Year 11 Mathematics Extension 1

Topic: Further Trigonometry (3 of 4)

Review Questions: Further Trigonometric Equations

Term 3, 2026

How to use this review set

These are real past-paper questions, grouped by question pattern. The grey label on each question shows where it is from and its marks.

- The questions are grouped by the pattern that keeps coming up in exams, a few of each in a row, running from the plainer patterns into the harder ones near the end. Work through them and you will meet every type an exam can throw at you.
- This set **is your diagnostic**. Working through it shows you exactly which patterns are solid and which need work. Attempt every question properly in your own book, and go back to your notes to help you work through tough questions if needed.
- Every question you get wrong, need help with or struggle with goes in your **Mistakes Log**, which you should be working on daily to get you exam-ready.

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4. Counting and interpreting solutions

Solving trigonometric equations

1. Solving equations reducible to a quadratic in one ratio

Q1 2024 Caringbah Trial Q8 · **1 mark**

What is the solution of the trigonometric equation $3\sin^2 x - 4\cos x + 1 = 0$ in the domain $0 \leq x \leq \pi$?

(A) $x = \dfrac{2\pi}{3}$

(B) $x = \dfrac{2\pi}{5}$

(C) $x \approx 0.841$

(D) $x \approx 1.969$

Q2 2025 Merewether Trial Q11f · **2 marks**

Solve $\cos 2\theta = \cos^2\theta + \cos\theta + 1$ for $0 \leq \theta \leq 2\pi$.

Q3 2020 Gosford Prelim Q11(c) · **3 marks**

Solve the equation $\sin^2\theta = \cos\theta + 1$ where $0 \leq \theta \leq 2\pi$.

Q4 2023 Sydney Grammar Prelim Q12(a) · **3 marks**

Solve $2\cos^2 x - 3\sin x = 0$, for $0 \leq x \leq 2\pi$. Give your answers in exact form.

Q5 2023 Hurlstone Prelim Q13(c) · **3 marks**

Solve, correct to the nearest degree: $\sin\theta + 6\cos^2\theta = 5$ where $0° \leq \theta \leq 360°$.

Q6 2024 Gosford Trial Q14a · **3 marks**

Solve $\cos 2x + \sin x - 1 = 0$ for $0 \leq x \leq 2\pi$.

Q7 2020 Sydney Tech Prelim Q13(b) · **3 marks**

Solve the following for $0 \leq \theta \leq 2\pi$: $\tan^2\theta = 2\sec^2\theta - 3$.

Q8 2021 HSC Q11(g) · **3 marks**

By factorising, or otherwise, solve $2\sin^3 x + 2\sin^2 x - \sin x - 1 = 0$ for $0 \leq x \leq 2\pi$.

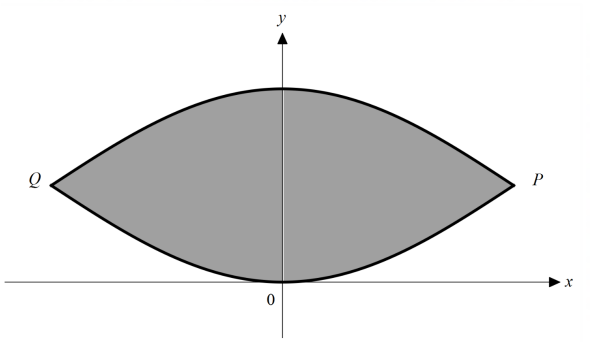
Q9 2022 Hurlstone Prelim Q12(d) · **3 marks**

Solve $3\cos^2\theta - \sin^2\theta - 6\cos\theta = 3$, for $0° \leq \theta \leq 360°$.

2. Solving by factorisation and identity substitution

Q10 2020 Abbotsleigh Trial Q13(d)(i) · **1 mark**

A design uses the shaded area enclosed between the curves $y = \cos^2\frac{x}{3}$ and $y = \sin^2\frac{x}{3}$. Find the x-coordinates of the points P and Q where the two curves first meet for $x > 0$.



Q11 2023 Hurlstone Prelim Q6 · **1 mark**

The t-results are to be used to solve the equation $5\sin x - 2\cos x = 2$, for values of x such that $0° \leq x \leq 360°$. Which value of x needs to be considered separately because it will not be "captured" using this method?

