Year 11 Mathematics Extension 1

Topic: Further Trigonometry (2 of 4)
Review Questions: Further Trigonometric Identities
Term 3, 2026

How to use this review set

These are real past-paper questions, grouped by question pattern. The grey label on each question shows where it is from and its marks.

- The questions are grouped by the pattern that keeps coming up in exams, a few of each in a row, running from the plainer patterns into the harder ones near the end. Work through them and you will meet every type an exam can throw at you.
- This set **is your diagnostic**. Working through it shows you exactly which patterns are solid and which need work. Attempt every question properly in your own book, and go back to your notes to help you work through tough questions if needed.
- Every question you get wrong, need help with or struggle with goes in your **Mistakes Log**, which you should be working on daily to get you exam-ready.

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Expanding with the compound and double angle formulae

1. Compound-angle (sum and difference) identities

Q1 2020 Abbotsleigh Trial Q6 · 1 mark

Both A and B are acute angles such that $\tan A = \frac{3}{5}$ and $\tan B = 4$. What is the size of angle $(A - B)$?

(A) $-135°$

(B) $45°$

(C) $135°$

(D) $-45°$

Q2 2021 Caringbah Trial Q4 · 1 mark

Given that α and β are both acute angles, evaluate $\sin(\alpha + \beta)$ if $\sin\alpha = \frac{8}{17}$ and $\sin\beta = \frac{4}{5}$.

(A) $\frac{108}{85}$

(B) $\frac{84}{85}$

(C) $\frac{36}{85}$

(D) $\frac{28}{85}$

Q3 2020 Hornsby Girls Prelim Q1 · 1 mark

$\sin(A + 3B) =$

(A) $\sin A \cos 3B - \cos A \sin 3B$

(B) $\sin 3B \cos A + \cos A \sin 3B$

(C) $\sin A \cos 3B + \cos A \sin 3B$

(D) $\sin A \cos 3B + \cos 3B \sin A$

Q4 2025 Normanhurst Boys Trial Q2 · **1 mark**

Given $\tan\theta = \dfrac{1}{2}$, what is the exact value of $\tan\left(\theta - \dfrac{\pi}{6}\right)$?

(A) $8 + 5\sqrt{3}$

(B) $8 - 5\sqrt{3}$

(C) $\dfrac{8 + 5\sqrt{3}}{11}$

(D) $\dfrac{8 - 5\sqrt{3}}{11}$

Q5 2023 SCEGGS Prelim Q6 · **1 mark**

Given $\tan\theta = A$, what is the correct expression for $\tan\left(\dfrac{\pi}{4} - \theta\right)$?

(A) A

(B) $\dfrac{1}{A}$

(C) $1 - A$

(D) $\dfrac{1 - A}{1 + A}$

Q6 2025 Baulkham Hills Prelim Q8 · **1 mark**

In triangle ABC, if $\cos A = \dfrac{1}{3}$, then which of the following equals $\cot\dfrac{A}{2}$?

(A) $\sqrt{2}$

(B) $-\sqrt{2}$

(C) $\dfrac{1}{2}$

(D) $\dfrac{1}{\sqrt{2}}$

Q7 2020 Sydney Girls Trial Q8 · **1 mark**

Given that $\sin\left(x-\frac{\pi}{3}\right)=3\cos\left(x-\frac{\pi}{4}\right)$, find the exact value of $\tan x$.

(A) $\dfrac{\sqrt{6}+6}{6-\sqrt{2}}$

(B) $\dfrac{\sqrt{6}+6}{\sqrt{2}-6}$

(C) $\dfrac{\sqrt{2}-6}{\sqrt{6}+6}$

(D) $\dfrac{6-\sqrt{2}}{\sqrt{6}+6}$

Q8 2023 North Sydney Boys Trial Q12c · **2 marks**

By equating coefficients, find the values of P and Q in the identity

$$P(2\sin x+\cos x)+Q(2\cos x-\sin x)\equiv 7\sin x+11\cos x.$$

Q9 2020 Caringbah Prelim Q15d · **2 marks**

If $\sec x+\tan x=k$, find an expression for $\sec x$ in terms of k.

