Year 11 Mathematics Extension 1

Topic: Further Functions (4 of 5)

Review Questions (Worked Solutions): Parametric Form

Term 3, 2026

Converting between parametric and Cartesian

1. Eliminate the parameter: lines and parabolas

Q1 2023 HSC Q11(a) · **2 marks**

The parametric equations of a line are $x = 1 + 3t$, $y = 4t$. Find the Cartesian equation of this line in the form $y = mx + c$.

Criteria	**Marks**
Eliminates the parameter t	1
Correct Cartesian equation in the form $y = mx + c$	1

Solution.

From $x = 1 + 3t$ we make t the subject: $t = \dfrac{x-1}{3}$. Substituting into $y = 4t$,

$$y = 4 \cdot \frac{x-1}{3} = \frac{4}{3}(x-1) = \frac{4x-4}{3}.$$

So $y = \dfrac{4}{3}x - \dfrac{4}{3}$, which is of the required form $y = mx + c$ with $m = \dfrac{4}{3}$ and $c = -\dfrac{4}{3}$.

Q2 2023 Penrith Prelim Q1 · **1 mark**

Find the Cartesian equation for the curve with the parametric equations $x = 4p - 6$, $y = 3p$.

(A) $y = \frac{1}{12}x + \frac{1}{2}$

(B) $y = \frac{3}{4}x + \frac{9}{2}$

(C) $y = \frac{1}{12}x - \frac{1}{2}$

(D) $y = \frac{3}{4}x + 6$

Criteria	Marks
Correct option (B)	1

Solution.

From $x = 4p - 6$ we solve for p: $p = \frac{x+6}{4}$. Substituting into $y = 3p$,

$$y = 3 \cdot \frac{x+6}{4} = \frac{3}{4}(x+6) = \frac{3}{4}x + \frac{9}{2}.$$

The answer is (B).

Q3 2021 Girraween Prelim Q3(c) · **2 marks**

A function is described by the parametric equations $x = t + 1$, $y = t^2 - 1$. Find the Cartesian equation of this function.

Criteria	Marks
Eliminates the parameter t	1
Correct Cartesian equation	1

Solution.

From $x = t + 1$ we have $t = x - 1$. Substituting into $y = t^2 - 1$,

$$y = (x-1)^2 - 1 = x^2 - 2x + 1 - 1 = x^2 - 2x.$$

So the Cartesian equation is $y = (x-1)^2 - 1$, equivalently $y = x^2 - 2x$.

Q4 2024 Baulkham Hills Prelim Q2 · **1 mark**

A parabola has the parametric equations $x = t - 2$ and $y = 2t^2 - 1$. What is the Cartesian equation of the parabola?

(A) $y = 2x^2 + 4x - 1$

(B) $y = 2x^2 + 4x + 1$

(C) $y = 2x^2 + 8x - 7$

(D) $y = 2x^2 + 8x + 7$

Criteria	Marks
Correct option (D)	1

Solution.

From $x = t - 2$ we have $t = x + 2$. Substituting into $y = 2t^2 - 1$,

$$y = 2(x + 2)^2 - 1 = 2(x^2 + 4x + 4) - 1 = 2x^2 + 8x + 8 - 1 = 2x^2 + 8x + 7.$$

The answer is (D).

Q5 2020 Baulkham Hills Prelim Q11(a) · **2 marks**

The parametric equations of a curve are $x = 4 - t$, $y = (1 - t)^2$. Find the Cartesian equation of the curve, simplifying your answer.

Criteria	Marks
Eliminates the parameter t	1
Correct simplified Cartesian equation	1

Solution.

From $x = 4 - t$ we have $t = 4 - x$. Then

$$1 - t = 1 - (4 - x) = x - 3,$$

so substituting into $y = (1 - t)^2$ gives

$$y = (x - 3)^2.$$

(Expanded, $y = x^2 - 6x + 9$.)

Q6 2024 Sydney Girls Trial Q4 · **1 mark**

The parametric equations of a curve are $x = 1 + t$ and $y = \dfrac{1 - t}{1 + t}$. What is the Cartesian equation of the curve?

(A) $y = \dfrac{2}{x} - 1$

(B) $y = \dfrac{1}{x} - 2$

(C) $y = \dfrac{1}{x} + 2$

(D) $y = \dfrac{2}{x} + 1$

Criteria	Marks
Correct option (A)	1

Solution.

From $x = 1 + t$ we have $t = x - 1$. Substituting into $y = \dfrac{1 - t}{1 + t}$,

$$y = \frac{1 - (x - 1)}{1 + (x - 1)} = \frac{2 - x}{x} = \frac{2}{x} - 1.$$

The answer is (A).

Q7 2025 Baulkham Hills Trial Q12(b) · **2 marks**

The parametric equations of a curve are $x = t^2 + 6t - 16$, $y = 6\ln(t + 3)$. Find the Cartesian equation of this curve in the form $y = a\ln(x + b)$.

Criteria	Marks
Recognises $x + 25 = (t + 3)^2$, so $t + 3 = \sqrt{x + 25}$	1
Correct Cartesian equation $y = 3\ln(x + 25)$	1

Solution.

Complete the square on x to bring out $(t + 3)^2$:

$$x = t^2 + 6t - 16 = (t^2 + 6t + 9) - 25 = (t + 3)^2 - 25,$$

so $(t + 3)^2 = x + 25$ and hence $t + 3 = \sqrt{x + 25}$ (taking $t + 3 > 0$ so that $\ln(t + 3)$ is defined). Then

$$y = 6\ln(t + 3) = 6\ln\left(\sqrt{x + 25}\right) = 6 \cdot \tfrac{1}{2}\ln(x + 25) = 3\ln(x + 25).$$

So $y = 3\ln(x + 25)$, that is $a = 3$ and $b = 25$.

2. Parametric circles

Q8 2022 Gosford Prelim Q3(d) · 1 mark

Find the Cartesian equation of the circle with parametric equations $x = 13\cos t$, $y = 13\sin t$.

