Year 11 Mathematics Extension 1

Topic: The Binomial Theorem (1 of 2)
Review Questions
Term 3, 2026

How to use this review set

These are real past-paper questions, grouped by question pattern. The grey label on each question shows where it is from and its marks.

- The questions are grouped by the pattern that keeps coming up in exams, a few of each in a row, running from the plainer patterns into the harder ones near the end. Work through them and you will meet every type an exam can throw at you.
- This set **is your diagnostic**. Working through it shows you exactly which patterns are solid and which need work. Attempt every question properly in your own book, and go back to your notes to help you work through tough questions if needed.
- Every question you get wrong, need help with or struggle with goes in your **Mistakes Log**, which you should be working on daily to get you exam-ready.
- Questions marked ★ reach into a **Year 12** topic: they are genuine cross-topic exam questions, flagged so you know.

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Expanding and finding terms

1. Expanding a binomial

Q1 2025 Smiths Hill Trial Q2 · **1 mark**

Which of the following is the correct expansion of $(3x-1)^3$?

(A) $27x^3+27x^2+9x+1$

(B) $9x^3-27x^2+12x-1$

(C) $27x^3-27x^2+9x-1$

(D) $27x^3-27x^2+12x-9$

Q2 2021 Kings Trial Q2 · **1 mark**

Expand $(2x-1)^3$.

Q3 2025 Sydney Grammar Trial Q11(b) · **2 marks**

Expand and simplify $(x-2)^3$.

Q4 2021 HSC Q11(b) · **2 marks**

Expand and simplify $(2a-b)^4$.

Q5 2023 Girraween Prelim Q10(a) · **2 marks**

Write the expansion of $(2x-y)^5$.

Q6 2021 Sydney Grammar Prelim Q2(c) · **3 marks**

Use a binomial expansion to find integers a and b such that $(1-\sqrt{2})^5 = a+b\sqrt{2}$.

Q7 2023 Blacktown Boys Prelim Q11(b) · **2 marks**

Show through expansion that $(3-\sqrt{5})^4 = 376 - 168\sqrt{5}$.

2. Finding a specific term

Q8 2023 Girraween Prelim Q5 · **1 mark**

What is the seventh term in the expansion of $(x+3y)^{11}$?

(A) $\binom{11}{7}(3)^7x^7y^6$

(B) $\binom{11}{7}(3)^7x^6y^7$

(C) $\binom{11}{6}(3)^6x^6y^5$

(D) $\binom{11}{6}(3)^6x^5y^6$

Q9 2024 Glenwood Prelim Q3 · **1 mark**

What is the sixth term in the expansion of $(2x+3)^8$?

(A) $\binom{8}{4}(2x)^4(3)^4$

(B) $\binom{8}{4}(2x)^3(3)^5$

(C) $\binom{8}{5}(2x)^2(3)^7$

(D) $\binom{8}{5}(2x)^3(3)^5$

Q10 2023 North Sydney Boys Prelim QE1 · **2 marks**

Find the term in x^5 in the expansion of $\left(2x-\frac{1}{x^2}\right)^8$.

Q11 2023 North Sydney Boys Trial Q2 · **1 mark**

Find the fifth term in the expansion of $\left(3x^2 - \frac{1}{2x}\right)^8$.

(A) $\binom{8}{4}\left(\frac{3}{2}\right)^4 x^4$

(B) $\binom{4}{8}\left(\frac{2}{3}\right)^4 x^{-4}$

(C) $\binom{8}{4}\left(\frac{2}{3}\right)^4 x^{-4}$

(D) $\binom{4}{8}\left(\frac{3}{2}\right)^4 x^4$

Q12 2025 Baulkham Hills Prelim Q4 · **1 mark**

Which of the following is the coefficient of the fourth term in the expansion of $(3x - 4)^6$?

(A) -34560

(B) 34560

(C) 25920

(D) -25920

Q13 2022 Hurlstone Trial Q12(c) · **3 marks**

Consider the expansion of $\left(x + \frac{1}{x}\right)^8$.