Q10 2021 Moriah Prelim Q6c · **3 marks**

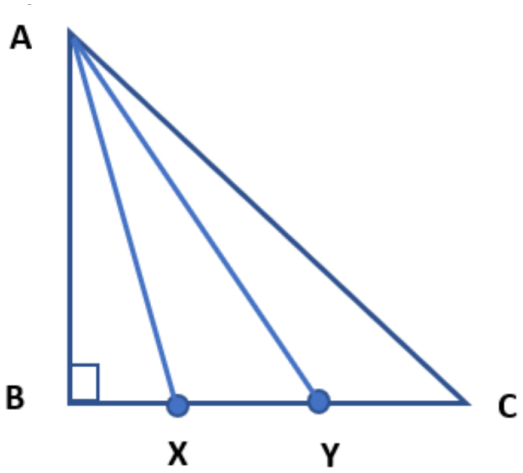
You are given that $\cos\theta=\frac{1}{2}$ and $\cos\beta=\frac{3}{5}$, and that θ and β are both acute angles.

(i) Find the exact value of $\cos(\theta+\beta)$.

(ii) If neither θ nor β is an acute angle, then the answer in part (i) is unchanged. Briefly explain why this is so.

Q11 2023 Glenwood Prelim Q9e · **4 marks**

In the isosceles triangle below, $AB = BC$, $\angle ABC = 90°$ and points X and Y are placed on BC such that $BX = XY = YC = 1$.



Let $\angle BAX = \alpha$, $\angle XAY = \beta$ and $\angle YAC = \gamma$. Find the exact values of $\tan\alpha$, $\tan\beta$ and $\tan\gamma$.

2. Double-angle formulae

Q12 2020 Sydney Grammar Trial Q5 · **1 mark**

Which expression is equivalent to $1 - \cos 2x$?

(A) $\sin^2 x - \cos^2 x$

(B) $2(\sin^2 x + 1)$

(C) $2\sin^2 x$

(D) $2(1 + \cos^2 x)$

Q13 2020 Cheltenham Girls Trial Q1 · 1 mark

What is the value of $\sin 2x$ given that $\sin x = \dfrac{2\sqrt{3}}{4}$ and x is obtuse?

(A) $\dfrac{\sqrt{3}}{4}$

(B) $\dfrac{\sqrt{3}}{2}$

(C) $-\dfrac{\sqrt{3}}{4}$

(D) $-\dfrac{\sqrt{3}}{2}$

Q14 2021 Glenwood Trial Q8 · 1 mark

It is known that $\sin x = \dfrac{1}{3}$, where $\dfrac{\pi}{2} < x < \pi$. What is the value of $\cos 2x$?

(A) $-\dfrac{2}{9}$

(B) $\dfrac{7}{9}$

(C) $\dfrac{2}{3}$

(D) $\dfrac{4\sqrt{2}}{3}$

Q15 2023 Hurlstone Prelim Q4 · 1 mark

It is given that $\cos\theta = \dfrac{3}{5}$ for $\dfrac{3\pi}{2} \leq \theta \leq 2\pi$. What is the value of $\sin 2\theta$ equivalent to?

(A) $-\dfrac{24}{25}$

(B) $\dfrac{24}{25}$

(C) $-\dfrac{4}{5}$

(D) $\dfrac{4}{5}$

Q16 2022 Moriah Trial Q2 · 1 mark

Which of the following is equivalent to $\dfrac{2\tan\theta}{1+\tan^2\theta}$?

(A) $\cos 2\theta$

(B) $\sin 2\theta$

(C) $\tan 2\theta$

(D) $\cot 2\theta$

Q17 2020 Sydney Girls Trial Q2 · 1 mark

What is the value of $\sin 2\theta$ given that $\sin\theta = \dfrac{\sqrt{3}}{4}$ and θ is obtuse?

(A) $-\dfrac{\sqrt{39}}{8}$

(B) $-\dfrac{\sqrt{3}}{2}$

(C) $\dfrac{\sqrt{3}}{2}$

(D) $\dfrac{\sqrt{39}}{8}$

Q18 2021 Kings Trial Q11(c) · 2 marks

Given $\cos x = -\dfrac{1}{3}$ and $\sin x > 0$, find the exact value of $\sin 2x$.

Q19 2022 Moriah Trial Q11(d) · 3 marks

You are given that $\sin\theta = \dfrac{4}{5}$ where $90^\circ < \theta < 180^\circ$.

(i) Find the exact value of $\tan\theta$.

(ii) Find the exact value of $\tan 2\theta$.

Q20 2022 Shore Trial Q12(b) · 3 marks

Given that $\sin\theta = \dfrac{3}{4}$ and $\dfrac{\pi}{2} \le \theta \le \pi$, determine the exact value of $\tan 2\theta$.