Criteria	Marks
Correct Cartesian equation $x^2 + y^2 = 169$	1

Solution.

From the parametric equations, $\cos t = \dfrac{x}{13}$ and $\sin t = \dfrac{y}{13}$. Using $\cos^2 t + \sin^2 t = 1$,

$$\left(\frac{x}{13}\right)^2 + \left(\frac{y}{13}\right)^2 = 1 \quad \Longrightarrow \quad x^2 + y^2 = 169.$$

Q9 2020 Baulkham Hills Prelim Q2 · 1 mark

The parametric equations $x = 3 + 4\cos\theta$, $y = 4\sin\theta - 4$ represent a circle, with centre C and radius r. Which of the following shows the correct centre and radius?

(A) $C(-3, 4)$ and $r = 4$

(B) $C(3, -4)$ and $r = 4$

(C) $C(-3, 4)$ and $r = 16$

(D) $C(3, -4)$ and $r = 16$

Criteria	Marks
Correct option (B)	1

Solution.

Rearrange so the trig terms are isolated: $x - 3 = 4\cos\theta$ and $y + 4 = 4\sin\theta$. Then

$$(x-3)^2 + (y+4)^2 = 16\cos^2\theta + 16\sin^2\theta = 16.$$

This is a circle of centre $(3, -4)$ and radius $\sqrt{16} = 4$. The answer is (B).

Q10 2024 Girraween Trial Q9 · **1 mark**

A curve is represented by the parametric equations $x = 2\cos\theta$, $y = 2\sin\theta$. Which of the following is the Cartesian equation of the curve?

(A) $x + y = 2$

(B) $x + y = 4$

(C) $x^2 + y^2 = 2$

(D) $x^2 + y^2 = 4$

Criteria	Marks
Correct option (D)	1

Solution.

From the parametric equations, $\cos\theta = \dfrac{x}{2}$ and $\sin\theta = \dfrac{y}{2}$. Using $\cos^2\theta + \sin^2\theta = 1$,

$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{2}\right)^2 = 1 \quad \Longrightarrow \quad x^2 + y^2 = 4.$$

The answer is (D).

Q11 2025 Penrith Trial Q14(a) · **2 marks**

Find the Cartesian form of the equation for $x = -2 + 5\cos\theta$, $y = 4 + 5\sin\theta$ and describe its shape.

Criteria	Marks
Correct Cartesian equation $(x+2)^2 + (y-4)^2 = 25$	1
Describes the shape fully: a circle, centre $(-2, 4)$, radius 5	1

Solution.

Isolate the trig terms: $x + 2 = 5\cos\theta$ and $y - 4 = 5\sin\theta$. Then

$$(x+2)^2 + (y-4)^2 = 25\cos^2\theta + 25\sin^2\theta = 25.$$

So the Cartesian equation is $(x+2)^2 + (y-4)^2 = 25$. This is a circle with centre $(-2, 4)$ and radius 5. (The marking scheme did not award the mark for saying only that the shape is a circle without the centre and radius.)

Q12 2022 Glenwood Prelim Q7(c) · **3 marks**

Find the Cartesian equation of the circle with the parametric equations $x = 10 + 4\cos\theta$ and $y = -8 + 4\sin\theta$.

Criteria	**Marks**
Isolates the trig terms: $x - 10 = 4\cos\theta$ and $y + 8 = 4\sin\theta$	1
Applies $\cos^2\theta + \sin^2\theta = 1$	1
Correct Cartesian equation $(x-10)^2 + (y+8)^2 = 16$	1

Solution.

Rearrange each equation so the trig term is alone:

$$x - 10 = 4\cos\theta, \qquad y + 8 = 4\sin\theta.$$

Squaring and adding, then using $\cos^2\theta + \sin^2\theta = 1$,

$$(x-10)^2 + (y+8)^2 = 16\cos^2\theta + 16\sin^2\theta = 16.$$

So the Cartesian equation is $(x-10)^2 + (y+8)^2 = 16$, a circle of centre $(10, -8)$ and radius 4.

Q13 2020 Fort St Prelim Q8(c) · **4 marks**

A curve has parametric equations $x = 4\sin t + 3$ and $y = 4\cos t - 1$.

(i) Eliminate the parameter t and show that the curve is a circle and give its Cartesian equation.

(ii) Find the exact co-ordinates of the point on the curve where $t = \dfrac{\pi}{4}$.

Criteria	**Marks**
(i) Isolates $\sin t = \dfrac{x-3}{4}$, $\cos t = \dfrac{y+1}{4}$ and applies $\sin^2 t + \cos^2 t = 1$	1
(i) Correct Cartesian equation $(x-3)^2 + (y+1)^2 = 16$	1
(ii) Substitutes $t = \dfrac{\pi}{4}$ into both equations	1
(ii) Correct exact coordinates $\left(2\sqrt{2}+3,\ 2\sqrt{2}-1\right)$	1

Solution.

(i) Rearrange so each trig term is alone: $\sin t = \dfrac{x-3}{4}$ and $\cos t = \dfrac{y+1}{4}$. Then, by $\sin^2 t + \cos^2 t = 1$,

$$\left(\frac{x-3}{4}\right)^2 + \left(\frac{y+1}{4}\right)^2 = 1 \quad \Longrightarrow \quad (x-3)^2 + (y+1)^2 = 16.$$

This is a circle of centre $(3, -1)$ and radius 4.

(ii) At $t = \dfrac{\pi}{4}$ we have $\sin\dfrac{\pi}{4} = \cos\dfrac{\pi}{4} = \dfrac{1}{\sqrt{2}}$. Substituting,

$$x = 4\cdot\frac{1}{\sqrt{2}} + 3 = 2\sqrt{2} + 3, \qquad y = 4\cdot\frac{1}{\sqrt{2}} - 1 = 2\sqrt{2} - 1.$$

So the exact coordinates are $\left(2\sqrt{2}+3,\ 2\sqrt{2}-1\right)$.