(i) Find the general term of $\left(x + \frac{1}{x}\right)^8$.

(ii) Find the coefficient of x^2 in $\left(x + \frac{1}{x}\right)^8$.

3. Finding a coefficient

Q14 2022 Abbotsleigh Trial Q11(c) · **1 mark**

Find the coefficient of y^4 in $(1 + 2y)^6$.

Q15 2022 Barker Trial Q11(b) · **2 marks**

Determine the coefficient of x^3 in the expansion of $(2-3x)^8$.

Q16 2024 Knox Trial Q11(a) · **2 marks**

Find the coefficient of x^2 in the expansion of $\left(2x+\frac{1}{x}\right)^{10}$.

Q17 2025 Killara Trial Q5 · **1 mark**

The coefficient of a^5b^4 in the expansion of $\left(3a-\frac{b}{3}\right)^9$ is:

(A) -42

(B) 3

(C) 42

(D) 378

Q18 2022 Cheltenham Girls Trial Q13(a) · **2 marks**

Determine the coefficient of x^4 in the expansion of $\left(\frac{2}{x}+3x\right)^6$.

Q19 2022 North Sydney Boys Prelim C1 · **2 marks**

Find the coefficient of x^2 in the expansion of $\left(x^2+\frac{2}{x}\right)^7$.

Q20 2021 North Sydney Boys Trial Q11(d) · **3 marks**

Find the coefficient of x^8 in the expansion of $\left(\frac{x}{4}-\frac{4}{x}\right)^{14}$.

Q21 2022 North Sydney Boys Trial Q9 · **1 mark**

Let n be a positive integer. The coefficient of x^3y^5 in the expansion of $(1+xy+y^2)^n$ equals:

(A) n

(B) $2n$

(C) $\binom{n}{3}\binom{n}{5}$

(D) $4\binom{n}{4}$

4. The term independent of x

Q22 2025 Hurlstone Trial Q1 · **1 mark**

What is the term independent of x in the expansion of $\left(x+\frac{2}{x}\right)^6$?

(A) 160

(B) 40

(C) 80

(D) 20

Q23 2025 St George Girls Trial Q11(b) · **2 marks**

Find the constant term in the expansion of $\left(x^2+\frac{2}{x}\right)^6$.

Q24 2025 Abbotsleigh Trial Q11(d) · **2 marks**

Find the constant term in the expansion of $\left(2x-\frac{1}{x}\right)^{10}$.

Q25 2023 Glenwood Prelim Q10(a) · **3 marks**

Find the term independent of x in the expansion of $\left(3x+\frac{1}{x^2}\right)^9$.

Q26 2022 Manly Trial Q11(c) · **3 marks**

Find the term independent of x in the expansion of $\left(3x^2 - \frac{1}{x}\right)^{12}$.

Q27 2020 North Sydney Boys Trial Q11(b) · **3 marks**

Find the term independent of x in the expansion of $\left(x^2 - \frac{1}{x^2}\right)^{10}$.

Q28 2020 James Ruse Prelim Q1 · **1 mark**

For what positive values of n does the expansion of $\left(x^2 - \frac{2}{x^3}\right)^n$ have a constant term?

(A) Odd values of n

(B) Even values of n

(C) Multiples of 3

(D) Multiples of 5

5. Coefficients in a product

Q29 2022 Blacktown Boys Prelim Q12(c) · **2 marks**

Find the coefficient of x^8 in the expansion of $(3 - 4x)(1 + x)^{10}$.

Q30 2023 Ascham Trial Q12(b) · **2 marks**

Find the coefficient of x^2 in the expansion of $(2 + x^2)(3 - 2x)^5$.

Q31 2023 Sydney Grammar Prelim Q13(a) · **2 marks**

Find the coefficient of x^2 in the expansion of $(1 + x + x^2)(2 + x)^5$.

