Year 11 Mathematics Advanced

Topic: Graph Transformations (1 of 2)
Review Questions (Worked Solutions)
Term 3, 2026

Applying transformations to points and equations

1. Find the image of a point or feature under a transformation

Q1 Presbyterian 2021 Trial Q5 · **1 mark**

The point $P(8, -3)$ lies on the graph of $y = f(x)$. What are the coordinates of the image of P if the function is transformed to $y = -2f(x + 7) + 5$?

(A) $(1, 11)$

(B) $(1, -1)$

(C) $(15, 11)$

(D) $(15, -1)$

Criteria	Marks
Correct option (A)	1

Solution.

Read the horizontal and vertical changes off $y = -2f(x + 7) + 5$ separately. The $+7$ inside shifts the graph 7 left, so the x-coordinate becomes $8 - 7 = 1$. The outside operations act on the y-coordinate: multiply by -2, then add 5, giving $-2(-3) + 5 = 11$. Hence

$$P' = (1, 11).$$

The answer is (A).

Q2 Ascham 2023 Trial Q5 · **1 mark**

The point $P(-2, 3)$ lies on $y = f(x)$. What are the coordinates of the image point P' on the graph of $y = 3f(x - 4) + 1$?

(A) $(2, 10)$

(B) $(-6, 10)$

(C) $(-6, 2)$

(D) $(2, 0)$

Criteria	**Marks**
Correct option (A)	1

Solution.

The -4 inside shifts the graph 4 to the right, so the x-coordinate becomes $-2 + 4 = 2$. Outside, multiply the y-coordinate by 3 then add 1: $3(3) + 1 = 10$. Hence

$$P' = (2, 10).$$

The answer is (A).

Q3 North Sydney Girls 2022 Trial Q9 · **1 mark**

The only stationary point on the graph $y = f(x)$ is the point (a, b). What are the coordinates of the only stationary point on the graph $y = -f(2x)$?

(A) $\left(\frac{1}{2}a,\ -b\right)$

(B) $(2a,\ -b)$

(C) $\left(-\frac{1}{2}a,\ b\right)$

(D) $(-2a,\ b)$

Criteria	**Marks**
Correct option (A)	1

Solution.

The $2x$ inside is a horizontal dilation by factor $\frac{1}{2}$, so the x-coordinate of the stationary point becomes $\frac{1}{2}a$ (solve $2x = a$). The leading -1 reflects in the x-axis, so the y-coordinate becomes $-b$. Hence the stationary point is

$$\left(\tfrac{1}{2}a,\ -b\right).$$

The answer is (A).

Q4 Hornsby Girls 2021 Trial Q10 · **1 mark**

The graph of $y = f(x)$ has a vertical asymptote at $x = 1$ and a horizontal asymptote at $y = 2$. The graph of $y = -f(-x)$ has a vertical asymptote and horizontal asymptote respectively:

(A) $x = 2,\ y = 1$

(B) $x = -1,\ y = 2$

(C) $x = -1,\ y = -2$

(D) $x = 1,\ y = -2$

Criteria	**Marks**
Correct option (C)	1

Solution.

$y = -f(-x)$ reflects the graph in both axes. The $-x$ inside reflects in the y-axis, sending the vertical asymptote $x = 1$ to $x = -1$. The leading -1 reflects in the x-axis, sending the horizontal asymptote $y = 2$ to $y = -2$. Hence

$$x = -1, \quad y = -2.$$

The answer is (C).

Q5 James Ruse 2023 Trial Q14 · **3 marks**

The point $P(-1, 3)$ lies on the curve with equation $y = f(x)$. State the coordinates of the image of point P (called P') after each of the following transformations has taken place:

(i) $y = f(x + 3) + 3$

(ii) $y = 2f(x) + 3$

(iii) $y = f(2x + 3)$

Criteria	**Marks**
Part (i) correct: $(-4, 6)$	1
Part (ii) correct: $(-1, 9)$	1
Part (iii) correct: $(-2, 3)$	1

Solution.

In each part, treat the inside operation as acting on x and the outside operation as acting on y.