3. Expressing a curve in parametric form

Q14 2023 Penrith Trial Q12(c) · 3 marks

A circle has the equation $x^2 - 4x + y^2 + 8y + 11 = 0$.

(i) Show that it can be written in the form $(x-2)^2 + (y+4)^2 = 9$. **(1 mark)**

(ii) Express the circle in parametric form. **(2 marks)**

Criteria	Marks
(i) Completes the square in both x and y to reach $(x-2)^2 + (y+4)^2 = 9$	1
(ii) Correct x-equation $x = 2 + 3\cos\theta$	1
(ii) Correct y-equation $y = -4 + 3\sin\theta$	1

Solution.

(i) Group and complete the square:

$$\left(x^2 - 4x\right) + \left(y^2 + 8y\right) + 11 = 0 \implies (x-2)^2 - 4 + (y+4)^2 - 16 + 11 = 0,$$

so $(x-2)^2 + (y+4)^2 = 9$. This is a circle with centre $(2, -4)$ and radius 3.

(ii) A circle $(x-h)^2 + (y-k)^2 = r^2$ has parametric form $x = h + r\cos\theta$, $y = k + r\sin\theta$. Here $h = 2$, $k = -4$, $r = 3$, so

$$x = 2 + 3\cos\theta, \qquad y = -4 + 3\sin\theta, \qquad 0 \le \theta < 2\pi.$$

Q15 2022 Manly Trial Q1 · **1 mark**

Which of the following parametric equations represents a circle that passes through the origin?

(A) $x = 3 + 3\cos\theta, \quad y = 4 + 3\sin\theta$

(B) $x = 3 + 4\cos\theta, \quad y = 4 + 4\sin\theta$

(C) $x = 3 + 5\cos\theta, \quad y = 4 + 5\sin\theta$

(D) $x = 3 + 7\cos\theta, \quad y = 4 + 7\sin\theta$

Criteria	Marks
Correct option (C)	1

Solution.

Each option is a circle with centre $(3, 4)$ and radius equal to the coefficient of $\cos\theta$. A circle passes through the origin when its radius equals the distance from the centre to the origin: $\sqrt{3^2 + 4^2} = 5$. Only radius 5 works, giving $x = 3 + 5\cos\theta$, $y = 4 + 5\sin\theta$. This is option **(C)**.

Q16 2023 Penrith Prelim Q10 · **1 mark**

The Cartesian equation $y = 2x^2 - 5x + 3$ is being converted to parametric equations with $x = 5 - 3m$. Find the parametric equation for y in terms of m.

(A) $y = 18m^2 - 15m + 28$

(B) $y = 18m^2 - 45m + 28$

(C) $y = 18m^2 + 15m + 28$

(D) $y = 18m^2 + 45m + 28$

Criteria	Marks
Correct option (B)	1

Solution.

Substitute $x = 5 - 3m$ into $y = 2x^2 - 5x + 3$:

$$y = 2(5 - 3m)^2 - 5(5 - 3m) + 3 = 2\left(25 - 30m + 9m^2\right) - 25 + 15m + 3.$$

Expanding, $y = 50 - 60m + 18m^2 - 25 + 15m + 3 = 18m^2 - 45m + 28$. This is option **(B)**.

Q17 2020 James Ruse Prelim Q4 · **1 mark**

A possible set of parametric equations for the curve $4x^2 + y^2 = 24x - 2y - 33$ is

(A) $x = 3 - \sin t, \quad y = 1 + 2\cos t$

(B) $x = 3 + \sin t, \quad y = -1 + 2\cos t$

(C) $x = 2 + \sin t, \quad y = 1 + 2\cos t$

(D) $x = -3 + \sin t, \quad y = 1 + 2\cos t$

Criteria	**Marks**
Correct option (B)	1

Solution.

Rearrange and complete the square: $4x^2 - 24x + y^2 + 2y + 33 = 0$, so

$$4(x-3)^2 - 36 + (y+1)^2 - 1 + 33 = 0 \implies 4(x-3)^2 + (y+1)^2 = 4.$$

Dividing by 4 gives $(x-3)^2 + \dfrac{(y+1)^2}{4} = 1$, an ellipse with $x - 3 = \sin t$ and $y + 1 = 2\cos t$. Hence $x = 3 + \sin t$, $y = -1 + 2\cos t$, which is option **(B)**. (Substituting option B back into the original equation gives 0 for all t; the other options do not.)

Q18 2024 Knox Trial Q3 · **1 mark**

Which of the following parametric equations represents a parabola with a minimum at $(0, 2)$?

(A) $x = t - \dfrac{1}{t}, \quad y = 2t^2 + \dfrac{2}{t^2}$

(B) $x = 2t + \dfrac{2}{t}, \quad y = t^2 + \dfrac{1}{t^2}$

(C) $x = 2t - \dfrac{2}{t}, \quad y = t^2 + \dfrac{1}{t^2}$

(D) $x = 2t - \dfrac{2}{t}, \quad y = t^2 - \dfrac{1}{t^2}$

Criteria	**Marks**
Correct option (C)	1

Solution.

For option C, $x = 2t - \dfrac{2}{t}$, so $x^2 = 4t^2 - 8 + \dfrac{4}{t^2} = 4\left(t^2 + \dfrac{1}{t^2}\right) - 8 = 4y - 8$. Hence $y = \dfrac{x^2}{4} + 2$, an upward parabola with vertex (minimum) at $(0, 2)$. Checking the others: option B gives $x^2 = 4y + 8$ (minimum at $(0, -2)$); option D gives $x^2 = \left(2t - \frac{2}{t}\right)^2 = 4t^2 - 8 + \frac{4}{t^2}$ while $y = t^2 - \frac{1}{t^2}$ is not expressible as a function of x alone in this form; option A gives a minimum at a different point. The answer is option **(C)**.