3. Exact values of special angles

Q21 2020 Glenwood Prelim Q3 · 1 mark

What is the exact value of $\cos 15°$?

(A) $\dfrac{\sqrt{6}}{2}$

(B) $\dfrac{\sqrt{3}}{2}$

(C) $\dfrac{\sqrt{6}-\sqrt{2}}{4}$

(D) $\dfrac{\sqrt{6}+\sqrt{2}}{4}$

Q22 2024 Baulkham Hills Prelim Q11a · 2 marks

Use a compound-angle formula to find the exact value of $\cos 75°$.

Q23 2023 Riverview Trial Q11b(i) · 2 marks

Find the exact value of $\sin \dfrac{23\pi}{12}$. Express your answer as a single fraction with a rational denominator.

Q24 2023 Fort St Trial Q11f · 3 marks

Find the exact value of $\cos \dfrac{\pi}{8}$, giving your answer in simplest form.

Q25 2021 Presbyterian Prelim Q7e · 3 marks

(i) Write $\tan 2A$ in terms of $\tan A$.

(ii) Hence find the exact value of $\tan 67\frac{1}{2}°$.

Q26 2022 Ascham Trial Q13e · 4 marks

(i) Show that $\dfrac{1-\cos 2x}{\sin 2x} + \dfrac{\sin 2x}{1+\cos 2x} = 2\tan x$.

(ii) Hence, show that $\tan \dfrac{\pi}{12} = 2-\sqrt{3}$.

Q27 2025 Caringbah Trial Q14b · **4 marks**

It is known that $\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$ (DO NOT PROVE THIS).

(i) Find the exact values of the solutions to the equation $8x^4 - 8x^2 + 1 = 0$, for $0 < \theta < \pi$.

(ii) Hence, or otherwise, find the exact value of $\cos\dfrac{\pi}{8}\cos\dfrac{3\pi}{8}$.

Products, sums and simplifying

4. Products to sums and sums to products

Q28 2024 Blacktown Boys Trial Q13a · **1 mark**

By expanding the left-hand side, show that

$$\sin(8x + 5x) + \sin(8x - 5x) = 2\sin 8x \cos 5x.$$

Q29 2020 Hunters Hill Trial Q6 · **1 mark**

Which of the following expressions is equivalent to $2\sin 5\theta \cos 3\theta$?

(A) $\sin 8\theta + \sin 2\theta$

(B) $\cos 8\theta + \cos 2\theta$

(C) $\sin 8\theta - \sin 2\theta$

(D) $\cos 8\theta - \cos 2\theta$

Q30 2024 Newington Trial Q2 · **1 mark**

Which of the following is equivalent to $2\cos 5x \sin x$?

(A) $\cos 6x + \sin 4x$

(B) $\cos 6x - \sin 4x$

(C) $\sin 6x + \sin 4x$

(D) $\sin 6x - \sin 4x$

Q31 2024 Sydney Boys Trial Q3 · 1 mark

Which of the following is equal to $\cos\alpha - \cos\beta$?

(A) $2\sin\left(\dfrac{\alpha+\beta}{2}\right)\cos\left(\dfrac{\alpha-\beta}{2}\right)$

(B) $2\sin\left(\dfrac{\alpha+\beta}{2}\right)\sin\left(\dfrac{\alpha-\beta}{2}\right)$

(C) $2\cos\left(\dfrac{\alpha+\beta}{2}\right)\cos\left(\dfrac{\alpha-\beta}{2}\right)$

(D) $-2\sin\left(\dfrac{\alpha+\beta}{2}\right)\sin\left(\dfrac{\alpha-\beta}{2}\right)$

Q32 2023 Gosford Trial Q10 · 1 mark

Which of the following is equal to $\cos 3\alpha \sin 2\alpha$?

