - \dfrac{2(1-\tan^2\theta)}{2\tan\theta}$ and simplifies to $\tan\theta$	2
(ii) Applies part (i) at θ, 2θ, 4θ to write each tangent as a difference of cotangents	1
(ii) Telescopes the sum to $\cot\theta - 8\cot 8\theta$	1

Solution.

(i) Using $\cot\theta = \dfrac{1}{\tan\theta}$ and $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$ so $\cot 2\theta = \dfrac{1-\tan^2\theta}{2\tan\theta}$,

$$\cot\theta - 2\cot 2\theta = \frac{1}{\tan\theta} - 2\cdot\frac{1-\tan^2\theta}{2\tan\theta} = \frac{1-(1-\tan^2\theta)}{\tan\theta} = \frac{\tan^2\theta}{\tan\theta} = \tan\theta.$$

(ii) Part (i) applied at θ, 2θ and 4θ gives

$$\tan\theta = \cot\theta - 2\cot 2\theta, \qquad \tan 2\theta = \cot 2\theta - 2\cot 4\theta, \qquad \tan 4\theta = \cot 4\theta - 2\cot 8\theta.$$

Therefore

$$\begin{aligned}\tan\theta + 2\tan 2\theta + 4\tan 4\theta &= (\cot\theta - 2\cot 2\theta) + 2(\cot 2\theta - 2\cot 4\theta) + 4(\cot 4\theta - 2\cot 8\theta)\\ &= \cot\theta + (-2+2)\cot 2\theta + (-4+4)\cot 4\theta - 8\cot 8\theta\\ &= \cot\theta - 8\cot 8\theta.\end{aligned}$$

Q6 2022 Sydney Girls Trial Q12(b) · 4 marks

(i) Express $f(x) = \sqrt{3}\cos x - \sin x$ in the form $R\cos(x+\alpha)$, where $0 < \alpha < \dfrac{\pi}{2}$.

(ii) Sketch $y = f(x)$ for $0 \le x \le 2\pi$, showing clearly the location of all stationary points and the end-points.

Criteria	Marks
(i) Equates $R\cos\alpha = \sqrt{3}$ and $R\sin\alpha = 1$ to find $R = 2$	1
(i) Finds $\alpha = \frac{\pi}{6}$, giving $f(x) = 2\cos\left(x + \frac{\pi}{6}\right)$	1
(ii) Marks the stationary points: minimum $\left(\frac{5\pi}{6}, -2\right)$ and maximum $\left(\frac{11\pi}{6}, 2\right)$	1
(ii) Marks the end-points $\left(0, \sqrt{3}\right)$ and $\left(2\pi, \sqrt{3}\right)$ and draws the cosine curve	1

Solution.

(i) Expanding $R\cos(x+\alpha) = R\cos\alpha\cos x - R\sin\alpha\sin x$ and matching to $\sqrt{3}\cos x - \sin x$:

$$R\cos\alpha = \sqrt{3}, \qquad R\sin\alpha = 1.$$

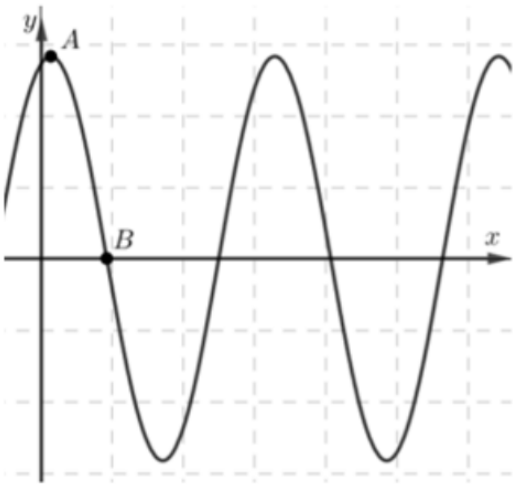
Then $R^2 = 3 + 1 = 4$ so $R = 2$, and $\tan\alpha = \dfrac{1}{\sqrt{3}}$ with α in the first quadrant, so $\alpha = \dfrac{\pi}{6}$. Hence

$$f(x) = 2\cos\left(x + \frac{\pi}{6}\right).$$

(ii) The curve is a cosine of amplitude 2 shifted left by $\dfrac{\pi}{6}$. Stationary points occur where $x + \dfrac{\pi}{6} = \pi, 2\pi$, i.e. a minimum at $\left(\dfrac{5\pi}{6}, -2\right)$ and a maximum at $\left(\dfrac{11\pi}{6}, 2\right)$. The end-points are $f(0) = 2\cos\dfrac{\pi}{6} = \sqrt{3}$ and $f(2\pi) = \sqrt{3}$, giving $\left(0, \sqrt{3}\right)$ and $\left(2\pi, \sqrt{3}\right)$. Sketch the smooth cosine curve through these points.

Q7 2024 Manly Trial Q14(e) · **4 marks**

The graph of $y = \sqrt{3}(\cos 2x + \sin 2x) - (\sin 2x - \cos 2x)$ is shown.



(i) Find the coordinates of A and B as labelled on the graph, given A is the maximum point on the graph and B is where the graph intersects the x-axis, without using calculus.

