Year 11 Mathematics Extension 1

Topic: Further Functions (2 of 5)

Review Questions (Worked Solutions): Inequalities

Term 3, 2026

Solving inequalities by type

1. Quadratic inequalities

Q1 2020 Girraween Prelim Q1 · **1 mark**

The solution of $x^2 - 5x + 6 \leq 0$ is

(A) $2 < x < 3$

(B) $2 \leq x \leq 3$

(C) $2 < x \leq 3$

(D) $2 \leq x < 3$

Criteria	Marks
Correct option (B)	1

Solution.

Factorise: $x^2 - 5x + 6 = (x - 2)(x - 3)$, so the roots are $x = 2$ and $x = 3$. The parabola opens upwards, so it is at or below zero between the roots (inclusive, since the inequality is $\leq$):

$$(x - 2)(x - 3) \leq 0 \quad \Longrightarrow \quad 2 \leq x \leq 3.$$

Both endpoints are included, so the answer is (B).

Q2 2021 Pymble Prelim Q3 · **1 mark**

What is the solution of the inequality $x^2 - 4x - 12 \leq 0$?

(A) $x \geq 12$ or $x \leq -12$

(B) $-12 \leq x \leq 12$

(C) $x \geq 6$ or $x \leq -2$

(D) $-2 \leq x \leq 6$

Criteria	**Marks**
Correct option (D)	1

Solution.

Factorise: $x^2 - 4x - 12 = (x - 6)(x + 2)$, so the roots are $x = 6$ and $x = -2$. The upward parabola is at or below zero between the roots:

$$(x - 6)(x + 2) \leq 0 \quad \Longrightarrow \quad -2 \leq x \leq 6.$$

The answer is (D).

Q3 2024 Penrith Prelim Q2 · **1 mark**

Which of the following represents the solution to the inequality $x^2 \leq 8 - 2x$?

(A) $(-4, 2)$

(B) $[-4, 2]$

(C) $(-\infty, -4) \cup (2, \infty)$

(D) $(-\infty, -4] \cup [2, \infty)$

Criteria	**Marks**
Correct option (B)	1

Solution.

Bring everything to one side:

$$x^2 \leq 8 - 2x \quad \Longrightarrow \quad x^2 + 2x - 8 \leq 0 \quad \Longrightarrow \quad (x + 4)(x - 2) \leq 0.$$

The roots are $x = -4$ and $x = 2$, and the upward parabola is at or below zero between them, endpoints included:

$$-4 \leq x \leq 2, \qquad \text{i.e. the closed interval } [-4, 2].$$

The answer is (B). (Because the inequality is $\leq$, both endpoints belong to the solution, so the closed interval $[-4, 2]$ is correct, not the open interval $(-4, 2)$.)

Q4 2022 Hurlstone Prelim Q11(a)(i) · **2 marks**

Solve the quadratic inequality $(5 - x)(2x + 1) \geq 0$.

Criteria	**Marks**
Correct critical values $x = 5$ and $x = -\frac{1}{2}$	1
Correct solution set $-\frac{1}{2} \leq x \leq 5$ with both boundaries included	1

Solution.

The critical values are where each factor is zero: $5 - x = 0$ gives $x = 5$, and $2x + 1 = 0$ gives $x = -\frac{1}{2}$. Expanding, $(5-x)(2x+1) = -2x^2 + 9x + 5$, a **downward** parabola, so the product is at or above zero between the two critical values:

$$-\frac{1}{2} \leq x \leq 5.$$

Both boundaries are included because the inequality is $\geq$.

Q5 2021 Sydney Grammar Prelim Q1(a) · **2 marks**

Solve the inequality $x^2 - 2x - 3 > 0$.

Criteria	**Marks**
Factorises and finds critical values $x = 3, x = -1$	1
Correct solution set $x < -1$ or $x > 3$ (strict)	1

Solution.

Factorise: $x^2 - 2x - 3 = (x - 3)(x + 1)$, so the roots are $x = 3$ and $x = -1$. The upward parabola is strictly above zero **outside** the roots:

$$x < -1 \quad \text{or} \quad x > 3.$$

The endpoints are excluded because the inequality is strict ($>$).

Q6 2022 Presbyterian Prelim Q11(e) · **2 marks**

Solve $3x^2 - 8x - 7 \geq 2x^2 - 2x + 9$.

Criteria	**Marks**
Collects to one side: $x^2 - 6x - 16 \geq 0$ and factorises	1
Correct solution set $x \leq -2$ or $x \geq 8$	1

Solution.

Move all terms to the left-hand side:

$$3x^2 - 8x - 7 - (2x^2 - 2x + 9) \geq 0 \quad \Longrightarrow \quad x^2 - 6x - 16 \geq 0.$$

Factorise: $x^2 - 6x - 16 = (x - 8)(x + 2)$, so the roots are $x = -2$ and $x = 8$. The upward parabola is at or above zero **outside** the roots, endpoints included:

$$x \leq -2 \quad \text{or} \quad x \geq 8.$$

2. Absolute value inequalities

Q7 2020 Hunters Hill Trial Q11(b) · 1 mark

Solve $|2x + 3| \leq 7$.

Criteria	Marks
Correct solution $-5 \leq x \leq 2$	1

Solution.

An inequality of the form $|A| \leq k$ (with $k > 0$) is equivalent to $-k \leq A \leq k$. So

$$-7 \leq 2x + 3 \leq 7.$$

Subtract 3 throughout: $-10 \leq 2x \leq 4$. Divide by 2:

$$-5 \leq x \leq 2.$$

Q8 2020 St George Girls Prelim Q5 · 1 mark

Solve $|x + 5| < 6$.

(A) $x < 1$

(B) $x > -11$

(C) $x < -11, x > 1$

(D) $-11 < x < 1$

Criteria	Marks
Correct option (D)	1

Solution.

An inequality of the form $|A| < k$ is equivalent to $-k < A < k$. So

$$-6 < x + 5 < 6.$$

Subtract 5 throughout:

$$-11 < x < 1.$$

The answer is (D).

Q9 2021 Presbyterian Prelim Q1 · **1 mark**

Which of the following is the solution to $|x-3|>4$?

(A) $-1<x<7$

(B) $x<-1$ or $x>7$

(C) $x<-7$ or $x>7$

(D) $x>1$ or $x<-1$

Criteria	Marks
Correct option (B)	1

Solution.

An inequality of the form $|A|>k$ (with $k>0$) splits into two cases: $A<-k$ or $A>k$. So

$$x-3<-4 \quad \text{or} \quad x-3>4.$$

Adding 3 to each: $x<-1$ or $x>7$. The answer is (B).

Q10 2025 HSC Q1 · **1 mark**

What is the solution to $|2x+3|<5$?

(A) $-4<x<1$

(B) $x<-4$ or $x>1$

(C) $-1<x<4$

(D) $x<-1$ or $x>4$

Criteria	Marks
Correct option (A)	1

Solution.