(A) $\dfrac{1}{2}(\sin 5\alpha + \sin\alpha)$

(B) $\dfrac{1}{2}(\sin 5\alpha - \sin\alpha)$

(C) $\dfrac{1}{2}\left(\sin\dfrac{5\alpha}{2} + \sin\dfrac{\alpha}{2}\right)$

(D) $\dfrac{1}{2}\left(\sin\dfrac{5\alpha}{2} - \sin\dfrac{\alpha}{2}\right)$

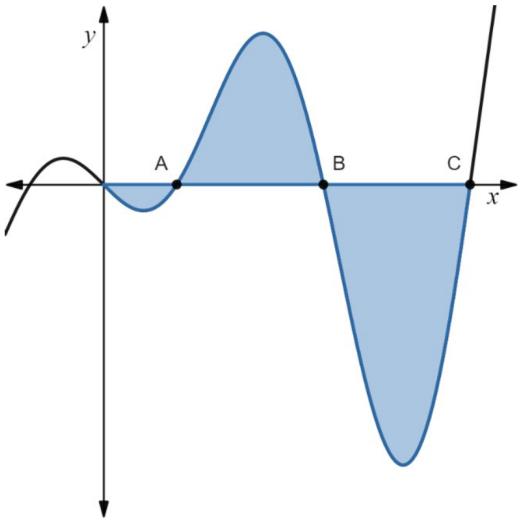
Q33 2020 Hornsby Girls Trial Q12(b)(i) · 2 marks

Use the identity $\sin A \sin B = \dfrac{1}{2}\big[\cos(A-B) - \cos(A+B)\big]$ to show that

$$\sin^2\left(\frac{x}{8}\right) = \frac{1}{2}\left(1 - \cos\frac{x}{4}\right).$$

Q34 2022 Sydney Girls Trial Q14c(i) · **2 marks**

The diagram represents the graph of the function $y = \sin 5x - \sin 6x$.



Determine the exact coordinates of the point C.

Q35 2022 Baulkham Hills Trial Q14(d) · **4 marks**

(i) Show that $2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = \cos x + \cos y$.

(ii) Hence find all solutions to

$$\cos 4\theta + \cos 3\theta + \cos 2\theta + \cos\theta = 0 \qquad \text{for } 0 \leq \theta \leq 2\pi.$$

5. Simplifying trigonometric expressions

Q36 2024 Baulkham Hills Prelim Q5 · **1 mark**

Which of the following is equivalent to $\dfrac{1-\cos 2x}{\sin 2x}$?

(A) $1 - \cot 2x$

(B) 1

(C) $\cot x$

(D) $\tan x$

Q37 2023 Hurlstone Prelim Q3 · 1 mark

Which of the following is equivalent to $\tan\left(\frac{\pi}{4}+x\right)$?

(A) $\dfrac{\sec^2 x}{1-\tan^2 x}$

(B) $\dfrac{\cos x-\sin x}{\cos x+\sin x}$

(C) $\dfrac{\sin x+\cos x}{\sin x-\cos x}$

(D) $\dfrac{\cos x+\sin x}{\cos x-\sin x}$

Q38 2025 North Sydney Boys Trial Q2 · 1 mark

Which of the following is equivalent to the expression $\sqrt{\dfrac{4+4\cos 2x}{1-\cos 2x}}$?

(A) $2\cot^2 x$

(B) $2\cot x$

(C) $2\tan x$

(D) $2\tan^2 x$

Q39 2020 St George Girls Trial Q8 · 1 mark

Which expression is equivalent to $\cos 5x\cos 2x-\sin 6x\sin 3x$?

(A) $\cos 7x-\sin 9x$

(B) $\cos 3x-\sin 3x$

(C) $\sin 8x\sin x$

(D) $\cos 8x\cos x$

Q40 2021 Scots Trial Q1 · 2 marks

(a) Which of the following is equivalent to $\cos(2x - y)\cos x + \sin(2x - y)\sin x$?

(A) $\cos(x - y)$

(B) $\cos(3x - y)$

(C) $\sin(x - y)$

(D) $\sin(3x - y)$

(b) Which of the following is an equivalent expression for $\dfrac{\tan(x - y) + \tan y}{1 - \tan(x - y)\tan y}$?

(A) $\tan x$

(B) $\tan(x - y)$

(C) $\tan(x - 2y)$

(D) $\dfrac{2\tan x - \tan y}{1 - \tan x \tan y + \tan^2 y}$

Q41 2021 Sydney Grammar Prelim Q5(a) · 3 marks

(i) Expand and simplify $(\sin\theta + \cos\theta)^2$.

(ii) Hence simplify $\dfrac{\cos\theta - \tan\theta\sin\theta}{1 + 2\sin\theta\cos\theta} + \dfrac{\tan\theta}{\sin\theta + \cos\theta}$.

The t-formula and proving identities

6. The t-formula (half-angle) substitution

Q42 2022 Blacktown Boys Prelim Q2 · **1 mark**

Which of the following gives the correct expression for $3\sin\theta - 2\cos\theta$ if $t = \tan\dfrac{\theta}{2}$?