(i) $y = f(x + 3) + 3$: shift 3 left ($x = -1 - 3 = -4$), then up 3 ($y = 3 + 3 = 6$):

$$P' = (-4, 6).$$

(ii) $y = 2f(x)+3$: x is unchanged; the y-coordinate is multiplied by 2 then increased by 3, giving $2(3)+3 = 9$:

$$P' = (-1, 9).$$

(iii) $y = f(2x+3)$: write $2x+3 = 2\left(x+\frac{3}{2}\right)$; solve $2x+3 = -1$ to get $x = -2$. The y-coordinate is unchanged:

$$P' = (-2, 3).$$

Q6 Abbotsleigh 2020 Trial Q7 · **1 mark**

The graph of $y = f(x)$ has a stationary point at $(2, -3)$. Consequently, which of the following is a stationary point of $y = -f\left(\frac{x}{2}\right) - 5$?

(A) $(4, 2)$

(B) $(4, -2)$

(C) $(1, 2)$

(D) $(1, -2)$

Criteria	Marks
Correct option (B)	1

Solution.

The $\frac{x}{2}$ inside is a horizontal dilation by factor 2, so the x-coordinate becomes $2 \times 2 = 4$ (solve $\frac{x}{2} = 2$). Outside, reflect in the x-axis then subtract 5: $-(-3) - 5 = -2$. Hence the stationary point is

$$(4, -2).$$

The answer is (B).

Q7 Ascham 2022 Trial Q24 · **2 marks**

The function $y = g(x)$ is transformed to the function $y = \frac{1}{2}g\big(3(x-1)\big) + 4$. Find the point $P(x, y)$ on $y = g(x)$, if its image point on the transformed function is $P'(6, 5)$.

Criteria	Marks
Sets up and solves for the pre-image x: $3(6-1) = 15$	1
Correct pre-image point $P(15, 2)$ (from $\frac{1}{2}g(15) + 4 = 5$)	1

Solution.

Work backwards from the image $P'(6, 5)$. A point $\big(t, g(t)\big)$ on $y = g(x)$ maps to the image whose x-coordinate satisfies $3(x-1) = t$, so at $x = 6$ we have $t = 3(6-1) = 15$. Thus the pre-image x-coordinate is 15.

For the y-coordinate, substitute the image into the rule:

$$5 = \tfrac{1}{2}g(15) + 4 \implies \tfrac{1}{2}g(15) = 1 \implies g(15) = 2.$$

So the point on $y = g(x)$ has $y = g(15) = 2$. Hence

$$P = (15, 2).$$

2. Find the equation of a curve after transformations

Q8 Barker 2020 Trial Q5 · 1 mark

The function $f(x) = x^2 - 5x + 1$ is reflected about the y-axis. What is the equation of the new function? (multiple choice).

Criteria	Marks
Correct option: $x^2 + 5x + 1 \rightarrow$ (A)	1

Solution.

Reflecting in the y-axis replaces x with $-x$:

$$f(-x) = (-x)^2 - 5(-x) + 1 = x^2 + 5x + 1.$$

The answer is **(A)**.

Q9 Pymble 2022 Trial Q1 · 1 mark

The function $f(x) = \dfrac{1}{x}$ is translated 3 units up and 2 units right to produce $y = g(x)$. Which of the following is the equation of the translated function $g(x)$? (multiple choice).

Criteria	Marks
Correct option: $\dfrac{1}{x-2} + 3 \rightarrow$ (C)	1

Solution.

A translation 2 units right replaces x with $x - 2$, and 3 units up adds 3:

$$g(x) = \frac{1}{x-2} + 3.$$

The answer is **(C)**.

Q10 Trinity Grammar 2022 Trial Q24 · **2 marks**

The function $f(x) = (x-2)^3 - 2$ is translated 5 units to the right and 4 units up. Find the equation of the transformed function.

Criteria	Marks
Applies right 5 ($x \to x-5$) and up 4 (+4)	1
Correct equation $y = (x-7)^3 + 2$	1

Solution.

Translating 5 units right replaces x with $x-5$, and 4 units up adds 4:

$$y = \big((x-5)-2\big)^3 - 2 + 4 = (x-7)^3 + 2.$$

Q11 Barker 2022 Trial Q4 · **1 mark**

Which of the following equations is the reflection of the curve $y = 3^x + 2$ in the x-axis, then translated 2 units downwards? (multiple choice).