Q32 2022 St George Girls Trial Q11(b) · **3 marks**

Find the coefficient of x^9 in the expansion of $(1 + 2x)(2 + x)^{12}$.

Q33 2023 Girraween Prelim Q10(e) · **3 marks**

Find the coefficient of x^5 in the expansion of $(1 - 3x + 2x^3)(1 - 2x)^6$.

Q34 2025 Baulkham Hills Trial Q12(c) · **2 marks**

Find the coefficient of x^2y^6 in the expansion of $\left(1-\frac{y}{x}\right)(x+y)^8$.

Q35 2024 James Ruse Trial Q12(b) · **3 marks**

Find the term independent of x in the expansion of $\left(x+\frac{1}{x}\right)^5\left(2x^2-\frac{3}{x}\right)^6$.

6. Finding an unknown from a coefficient condition

Q36 2021 Girraween Trial Q2 · **1 mark**

In the expansion of $(2x+k)^6$, the coefficients of x and x^2 are equal. What is the value of k?

(A) 5

(B) 6

(C) 11

(D) 12

Q37 2020 North Sydney Boys Trial Q2 · **1 mark**

The coefficient of x^3 in the expansion of $(a+x)^6$ is 1280. What is the value of a?

(A) 64

(B) 4

(C) 40

(D) $\sqrt[3]{32}$

Q38 2023 Hunters Hill Trial Q11(a) · **3 marks**

Find the value of the constant a such that $8750x^4$ is a term in the binomial expansion of $(a+2x)^7$.

Q39 2021 Sydney Grammar Prelim Q6(a) · **3 marks**

In the expansion of $(2+x)^n$, the coefficients of x^5 and x^6 are equal. Find the value of n.

Q40 2019 HSC Q13(b) · **3 marks**

In the expansion of $(5x + 2)^{20}$, the coefficients of x^k and x^{k+1} are equal. What is the value of k?

Q41 2025 Caringbah Trial Q14(a) · **3 marks**

Given that k is a non-zero constant and n is a positive integer, then

$$(1 + kx)^n = 1 + 40x + 120k^2x^2 + \cdots$$

Find the values of k and n.

Q42 2023 Hornsby Girls Trial Q11(d) · **3 marks**

In the expansion of $(1 + 2x)^n$, the coefficients of x^6 and x^8 are in the ratio $7 : 3$. Find the value of n.

Q43 2021 Knox Prelim Q13(d) · **3 marks**

In the expansion of $(ax + b)^6$, where a and b are constants, the coefficient of x^4 is 2160 and the coefficient of x^5 is -576. Find a and b.

Coefficients and identities

7. Manipulating binomial coefficients

Q44 2023 SCEGGS Prelim Q10(a) · **2 marks**

Prove Pascal's identity, $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$, given that n and k are natural numbers with $n > k$.

Q45 2021 SCEGGS Prelim Q5(e) · **2 marks**

Use factorial notation to show that $\binom{8}{5} = \binom{7}{4} + \binom{7}{5}$.

Q46 2021 Presbyterian Prelim Q7(c) · **2 marks**

Show that $\binom{n+1}{r} \div \binom{n}{r-1} = \dfrac{n+1}{r}$.

Q47 2023 Barker Prelim Q12(d) · **2 marks**

Prove that $\binom{2n}{2} - 2\binom{n}{2} = n^2$.

Q48 2023 HSC Q12(d) · **2 marks**

It is known that $\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$ for all integers such that $1 \leq r \leq n-1$. (Do NOT prove this.)