An inequality of the form $|A|<k$ is equivalent to $-k<A<k$. So

$$-5<2x+3<5.$$

Subtract 3 throughout: $-8<2x<2$. Divide by 2:

$$-4<x<1.$$

The answer is (A).

Q11 2021 Pymble Prelim Q5 · **1 mark**

What is the solution of the inequality $|2x - 4| > 6$?

(A) $-1 < x < 5$

(B) $x < -1$ or $x > 5$

(C) $-6 < x < 6$

(D) $x < -6$ or $x > 6$

Criteria	**Marks**
Correct option (B)	1

Solution.

An inequality of the form $|A| > k$ splits into $A < -k$ or $A > k$. So

$$2x - 4 < -6 \quad \text{or} \quad 2x - 4 > 6.$$

From the first, $2x < -2$, so $x < -1$. From the second, $2x > 10$, so $x > 5$. Hence

$$x < -1 \quad \text{or} \quad x > 5.$$

The answer is (B).

Q12 2024 Ascham Trial Q14(a) · **3 marks**

Solve $\dfrac{2}{|x-1|} \geq 3$.

Criteria	**Marks**
Notes $\lvert x-1\rvert > 0$ (so $x \neq 1$) and rearranges to $\lvert x-1\rvert \leq \dfrac{2}{3}$	1
Removes the modulus: $-\dfrac{2}{3} \leq x-1 \leq \dfrac{2}{3}$	1
Correct solution set $\dfrac{1}{3} \leq x \leq \dfrac{5}{3}, x \neq 1$	1

Solution.

First, $|x-1|$ must be positive for the fraction to be defined, so $x \neq 1$; and since $|x-1| > 0$, we may multiply both sides by it without reversing the inequality:

$$\frac{2}{|x-1|} \geq 3 \quad \Longrightarrow \quad 2 \geq 3\,|x-1| \quad \Longrightarrow \quad |x-1| \leq \frac{2}{3}.$$

Remove the modulus using $|A| \leq k \iff -k \leq A \leq k$:

$$-\frac{2}{3} \leq x-1 \leq \frac{2}{3}.$$

Add 1 throughout:

$$\frac{1}{3} \leq x \leq \frac{5}{3}.$$

We must still exclude $x = 1$ (where the original expression is undefined). So the solution is

$$\frac{1}{3} \leq x \leq \frac{5}{3}, \quad x \neq 1, \qquad \text{that is} \qquad \left[\tfrac{1}{3}, 1\right) \cup \left(1, \tfrac{5}{3}\right].$$

Q13 2024 Riverview Trial Q1 · **1 mark**

How many integer solutions are there for the inequality $\dfrac{1}{|x-2|} > \dfrac{1}{4}$?

(A) 7

(B) 6

(C) 5

(D) 4

Criteria	**Marks**
Correct option (B)	1

Solution.

Since $|x-2| > 0$ (which also requires $x \neq 2$), taking reciprocals of $\dfrac{1}{|x-2|} > \dfrac{1}{4}$ reverses the inequality:

$$|x-2| < 4.$$

Remove the modulus: $-4 < x-2 < 4$, so $-2 < x < 6$, together with $x \neq 2$. The integers in $-2 < x < 6$ are $-1, 0, 1, 2, 3, 4, 5$; removing $x = 2$ leaves

$$-1,\ 0,\ 1,\ 3,\ 4,\ 5,$$

which is 6 integer solutions. The answer is (B).

3. Rational inequalities: linear over linear

Q14 2024 Fort St Trial Q3 · **1 mark**

The solution to the inequality $\dfrac{3}{x-2} \leq 4$ is given by:

(A) $x < -2$ and $x \geq -\dfrac{11}{4}$

(B) $x > -2$ and $x \leq -\dfrac{11}{4}$

(C) $x < 2$ and $x \geq \dfrac{11}{4}$

(D) $x > 2$ and $x \leq \dfrac{11}{4}$

Criteria	Marks
Correct option (C)	1

Solution.

Multiply both sides by $(x-2)^2 > 0$ (valid for $x \neq 2$), which preserves the inequality:

$$3(x-2) \leq 4(x-2)^2.$$

Bring everything to one side:

$$4(x-2)^2 - 3(x-2) \geq 0 \implies (x-2)\big[4(x-2)-3\big] \geq 0 \implies (x-2)(4x-11) \geq 0.$$

The critical values are $x = 2$ and $x = \dfrac{11}{4}$. This upward parabola is ≥ 0 outside the roots: $x \leq 2$ or $x \geq \dfrac{11}{4}$. Excluding $x = 2$ (undefined) gives $x < 2$ or $x \geq \dfrac{11}{4}$. This is option **(C)**.

Q15 2023 Abbotsleigh Trial Q6 · **1 mark**

Which of the following is the solution set of the inequation $\dfrac{2}{x-1} \leq 3$?

(A) $x < 1$ and $x \geq \dfrac{5}{3}$

(B) $1 < x \leq \dfrac{5}{3}$

(C) $x \leq \dfrac{1}{3}$ and $x \geq \dfrac{5}{3}$

(D) $\dfrac{1}{3} \leq x \leq \dfrac{5}{3}$

Criteria	Marks
Correct option (A)	1

Solution.

Multiply by $(x-1)^2 > 0$ (for $x \neq 1$):

$$2(x-1) \leq 3(x-1)^2 \implies 3(x-1)^2 - 2(x-1) \geq 0 \implies (x-1)\big[3(x-1)-2\big] \geq 0,$$

so $(x-1)(3x-5) \geq 0$. Critical values $x = 1$ and $x = \dfrac{5}{3}$; the expression is ≥ 0 outside the roots. Excluding $x = 1$ gives $x < 1$ or $x \geq \dfrac{5}{3}$. This is option **(A)**.

Q16 2020 Glenwood Trial Q11(a) · **2 marks**

Solve $\dfrac{x+2}{x-5} \geq 0$.

Criteria	Marks
Identifies critical values $x = -2$ and $x = 5$ (with $x = 5$ excluded)	1
Correct solution $x \leq -2$ or $x > 5$	1

Solution.

A quotient is ≥ 0 when numerator and denominator have the same sign (or the numerator is 0), with $x \neq 5$. Multiply by $(x-5)^2 > 0$:

$$(x+2)(x-5) \geq 0.$$

Critical values are $x = -2$ and $x = 5$; the upward parabola is ≥ 0 outside the roots, so $x \leq -2$ or $x \geq 5$. Since $x = 5$ makes the original expression undefined, exclude it:

$$x \leq -2 \quad \text{or} \quad x > 5.$$

Q17 2022 HSC Q11(f) · **3 marks**

Solve $\dfrac{x}{2-x} \geq 5$.