Q19 2025 Barker Trial Q8 · **1 mark**

The diagram shows the interval from $(0, 2)$ to $(2, 0)$. Which pair of parametric equations does **not** represent the interval shown?

(A) $x = 2(1 - t^2), \quad y = 2t^2 \quad$ for $0 \leq t \leq 1$

(B) $x = e^t - 1, \quad y = 3 - e^t \quad$ for $0 \leq t \leq \ln 2$

(C) $x = 2\sin t, \quad y = 2 - 2\sqrt{1 - \cos^2 t} \quad$ for $0 \leq t \leq \dfrac{\pi}{2}$

(D) $x = 2 - \dfrac{2}{\pi}\cos^{-1} t, \quad y = \dfrac{2}{\pi}\cos^{-1} t \quad$ for $-1 \leq t \leq 1$

Criteria	**Marks**
Correct option (B)	1

Solution.

The interval shown is the segment of the line $x + y = 2$ running from $(0, 2)$ to $(2, 0)$, so $0 \leq x \leq 2$. Every option satisfies $x + y = 2$, so the deciding factor is which values of x (and y) are actually covered.

- **A:** $x + y = 2(1 - t^2) + 2t^2 = 2$; as t runs $0 \to 1$, x runs $2 \to 0$: the whole interval.
- **C:** $y = 2 - 2|\sin t| = 2 - 2\sin t$ on $[0, \frac{\pi}{2}]$, and $x = 2\sin t$, so $x + y = 2$; as t runs $0 \to \frac{\pi}{2}$, x runs $0 \to 2$: the whole interval.
- **D:** $x + y = 2$; as t runs $-1 \to 1$, $\cos^{-1} t$ runs $\pi \to 0$, so x runs $0 \to 2$: the whole interval.
- **B:** $x + y = (e^t - 1) + (3 - e^t) = 2$, but as t runs $0 \to \ln 2$, e^t runs $1 \to 2$, so $x = e^t - 1$ runs only $0 \to 1$. This covers only the half of the interval from $(0, 2)$ to $(1, 1)$.

Option **(B)** traces only half the interval, so it does not represent the full interval shown. The answer is **(B)**.

4. Harder eliminations: trig identities and reciprocals

Q20 2020 Hornsby Girls Trial Q9 · 1 mark

A curve C has parametric equations $x = \cos^2 t$ and $y = 4\sin^2 t$ for $t \in \mathbb{R}$. What is the Cartesian equation of C?

(A) $y = 1 - x$ for $0 \leq x \leq 1$

(B) $y = 4 - 4x$ for $t \in \mathbb{R}$

(C) $y = 4 - 4x$ for $0 \leq x \leq 1$

(D) $y = 1 - x$ for $t \in \mathbb{R}$

Criteria	Marks
Correct option (C)	1

Solution.

Using $\sin^2 t = 1 - \cos^2 t$,

$$y = 4\sin^2 t = 4\left(1 - \cos^2 t\right) = 4(1 - x) = 4 - 4x.$$

Since $x = \cos^2 t$ and $\cos^2 t$ takes every value in $[0, 1]$, the domain is $0 \leq x \leq 1$. This is option **(C)**.

Q21 2024 Glenwood Prelim Q8(b)(i) · 1 mark

The parametric equations of a particle are $x = \sin t$ and $y = \cos 2t$, where t is the time in seconds. Find the Cartesian equation.

Criteria	Marks
Correct Cartesian equation $y = 1 - 2x^2$ (with domain $-1 \leq x \leq 1$)	1

Solution.

Use the double-angle identity $\cos 2t = 1 - 2\sin^2 t$. Since $x = \sin t$,

$$y = \cos 2t = 1 - 2\sin^2 t = 1 - 2x^2.$$

As $x = \sin t$, the domain is $-1 \leq x \leq 1$.

Q22 2021 North Sydney Boys Trial Q11(a) · 1 mark

Find the Cartesian equation represented by $x = \cos t + \sin t$ and $y = \cos t - \sin t$.

Criteria	Marks
Correct Cartesian equation $x^2 + y^2 = 2$	1

Solution.

Square and add:

$$x^2 + y^2 = (\cos t + \sin t)^2 + (\cos t - \sin t)^2 = 2\cos^2 t + 2\sin^2 t = 2.$$

So the Cartesian equation is $x^2 + y^2 = 2$ (a circle of radius $\sqrt{2}$ centred at the origin).

Q23 2023 Sydney Tech Trial Q7 · 1 mark

The parametric equation of a function is given by $x = \sin t$ and $y = \sin 2t$. What is its Cartesian equation?

(A) $y = 2x\sqrt{1 - x^2} \quad \{0 \leq x \leq 1\}$

(B) $y = 2x^2 - 1 \quad \{0 \leq x \leq 1\}$

(C) $y = 1 - 2x^2 \quad \{0 \leq x \leq 1\}$

(D) $y = \dfrac{x + 1}{2} \quad \{0 \leq x \leq 1\}$

Criteria	Marks
Correct option (A)	1

Solution.

Using the double-angle identity $\sin 2t = 2\sin t \cos t$ and $x = \sin t$,

$$y = 2\sin t \cos t = 2x\sqrt{1 - x^2} \qquad (\text{taking } \cos t = \sqrt{1 - \sin^2 t}).$$

This matches option **(A)**, $y = 2x\sqrt{1 - x^2}$ with $0 \leq x \leq 1$.

Q24 2020 James Ruse Prelim Q3 · 1 mark

A function is given by the parametric equations $x = t + \dfrac{1}{t}$, $y = t^3 + \dfrac{1}{t^3}$. Its Cartesian equation is

(A) $y = x^3 - 3x$

(B) $y = x^3 + 3x$

(C) $y = x^2 - x$

(D) $y = x^3$

Criteria	Marks
Correct option (A)	1

Solution.