Find ONE possible set of values for p and q such that

$$\binom{2022}{80} + \binom{2022}{81} + \binom{2023}{1943} = \binom{p}{q}.$$

Q49 2025 HSC Q13(e) · **3 marks**

The Pascal's triangle relation can be expressed as

$$\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}. \qquad \text{(Do NOT prove this.)}$$

(i) Show that $\binom{m}{R} = \binom{m+1}{R+1} - \binom{m}{R+1}$. (1 mark)

(ii) Hence, or otherwise, prove that

$$\binom{2000}{2000} + \binom{2001}{2000} + \binom{2002}{2000} + \cdots + \binom{2050}{2000} = \binom{2051}{2001}.$$ (2 marks)

8. Summing and evaluating coefficients by substitution

Q50 2020 North Sydney Boys Trial Q14(a) · **2 marks**

Show that

$$\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n} = 2^n.$$

Q51 2021 Sydney Girls Prelim Q3(a) · **1 mark**

Given the binomial expansion $(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \binom{n}{3}x^3 + \cdots + \binom{n}{n}x^n$, find the value of

$$\binom{100}{0} + \binom{100}{1} + \binom{100}{2} + \cdots + \binom{100}{100}.$$

Q52 2023 Blacktown Boys Prelim Q10 · **1 mark**

By considering the binomial expansion of $(1+x)^{10}$, what is the value of

$$5\binom{10}{1} + 5^2\binom{10}{2} + 5^3\binom{10}{3} + \cdots + 5^{10}\binom{10}{10}?$$

(A) 4^{10}

(B) 6^{10}

(C) $4^{10} - 1$

(D) $6^{10} - 1$

Q53 2024 Girraween Prelim Q11(c) · **5 marks**

(i) Write down the binomial expansion of $(1+x)^n$.

(ii) Hence, show that $\binom{n}{1} + \binom{n}{2} + \binom{n}{3} + \cdots + \binom{n}{n} = 2^n - 1$.

(iii) By substituting a suitable value for x in the expansion $(1+x)^n$, show that

$$1 + 2\binom{n}{1} + 4\binom{n}{2} + \cdots + 2^{n-1}\binom{n}{n-1} = 3^n - 2^n.$$

Q54 2021 Scots Prelim Q10(b) · **3 marks**

(i) Using the binomial theorem, obtain an expression for $(1+x)^{2n} + (1-x)^{2n}$.

(ii) Hence, show that $1 + \binom{20}{2} + \binom{20}{4} + \cdots + \binom{20}{20} = 2^{19}$.

Q55 2023 Sydney Tech Trial Q13(e) · **3 marks**

Given that

$$(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \cdots + \binom{n}{n}x^n \qquad \text{(do not show this),}$$

and by selecting two appropriate values of x, show that

$$2^{n-1} = \binom{n}{0} + \binom{n}{2} + \cdots + \binom{n}{n-1} \qquad \text{when } n \text{ is odd.}$$

Q56 2024 Cheltenham Girls Trial Q14(d) · **3 marks**

By considering the binomial expansion $(1+x)^n$, prove that

$$\binom{n}{1} + 3\binom{n}{2} + 9\binom{n}{3} + \cdots + 3^{n-1}\binom{n}{n} = \frac{1}{3}\left(2^{2n} - 1\right).$$

9. Identities via differentiation and integration

Q57 2023 Penrith Trial Q14(d) · **2 marks**

By differentiating both sides of the expansion of $(1+x)^n$ with respect to x, prove that

$$\binom{n}{1} + 2\binom{n}{2} + 3\binom{n}{3} + \cdots + n\binom{n}{n} = n\,2^{n-1}.$$

Q58 2021 Knox Trial Q14(c)(i) · **4 marks**

By considering the expansion of $(1+x)^{2n}$, show that

$$\binom{2n}{1} + 2\binom{2n}{2} + 3\binom{2n}{3} + \cdots + 2n\binom{2n}{2n} = n\,2^{2n}.$$

Q59 2022 Baulkham Hills Prelim Q14(c) · **4 marks**

By considering the expansion of $(1+x)^n$ and differentiating, show the following identity, given that n is even.

$$n\,2^{n-1} = 2\left[\binom{n}{1} + 3\binom{n}{3} + 5\binom{n}{5} + \cdots + (n-1)\binom{n}{n-1}\right].$$