Criteria	Marks
Multiplies by the squared denominator and forms a correct quadratic inequality	1
Correct critical values $x = \dfrac{5}{3}$ and $x = 2$ ($x = 2$ excluded)	1
Correct solution $\dfrac{5}{3} \leq x < 2$	1

Solution.

Multiply both sides by $(2-x)^2 > 0$ (for $x \neq 2$):

$$x(2-x) \geq 5(2-x)^2.$$

Bring to one side:

$$5(2-x)^2 - x(2-x) \leq 0 \;\Rightarrow\; (2-x)\big[5(2-x) - x\big] \leq 0 \;\Rightarrow\; (2-x)(10-6x) \leq 0.$$

Divide by 2: $(2-x)(5-3x) \leq 0$. The critical values are $x = 2$ and $x = \dfrac{5}{3}$. Writing it as $(x-2)(3x-5) \leq 0$ (multiplying by $+1$ after factoring out the two negatives), the expression is ≤ 0 between the roots: $\dfrac{5}{3} \leq x \leq 2$. Excluding $x = 2$:

$$\frac{5}{3} \leq x < 2.$$

Q18 2021 Manly Trial Q11(a) · **2 marks**

Solve for x given $\dfrac{x}{x-3} \geq 4$.

Criteria	Marks
Multiplies by the squared denominator to reach a correct quadratic inequality	1
Correct solution $3 < x \leq 4$	1

Solution.

Multiply by $(x-3)^2 > 0$ (for $x \neq 3$):

$$x(x-3) \geq 4(x-3)^2 \;\Rightarrow\; 4(x-3)^2 - x(x-3) \leq 0 \;\Rightarrow\; (x-3)\big[4(x-3) - x\big] \leq 0,$$

so $(x-3)(3x-12) \leq 0$, i.e. $3(x-3)(x-4) \leq 0$. Critical values $x = 3$ and $x = 4$; the expression is ≤ 0 between the roots: $3 \leq x \leq 4$. Excluding $x = 3$:

$$3 < x \leq 4.$$

Q19 2024 North Sydney Girls Trial Q11(a) · **3 marks**

Solve $\dfrac{x+1}{x-3} \leq 2$.

Criteria	Marks
Multiplies by the squared denominator and forms a correct quadratic inequality	1
Correct critical values $x = 3$ and $x = 7$ ($x = 3$ excluded)	1
Correct solution $x < 3$ or $x \geq 7$	1

Solution.

Multiply by $(x-3)^2 > 0$ (for $x \neq 3$):

$$(x+1)(x-3) \leq 2(x-3)^2 \implies 2(x-3)^2 - (x+1)(x-3) \geq 0.$$

Factor out $(x-3)$:

$$(x-3)\big[2(x-3) - (x+1)\big] \geq 0 \implies (x-3)(x-7) \geq 0.$$

Critical values $x = 3$ and $x = 7$; the upward parabola is ≥ 0 outside the roots, so $x \leq 3$ or $x \geq 7$. Excluding $x = 3$:

$$x < 3 \quad \text{or} \quad x \geq 7.$$

Q20 2021 Killara Trial Q11(a) · **2 marks**

Solve for x: $\dfrac{3x}{x-2} < 4$.

Criteria	Marks
Multiplies by the squared denominator to reach a correct quadratic inequality	1
Correct solution $x < 2$ or $x > 8$	1

Solution.

Multiply by $(x-2)^2 > 0$ (for $x \neq 2$):

$$3x(x-2) < 4(x-2)^2 \implies 4(x-2)^2 - 3x(x-2) > 0.$$

Factor out $(x-2)$:

$$(x-2)\big[4(x-2) - 3x\big] > 0 \implies (x-2)(x-8) > 0.$$

Critical values $x = 2$ and $x = 8$; the upward parabola is > 0 outside the roots:

$$x < 2 \quad \text{or} \quad x > 8.$$

4. Rational inequalities: quadratic numerator or denominator

Q21 2021 Newington Trial Q1 · 1 mark

What is the solution set of the inequality $\dfrac{x^2-6}{x} \leq 1$?

(A) $(-\infty, -2\,] \cup (0, 3\,]$

(B) $[-2, 0) \cup [3, \infty)$

(C) $[-2, 3\,]$

(D) $(-\infty, -2\,] \cup [0, 3\,]$

Criteria	Marks
Correct option (A)	1

Solution.

Multiply by $x^2 > 0$ (for $x \neq 0$), which preserves the inequality:

$$x(x^2-6) \leq x^2 \;\Rightarrow\; x^3 - x^2 - 6x \leq 0 \;\Rightarrow\; x\big(x^2 - x - 6\big) \leq 0 \;\Rightarrow\; x(x-3)(x+2) \leq 0.$$

The zeros are $x = -2,\ 0,\ 3$. Testing signs of the cubic $x(x-3)(x+2)$: it is ≤ 0 on $(-\infty, -2\,]$ and on $[\,0, 3\,]$. Since $x = 0$ is excluded (original undefined there), the solution is

$$x \leq -2 \quad \text{or} \quad 0 < x \leq 3,$$

that is $(-\infty, -2\,] \cup (0, 3\,]$. This is option **(A)**.

Q22 2021 Sydney Girls Trial Q11(c) · **2 marks**

Solve $\dfrac{x^2-9}{3x} > 0$.

Criteria	Marks
Correct critical values $x = -3,\ 0,\ 3$ (from numerator and denominator)	1
Correct solution $-3 < x < 0$ or $x > 3$	1

Solution.

Since $3 > 0$, the sign of $\dfrac{x^2-9}{3x}$ is the sign of $\dfrac{(x-3)(x+3)}{x}$. Multiply by $x^2 > 0$ (for $x \neq 0$):

$$x(x-3)(x+3) > 0.$$

The zeros are $x = -3,\ 0,\ 3$. A sign diagram for this cubic (positive leading coefficient) gives positive on $(-3, 0)$ and on $(3, \infty)$:

$$-3 < x < 0 \quad \text{or} \quad x > 3.$$

Q23 2024 Penrith Trial Q11(a) · **3 marks**

Solve $\dfrac{4-x^2}{x-1} \leq 0$.

Criteria	Marks
Factorises to $\dfrac{(2-x)(2+x)}{x-1}$ and identifies critical values $x = -2,\ 1,\ 2$	1
Correct sign analysis about the three critical values ($x = 1$ excluded)	1
Correct solution $-2 \leq x < 1$ or $x \geq 2$	1

Solution.