Cube $x = t + \dfrac{1}{t}$:

$$x^3 = \left(t + \frac{1}{t}\right)^3 = t^3 + 3t + \frac{3}{t} + \frac{1}{t^3} = \left(t^3 + \frac{1}{t^3}\right) + 3\left(t + \frac{1}{t}\right) = y + 3x.$$

Hence $y = x^3 - 3x$, which is option **(A)**.

Q25 2022 Trinity Grammar Trial Q9 · 1 mark

A curve is defined parametrically by $x = \sec t + 1$ and $y = \tan t$ where $t \in \left[0, \dfrac{\pi}{2}\right)$. Which of the following statements is correct for the Cartesian equation and domain of the curve?

(A) $y = \sqrt{x^2 - 2x},\ x \in [2, \infty)$

(B) $y = \sqrt{x^2 - 2x},\ x \in (-\infty, 0] \cup [2, \infty)$

(C) $y^2 = x^2 - 2x,\ x \in [2, \infty)$

(D) $y^2 = x^2 - 2x,\ x \in (-\infty, 0] \cup [2, \infty)$

Criteria	Marks
Correct option (A)	1

Solution.

Since $x = \sec t + 1$, we have $\sec t = x - 1$, and $y = \tan t$. Using $\sec^2 t - \tan^2 t = 1$,

$$(x-1)^2 - y^2 = 1 \implies y^2 = (x-1)^2 - 1 = x^2 - 2x.$$

On $t \in \left[0, \dfrac{\pi}{2}\right)$: $\sec t \geq 1$, so $x - 1 \geq 1$, giving $x \geq 2$; and $\tan t \geq 0$, so $y \geq 0$ and we take the positive root $y = \sqrt{x^2 - 2x}$. Thus the Cartesian equation is $y = \sqrt{x^2 - 2x}$ with $x \in [2, \infty)$, which is option **(A)**.

Q26 2022 Girraween Prelim Q6(b) · **4 marks**

A curve is represented by the following parametric equations $x = 5\tan\theta - 3$, $y = 9 + 4\sec\theta$. Find the Cartesian equation of the curve.

Criteria	**Marks**
Makes $\tan\theta$ the subject: $\tan\theta = \dfrac{x+3}{5}$	1
Makes $\sec\theta$ the subject: $\sec\theta = \dfrac{y-9}{4}$	1
Applies the identity $\sec^2\theta - \tan^2\theta = 1$	1
Correct Cartesian equation $\left(\dfrac{y-9}{4}\right)^2 - \left(\dfrac{x+3}{5}\right)^2 = 1$	1

Solution.

Make $\tan\theta$ and $\sec\theta$ the subjects:

$$x = 5\tan\theta - 3 \implies \tan\theta = \frac{x+3}{5}, \qquad y = 9 + 4\sec\theta \implies \sec\theta = \frac{y-9}{4}.$$

Substitute into the Pythagorean identity $\sec^2\theta - \tan^2\theta = 1$:

$$\left(\frac{y-9}{4}\right)^2 - \left(\frac{x+3}{5}\right)^2 = 1.$$

This is a hyperbola centred at $(-3, 9)$.

Graphing and using parametric curves

5. Matching a parametric curve to its graph

Q27 2022 HSC Q5 · 1 mark

A curve is defined in parametric form by $x = 2 + t$ and $y = 3 - 2t^2$ for $-1 \leq t \leq 0$. Which diagram best represents this curve?

Criteria	Marks
Correct diagram (arc of a concave-down parabola from $(1, 1)$ up to $(2, 3)$)	1

Solution.

Eliminate the parameter. From $x = 2 + t$ we get $t = x - 2$, so

$$y = 3 - 2(x - 2)^2,$$

a concave-down parabola with vertex $(2, 3)$. The parameter runs over $-1 \leq t \leq 0$, i.e. $1 \leq x \leq 2$. The endpoints are: at $t = -1$, $(x, y) = (1, 1)$; at $t = 0$, $(x, y) = (2, 3)$. As t increases from -1 to 0, the point moves from $(1, 1)$ up to the vertex $(2, 3)$, tracing the left arm of the parabola. The correct diagram is the one showing this rising arc from $(1, 1)$ to $(2, 3)$.

Q28 2022 Ascham Trial Q1 · 1 mark

What is the graph of the function which has parametric equations $x = -\dfrac{p}{2} - 1$ and $y = p + 1$?

Criteria	Marks
Correct diagram (straight line $y = -2x - 1$)	1

Solution.

Eliminate the parameter p. From $x = -\dfrac{p}{2} - 1$ we get $p = -2x - 2$, and substituting into $y = p + 1$ gives

$$y = -2x - 1.$$

This is a straight line with gradient -2, y-intercept $(0, -1)$ and x-intercept $\left(-\frac{1}{2}, 0\right)$. The correct diagram is the one showing this line.

Q29 2022 Hunters Hill Trial Q4 · 1 mark

A function is defined by the parametric equations $x = 3t^2$ and $y = 9t$, for $t > 0$. Which of the following sketches represents this function?

Criteria	Marks
Correct option (B): the upper branch of $y^2 = 27x$ through $\left(2, 3\sqrt{6}\right)$	1

Solution.

Eliminate the parameter. From $y = 9t$ we get $t = \dfrac{y}{9}$, so

$$x = 3t^2 = 3 \cdot \frac{y^2}{81} = \frac{y^2}{27}, \qquad \text{i.e.} \qquad y^2 = 27x.$$

This is a parabola opening to the right. Since $y = 9t$ and $t > 0$, we have $y > 0$, so only the upper half is traced (and $x = 3t^2 > 0$). The labelled point in option (B) is $(2, 3\sqrt{6})$: at $x = 2$, $t^2 = \frac{2}{3}$, so $t = \sqrt{\frac{2}{3}}$ and $y = 9\sqrt{\frac{2}{3}} = 3\sqrt{6}$, which checks. The answer is **(B)**.

Q30 2023 Blacktown Boys Prelim Q3 · 1 mark

The graph of the function which has parametric equations $x = -2t$ and $y = 1 + t$ is

Criteria	Marks
Correct diagram (straight line $y = 1 - \frac{1}{2}x$, negative gradient)	1

Solution.