Criteria	Marks
Correct option: $-3^x - 4 \to$ (D)	1

Solution.

Reflecting in the x-axis replaces y with $-y$, giving $y = -(3^x + 2) = -3^x - 2$. Translating 2 units down subtracts 2:

$$y = -3^x - 2 - 2 = -3^x - 4.$$

The answer is **(D)**.

Q12 Killara 2020 Trial Q2 · **1 mark**

The function $y = \ln x$ is dilated vertically by a scale factor of 3 and then translated 4 units to the left. Which of the following is the equation of the transformed function? (multiple choice).

Criteria	Marks
Correct option: $3\ln(x+4) \to$ (D)	1

Solution.

A vertical dilation by factor 3 multiplies the function by 3, giving $y = 3\ln x$. Translating 4 units left replaces x with $x+4$:

$$y = 3\ln(x+4).$$

The answer is **(D)**.

Q13 Glenwood 2021 Trial Q28 · **2 marks**

The graph of $y = \sqrt{x}$ undergoes: translation 3 units right, horizontal dilation by factor $\frac{1}{2}$, translation 1 unit down, and vertical dilation by factor 4. Find the rule of the resulting graph.

Criteria	**Marks**
Correctly applies the two horizontal transformations (right 3 then horizontal dilation by $\frac{1}{2}$, i.e. $x \to 2x - 3$)	1
Correctly applies the vertical dilation ($\times 4$) and vertical translation (-1) to obtain $y = 4\sqrt{2x-3} - 1$	1

Solution.

Apply the transformations in order. Right 3: $y = \sqrt{x-3}$. Horizontal dilation by factor $\frac{1}{2}$ replaces x with $2x$: $y = \sqrt{2x-3}$. Down 1: $y = \sqrt{2x-3} - 1$. Vertical dilation by factor 4 multiplies by 4:

$$y = 4\sqrt{2x-3} - 1.$$

Q14 Shore 2023 Trial Q25 · **2 marks**

The graph $y = f(x)$ of $f(x) = x^3$ is translated 3 units right and 5 units up, then horizontally dilated by a scale factor of $\frac{1}{2}$ to produce $y = g(x)$. Find the equation of the transformed function $g(x)$.

Criteria	**Marks**
Applies right 3 and up 5 to reach $(x-3)^3 + 5$	1
Applies horizontal dilation by $\frac{1}{2}$ ($x \to 2x$) to obtain $g(x) = (2x-3)^3 + 5$	1

Solution.

Translating 3 right and 5 up gives $y = (x-3)^3 + 5$. A horizontal dilation by scale factor $\frac{1}{2}$ replaces x with $2x$:

$$g(x) = (2x-3)^3 + 5.$$

Q15 Girraween 2025 Trial Q19 · **3 marks**

Find the equation of the new function if $f(x) = x^3$ is transformed in order: (1) horizontal dilation by factor 3, (2) reflection across the y-axis, (3) translation left by 1 unit.

Criteria	Marks
Horizontal dilation by factor 3 ($x \to \frac{x}{3}$) gives $y = \left(\frac{x}{3}\right)^3$	1
Reflection across the y-axis ($x \to -x$) gives $y = -\left(\frac{x}{3}\right)^3$	1
Translation left 1 ($x \to x+1$) gives $y = -\left(\dfrac{x+1}{3}\right)^3$	1

Solution.

Apply each transformation in order. Horizontal dilation by factor 3 replaces x with $\frac{x}{3}$: $y = \left(\frac{x}{3}\right)^3$. Reflecting across the y-axis replaces x with $-x$: $y = \left(\frac{-x}{3}\right)^3 = -\left(\frac{x}{3}\right)^3$. Translating left 1 replaces x with $x+1$:

$$y = -\left(\frac{x+1}{3}\right)^3.$$

Q16 Shore 2024 Trial Q19(a) · **2 marks**

The graph of $y = \dfrac{1}{x+6}$ is dilated horizontally by a factor of $\frac{1}{3}$ and then translated 2 units to the right. Find the equation of the transformed function $y = g(x)$.