Factor the numerator: $\dfrac{(2-x)(2+x)}{x-1} \leq 0$. Multiply by $(x-1)^2 > 0$ (for $x \neq 1$):

$$(2-x)(2+x)(x-1) \leq 0.$$

Multiply through by -1 (from $2 - x = -(x-2)$) to write it with positive leading terms:

$$(x-2)(x+2)(x-1) \geq 0.$$

The zeros are $x = -2,\ 1,\ 2$. This cubic (positive leading coefficient) is ≥ 0 on $[-2, 1]$ and on $[2, \infty)$. Excluding $x = 1$ (undefined):

$$-2 \leq x < 1 \quad \text{or} \quad x \geq 2.$$

Q24 2022 SCEGGS Prelim Q9(c) · 3 marks

Solve $\dfrac{x^2+6}{x} \geq 5$.

Criteria	Marks
Multiplies by $x^2 > 0$ and forms a correct cubic (or reasons via a sign diagram)	1
Correct critical values $x = 0,\ 2,\ 3$ ($x = 0$ excluded)	1
Correct solution $0 < x \leq 2$ or $x \geq 3$	1

Solution.

Multiply by $x^2 > 0$ (for $x \neq 0$):

$$x(x^2+6) \geq 5x^2 \implies x^3 - 5x^2 + 6x \geq 0 \implies x\left(x^2 - 5x + 6\right) \geq 0 \implies x(x-2)(x-3) \geq 0.$$

The zeros are $x = 0,\ 2,\ 3$. A sign diagram for this cubic (positive leading coefficient) gives ≥ 0 on $[\,0, 2\,]$ and on $[\,3, \infty)$. Excluding $x = 0$ (undefined):

$$0 < x \leq 2 \quad \text{or} \quad x \geq 3.$$

Q25 2022 St George Girls Trial Q11(d) · 3 marks

Solve the inequality $\dfrac{x^2}{x-2} \geq -1$.

Criteria	Marks
Multiplies by $(x-2)^2 > 0$ and forms a correct cubic inequality	1
Correct critical values $x = -2,\ 1,\ 2$ ($x = 2$ excluded)	1
Correct solution $-2 \leq x \leq 1$ or $x > 2$	1

Solution.

Multiply by $(x-2)^2 > 0$ (for $x \neq 2$):

$$x^2(x-2) \geq -(x-2)^2 \implies x^2(x-2) + (x-2)^2 \geq 0.$$

Factor out $(x-2)$:

$$(x-2)\left[x^2 + (x-2)\right] \geq 0 \implies (x-2)\left(x^2 + x - 2\right) \geq 0 \implies (x-2)(x+2)(x-1) \geq 0.$$

The zeros are $x = -2,\ 1,\ 2$. This cubic (positive leading coefficient) is ≥ 0 on $[-2, 1\,]$ and on $[\,2, \infty)$. Excluding $x = 2$ (undefined):

$$-2 \leq x \leq 1 \quad \text{or} \quad x > 2.$$

Q26 2020 Baulkham Hills Prelim Q11(b)(ii) · **3 marks**

Solve the inequality $\dfrac{7}{9-x^2} > -1$.

Criteria	**Marks**
Multiplies by $(9-x^2)^2 > 0$ and forms a correct quartic inequality	1
Factorises to critical values $x = -4,\ -3,\ 3,\ 4$ ($x = \pm 3$ excluded)	1
Correct solution $x < -4$ or $-3 < x < 3$ or $x > 4$	1

Solution.

Note $9 - x^2 = (3-x)(3+x)$, so $x \neq \pm 3$. Multiply both sides by $(9-x^2)^2 > 0$:

$$7(9-x^2) > -(9-x^2)^2 \implies 7(9-x^2) + (9-x^2)^2 > 0.$$

Let $u = 9 - x^2$: $u(7+u) > 0$, i.e. $(9-x^2)(16-x^2) > 0$. Factor:

$$(3-x)(3+x)(4-x)(4+x) > 0 \implies (x-3)(x+3)(x-4)(x+4) > 0$$

(the two sign changes from $3-x$ and $4-x$ cancel). The zeros are $x = -4,\ -3,\ 3,\ 4$. This quartic (positive leading coefficient) is > 0 outside $[-4,-3]$ and outside $[3,4]$, and positive on the middle interval $(-3,3)$. Reading the sign diagram:

$$x < -4 \quad \text{or} \quad -3 < x < 3 \quad \text{or} \quad x > 4.$$

(The excluded points $x = \pm 3$ do not appear in the solution.)

5. Rational inequalities with a variable on both sides

Q27 2025 Barker Trial Q11(a) · **3 marks**

Solve $\dfrac{2}{x-1} < x$.

Criteria	Marks
Multiplies through by $(x-1)^2 > 0$ and rearranges to a factored cubic	1
Correctly factorises to $(x+1)(x-1)(x-2) > 0$ and sign-analyses (noting $x \neq 1$)	1
Correct solution $-1 < x < 1$ or $x > 2$	1

Solution.

Since $x = 1$ makes the left side undefined, $x \neq 1$. Multiply both sides by $(x-1)^2$, which is positive for all $x \neq 1$ so the inequality sign is preserved:

$$2(x-1) < x(x-1)^2.$$

Bring everything to one side:

$$x(x-1)^2 - 2(x-1) > 0 \implies (x-1)\big[x(x-1) - 2\big] > 0 \implies (x-1)(x^2 - x - 2) > 0,$$

and $x^2 - x - 2 = (x-2)(x+1)$, so

$$(x+1)(x-1)(x-2) > 0.$$

The roots are $x = -1,\ 1,\ 2$ (all simple). With a positive leading coefficient the cubic is negative on $(-\infty, -1)$, positive on $(-1, 1)$, negative on $(1, 2)$, and positive on $(2, \infty)$. We need it strictly positive, and $x = 1$ is excluded from the domain anyway:

$$-1 < x < 1 \quad \text{or} \quad x > 2.$$

Q28 2025 Caringbah Prelim Q7(a) · **3 marks**

Solve $\dfrac{2}{x-1} \geq x$.

Criteria	**Marks**
Multiplies through by $(x-1)^2 > 0$ and rearranges to a factored cubic	1
Correctly factorises to $(x+1)(x-1)(x-2) \leq 0$ and sign-analyses (noting $x \neq 1$)	1
Correct solution $x \leq -1$ or $1 < x \leq 2$	1

Solution.

Here $x = 1$ makes the left side undefined, so $x \neq 1$. Multiply both sides by $(x-1)^2 > 0$ (which keeps the inequality direction):

$$2(x-1) \geq x(x-1)^2.$$

Rearrange to one side:

$$x(x-1)^2 - 2(x-1) \leq 0 \implies (x-1)(x^2 - x - 2) \leq 0 \implies (x+1)(x-1)(x-2) \leq 0.$$

The roots are $x = -1,\ 1,\ 2$. With a positive leading coefficient the cubic is negative on $(-\infty, -1)$, positive on $(-1, 1)$, negative on $(1, 2)$, and positive on $(2, \infty)$. We want it ≤ 0, including the roots $x = -1$ and $x = 2$ (both give equality in the original), but $x = 1$ must be excluded because the original expression is undefined there:

$$x \leq -1 \quad \text{or} \quad 1 < x \leq 2.$$

Q29 2022 Hunters Hill Trial Q11(a) · **3 marks**

Solve $\dfrac{4}{x+3} \geq x$.

