Year 11 Mathematics Extension 1

Topic: Further Functions (5 of 5)

Review Questions (Worked Solutions): Mixed One-Off Exam Questions

Term 3, 2026

Mixed one-off exam questions

Q1 2022 Killara Trial Q6 · 1 mark

Find the Cartesian equation for the graph formed by the parametric equations $x = 4 + 4\cos t$, $y = 5 + 3\sin t$.

(A) $\dfrac{(x+4)^2}{16} + \dfrac{(y+5)^2}{9} = 1$

(B) $\dfrac{(x-4)^2}{16} + \dfrac{(y-5)^2}{9} = 1$

(C) $(x-4)^2 + (y-5)^2 = 1$

(D) $\dfrac{(x-5)^2}{9} + \dfrac{(y-4)^2}{16} = 1$

Criteria	Marks
Correct option (B)	1

Solution.

Isolate the trig terms: $\cos t = \dfrac{x-4}{4}$ and $\sin t = \dfrac{y-5}{3}$. Using $\cos^2 t + \sin^2 t = 1$,

$$\left(\frac{x-4}{4}\right)^2 + \left(\frac{y-5}{3}\right)^2 = 1 \quad \Longrightarrow \quad \frac{(x-4)^2}{16} + \frac{(y-5)^2}{9} = 1.$$

This is an ellipse centred at $(4, 5)$. The answer is (B).

Q2 2024 James Ruse Trial Q4 · **1 mark**

Let $f(x) = x^3$ where $x \in \{0, 1, 2, 3\}$. Which of the following is the domain of $f^{-1}(x)$?

(A) $\{0,\ 1,\ \sqrt{8},\ \sqrt{27}\}$

(B) $\left\{0,\ 1,\ \frac{1}{\sqrt[3]{8}},\ \frac{1}{\sqrt[3]{27}}\right\}$

(C) $\{0,\ 1,\ 8,\ 27\}$

(D) $\left\{0,\ 1,\ \frac{1}{8},\ \frac{1}{27}\right\}$

Criteria	Marks
Correct option (C)	1

Solution.

The domain of f^{-1} is the range of f. Applying $f(x) = x^3$ to each element of $\{0, 1, 2, 3\}$,

$$f(0) = 0, \quad f(1) = 1, \quad f(2) = 8, \quad f(3) = 27,$$

so the range of f, and hence the domain of f^{-1}, is $\{0, 1, 8, 27\}$. The answer is (C).

Q3 2025 Sydney Grammar Trial Q6 · **1 mark**

The function $f(x)$ has an inverse function, $f^{-1}(x)$. Which of the following is always a correct expression for the inverse of $y = f(3x)$?

(A) $f^{-1}\left(\frac{x}{3}\right)$

(B) $f^{-1}(3x)$

(C) $\frac{1}{3}f^{-1}(x)$

(D) $3f^{-1}(x)$

Criteria	Marks
Correct option (C)	1

Solution.

To find the inverse of $y = f(3x)$, swap x and y and solve for y:

$$x = f(3y).$$

Apply f^{-1} to both sides: $f^{-1}(x) = 3y$, so $y = \frac{1}{3}f^{-1}(x)$. The answer is (C).

Q4 2022 Presbyterian Prelim Q12(d)(ii) · **3 marks**

The graph of $y = f(x)$ is given, where $f(x) = x + 2$ for $x < 0$ and $f(x) = |x - 2|$ for $x \geq 0$. Sketch the graph of $y = f(x) + f(-x) + 1$, carefully showing any intercepts on the co-ordinate axes.

Criteria	**Marks**
Recognises that $g(x) = f(x) + f(-x) + 1$ is even (symmetric in the y-axis)	1
Correct central section: the tent from $(-2, 1)$ up to $(0, 5)$ down to $(2, 1)$	1
Correct outer sections: the constant $y = 1$ for $\lvert x\rvert \geq 2$	1

Solution.