Criteria	Marks
Applies the horizontal dilation by factor $\frac{1}{3}$ ($x \to 3x$) to reach $\dfrac{1}{3x+6}$	1
Applies the translation 2 units right ($x \to x-2$) to obtain $g(x) = \dfrac{1}{3x}$	1

Solution.

A horizontal dilation by a factor of $\frac{1}{3}$ replaces x with $3x$:

$$y = \frac{1}{3x+6}.$$

Translating 2 units to the right replaces x with $x-2$:

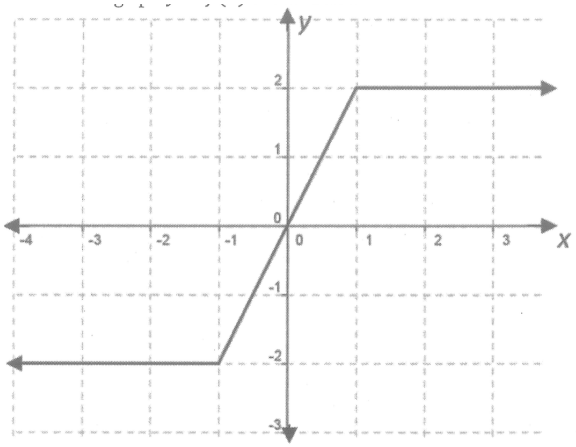
$$g(x) = \frac{1}{3(x-2)+6} = \frac{1}{3x-6+6} = \frac{1}{3x}.$$

Sketching, describing and identifying transformed graphs

3. Sketch the graph after transformations

Q17 Sydney Tech 2024 Trial Q14 · **2 marks**

Consider the graph of $y = f(x)$ shown. Sketch (a) $y = -f(x)$ and (b) $y = f(-x)$.



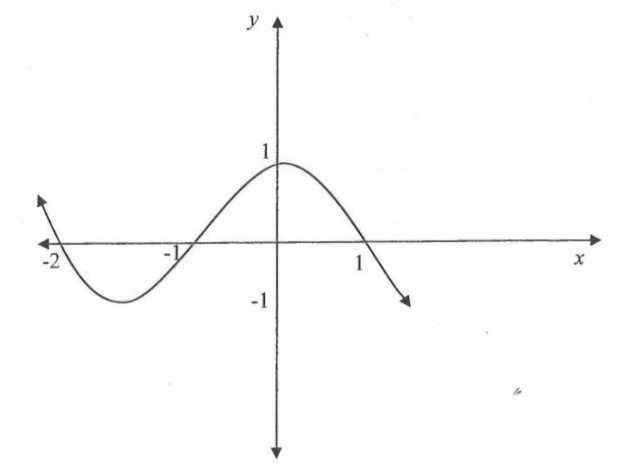
Criteria	Marks
(a) $y = -f(x)$: reflect in the x-axis, key points correct	1
(b) $y = f(-x)$: reflect in the y-axis, key points correct	1

Solution.

The given $f(x)$ runs along $y = -2$ for $-3 \le x \le -1$, rises as a straight segment through the corner $(-1, -2)$ and the origin $(0, 0)$ to $(1, 2)$, then continues horizontally along $y = 2$ for $x \ge 1$. **(a)** $y = -f(x)$ reflects every point in the x-axis, $(x, y) \mapsto (x, -y)$: the left arm becomes $y = 2$ for $-3 \le x \le -1$, the segment falls from $(-1, 2)$ through $(0, 0)$ to $(1, -2)$, then runs along $y = -2$ for $x \ge 1$. **(b)** $y = f(-x)$ reflects every point in the y-axis, $(x, y) \mapsto (-x, y)$: the horizontal arm at $y = 2$ now runs to the left for $x \le -1$, the segment falls from $(-1, 2)$ through $(0, 0)$ to $(1, -2)$, and the arm at $y = -2$ now runs from $(1, -2)$ to $(3, -2)$.

Q18 Riverview 2022 Trial Q29 · **2 marks**

The graph of $y = f(x)$ is shown. Draw $y = f(x-1)+1$, showing all important features.