Eliminate the parameter. From $x = -2t$ we get $t = -\dfrac{x}{2}$, so

$$y = 1 + t = 1 - \frac{x}{2}.$$

This is a straight line with gradient $-\frac{1}{2}$, y-intercept $(0, 1)$ and x-intercept $(2, 0)$. The correct diagram is the one showing this line with negative gradient through $(0, 1)$ and $(2, 0)$.

Q31 2021 HSC Q8 · **1 mark**

The diagram shows a semicircle. Which pair of parametric equations represents the semicircle shown? (A) $x = 3 + \sin t,\ y = 2 + \cos t$ (B) $x = 3 + \cos t,\ y = 2 + \sin t$ (C) $x = 3 - \sin t,\ y = 2 - \cos t$ (D) $x = 3 - \cos t,\ y = 2 - \sin t$, each for $-\dfrac{\pi}{2} \leq t \leq \dfrac{\pi}{2}$.

Criteria	Marks
Correct option (C)	1

Solution.

The semicircle is the lower half of a circle of radius 1 centred at $(3, 2)$: its diameter runs along $y = 2$ from $(2, 2)$ to $(4, 2)$, with the lowest point of the arc at $(3, 1)$. Test each option over $-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$ at $t = -\frac{\pi}{2}$, 0, $\frac{\pi}{2}$:

- (C) $x = 3 - \sin t,\ y = 2 - \cos t$ gives $(4, 2),\ (3, 1),\ (2, 2)$: the lower semicircle. Correct.
- (A) $x = 3 + \sin t,\ y = 2 + \cos t$ gives $(2, 2),\ (3, 3),\ (4, 2)$: the upper semicircle, not the shown arc.

Option (C) traces the endpoints $(2, 2)$ and $(4, 2)$ with the lowest point $(3, 1)$, matching the diagram. The answer is **(C)**.

Q32 2023 St George Girls Trial Q8 · **1 mark**

A parametric function is graphed for $-1 \leq t \leq 2$. Which of the following shows the parametric equations of the curve? (A) $x = t - 1,\ y = 3t - t^3$ (B) $x = t + 1,\ y = 3t - t^3$ (C) $x = t + 1,\ y = -3t + t^3$ (D) $x = t - 1,\ y = 3t + t^3$.

Criteria	Marks
Correct option (A)	1

Solution.

For $-1 \leq t \leq 2$, the graph shows x ranging over $-2 \leq x \leq 1$. Only $x = t - 1$ gives this range ($-2 \leq t - 1 \leq 1$), which eliminates (B) and (C). To separate (A) from (D), test $t = 2$: option (D) gives $y = 3(2) + 2^3 = 14$, so the point $(1, 14)$, which is not on the graph. Option (A) gives $y = 3(2) - 2^3 = -2$, consistent with the graph. The answer is **(A)**.

6. Sketching a parametric curve

Q33 2022 Gosford Prelim Q5(b)(i) · **2 marks**

Sketch the parabola whose parametric equations are $x = t$ and $y = t^2$, marking the points P and Q corresponding to $t = -1$ and $t = 2$.

Criteria	Marks
Correct curve $y = x^2$ (concave-up parabola, vertex at the origin)	1
Correct points $P(-1, 1)$ and $Q(2, 4)$ marked	1

Solution.

Since $x = t$ and $y = t^2$, eliminating t gives $y = x^2$, the standard concave-up parabola with vertex at the origin. The two marked points are found by substituting the parameter values:

$$t = -1 : \ (x, y) = (-1, 1) = P, \qquad t = 2 : \ (x, y) = (2, 4) = Q.$$

Sketch $y = x^2$ with its vertex at $(0, 0)$, and mark $P(-1, 1)$ on the left branch and $Q(2, 4)$ on the right branch.

Q34 2022 Hurlstone Prelim Q13(c)(i) · **2 marks**

The parametrisation of a circle is $x - 2 = 2\cos\theta$ and $y = 2\sin\theta$. Sketch the circle, labelling its centre and any axis intercepts.

Criteria	Marks
Correct circle: centre $(2, 0)$, radius 2	1
Correct axis intercepts labelled (x-intercepts $(0, 0)$ and $(4, 0)$; y-intercept $(0, 0)$)	1

Solution.

Write $x - 2 = 2\cos\theta$ and $y = 2\sin\theta$. Then $\cos\theta = \dfrac{x-2}{2}$ and $\sin\theta = \dfrac{y}{2}$, and using $\cos^2\theta + \sin^2\theta = 1$:

$$\left(\frac{x-2}{2}\right)^2 + \left(\frac{y}{2}\right)^2 = 1 \quad \Longrightarrow \quad (x-2)^2 + y^2 = 4.$$

This is a circle of centre $(2, 0)$ and radius 2. It meets the x-axis where $y = 0$: $(x-2)^2 = 4$, so $x = 0$ or $x = 4$, giving $(0, 0)$ and $(4, 0)$. It touches the y-axis only at the origin $(0, 0)$. Sketch the circle centred at $(2, 0)$ passing through $(0, 0)$, $(4, 0)$, $(2, 2)$ and $(2, -2)$, with the centre and the intercepts $(0, 0)$ and $(4, 0)$ labelled.

Q35 2023 Presbyterian Ladies College Prelim Q11(d) · 5 marks

A curve is defined by $x = 8t$ and $y = 4t^2$. **(i)** Draw the curve for $-3 \le t \le 3$, showing the endpoints and any intercepts. **(ii)** For what value(s) of t does the curve meet the line $y = 10 - \frac{3}{4}x$?

Criteria	Marks
(i) Correct curve $y = \frac{1}{16}x^2$ (concave-up parabola, vertex at the origin)	1
(i) Correct endpoints $(-24, 36)$ and $(24, 36)$, with the intercept at the origin	1
(ii) Correct substitution to form $4t^2 = 10 - 6t$ (i.e. $2t^2 + 3t - 5 = 0$)	1
(ii) Correct value $t = 1$	1
(ii) Correct value $t = -\frac{5}{2}$	1

Solution.