Write $g(x) = f(x) + f(-x) + 1$. Because $g(-x) = f(-x) + f(x) + 1 = g(x)$, the graph is even and is symmetric about the y-axis, so it is enough to work out g for $x \geq 0$.

For $0 \leq x \leq 2$: here $f(x) = |x - 2| = 2 - x$, and since $-x \leq 0$ we have $f(-x) = -x + 2 = 2 - x$. So

$$g(x) = (2 - x) + (2 - x) + 1 = 5 - 2x, \qquad 0 \leq x \leq 2,$$

a line from $(0, 5)$ down to $(2, 1)$.

For $x \geq 2$: here $f(x) = |x - 2| = x - 2$ and $f(-x) = -x + 2 = 2 - x$, so

$$g(x) = (x - 2) + (2 - x) + 1 = 1, \qquad x \geq 2,$$

the horizontal line $y = 1$.

Reflecting in the y-axis, the full graph is: the constant $y = 1$ for $x \leq -2$, a straight rise from $(-2, 1)$ up to the peak $(0, 5)$, a straight fall from $(0, 5)$ down to $(2, 1)$, then the constant $y = 1$ for $x \geq 2$. The y-intercept is $(0, 5)$; there are no x-intercepts (the graph never drops below $y = 1$).

Q5 2024 Penrith Prelim Q13(a) · **3 marks**

The "Folium of Descartes" is defined by $x = \dfrac{3t}{1+t^3}$, $y = \dfrac{3t^2}{1+t^3}$. (i) Find $\dfrac{y}{x}$ in terms of t, in simplest form. (ii) Deduce that the Cartesian equation of the curve is $x^3 + y^3 = 3xy$.

Criteria	Marks
(i) Correct simplification $\dfrac{y}{x} = t$	1
(ii) Expresses $x^3 + y^3$ (or $3xy$) using the parametric forms	1
(ii) Completes the deduction to $x^3 + y^3 = 3xy$	1

Solution.

(i) Dividing y by x, the common denominator $1 + t^3$ cancels:

$$\frac{y}{x} = \frac{\dfrac{3t^2}{1+t^3}}{\dfrac{3t}{1+t^3}} = \frac{3t^2}{3t} = t.$$

(ii) From part (i), $t = \dfrac{y}{x}$. Substitute this into $x = \dfrac{3t}{1+t^3}$:

$$x = \frac{3 \cdot \dfrac{y}{x}}{1 + \dfrac{y^3}{x^3}} = \frac{\dfrac{3y}{x}}{\dfrac{x^3+y^3}{x^3}} = \frac{3y}{x} \cdot \frac{x^3}{x^3+y^3} = \frac{3x^2y}{x^3+y^3}.$$

Multiplying both sides by $x^3 + y^3$ and then dividing by x (with $x \neq 0$),

$$x(x^3 + y^3) = 3x^2y \quad \Longrightarrow \quad x^3 + y^3 = 3xy,$$

as required.

Q6 2023 HSC Q8 · **1 mark**

The diagram shows the graph of a function (an even, "W"-shaped graph with value 1 at $x = 0$, peaks of height 2 at $x = \pm 1$, and x-intercepts at $x = \pm 3$). Which of the following is the equation of the function?

(A) $y = \big|1 - \big||x| - 2\big|\big|$

(B) $y = \big|2 - \big||x| - 1\big|\big|$

(C) $y = \big|1 - |x - 2|\big|$

(D) $y = \big|2 - |x - 1|\big|$

Criteria	**Marks**
Correct option (B)	1

Solution.