(A) $x = 90°$

(B) $x = 180°$

(C) $x = 0°$

(D) $x = 270°$

Q12 2020 Fort St Trial Q14(a) · **3 marks**

Use the t-formula to solve the equation $\cos x - \sin x = 1$, where $0 \leq x \leq 2\pi$.

Q13 2022 Hunters Hill Trial Q12(a) · **3 marks**

Find all values of θ, such that $-\pi \leq \theta \leq \pi$, that satisfy the equation

$$3\sin\theta = \sqrt{3}\sin 2\theta.$$

Q14 2023 Manly Trial Q13(c) · 3 marks

Use sums to products of trigonometric ratios to solve the equation

$$\cos x + \cos 2x + \cos 3x = 0 \qquad \text{where } 0 \leq x \leq 2\pi.$$

Q15 2021 Riverview Trial Q1(b) · 3 marks

Solve the equation $\sin 2x = \tan x$ for the domain $[0, \pi)$.

Q16 2023 Ascham Trial Q13(a) · 4 marks

(i) Show that $\tan 2x + \tan x = \dfrac{\sin 3x}{\cos 2x \cos x}$.

(ii) Hence solve the equation $\tan 2x + \tan x = 0$ for $0 \leq x \leq \dfrac{\pi}{2}$.

Q17 2024 Hunters Hill Trial Q12(d) · 4 marks

Solve $\sin\left(x + \dfrac{\pi}{3}\right)\cos\left(x - \dfrac{\pi}{3}\right) = \dfrac{1+\sqrt{3}}{4}$, for $0 \leq x \leq 2\pi$.

Q18 2021 Ascham Trial Q14(c) · 4 marks

(i) Show that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$.

(ii) Let $x = \cos\theta$. Find three distinct roots of $8x^3 - 6x - 1 = 0$.

Q19 2023 North Sydney Boys Prelim Q4 · 5 marks

(i) Use the substitution $t = \tan\dfrac{x}{2}$ to show that

$$2\sin x - 3\cos x = \frac{3t^2 + 4t - 3}{1 + t^2}.$$

(ii) Hence solve the equation $2\sin x - 3\cos x = 2$ for $0° \leq x \leq 360°$. Give your answer correct to the nearest degree.

Q20 2024 James Ruse Prelim Q9(a) · 7 marks

(i) Using compound angle results, show that $\sin 3x = 3\sin x - 4\sin^3 x$.

(ii) Consider the equation $\sin 2x = \sin 3x$. You are given that $x = \dfrac{\pi}{5}$ is a solution to this equation. Show that $x = \dfrac{3\pi}{5}$ is another solution to this equation.

(iii) Hence, find the exact value of $\cos\dfrac{3\pi}{5}$.

The auxiliary-angle method

3. Expressing and solving $a\cos x + b\sin x$

Q21 2024 Sydney Girls Trial Q3 · **1 mark**

Which of the following is equivalent to $\sin x - \sqrt{3}\cos x$?

(A) $2\sin\left(x - \frac{\pi}{3}\right)$

(B) $2\sin\left(x + \frac{\pi}{3}\right)$

(C) $2\sin\left(x - \frac{\pi}{6}\right)$

(D) $2\sin\left(x + \frac{\pi}{6}\right)$

Q22 2021 Hurlstone Trial Q5 · **1 mark**

Which of the following is an expression for $y = \sin x + \sqrt{3}\cos x$ in the form $R\sin(x + \alpha)$?

(A) $2\sin\left(x + \frac{\pi}{4}\right)$

(B) $2\sin\left(x + \frac{\pi}{3}\right)$

(C) $2\sin\left(x - \frac{\pi}{4}\right)$

(D) $2\sin\left(x - \frac{\pi}{3}\right)$

Q23 2025 Smiths Hill Trial Q1 · **1 mark**

Which of the following expressions is equivalent to $\cos x + \sin x$?

(A) $\sqrt{2}\cos\left(x + \frac{\pi}{4}\right)$

(B) $\sqrt{2}\cos\left(x - \frac{\pi}{4}\right)$

(C) $\sqrt{2}\sin\left(x - \frac{\pi}{4}\right)$

(D) $\sqrt{2}\sin\left(x + \frac{\pi}{4}\right)$

Q24 2020 Normanhurst Boys Trial Q3 · **1 mark**

What is the smallest positive value of x for which $\sin x + \cos x$ is at its maximum?