(i) Eliminate the parameter: from $x = 8t$, $t = \dfrac{x}{8}$, so $y = 4t^2 = 4 \cdot \dfrac{x^2}{64} = \dfrac{x^2}{16}$. This is a concave-up parabola with vertex at the origin. The endpoints are at $t = \pm 3$: $t = -3$ gives $(-24, 36)$ and $t = 3$ gives $(24, 36)$. The only intercept is the vertex at the origin $(0, 0)$. Sketch $y = \frac{1}{16}x^2$ from $(-24, 36)$ to $(24, 36)$.

(ii) Substitute $x = 8t$ and $y = 4t^2$ into $y = 10 - \frac{3}{4}x$:

$$4t^2 = 10 - \frac{3}{4}(8t) = 10 - 6t.$$

So $4t^2 + 6t - 10 = 0$, i.e. $2t^2 + 3t - 5 = 0$, which factorises as $(2t + 5)(t - 1) = 0$. Hence

$$t = 1 \quad \text{or} \quad t = -\frac{5}{2}.$$

(These correspond to the points $(8, 4)$ and $(-20, 25)$.)

Q36 2020 International Grammar Prelim Q12(d) · 5 marks

A curve is defined by $x = 4t^2$ and $y = 8t$. **(i)** Draw the curve for $-3 \leq t \leq 3$, showing the beginning and ending coordinates and any intercepts. **(ii)** For what value(s) of t does the parabola meet the line $y = 6 - 2x$?

Criteria	Marks
(i) Correct curve $y^2 = 16x$ (parabola opening to the right, vertex at the origin)	1
(i) Correct endpoints $(36, -24)$ (at $t = -3$) and $(36, 24)$ (at $t = 3$), with the intercept at the origin	1
(ii) Correct substitution to form $8t^2 + 8t - 6 = 0$ (i.e. $4t^2 + 4t - 3 = 0$)	1
(ii) Correct value $t = \frac{1}{2}$	1
(ii) Correct value $t = -\frac{3}{2}$	1

Solution.

(i) Eliminate the parameter: from $y = 8t$, $t = \dfrac{y}{8}$, so $x = 4t^2 = 4 \cdot \dfrac{y^2}{64} = \dfrac{y^2}{16}$, i.e. $y^2 = 16x$. This is a parabola opening to the right with vertex at the origin. The endpoints are at $t = \pm 3$: $t = -3$ gives $(36, -24)$ and $t = 3$ gives $(36, 24)$. The only intercept is the vertex at the origin $(0, 0)$. Sketch $y^2 = 16x$ between these two endpoints.

(ii) Substitute $x = 4t^2$ and $y = 8t$ into $y = 6 - 2x$:

$$8t = 6 - 2(4t^2) = 6 - 8t^2.$$

So $8t^2 + 8t - 6 = 0$, i.e. $4t^2 + 4t - 3 = 0$, which factorises as $(2t - 1)(2t + 3) = 0$. Hence

$$t = \frac{1}{2} \quad \text{or} \quad t = -\frac{3}{2}.$$

(These correspond to the points $(1, 4)$ and $(9, -12)$.)

Q37 2024 Ascham Trial Q14(b) · 3 marks

Sketch the function represented by $x = 2t - 1$ and $y = t^2 + t$ for $-1 \leq t \leq 1$.

Criteria	Marks
Correct Cartesian form $y = \frac{1}{4}(x+1)(x+3)$ (concave-up parabola)	1
Correct endpoints $(-3, 0)$ (at $t = -1$) and $(1, 2)$ (at $t = 1$)	1
Correct sketch of the arc between the endpoints, showing the minimum $\left(-2, -\frac{1}{4}\right)$	1

Solution.

Eliminate the parameter. From $x = 2t - 1$, $t = \dfrac{x+1}{2}$. Substituting into $y = t^2 + t$:

$$y = \left(\frac{x+1}{2}\right)^2 + \frac{x+1}{2} = \frac{(x+1)^2}{4} + \frac{x+1}{2} = \frac{x^2 + 2x + 1 + 2x + 2}{4} = \frac{x^2 + 4x + 3}{4} = \frac{(x+1)(x+3)}{4}.$$

This is a concave-up parabola. The parameter runs $-1 \leq t \leq 1$, giving $-3 \leq x \leq 1$. The endpoints are: at $t = -1$, $(x, y) = (-3, 0)$; at $t = 1$, $(x, y) = (1, 2)$. The turning point of $y = t^2 + t$ is at $t = -\frac{1}{2}$, giving the minimum $\left(-2, -\frac{1}{4}\right)$ (which lies in the interval). Sketch the arc of $y = \frac{1}{4}(x+1)(x+3)$ from $(-3, 0)$, dipping to the minimum $\left(-2, -\frac{1}{4}\right)$, and rising to $(1, 2)$.

7. Points and intersections on a parametric curve

Q38 Hurlstone 2025 Prelim Q1 · **1 mark**

Consider the parametric equations $x = 3t$, $y = -3t^2$. The point $(-6, m)$ lies on the curve. Which of the following gives the value of m?

(A) $m = -12$

(B) $m = -2$

(C) $m = 2$

(D) $m = 12$

Criteria	Marks
Correct option (A)	1

Solution.

The point $(-6, m)$ must satisfy both parametric equations for a single value of the parameter t. Use the x-coordinate to find t:

$$x = 3t = -6 \implies t = -2.$$

Substitute $t = -2$ into $y = -3t^2$:

$$m = -3(-2)^2 = -3(4) = -12.$$

So $(-6, -12)$ lies on the curve and $m = -12$. The answer is **(A)**.