(A) $\dfrac{6t - 2 - 2t^2}{1 + t^2}$

(B) $\dfrac{6t - 2 + 2t^2}{1 + t^2}$

(C) $\dfrac{6t - 2 - 2t^2}{1 - t^2}$

(D) $\dfrac{6t + 2 - 2t^2}{1 + t^2}$

Q43 2021 Knox Prelim Q3 · **1 mark**

Given that $t = \tan\dfrac{x}{2}$, which of the following is an expression for $1 + \sec x$?

(A) $\dfrac{2t}{1 + t^2}$

(B) $\dfrac{2t}{1 - t^2}$

(C) $\dfrac{2}{1 + t^2}$

(D) $\dfrac{2}{1 - t^2}$

Q44 2024 Girraween Prelim Q9(c) · **2 marks**

By making the substitution $t = \tan\dfrac{\theta}{2}$, show that

$$\operatorname{cosec}\theta + \cot\theta = \cot\frac{\theta}{2}.$$

Q45 2024 Hurlstone Prelim Q12(b) · **2 marks**

With the use of t-results, prove that

$$\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}=\tan\frac{\theta}{2}.$$

Q46 2022 Baulkham Hills Prelim Q11(e) · **3 marks**

Using $t=\tan\dfrac{x}{2}$, prove that

$$\frac{1+\sin x+\cos x}{1-\sin x+\cos x}=\tan x+\sec x.$$

Q47 2020 Knox Prelim Q13(f) · **3 marks**

By using $t=\tan\dfrac{\theta}{2}$, solve the equation $\sqrt{3}\sin\theta+\cos\theta=1$ for $0\le\theta\le 2\pi$.

Q48 2020 Barker Trial Q12(d) · **5 marks**

(i) If $t=\tan A$, find an expression in terms of t for $\cos 2A$.

(ii) Use t formulas with the substitution $t=\tan x$ to prove that

$$\sec 2x+\tan 2x=\tan(x+45^\circ).$$

(iii) Hence or otherwise solve $\sec 2x+\tan 2x=\dfrac{4}{5}$ for $0^\circ\le x\le 360^\circ$. Answer to the nearest degree.

7. Proving trigonometric identities

Q49 2020 Cherrybrook Tech Trial Q12a · **1 mark**

Show that $\sin(A+B)+\sin(A-B)=2\sin A\cos B$.

Q50 2024 Gosford Trial Q12d · **2 marks**

Show that $\dfrac{\sin 2x}{1+\cos 2x}=\tan x$.

Q51 2024 Caringbah Trial Q14a · **2 marks**

Use trigonometric identities to prove that

$$\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \frac{1}{\cos\left(\frac{\pi}{2}-\theta\right)} - \cot\theta.$$

Q52 2022 Hurlstone Prelim Q12a · **2 marks**

Prove that $\dfrac{1+\cot^2\theta}{2\cot\theta} = \operatorname{cosec} 2\theta$.

Q53 2022 SCEGGS Prelim Q9e · **2 marks**

Show that $\operatorname{cosec} 2\theta - \cot 2\theta = \tan\theta$.

Q54 2024 Hurlstone Prelim Q13d · **2 marks**

Show, without using a calculator, that

$$2\cos 95^\circ \cos 55^\circ - 2\sin 50^\circ \sin 10^\circ = \tfrac{1}{2}\left(1-\sqrt{3}\right).$$

Q55 2020 Hurlstone Trial Q13b · **3 marks**

(i) Show that $\sin(\alpha+\beta) + \sin(\alpha-\beta) = 2\sin\alpha\cos\beta$.

(ii) Hence, if α, β and γ are the angles of $\triangle ABC$ and $\sin\gamma = 2\sin\alpha\cos\beta$, show that $\triangle ABC$ has two equal angles.

Q56 2023 Trinity Grammar Trial Q12e · **3 marks**

Given that $\cos 2x = a + b$ and $\sin 2x = a - b$, show that $a^2 + b^2 = \dfrac{1}{2}$.

Q57 2021 Scots Prelim Q10e · **3 marks**

Given that $(x+y)(x^2 - xy + y^2) = x^3 + y^3$, prove the trigonometric identity

$$\frac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta} + \tfrac{1}{2}\sin 2\theta = 1.$$

Q58 2021 St George Girls Trial Q15b · 4 marks

(i) By using $\cos 4\theta = 2\cos^2 2\theta - 1$, show that

$$8\cos^4\theta - 8\cos^2\theta + 1 - \cos 4\theta = 0.$$

(ii) Using part (i) and by letting $x = \cos\theta$ in the equation $16x^4 - 16x^2 + 2 - \sqrt{2} = 0$, show that $\cos 4\theta = \dfrac{\sqrt{2}}{2}$.