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{4}$

(C) π

(D) $\frac{\pi}{3}$

Q25 2023 St George Girls Trial Q11d · **2 marks**

Express $9\sin x + 40\cos x$ in the form $R\sin(x+\alpha)$ where $0 \leq \alpha \leq \frac{\pi}{2}$. Leave α correct to 3 significant figures.

Q26 2021 Sydney Girls Trial Q13a · **2 marks**

A particle is moving in a straight line and is oscillating backwards and forwards. At time t seconds, it has displacement x metres from a fixed point O on the line, where $x = A\cos\left(\frac{\pi}{4}t + \alpha\right)$, $A > 0$, and $0 < \alpha < \frac{\pi}{2}$. After 1 second the particle is 2 metres to the right of O, and after 3 seconds it is 4 metres to the left of O.

Given that $A\cos\alpha - A\sin\alpha = 2\sqrt{2}$ and $A\cos\alpha + A\sin\alpha = 4\sqrt{2}$, solve the equations simultaneously to find the values of A and α in exact form.

Q27 2022 Trinity Grammar Trial Q14(b)(iii) · **2 marks**

An object is fired from a point O at an angle of elevation θ to the horizontal. In an earlier part it was shown that θ satisfies

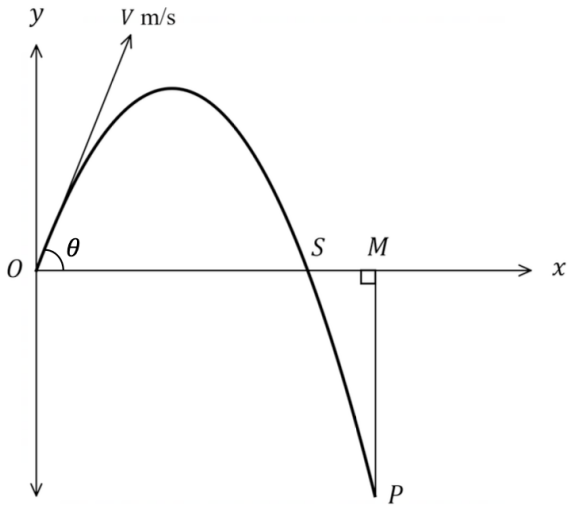


diagram not to scale

$$\sin 2\theta - 3\cos 2\theta = 3.$$

Given that θ is an angle of elevation, find the value of θ.

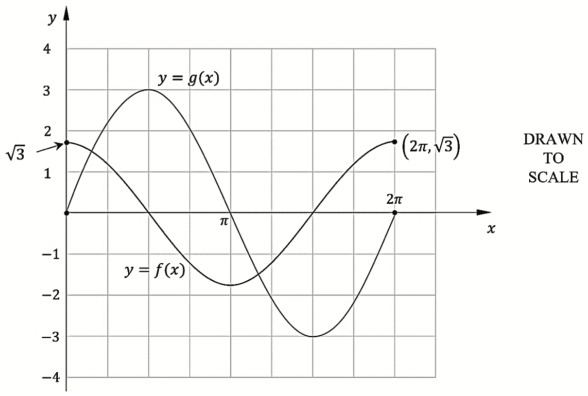
Q28 2025 Gosford Trial Q11a · **3 marks**

(i) Express $2\cos x - \sin x$ in the form $R\cos(x+\alpha)$, where $R > 0$ and $0° < \alpha < 360°$.

(ii) Hence find the range of the function $g(x) = (2\cos x - \sin x)^2$.

Q29 2025 St George Girls Trial Q12a · **3 marks**

The graphs of $y = f(x)$ and $y = g(x)$ are shown below, where $f(x)$ is a cosine function and $g(x)$ is a sine function. It is given that $f(x) + g(x) = R\cos(x - \theta)$, where $0 \leq \theta \leq \dfrac{\pi}{2}$. Find R and θ, where θ is in radians.



Q30 2025 Hills Grammar Trial Q12(a) · **3 marks**

It is given that $3\cos x + b\sin x = 5\cos(x + \theta)$, where $b > 0$ and $-90° < \theta < 90°$. Find the values of b and θ, giving your answer for θ correct to the nearest minute.

Q31 2020 North Sydney Boys Trial Q13a · **4 marks**

(i) Express $\cos\theta - \sqrt{3}\sin\theta$ in the form $R\sin(\theta + \alpha)$ where $R > 0$.