Criteria	**Marks**
Multiplies through by $(x+3)^2 > 0$ and rearranges to a factored cubic	1
Correctly factorises to $(x+4)(x+3)(x-1) \leq 0$ and sign-analyses (noting $x \neq -3$)	1
Correct solution $x \leq -4$ or $-3 < x \leq 1$	1

Solution.

The left side is undefined at $x = -3$, so $x \neq -3$. Multiply both sides by $(x+3)^2 > 0$:

$$4(x+3) \geq x(x+3)^2.$$

Rearrange:

$$x(x+3)^2 - 4(x+3) \leq 0 \implies (x+3)\big[x(x+3) - 4\big] \leq 0 \implies (x+3)(x^2+3x-4) \leq 0,$$

and $x^2 + 3x - 4 = (x+4)(x-1)$, so

$$(x+4)(x+3)(x-1) \leq 0.$$

The roots are $x = -4,\ -3,\ 1$. With a positive leading coefficient the cubic is negative on $(-\infty, -4)$, positive on $(-4, -3)$, negative on $(-3, 1)$, and positive on $(1, \infty)$. We want it ≤ 0, keeping the roots $x = -4$ and $x = 1$, but excluding $x = -3$ (undefined in the original):

$$x \leq -4 \quad \text{or} \quad -3 < x \leq 1.$$

Q30 2020 Sydney Tech Trial Q11(f) · 3 marks

Solve $\dfrac{2x}{x-3} \geq x+4$.

Criteria	Marks
Multiplies through by $(x-3)^2 > 0$ and rearranges to a factored cubic	1
Correctly factorises to $(x+3)(x-3)(x-4) \leq 0$ and sign-analyses (noting $x \neq 3$)	1
Correct solution $x \leq -3$ or $3 < x \leq 4$	1

Solution.

The left side is undefined at $x = 3$, so $x \neq 3$. Multiply both sides by $(x-3)^2 > 0$:

$$2x(x-3) \geq (x+4)(x-3)^2.$$

Rearrange to one side:

$$(x+4)(x-3)^2 - 2x(x-3) \leq 0 \implies (x-3)\big[(x+4)(x-3) - 2x\big] \leq 0.$$

Now $(x+4)(x-3) - 2x = x^2 + x - 12 - 2x = x^2 - x - 12 = (x-4)(x+3)$, so

$$(x+3)(x-3)(x-4) \leq 0.$$

The roots are $x = -3,\ 3,\ 4$. With a positive leading coefficient the cubic is negative on $(-\infty, -3)$, positive on $(-3, 3)$, negative on $(3, 4)$, and positive on $(4, \infty)$. We want it ≤ 0, keeping the roots $x = -3$ and $x = 4$, but excluding $x = 3$ (undefined in the original):

$$x \leq -3 \quad \text{or} \quad 3 < x \leq 4.$$

Q31 2025 Sydney Boys Trial Q11(d) · 3 marks

Solve $\dfrac{1+x}{x} \geq 2x+1$ for x.

Criteria	Marks
Multiplies through by $x^2 > 0$ and rearranges to a factored cubic	1
Correctly factorises to $x(2x^2-1) \leq 0$ and sign-analyses (noting $x \neq 0$)	1
Correct solution $x \leq -\frac{1}{\sqrt{2}}$ or $0 < x \leq \frac{1}{\sqrt{2}}$	1

Solution.

The left side is undefined at $x = 0$, so $x \neq 0$. Multiply both sides by $x^2 > 0$ (which preserves the inequality):

$$(1+x)x \geq (2x+1)x^2.$$

Rearrange to one side:

$$(2x+1)x^2 - (1+x)x \leq 0 \implies x\big[(2x+1)x - (1+x)\big] \leq 0 \implies x\big(2x^2+x-1-x\big) \leq 0,$$

which simplifies to

$$x(2x^2-1) \leq 0.$$

The roots are $x = 0$ and $2x^2 - 1 = 0 \Rightarrow x = \pm\dfrac{1}{\sqrt{2}}$. With a positive leading coefficient the cubic $x(2x^2-1)$ is negative on $\left(-\infty, -\frac{1}{\sqrt{2}}\right)$, positive on $\left(-\frac{1}{\sqrt{2}}, 0\right)$, negative on $\left(0, \frac{1}{\sqrt{2}}\right)$, and positive on $\left(\frac{1}{\sqrt{2}}, \infty\right)$. We want it ≤ 0, keeping the roots $x = \pm\frac{1}{\sqrt{2}}$, but excluding $x = 0$ (undefined in the original):

$$x \leq -\frac{1}{\sqrt{2}} \quad \text{or} \quad 0 < x \leq \frac{1}{\sqrt{2}}.$$

Q32 2021 Sydney Grammar Prelim Q4(d) · **4 marks**

Solve the inequality $\dfrac{6}{x-1} \geq x+4$.

Criteria	Marks
States $x \neq 1$ and multiplies through by $(x-1)^2 > 0$	1
Rearranges to one side and factorises to $(x+5)(x-1)(x-2) \leq 0$	1
Sign-analyses the cubic across all four intervals	1
Correct solution $x \leq -5$ or $1 < x \leq 2$ (with $x = 1$ excluded)	1

Solution.

The left side is undefined at $x = 1$, so $x \neq 1$. Multiply both sides by $(x-1)^2 > 0$ (which keeps the inequality direction):

$$6(x-1) \geq (x+4)(x-1)^2.$$

Bring everything to one side:

$$(x+4)(x-1)^2 - 6(x-1) \leq 0 \implies (x-1)\big[(x+4)(x-1) - 6\big] \leq 0.$$

Now $(x+4)(x-1) - 6 = x^2 + 3x - 4 - 6 = x^2 + 3x - 10 = (x+5)(x-2)$, so

$$(x+5)(x-1)(x-2) \leq 0.$$

The roots are $x = -5,\ 1,\ 2$. With a positive leading coefficient the cubic is negative on $(-\infty, -5)$, positive on $(-5, 1)$, negative on $(1, 2)$, and positive on $(2, \infty)$. We want it ≤ 0, keeping the roots $x = -5$ and $x = 2$ (both give equality in the original), but excluding $x = 1$ (undefined in the original):

$$x \leq -5 \quad \text{or} \quad 1 < x \leq 2.$$

6. Cubic and polynomial inequalities

Q33 2022 Girraween Prelim Q5 · 1 mark

Solve the inequality $(x+2)(2x+1)(x-3) \leq 0$. (A) $x \leq -2$ or $-0.5 \leq x \leq 3$ (B) $x \leq 2$ or $-0.5 \leq x \leq 3$ (C) $x \leq -2$ or $0.5 \leq x \leq 3$ (D) $x < 2$ or $-0.5 \leq x \leq 3$.