Q59 2021 James Ruse Prelim Q44 · 4 marks

Given that $m = a\cos^3 x + 3a\cos x\sin^2 x$ and $n = a\sin^3 x + 3a\sin x\cos^2 x$, prove that

$$(m+n)^{2/3} + (m-n)^{2/3} = 2a^{2/3}.$$

Q60 2023 Knox Prelim Q14c · 9 marks

(i) Show that $2\sin(k-1)\theta\cos\theta - \sin(k-2)\theta = \sin k\theta$.

(ii) Hence show that $16\sin^5\theta - 20\sin^3\theta + 5\sin\theta = \sin 5\theta$.

(iii) By letting $x = \sin\theta$ in the equation $16x^4 - 20x^2 + 5 = 0$ and by considering the roots of the equation formed, prove that

$$\sin^2\frac{\pi}{5} + \sin^2\frac{2\pi}{5} = \frac{5}{4}.$$

Triple-angle identities

8. Triple-angle identities and cubic-root problems

Q61 2023 Glenwood Trial Q13(b)(i) · 1 mark

Consider the identity $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$. By letting $x = \cos\theta$ in the cubic equation $8x^3 - 6x - 1 = 0$, show that $\cos 3\theta = \dfrac{1}{2}$.

Q62 2023 Manly Trial Q12(c)(i) · 2 marks

Show that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$.

Q63 2025 Pymble Trial Q13(e) · 3 marks

It is known that $\cos 3\theta = 4\cos\theta\,\cos\left(\dfrac{\pi}{3} - \theta\right)\cos\left(\dfrac{\pi}{3} + \theta\right)$. Using this result, or otherwise, solve

$$\sqrt{2}\,\cos\theta\left(\cos^2\theta - 3\sin^2\theta\right) = 1 \quad \text{for } 0 \le \theta \le 2\pi.$$

Q64 2020 Manly Trial Q12(c) · 4 marks

(i) Show that $\cos 3x = 4\cos^3 x - 3\cos x$.

(ii) Find all solutions to the equation $\cos 3x + 2\cos x = 0$ over the domain $[-2\pi, 2\pi]$.

Q65 2023 North Sydney Boys Prelim Q5 · 4 marks

(i) Use the fact that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$ to solve $8x^3 - 6x - 1 = 0$.

(ii) Hence, show that $\cos\dfrac{\pi}{9} = \cos\dfrac{2\pi}{9} + \cos\dfrac{4\pi}{9}$.

Q66 2022 Fort St Trial Q16(b) · 6 marks

(i) Prove the identity $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$.

(ii) Hence, find exact value expressions for three unique solutions of

$$8x^3 - 6x = -\sqrt{3}.$$

Q67 2020 Caringbah Trial Q13(c) · 7 marks

(i) Prove the trigonometric identity

$$\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$$

using the compound angle formula.

(ii) Hence find expressions for the exact values of the solutions to the equation

$$\frac{3x - x^3}{1 - 3x^2} = \sqrt{3}.$$

(iii) Hence show that $\tan\dfrac{\pi}{9}\tan\dfrac{4\pi}{9}\tan\dfrac{7\pi}{9} = -\sqrt{3}$.

Q68 2020 Riverview Trial Q13(c) · 7 marks

(i) Prove the trigonometric identity

$$\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}.$$

(ii) Use the identity from part (i) and let $x = \tan\theta$ to find the roots of the cubic equation $x^3 - 3x^2 - 3x + 1 = 0$, and hence find the exact value of $\tan\dfrac{\pi}{12}$.

Q69 2022 Normanhurst Boys Trial Q13(c) · 8 marks

You are given that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$ and $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$.

(i) Use this to show that

$$\sin^4\theta + \cos^4\theta = \frac{\cos 4\theta + 3}{4}.$$

(ii) By letting $x = \cos\theta$, show that the equation

$$8x^4 + 8\left(1 - x^2\right)^2 = 7$$

has roots $\pm\cos\dfrac{\pi}{12}$ and $\pm\cos\dfrac{5\pi}{12}$.

(iii) Deduce that $\cos\dfrac{\pi}{12}\cos\dfrac{5\pi}{12} = \dfrac{1}{4}$.