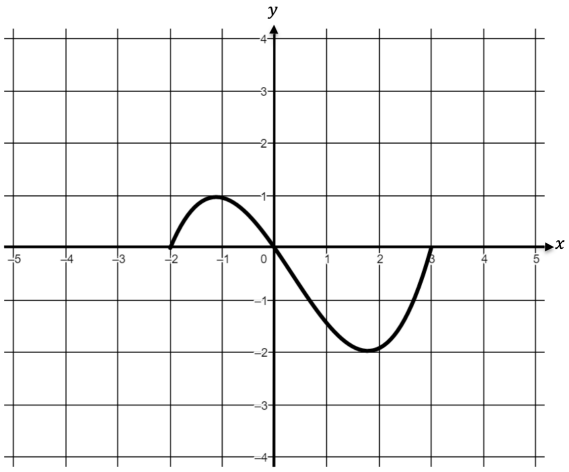
Criteria	**Marks**
Recognises translation 1 right and 1 up	1
Correctly translated key features (turning points and intercepts)	1

Solution.

$y = f(x-1)+1$ translates the whole curve 1 unit right and 1 unit up: $(x, y) \mapsto (x+1,\ y+1)$. The local minimum $(-1,-1)$ moves to $(0,0)$; the local maximum $(0,1)$ moves to $(1,2)$; the x-intercepts at $x=-2$ and $x=1$ move to $(-1,1)$ and $(2,1)$ respectively. Slide every point of the original curve one right and one up and join with the same shape.

Q19 St George Girls 2020 Trial Q21 · **2 marks**

The graph shows $y = f(x)$. Sketch $y = -2f(x-1)$ on the same axes.



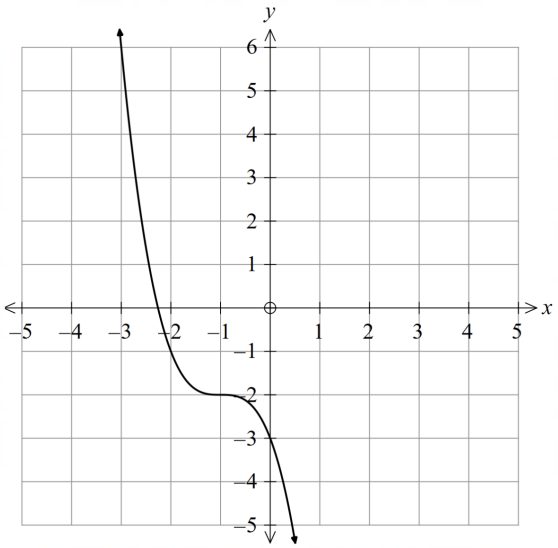
Criteria	Marks
Applies the three transformations (right 1, reflect in x-axis, vertical stretch 2)	1
Correctly transformed key features	1

Solution.

$y = -2f(x-1)$ is a translation 1 right, a reflection in the x-axis and a vertical stretch by factor 2: $(x, y) \mapsto (x+1, -2y)$. The local maximum $(-1, 1)$ becomes the local minimum $(0, -2)$; the local minimum $(1, -2)$ becomes the local maximum $(2, 4)$; the x-intercepts stay on the axis and shift right 1, so $x = -2 \to (-1, 0)$, the origin $(0, 0) \to (1, 0)$ and $x = 2 \to (3, 0)$. Turning points invert and double in height.

Q20 Barker 2023 Trial Q26 · **2 marks**

Given $y = f(x)$, draw $y = 2f(x) + 6$, showing intercepts and the point of inflection.



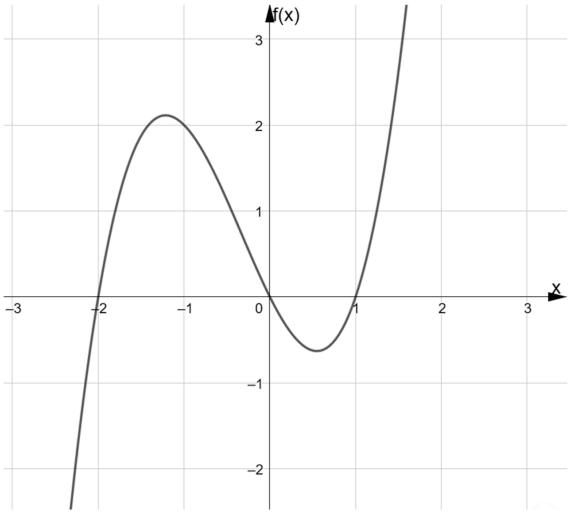
Criteria	Marks
Applies vertical stretch by 2 then shift up 6	1
Correct transformed inflection $(0, 2)$ and intercepts	1

Solution.