The graph is symmetric about the y-axis, so the equation must be even; this rules out (C) and (D), which contain $|x - 2|$ and $|x - 1|$ (not symmetric). Testing (B), $y = \big|2 - \big||x| - 1\big|\big|$:

$$x = 0: \ \big|2 - |0 - 1|\big| = |2 - 1| = 1, \quad x = \pm 1: \ \big|2 - |1 - 1|\big| = 2, \quad x = \pm 3: \ \big|2 - |3 - 1|\big| = 0.$$

These match the marked points $(0, 1)$, $(\pm 1, 2)$ and the intercepts $(\pm 3, 0)$. Option (A) gives $y = 1$ at $x = 0$ but height 1 (not 2) at $x = \pm 1$, so it fails. The answer is (B).

Q7 2023 Sydney Girls Trial Q13(c) · **3 marks**

(i) Expand $(2y-1)^3$. (ii) Hence, find the inverse function of $f(x) = 4x^3 - 6x^2 + 3x$.

Criteria	**Marks**
(i) Correct expansion $8y^3 - 12y^2 + 6y - 1$	1
(ii) Recognises $(2x-1)^3 = 2f(x) - 1$ and rearranges to make x the subject	1
(ii) Correct inverse $f^{-1}(x) = \dfrac{1+\sqrt[3]{2x-1}}{2}$	1

Solution.

(i) Using $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$ with $a = 2y$, $b = 1$:

$$(2y-1)^3 = (2y)^3 - 3(2y)^2 + 3(2y) - 1 = 8y^3 - 12y^2 + 6y - 1.$$

(ii) Let $y = f(x) = 4x^3 - 6x^2 + 3x$. From part (i) with x in place of y,

$$(2x-1)^3 = 8x^3 - 12x^2 + 6x - 1 = 2(4x^3 - 6x^2 + 3x) - 1 = 2y - 1.$$

So $(2x-1)^3 = 2y - 1$. To invert, swap x and y and solve for y: $(2y-1)^3 = 2x - 1$, hence

$$2y - 1 = \sqrt[3]{2x-1} \quad \Longrightarrow \quad y = \frac{1+\sqrt[3]{2x-1}}{2}.$$

Therefore $f^{-1}(x) = \dfrac{1+\sqrt[3]{2x-1}}{2}$.

Q8 2021 Frensham Trial Q12(c) · **4 marks**

The graphs of $g(x) = mx + b$ and $y = 3|x|^3 - 2|x|^2$ are shown. The solution to the inequality $mx + b < 3|x|^3 - 2|x|^2$ is $x < -2$ or $x > 1$. (i) Sketch $y = 3|x|^3 - 2|x|^2$. (ii) Hence, find the values of m and b.

Criteria	**Marks**
(i) Correct even graph of $y = 3\|x\|^3 - 2\|x\|^2$ (symmetric in the y-axis)	1
(ii) Identifies the boundary points as the intersections $(1, 1)$ and $(-2, 16)$	1
(ii) Sets up the simultaneous equations from these two points	1
(ii) Correct values $m = -5$ and $b = 6$	1

Solution.

(i) The curve $y = 3|x|^3 - 2|x|^2$ is even (only $|x|$ appears), so it is symmetric about the y-axis. For $x \geq 0$ it is $y = 3x^3 - 2x^2$, which passes through the origin, dips slightly below the axis and turns, then rises; the sketch is this branch together with its mirror image in the y-axis.

(ii) The line $g(x) = mx + b$ meets the curve exactly at the boundaries of the solution set, that is at $x = -2$ and $x = 1$. On the curve,

$$x = 1 : \ 3(1)^3 - 2(1)^2 = 1, \qquad x = -2 : \ 3|-2|^3 - 2|-2|^2 = 3(8) - 2(4) = 16,$$

so the line passes through $(1, 1)$ and $(-2, 16)$. Substituting into $y = mx + b$,

$$1 = m + b, \qquad 16 = -2m + b.$$

Subtracting the first from the second: $15 = -3m$, so $m = -5$. Then $b = 1 - m = 1 - (-5) = 6$. Hence $m = -5$ and $b = 6$ (the line is $y = -5x + 6$).