Q39 Merewether 2025 Trial Q8 · **1 mark**

The point $P(4, a)$ is on the graph of $x = \frac{t}{2}$, $y = 2\sqrt{t}$. There is another point $F(2, 0)$, such that $|PF|$ is which of the following?

(A) 4

(B) 5

(C) 6

(D) 7

Criteria	Marks
Correct option (C)	1

Solution.

First find the parameter t at P using the x-coordinate:

$$x = \frac{t}{2} = 4 \implies t = 8.$$

Then the y-coordinate is

$$a = 2\sqrt{t} = 2\sqrt{8} = 2 \cdot 2\sqrt{2} = 4\sqrt{2},$$

so $P = (4, 4\sqrt{2})$. The distance from P to $F(2, 0)$ is

$$|PF| = \sqrt{(4-2)^2 + \left(4\sqrt{2} - 0\right)^2} = \sqrt{4 + 32} = \sqrt{36} = 6.$$

The answer is **(C)**.

Q40 Cherrybrook Tech 2021 Trial Q7 · **1 mark**

A curve is defined parametrically by $x = -\ln t$, $y = \cos 2t$ for $t > 0$. At what approximate value of x does the curve cross the x-axis for the first time as t increases from zero?

(A) -1.7

(B) -1.37

(C) -0.86

(D) 0.24

Criteria	**Marks**
Correct option (D)	1

Solution.

The curve crosses the x-axis where $y = 0$, that is where $\cos 2t = 0$. Solving,

$$2t = \frac{\pi}{2},\ \frac{3\pi}{2},\ \ldots \implies t = \frac{\pi}{4},\ \frac{3\pi}{4},\ \ldots$$

As t increases from 0, the first crossing is at $t = \dfrac{\pi}{4}$. The corresponding x-value is

$$x = -\ln\left(\frac{\pi}{4}\right) \approx -\ln(0.7854) \approx 0.24.$$

The answer is **(D)**. (Note the tempting distractor -0.86 is the x-value at the second crossing $t = \frac{3\pi}{4}$, i.e. $x = -\ln\frac{3\pi}{4} \approx -0.86$, which is not the first.)

Q41 Presbyterian (PLC Sydney) 2022 Prelim Q13(f) · **3 marks**

A curve is defined by $x = 3\cos t + 4$ and $y = 3\sin t - 5$.

(i) The point $P(4, -2)$ lies on the curve. Find the value of t within the domain $[0, 2\pi]$ which corresponds to the point P. **(1 mark)**

(ii) Find the Cartesian equation which describes the curve. **(2 marks)**

Criteria	**Marks**
(i) Correct value $t = \dfrac{\pi}{2}$	1
(ii) Isolates $\cos t$ and $\sin t$ and applies $\cos^2 t + \sin^2 t = 1$	1
(ii) Correct Cartesian equation $(x-4)^2 + (y+5)^2 = 9$	1

Solution.

(i) Substitute the coordinates of $P(4, -2)$ into the parametric equations. From the x-coordinate:

$$3\cos t + 4 = 4 \implies \cos t = 0.$$

From the y-coordinate:

$$3\sin t - 5 = -2 \implies 3\sin t = 3 \implies \sin t = 1.$$

The only value in $[0, 2\pi]$ with $\cos t = 0$ and $\sin t = 1$ is $\boxed{t = \dfrac{\pi}{2}}$.

(ii) Isolate the trig terms and use the Pythagorean identity $\cos^2 t + \sin^2 t = 1$:

$$\cos t = \frac{x-4}{3}, \qquad \sin t = \frac{y+5}{3}.$$

Then

$$\left(\frac{x-4}{3}\right)^2 + \left(\frac{y+5}{3}\right)^2 = 1 \implies (x-4)^2 + (y+5)^2 = 9.$$

The curve is a circle of radius 3 centred at $(4, -5)$. (Check: at $t = \frac{\pi}{2}$, $x = 4$, $y = -2$, and $(4-4)^2 + (-2+5)^2 = 0 + 9 = 9$.)

Q42 Hurlstone 2021 Trial Q11(d) · 3 marks

Two boats start sailing at the same time. Boat A: $x = \dfrac{t}{2}$, $y = t + 1$. Boat B: $x = t - 2$, $y = 9 - 2t$, where t is the time in hours since they started.

(i) Find the Cartesian equation for the course of each boat and show that the courses intersect at $(1, 3)$. **(2 marks)**

(ii) Do the boats collide? Justify your answer supported by calculations. **(1 mark)**

Criteria	Marks
(i) Both Cartesian equations correct: $y = 2x + 1$ and $y = 5 - 2x$	1
(i) Solves them simultaneously to confirm intersection at $(1, 3)$	1
(ii) Finds the two arrival times ($t = 2$ and $t = 3$) and concludes no collision	1

Solution.

(i) Eliminate the parameter t from each course.

Boat A: from $x = \dfrac{t}{2}$ we get $t = 2x$, so

$$y = t + 1 = 2x + 1.$$

Boat B: from $x = t - 2$ we get $t = x + 2$, so

$$y = 9 - 2t = 9 - 2(x + 2) = 5 - 2x.$$

The courses meet where the two Cartesian equations are equal:

$$2x + 1 = 5 - 2x \implies 4x = 4 \implies x = 1, \qquad y = 2(1) + 1 = 3.$$

So the courses intersect at $(1, 3)$, as required. (Check on Boat B: $y = 5 - 2(1) = 3$.)

(ii) Crossing courses does not mean the boats are there at the same time. Find the time each boat reaches $(1, 3)$.

Boat A at $(1, 3)$: $x = \dfrac{t}{2} = 1 \Rightarrow t = 2$ (and $y = 2 + 1 = 3$ confirms). So Boat A is there after 2 hours.

Boat B at $(1, 3)$: $x = t - 2 = 1 \Rightarrow t = 3$ (and $y = 9 - 2(3) = 3$ confirms). So Boat B is there after 3 hours.

The boats pass through the common point at different times (2 hours versus 3 hours), so they do **not** collide.