The given $f(x)$ is an increasing cubic with a point of inflection at $(0, -2)$ (which is also its y-intercept) and an x-intercept at $(-2, 0)$. Applying $y = 2f(x) + 6$, i.e. $(x, y) \mapsto (x,\ 2y + 6)$: the point of inflection $(0, -2) \mapsto (0, 2)$, which is the new y-intercept; the original x-intercept $(-2, 0) \mapsto (-2, 6)$. The new x-intercept is where $2f(x) + 6 = 0$, i.e. $f(x) = -3$, which lies to the left of $x = -2$ (read from the given curve). Double every height then lift the whole curve 6 units up, keeping the same S-shape.

Q21 Cherrybrook Tech 2021 Trial Q20 · **2 marks**

The graph shows $y = f(x)$. Sketch $y = 2 - f(x)$ on the same graph.



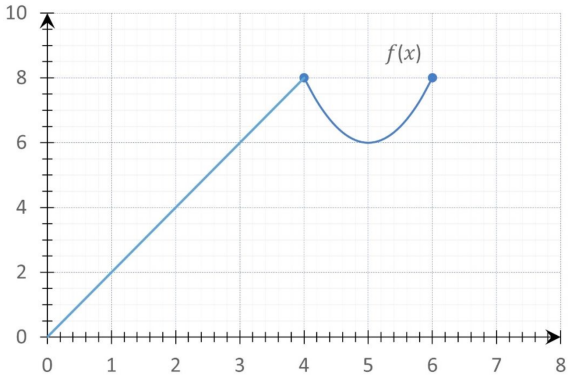
Criteria	Marks
Recognises reflection in the x-axis then shift up 2	1
Correctly transformed key features	1

Solution.

$y = 2 - f(x) = -f(x) + 2$ reflects in the x-axis then shifts up 2: $(x, y) \mapsto (x,\ 2 - y)$. Points where the original curve crosses the x-axis ($y = 0$) map to height $y = 2$: so $x = -2 \to (-2, 2)$ and the origin $(0, 0) \to (0, 2)$. The original local maximum (near $(-0.8, 2)$) reflects down to about $(-0.8, 0)$, and the original local minimum (near $(0.8, -0.8)$) reflects up to about $(0.8, 2.8)$, becoming the new maximum. Where f rises steeply on the right the new curve falls steeply. Turn each height into $2 - y$.

Q22 James Ruse 2023 Trial Q16 · **3 marks**

Given $y = f(x)$, sketch $g(x) = f(2x)$ and $h(x) = 0.5f(x)$ on the same axes.



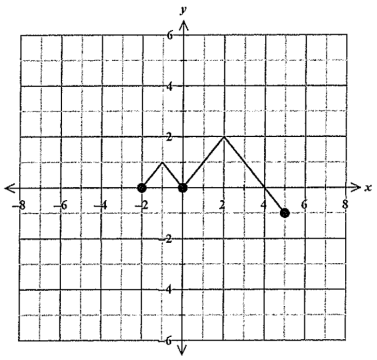
Criteria	Marks
$g(x) = f(2x)$: horizontal compression by $\frac{1}{2}$, key points correct	1
$h(x) = 0.5f(x)$: vertical compression by $\frac{1}{2}$, key points correct	1
Both curves labelled clearly	1

Solution.