Q60 ★ 2021 Cheltenham Girls Trial Q7(c) · **4 marks**

★ *Part (ii) uses integration, a Year 12 topic.*

Consider the binomial expansion

$$\binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \cdots + \binom{n}{r}x^r + \cdots + \binom{n}{n}x^n = (1+x)^n.$$

(i) Show that

$$1 - \binom{n}{1} + \binom{n}{2} - \cdots + (-1)^n\binom{n}{n} = 0.$$

(ii) Hence, by integrating both sides with respect to x, show that

$$1 - \frac{1}{2}\binom{n}{1} + \frac{1}{3}\binom{n}{2} - \cdots + \frac{(-1)^n}{n+1}\binom{n}{n} = \frac{1}{n+1}.$$

Q61 2021 James Ruse Prelim Q42(b) · **2 marks**

Use the binomial expansion of $(1+x)^n$ to prove that

$$\binom{n}{1} + 4\binom{n}{2} + 12\binom{n}{3} + \cdots + r\binom{n}{r}2^{r-1} + \cdots + n\binom{n}{n}2^{n-1} = n\,3^{n-1}.$$

10. Identities by comparing coefficients

Q62 2023 Cheltenham Girls Trial Q12(c) · **3 marks**

(i) Find the value of the coefficient of x^4 in the expansion of $(1+x)^3(1+x)^6$. (1 mark)

(ii) Using the expansion of $(1+x)^3(1+x)^6$, show that

$$\binom{3}{0}\binom{6}{6} + \binom{3}{1}\binom{6}{5} + \binom{3}{2}\binom{6}{4} + \binom{3}{3}\binom{6}{3} = \binom{9}{6}.$$ (2 marks)

Q63 2023 Glenwood Prelim Q10(c) · **3 marks**

By expanding both sides of the identity $(1+x)^{11} = (1+x)^8(1+x)^3$, prove that

$$\binom{11}{3} = \binom{8}{0}\binom{3}{3} + \binom{8}{1}\binom{3}{2} + \binom{8}{2}\binom{3}{1} + \binom{8}{3}\binom{3}{0}.$$

Q64 2021 Scots Trial Q13(b) · **3 marks**

(i) Given the identity $(1+x)^{m+n} = (1+x)^m(1+x)^n$, prove that

$$\binom{m+n}{4} = \binom{n}{4} + \binom{n}{3}\binom{m}{1} + \binom{n}{2}\binom{m}{2} + \binom{n}{1}\binom{m}{3} + \binom{m}{4}.$$ (2 marks)

(ii) Hence, by showing working, find the value of

$$\binom{16}{3}\binom{4}{1} + \binom{16}{2}\binom{4}{2} + \binom{16}{1}\binom{4}{3}.$$ (1 mark)

Q65 2023 Girraween Prelim Q11(b) · **3 marks**

By considering $(1+x)^{11} = (1+x)^3(1+x)^8$, show that

$$\binom{11}{5} = \binom{8}{5} + \binom{3}{1}\binom{8}{4} + \binom{3}{2}\binom{8}{3} + \binom{8}{2}.$$

Q66 2025 Baulkham Hills Prelim Q14(c) · **5 marks**

(i) Show that $x^n(1+x)^n\left(1+\frac{1}{x}\right)^n = (1+x)^{2n}$. (2 marks)

(ii) Hence show that

$$1 + \binom{n}{1}^2 + \binom{n}{2}^2 + \cdots + \binom{n}{n}^2 = \binom{2n}{n}.$$ (3 marks)

Q67 2020 James Ruse Prelim Q7(e) · **4 marks**

(i) Show that $(1+x)^m\left(1-\frac{1}{x}\right)^m = \left(x-\frac{1}{x}\right)^m$. (2 marks)

(ii) By considering the term(s) independent of x in the expansion of (i), show that

$$\binom{2n}{0}^2 - \binom{2n}{1}^2 + \binom{2n}{2}^2 - \cdots + \binom{2n}{2n}^2 = (-1)^n\binom{2n}{n}.$$ (2 marks)