(ii) Hence or otherwise, solve

$$\sqrt{3}(\cos 2x + \sin 2x) - (\sin 2x - \cos 2x) = 2\sqrt{2} \quad \text{for } 0 \le x \le 2\pi.$$

Criteria	**Marks**
(i) Collects terms as $(\sqrt{3}+1)\cos 2x + (\sqrt{3}-1)\sin 2x$ and writes it as $2\sqrt{2}\cos\left(2x - \frac{\pi}{12}\right)$	1
(i) States $A = \left(\frac{\pi}{24}, 2\sqrt{2}\right)$ (maximum) and $B = \left(\frac{7\pi}{24}, 0\right)$ (first x-intercept)	1
(ii) Reduces the equation to $\cos\left(2x - \frac{\pi}{12}\right) = 1$	1
(ii) Solves over $0 \le x \le 2\pi$ to give $x = \frac{\pi}{24}, \ \frac{25\pi}{24}$	1

Solution.

(i) Collecting like terms,

$$y = \sqrt{3}\cos 2x + \sqrt{3}\sin 2x - \sin 2x + \cos 2x = (\sqrt{3}+1)\cos 2x + (\sqrt{3}-1)\sin 2x.$$

Writing $y = R\cos(2x - \alpha) = R\cos\alpha\cos 2x + R\sin\alpha\sin 2x$ gives $R\cos\alpha = \sqrt{3}+1$ and $R\sin\alpha = \sqrt{3}-1$, so

$$R^2 = (\sqrt{3}+1)^2 + (\sqrt{3}-1)^2 = 8 \implies R = 2\sqrt{2}, \qquad \tan\alpha = \frac{\sqrt{3}-1}{\sqrt{3}+1} = 2 - \sqrt{3} \implies \alpha = \frac{\pi}{12}.$$

Hence $y = 2\sqrt{2}\cos\left(2x - \dfrac{\pi}{12}\right)$. The maximum value is the amplitude $2\sqrt{2}$, reached when $2x - \dfrac{\pi}{12} = 0$, i.e. $x = \dfrac{\pi}{24}$, so $A = \left(\dfrac{\pi}{24}, 2\sqrt{2}\right)$. The first x-intercept after A occurs when $\cos\left(2x - \dfrac{\pi}{12}\right) = 0$, i.e. $2x - \dfrac{\pi}{12} = \dfrac{\pi}{2}$,

giving $x = \dfrac{7\pi}{24}$, so $B = \left(\dfrac{7\pi}{24}, 0\right)$.

(ii) The equation becomes $2\sqrt{2}\cos\left(2x - \dfrac{\pi}{12}\right) = 2\sqrt{2}$, so $\cos\left(2x - \dfrac{\pi}{12}\right) = 1$. For $0 \leq x \leq 2\pi$, $2x - \dfrac{\pi}{12}$ ranges over $\left[-\dfrac{\pi}{12}, \dfrac{47\pi}{12}\right]$, so $2x - \dfrac{\pi}{12} = 0,\ 2\pi$, giving

$$x = \frac{\pi}{24}, \qquad x = \frac{25\pi}{24}.$$

Q8 2021 Hurlstone Trial Q14(e) · 5 marks

(i) Prove that

$$\tan(A+B+C)=\frac{\tan A+\tan B+\tan C-\tan A\tan B\tan C}{1-\tan A\tan B-\tan B\tan C-\tan C\tan A}.$$

(ii) Given A, B and C are acute angles of a triangle and

$$\frac{\tan A}{5}=\frac{\tan B}{6}=\frac{\tan C}{7}=k,$$

hence or otherwise show that $k=\sqrt{\dfrac{3}{35}}$ is the only solution to $\tan(A+B+C)=0$.

Criteria	Marks
(i) Applies $\tan(A+B+C)=\tan\big((A+B)+C\big)=\dfrac{\tan(A+B)+\tan C}{1-\tan(A+B)\tan C}$	1
(i) Substitutes $\tan(A+B)=\dfrac{\tan A+\tan B}{1-\tan A\tan B}$ and simplifies to the stated fraction	1
(ii) Sets $\tan A=5k$, $\tan B=6k$, $\tan C=7k$ and uses $\tan(A+B+C)=0$ so the numerator is 0	1
(ii) Reduces to $18k=210k^3$, i.e. $6k(35k^2-3)=0$	1
(ii) Rejects $k=0$ and the negative root, so $k=\sqrt{\frac{3}{35}}$ is the only valid solution	1

Solution.

(i) Group as $(A+B)+C$:

$$\tan(A+B+C)=\frac{\tan(A+B)+\tan C}{1-\tan(A+B)\tan C}.$$

Substituting $\tan(A+B)=\dfrac{\tan A+\tan B}{1-\tan A\tan B}$ and multiplying numerator and denominator by $1-\tan A\tan B$,

$$\tan(A+B+C)=\frac{(\tan A+\tan B)+\tan C(1-\tan A\tan B)}{(1-\tan A\tan B)-(\tan A+\tan B)\tan C}=\frac{\tan A+\tan B+\tan C-\tan A\tan B\tan C}{1-\tan A\tan B-\tan B\tan C-\tan C\tan A}.$$

(ii) Since A, B, C are the angles of a triangle, $A+B+C=\pi$, so $\tan(A+B+C)=\tan\pi=0$. From part (i), a fraction is zero only when its numerator is zero:

$$\tan A+\tan B+\tan C-\tan A\tan B\tan C=0.$$

With $\tan A=5k$, $\tan B=6k$, $\tan C=7k$,

$$5k+6k+7k=(5k)(6k)(7k)\implies 18k=210k^3\implies 6k(35k^2-3)=0.$$

So $k=0$ or $k^2=\dfrac{3}{35}$. As A, B, C are acute, $\tan A,\tan B,\tan C>0$, so $k>0$; this rules out $k=0$ and the negative root. Hence

$$k=\sqrt{\frac{3}{35}}$$

is the only solution.