Criteria	Marks
Correct option (A)	1

Solution.

The factors give simple roots $x = -2,\ -\frac{1}{2},\ 3$. With a positive leading coefficient, the product is negative before -2, positive on $(-2, -\frac{1}{2})$, negative on $(-\frac{1}{2}, 3)$, and positive beyond 3. We need ≤ 0 (the negative parts, plus the roots):

$$x \leq -2 \quad \text{or} \quad -\tfrac{1}{2} \leq x \leq 3.$$

The answer is **(A)**.

Q34 2022 Sydney Boys Trial Q2 · 1 mark

What are the values of x for which $(x+3)(x-1)(2-x) > 0$? (A) $x < -3$ or $1 < x < 2$ (B) $x > -3$ or $1 < x < 2$ (C) $-3 < x < 1$ or $x > 2$ (D) $-3 < x < 1$.

Criteria	Marks
Correct option (A)	1

Solution.

Same factored cubic as Q27 but with a strict inequality. Multiplying by -1: $(x+3)(x-1)(2-x) > 0$ is equivalent to $(x+3)(x-1)(x-2) < 0$. With roots $-3,\ 1,\ 2$ and a positive leading coefficient, the cubic is negative on $(-\infty, -3)$ and on $(1, 2)$. Hence

$$x < -3 \quad \text{or} \quad 1 < x < 2,$$

with strict inequalities (boundary points excluded). The answer is **(A)**.

Q35 2023 Hunters Hill Trial Q11(c) · **3 marks**

Solve $(x^2 - 1)(x + 3) < 0$.

Criteria	Marks
Fully factorises to $(x - 1)(x + 1)(x + 3)$ and identifies the roots $x = -3,\ -1,\ 1$	1
Correctly analyses the sign of the cubic across the intervals	1
Correct solution $x < -3$ or $-1 < x < 1$	1

Solution.

Factorise the difference of squares: $(x^2 - 1)(x + 3) = (x - 1)(x + 1)(x + 3)$. The roots are $x = -3,\ -1,\ 1$, all simple. The leading coefficient is positive, so $y = (x - 1)(x + 1)(x + 3)$ is negative on $(-\infty, -3)$, positive on $(-3, -1)$, negative on $(-1, 1)$, and positive on $(1, \infty)$. We need the product strictly less than 0:

$$x < -3 \quad \text{or} \quad -1 < x < 1.$$

Q36 2025 Blacktown Boys Trial Q5 · **1 mark**

Given that $P(x) = x^3 + x^2 - 8x - 12$ has a double root at $x = -2$, which of the following is the solution to $P(x) < 0$? (A) $x < -2$ (B) $x < 3$ (C) $-2 < x < 3$ (D) $x < -2,\ -2 < x < 3$.

Criteria	Marks
Correct option (D)	1

Solution.

Since $x = -2$ is a double root, $(x + 2)^2$ is a factor. Dividing (or matching the constant term), $P(x) = (x + 2)^2(x - 3)$. The factor $(x + 2)^2 \geq 0$ for all x and is 0 only at $x = -2$, so the sign of $P(x)$ is governed by $(x - 3)$ except at the double root, where $P = 0$. Thus $P(x) < 0$ requires $x - 3 < 0$ (so $x < 3$) while excluding $x = -2$ (where $P = 0$, not < 0):

$$x < -2 \quad \text{or} \quad -2 < x < 3.$$

The answer is **(D)**. (Note the sign does not change at the double root, which is why the point $x = -2$ is a single excluded value inside the interval $x < 3$.)

Q37 2024 Sydney Boys Trial Q11(a) · 3 marks

By first factorising $f(x)$, where $f(x) = -x^3 - 2x^2 + 4x + 8$, solve $f(x) < 0$.

Criteria	Marks
Factorises $f(x)$ (e.g. by grouping) to $-(x-2)(x+2)^2$	1
Identifies the double root at $x = -2$ and the simple root at $x = 2$, and analyses the sign	1
Correct solution $x > 2$	1

Solution.

Factorise by grouping. Take out -1 first: $f(x) = -\left(x^3 + 2x^2 - 4x - 8\right)$. Group:

$$x^3 + 2x^2 - 4x - 8 = x^2(x+2) - 4(x+2) = (x+2)(x^2-4) = (x+2)(x-2)(x+2) = (x+2)^2(x-2).$$

Hence $f(x) = -(x+2)^2(x-2)$. Now $(x+2)^2 \geq 0$ always (and $= 0$ only at $x = -2$), so

$$f(x) < 0 \iff -(x+2)^2(x-2) < 0 \iff (x+2)^2(x-2) > 0.$$

Since $(x+2)^2 > 0$ for $x \neq -2$, this needs $x - 2 > 0$, i.e.

$$x > 2.$$

(At $x = -2$, $f = 0$, not negative, so it is excluded, but it already lies outside $x > 2$.)

Graphical and reasoning

7. Solving an inequality using a graph

Q38 2024 Cheltenham Girls Trial Q3 · 1 mark

The graph shows $y = x^2$ with the lines $y = x + a$ and $y = a - x$ and their intersection points with the curve (constants a, m, n, p, q all positive). What is the solution to $x^2 + x - a < 0$? (A) $-n < x < -m$ (B) $-n < x < m$ (C) $-m < x < m$ (D) $-m < x < n$.

Criteria	Marks
Correct option (B)	1

Solution.

Rearrange the inequality to compare the two graphs shown: $x^2 + x - a < 0 \iff x^2 < a - x$, i.e. the solution is where the parabola $y = x^2$ lies below the line $y = a - x$. On the diagram, $y = x^2$ and $y = a - x$ meet at the two points whose x-coordinates are $x = -n$ (the left intersection) and $x = m$ (the right intersection). Between these two crossings the parabola is below the line, so

$$-n < x < m.$$

The answer is **(B)**.

Q39 2025 Hurlstone Prelim Q11(c) · 2 marks

The graph shows $y = x - 2$ and $y = \dfrac{1}{x}$, intersecting at two points. Solve $x - 2 > \dfrac{1}{x}$.

Criteria	Marks
Finds the intersection x-values $x = 1 \pm \sqrt{2}$ (and uses the asymptote $x = 0$)	1
Correct solution $1 - \sqrt{2} < x < 0$ or $x > 1 + \sqrt{2}$	1

Solution.