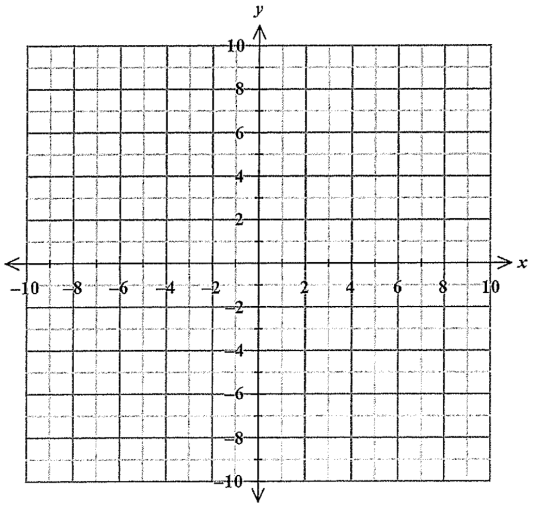
The given $f(x)$ starts at $(0, 0)$, rises to a local maximum near $(4, 8)$, dips to a local minimum near $(6, 6)$ and rises again to about $(8, 7)$. $g(x) = f(2x)$ compresses horizontally by factor $\frac{1}{2}$, $(x, y) \mapsto (\frac{x}{2}, y)$: heights are unchanged and x-values halve, so $(4, 8) \to (2, 8)$, $(6, 6) \to (3, 6)$, $(8, 7) \to (4, 7)$; the graph is squeezed into $0 \le x \le 4$. $h(x) = 0.5f(x)$ compresses vertically by factor $\frac{1}{2}$, $(x, y) \mapsto (x, \frac{y}{2})$: x-values unchanged and heights halve, so $(4, 8) \to (4, 4)$, $(6, 6) \to (6, 3)$, $(8, 7) \to (8, 3.5)$.

Q23 Killara 2024 Trial Q29 · **2 marks**

Given $f(x)$, sketch $y = 2f\left(-\frac{x}{2}\right)$ on the axes.



Sketch $2f\left(-\frac{1}{2}x\right)$ on the axis below.



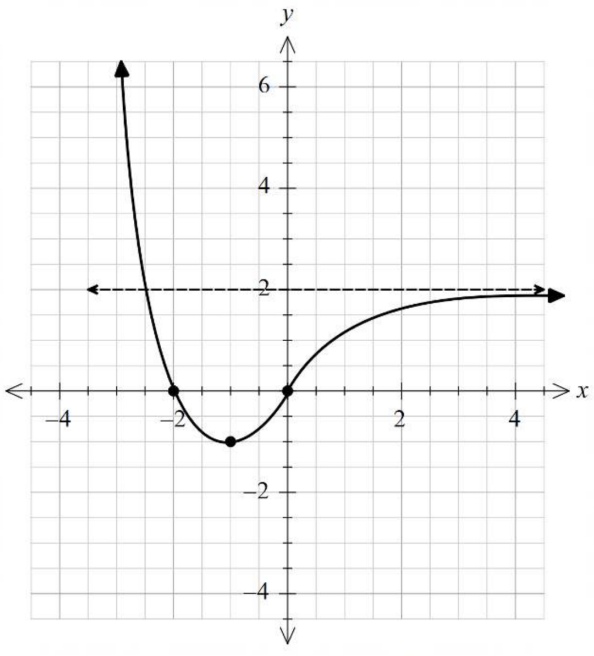
Criteria	Marks
Horizontal stretch by 2 and reflection in the y-axis	1
Vertical stretch by 2 with key points correct	1

Solution.

$y = 2f\left(-\frac{x}{2}\right)$ combines a vertical stretch by 2, a horizontal stretch by 2 and a reflection in the y-axis: $(x, y) \mapsto (-2x, 2y)$. From the given $f(x)$ (local minimum $(0, -1)$, local maximum $(2, 3)$, x-intercepts at $x = -2$ and $x = 4$): the minimum $(0, -1) \mapsto (0, -2)$; the maximum $(2, 3) \mapsto (-4, 6)$; the x-intercepts map to $(-2, 0) \mapsto (4, 0)$ and $(4, 0) \mapsto (-8, 0)$. Note the $\frac{x}{2}$ *widens* the graph (a common error is to compress it).

Q24 Pymble 2022 Trial Q16 · **3 marks**

Given $y = f(x)$, sketch $y = -2f(-2x)$, showing intercepts, turning points and asymptotes.



Criteria	Marks
Correct asymptotes of the transformed curve	1
Correct transformed turning point	1
Correct intercept and overall shape	1

Solution.