(ii) Hence, solve $\cos\theta - \sqrt{3}\sin\theta = 1$ for $0 \leq \theta \leq 2\pi$.

Q32 2022 Baulkham Hills Trial Q11b · **4 marks**

By expressing $2\cos x + 9\sin x$ in the form $A\cos(x - \theta)$, solve

$$2\cos x + 9\sin x = 1, \qquad 0 \leq x \leq 2\pi.$$

Give your answer correct to two decimal places.

Q33 2020 Gosford Trial Q11b · **4 marks**

(i) Express $6\cos\theta + 8\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 \leq \alpha < 2\pi$. Give α correct to 1 decimal place.

(ii) Hence calculate the maximum value of $\dfrac{4}{12 + 6\cos\theta + 8\sin\theta}$.

Q34 2021 North Sydney Boys Trial Q12b · **4 marks**

(i) Express $\sin x + \sqrt{3}\cos x$ in the form $R\sin(x+\alpha)$ where $R > 0$ and $0 \leq \alpha \leq 2\pi$.

(ii) Hence solve the inequality $\sin x + \sqrt{3}\cos x \geq 1$ in the domain $[0, 2\pi]$.

Q35 2024 Trinity Grammar Trial Q11b · **4 marks**

(i) Rewrite $\cos x - \sqrt{3}\sin x$ in the form $R\cos(x+\alpha)$, where $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$.

(ii) Hence, or otherwise, solve $\cos x - \sqrt{3}\sin x = 1$ for $0 \leq x \leq 2\pi$.

Q36 2020 Abbotsleigh Trial Q13c · **5 marks**

A machine produces a sound which can be modelled using the equation $y = \cos t - \sqrt{3}\sin t$, where y gives a measure of the sound at time t seconds after the machine is switched on. The sound is audible to humans when $y \geq 1$.

(i) Express $\cos t - \sqrt{3}\sin t$ in the form $R\cos(t+\alpha)$, where $R > 0$ and $0 \leq \alpha < \dfrac{\pi}{2}$.

(ii) Hence find the time intervals during which the noise is audible to humans for $0 < t \leq 4\pi$.

Q37 2020 Girraween Trial Q15a · **6 marks**

(i) Show that $2\sin 2x - 3\cos 2x - 3\sin x + 3 = \sin x\,(6\sin x + 4\cos x - 3)$.

(ii) Express $6\sin x + 4\cos x$ in the form $R\sin(x+\alpha)$ where $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$.

(iii) Hence, solve $2\sin 2x - 3\cos 2x - 3\sin x + 3 = 0$ for $0 \leq x < \pi$. Answer in radians, correct to 2 decimal places.

Counting and interpreting solutions

4. Counting and interpreting solutions

Q38 2020 Sydney Tech Prelim Q1 · 1 mark

How many solutions does $-3 = 3\sin x$ have when $0° < x < 360°$?

(A) 0

(B) 1

(C) 2

(D) 3

Q39 2022 Glenwood Trial Q4 · 1 mark

How many solutions does the equation $\sin 6x - \sin 2x = 0$ have for $0 \leq x \leq 2\pi$?

(A) 5

(B) 12

(C) 14

(D) 13

Q40 2025 HSC Q5 · 1 mark

How many distinct solutions are there to the equation $\cos 5x + \sin x = 0$ for $0 \leq x \leq 2\pi$?

(A) 5

(B) 6

(C) 9

(D) 10

Q41 2024 Hurlstone Prelim Q5 · **1 mark**

How many solutions are there to the equation $\tan 2x = 3$ when $-\pi \leq x \leq \pi$?

(A) 2

(B) 4

(C) 6

(D) 8

Q42 2020 Manly Trial Q8 · **1 mark**

The number of solutions to the equation $(\sin^2 x - 1)(\tan^2 x - 1) = 0$ in the domain $[0, 2\pi]$ is

(A) 2

(B) 4

(C) 6

(D) 8

Q43 2022 North Sydney Boys Trial Q7 · **1 mark**

In the range $0 \leq x \leq 2\pi$, the equation $2\sin^2 x + 2\cos^2 x = 2$

(A) has no solutions

(B) has 1 solution

(C) has 2 solutions

(D) holds for all values of x