The line $y = x - 2$ meets $y = \dfrac{1}{x}$ where $x - 2 = \dfrac{1}{x}$. Multiplying by x (noting $x \neq 0$) gives $x^2 - 2x = 1$, i.e. $x^2 - 2x - 1 = 0$, so

$$x = 1 \pm \sqrt{2}.$$

Reading the diagram, the line is above the hyperbola on two stretches. On the right branch ($x > 0$) the line is above the hyperbola once past the right intersection $x = 1 + \sqrt{2}$. On the left of the vertical asymptote at $x = 0$, the hyperbola drops to $-\infty$, so the line lies above it from the left intersection $x = 1 - \sqrt{2}$ up to (but not including) $x = 0$. Hence

$$1 - \sqrt{2} < x < 0 \quad \text{or} \quad x > 1 + \sqrt{2}.$$

(A quick test confirms this: at $x = -0.1$, $x - 2 = -2.1 > \frac{1}{x} = -10$; at $x = 3$, $x - 2 = 1 > \frac{1}{3}$.)

Q40 2025 Sydney Girls Trial Q13(b) · 3 marks

The curves $y = 1 + 3x - x^2$ and $y = \dfrac{3}{x}$ are sketched. Solve $1 + 3x - x^2 \leq \dfrac{3}{x}$.

Criteria	Marks
Finds the intersection x-values $x = -1,\ 1,\ 3$ (from $x^3 - 3x^2 - x + 3 = 0$)	1
Uses the graph / asymptote $x = 0$ to determine where the parabola is at or below the hyperbola	1
Correct solution $x \leq -1$ or $0 < x \leq 1$ or $x \geq 3$	1

Solution.

The curves meet where $1 + 3x - x^2 = \dfrac{3}{x}$. Multiplying by x (and noting $x \neq 0$):

$$x(1 + 3x - x^2) = 3 \implies x + 3x^2 - x^3 = 3 \implies x^3 - 3x^2 - x + 3 = 0.$$

Factor by grouping: $x^2(x - 3) - (x - 3) = (x - 3)(x^2 - 1) = (x - 3)(x - 1)(x + 1) = 0$, so the intersections are at

$$x = -1,\ 1,\ 3.$$

We want where the parabola $y = 1 + 3x - x^2$ is at or below the hyperbola $y = \dfrac{3}{x}$. The hyperbola has a vertical asymptote at $x = 0$: it is negative for $x < 0$ and positive for $x > 0$. Reading the sketch together with the crossing points:

- For $x \leq -1$: the (concave-down) parabola is at or below the negative branch of the hyperbola. Included.
- For $-1 < x < 0$: the parabola is above the hyperbola. Excluded.
- For $0 < x \leq 1$: the parabola is at or below the hyperbola. Included (with $x = 0$ excluded from the domain).
- For $1 < x < 3$: the parabola is above the hyperbola. Excluded.
- For $x \geq 3$: the parabola (now falling steeply) is at or below the hyperbola. Included.

Hence

$$x \leq -1 \quad \text{or} \quad 0 < x \leq 1 \quad \text{or} \quad x \geq 3.$$

Q41 2024 HSC Q12(e) · **3 marks**

The diagram shows the graph of $y = \dfrac{1}{|x-5|}$. For what values of x is $\dfrac{x}{6} \geq \dfrac{1}{|x-5|}$?

Criteria	**Marks**
Considers the two cases $x < 5$ and $x > 5$ (removing the absolute value)	1
Solves the resulting quadratics: boundary points $x = 2, 3$ (for $x < 5$) and $x = 6$ (for $x > 5$)	1
Correct solution $2 \leq x \leq 3$ or $x \geq 6$	1

Solution.

The line $y = \dfrac{x}{6}$ must be at or above $y = \dfrac{1}{|x-5|}$. Note $x = 5$ is excluded (the right side is undefined there), and since $\dfrac{1}{|x-5|} > 0$ we need $\dfrac{x}{6} > 0$, so only $x > 0$ can work. Split on the absolute value.

Case $x < 5$: $|x-5| = 5 - x$, so $\dfrac{x}{6} \geq \dfrac{1}{5-x}$. Here $5 - x > 0$, so multiply through:

$$x(5-x) \geq 6 \implies 5x - x^2 \geq 6 \implies x^2 - 5x + 6 \leq 0 \implies (x-2)(x-3) \leq 0,$$

giving $2 \leq x \leq 3$ (which lies inside $x < 5$).

Case $x > 5$: $|x-5| = x - 5$, so $\dfrac{x}{6} \geq \dfrac{1}{x-5}$. Here $x - 5 > 0$, so multiply through:

$$x(x-5) \geq 6 \implies x^2 - 5x - 6 \geq 0 \implies (x-6)(x+1) \geq 0,$$

giving $x \leq -1$ or $x \geq 6$; intersecting with $x > 5$ leaves $x \geq 6$.

Combining the two cases:

$$2 \leq x \leq 3 \quad \text{or} \quad x \geq 6.$$

Q42 2023 Hurlstone Prelim Q7 · **1 mark**

The graphs of $f(x) = |2x - 3|$ and $g(x) = |x + 1|$ are shown. Which of the following is the solution to $|2x - 3| + |x + 1| \leq 4$? (A) $[0, 2]$ (B) $(-\infty, 0] \cup [2, \infty)$ (C) $[2, \infty)$ (D) $(-\infty, 0]$.

Criteria	Marks
Correct option (A)	1

Solution.

Let $h(x) = |2x - 3| + |x + 1|$. The two "corners" are at $x = \frac{3}{2}$ and $x = -1$; test the boundary $h(x) = 4$ on each relevant piece.

For $-1 \leq x \leq \frac{3}{2}$: $|2x - 3| = 3 - 2x$ and $|x + 1| = x + 1$, so $h(x) = (3 - 2x) + (x + 1) = 4 - x$. Then $h(x) \leq 4 \iff 4 - x \leq 4 \iff x \geq 0$, giving $0 \leq x \leq \frac{3}{2}$.

For $x > \frac{3}{2}$: $h(x) = (2x - 3) + (x + 1) = 3x - 2$. Then $3x - 2 \leq 4 \iff x \leq 2$, giving $\frac{3}{2} < x \leq 2$.

For $x < -1$: $h(x) = (3 - 2x) - (x + 1) = 2 - 3x$, which exceeds 4 for all $x < -1$ (e.g. at $x = -1$, $h = 5$), so no solutions here.

Combining the pieces gives $0 \leq x \leq 2$, i.e. $[0, 2]$. The answer is **(A)**.

8. Reasoning about equivalent inequalities

Q43 Killara 2024 Trial Q5 · **1 mark**

Which inequality below has the same solution as $|x+5|+|x-1|=6$?

(A) $\dfrac{1}{x^2-4x-5} \leq 0$

(B) $x^2-4x-5<0$

(C) $|x+2| \leq 3$

(D) $|x+2| \geq 3$

Criteria	Marks
Correct option (C)	1

Solution.

Read $|x+5|+|x-1|$ as the sum of the distances from x to -5 and from x to 1. The distance between -5 and 1 is 6, so the sum of distances equals 6 exactly when x lies between them (inclusive):

$$|x+5|+|x-1|=6 \iff -5 \leq x \leq 1.$$

Now test each option against the target set $[-5,1]$.

(C) $|x+2| \leq 3 \iff -3 \leq x+2 \leq 3 \iff -5 \leq x \leq 1$. This is exactly the target set.

The other options do not match: $x^2-4x-5=(x-5)(x+1)$, so $x^2-4x-5<0$ gives $-1<x<5$ (option B), and option A gives the same open interval; option D gives $x \leq -5$ or $x \geq 1$. The answer is **(C)**.

Q44 Baulkham Hills 2020 Prelim Q9 · **1 mark**

Which inequality has the same solution as $|x+3|+|x-4|=7$?

(A) $|2x-1| \geq 7$

(B) $x^2 - x - 12 \leq 0$

(C) $\dfrac{7}{3-x} \geq 1$

(D) $\dfrac{1}{x-4} - \dfrac{1}{x+3} \leq 0$

Criteria	Marks
Correct option (B)	1

Solution.

As in Q38, $|x+3|+|x-4|$ is the sum of the distances from x to -3 and to 4. The distance between -3 and 4 is 7, so the sum equals 7 exactly when x lies between them (inclusive):

$$|x+3|+|x-4|=7 \iff -3 \leq x \leq 4.$$

The set $-3 \leq x \leq 4$ is precisely where $(x+3)(x-4) \leq 0$, and $(x+3)(x-4) = x^2 - x - 12$. So the equivalent inequality is

$$x^2 - x - 12 \leq 0,$$

which is option **(B)**.

Q45 Normanhurst Boys 2021 Trial Q2 · **1 mark**

Which of the following has the same solutions as the inequality $\dfrac{-x}{(x-3)(2x+1)} \leq \dfrac{1}{4}$?

(A) $(2x-3)(x+1)(x-3)(2x+1) \geq 0$

(B) $(2x-3)(x+1)(x-3)(2x+1) \leq 0$

(C) $(2x-3)(x+1) \geq 0$

(D) $(2x-3)(x+1) \leq 0$

Criteria	**Marks**
Correct option (A)	1

Solution.

Bring everything to one side so we can compare with zero:

$$\frac{1}{4} - \frac{-x}{(x-3)(2x+1)} \geq 0 \iff \frac{1}{4} + \frac{x}{(x-3)(2x+1)} \geq 0.$$

Combine over the common denominator $4(x-3)(2x+1)$:

$$\frac{4x + (x-3)(2x+1)}{4(x-3)(2x+1)} \geq 0 \iff \frac{2x^2 - x - 3}{4(x-3)(2x+1)} \geq 0 \iff \frac{(x+1)(2x-3)}{4(x-3)(2x+1)} \geq 0.$$

The standard technique for a rational inequality is to multiply both sides by the **square** of the denominator, which is positive and so preserves the direction. Multiplying by $\left[4(x-3)(2x+1)\right]^2 > 0$:

$$(x+1)(2x-3) \cdot 4(x-3)(2x+1) \geq 0,$$

and dropping the positive constant 4 gives

$$(2x-3)(x+1)(x-3)(2x+1) \geq 0.$$

This is option **(A)**. (The excluded values $x = 3$ and $x = -\frac{1}{2}$, where the original is undefined, are handled separately as usual.) The answer is **(A)**.

Q46 James Ruse 2024 Trial Q5 · **1 mark**

Which of the following has the same solution as that of $\dfrac{3}{2-x} > 2$?

(A) $2x - 1 \geq 0$

(B) $(x-2)(2x-1) > 0$

(C) $(x-2)(2x-1) < 0$

(D) None of the above.

Criteria	**Marks**
Correct option (C)	1

Solution.

The safe move is to multiply both sides by $(2-x)^2$, which is positive (so the inequality direction is preserved) whenever $x \neq 2$:

$$\frac{3}{2-x} > 2 \implies 3(2-x) > 2(2-x)^2 \implies 2(2-x)^2 - 3(2-x) < 0.$$

Expanding,

$$2(2-x)^2 - 3(2-x) = 2x^2 - 5x + 2 = (x-2)(2x-1),$$

so the inequality becomes $(x-2)(2x-1) < 0$, giving $\frac{1}{2} < x < 2$. This is exactly the solution set of the original (check: on $\frac{1}{2} < x < 2$ we have $2 - x > 0$ and $\frac{3}{2-x} > 2$), so option **(C)** has the same solution set. The answer is **(C)**.

Q47 Sydney Grammar 2020 Trial Q10 · 1 mark

In trying to solve $\dfrac{x-2}{x-3} < \dfrac{4}{\sqrt{x-2}}$ over its natural domain, three students produce:

$$\begin{aligned}
&\text{Student I} && (x-2)\sqrt{x-2} < 4(x-3)\\
&\text{Student II} && \sqrt{(x-2)^3(x-3)} < 4(x-3)^2\\
&\text{Student III} && (x-2)^2(x-3) < 4(x-3)^2\sqrt{x-2}
\end{aligned}$$

Which students are still on track to obtain the correct solution set?

(A) Just Student II

(B) Student III only

(C) Both Students II and III but not Student I

(D) All three students

Criteria	Marks
Correct option (B)	1

Solution.

First fix the natural domain. For $\sqrt{x-2}$ to be real and nonzero we need $x > 2$, and the left side needs $x \neq 3$. So the domain is $x > 2$, $x \neq 3$. The key principle: when you clear denominators, you may only multiply by a quantity that is **positive throughout the domain**, otherwise the inequality direction can flip and the solution set changes.

Student I multiplied both sides by $(x-3)$ (after multiplying by $\sqrt{x-2} > 0$):

$$\frac{(x-2)\sqrt{x-2}}{x-3} < 4 \xrightarrow{\times(x-3)} (x-2)\sqrt{x-2} < 4(x-3).$$

But $x-3$ is **negative** on $2 < x < 3$, so multiplying by it there reverses the true inequality. Student I therefore loses the entire branch $2 < x < 3$ and is off track.

Student II has effectively squared and rearranged in a way that changes the relation. On $2 < x < 3$ the quantity $(x-2)^3(x-3) < 0$, so $\sqrt{(x-2)^3(x-3)}$ is not even defined there, and elsewhere the resulting set does not match the original. Student II is off track.

Student III multiplied both sides by $(x-2)(x-3)^2$:

$$\frac{x-2}{x-3}\cdot(x-2)(x-3)^2 = (x-2)^2(x-3), \qquad \frac{4}{\sqrt{x-2}}\cdot(x-2)(x-3)^2 = 4(x-3)^2\sqrt{x-2}.$$

On the domain $x > 2$, $x \neq 3$ the multiplier $(x-2)(x-3)^2$ is strictly positive, so the inequality direction is preserved and the solution set is unchanged. Student III is on track.

Only Student III preserves the solution set, so the answer is **(B)**.