The given $f(x)$ has a vertical asymptote $x = -3$, a horizontal asymptote $y = 2$, a turning point at $(-1, -1)$ and an x-intercept at $x = 1$. The map $y = -2f(-2x)$ combines a horizontal compression by $\frac{1}{2}$, a reflection in the y-axis, a vertical stretch by 2 and a reflection in the x-axis: $(x, y) \mapsto (-\frac{x}{2}, -2y)$; an asymptote $x = k \mapsto x = -\frac{k}{2}$ and $y = c \mapsto y = -2c$. Hence the vertical asymptote $x = -3 \mapsto x = \frac{3}{2}$; the horizontal asymptote $y = 2 \mapsto y = -4$; the turning point $(-1, -1) \mapsto \left(\frac{1}{2}, 2\right)$ (a maximum); and the x-intercept $x = 1 \mapsto x = -\frac{1}{2}$. Draw the branch approaching $x = \frac{3}{2}$ and levelling off toward $y = -4$, with the peak at $\left(\frac{1}{2}, 2\right)$ and cutting the x-axis at $-\frac{1}{2}$.

4. Describe or state the transformations

Q25 2020 HSC Q2 · 1 mark

The function $f(x) = x^3$ is transformed to $g(x) = (x-2)^3 + 5$. Which row shows the directions of the two translations?

Criteria	Marks
Correct row: **B** (Right, Up)	1

Solution.

The replacement $x \mapsto x - 2$ shifts the graph 2 units to the **right**; the +5 shifts it 5 units **up**. Answer **B**.

Q26 Gosford 2023 Trial Q3 · 1 mark

$f(x) = x^2$, $g(x) = -1 - (x+3)^2$. Which transformations map f to g?

Criteria	Marks
Correct option: **A**	1

Solution.

Take $y = x^2$: translate 3 units **left** ($x \mapsto x + 3$) to get $(x+3)^2$; translate 1 unit **up** to get $(x+3)^2 + 1$; then **reflect across the x-axis** to get $-\left[(x+3)^2 + 1\right] = -1 - (x+3)^2 = g(x)$. Answer **A**.

Q27 Baulkham Hills 2022 Trial Q9 · 1 mark

For $y = \log_{10} x \to y = 3\log_{10}\left[2(x+5)\right] - 4$, which listed step is **not** correct?

Criteria	Marks
Identifies the incorrect step: **D**	1

Solution.

The four correct transformations are: horizontal dilation by a factor of $\frac{1}{2}$ (the coefficient 2 inside the log), translate 5 units left, vertical dilation by a factor of 3, and translate down 4 units. Option **D** states a horizontal dilation by a factor of 2, but the coefficient 2 produces a horizontal dilation by a factor of $\frac{1}{2}$ (a horizontal compression), so **D** is the step that is *not* correct. Answer **D**.

Q28 Hunters Hill 2020 Trial Q13 · **2 marks**

Describe the transformations applied to $y = x^2$ to give $y = (4x + 2)^2$.

Criteria	Marks
Identifies the horizontal dilation by a factor of $\frac{1}{4}$	1
Identifies the horizontal translation ($\frac{1}{2}$ unit left)	1

Solution.

Write $(4x + 2)^2 = \left(4(x + \frac{1}{2})\right)^2$. Apply a horizontal dilation by a factor of $\frac{1}{4}$ (i.e. $x \mapsto 4x$), giving $(4x)^2$, then a horizontal translation of $\frac{1}{2}$ unit to the **left** ($x \mapsto x + \frac{1}{2}$), giving $\left(4(x + \frac{1}{2})\right)^2 = (4x + 2)^2$. Equivalently: translate 2 units left first ($x \mapsto x + 2$, giving $(x + 2)^2$) then dilate horizontally by a factor of $\frac{1}{4}$; both orderings reach $(4x + 2)^2$.

Q29 Penrith 2020 Trial Q14 · **2 marks**

Describe a sequence of transformations from $f(x) = \dfrac{1}{x^2}$ to $g(x) = -\dfrac{1}{(x-1)^2} + 3$.

Criteria	Marks
Correct horizontal translation (1 right) and reflection in the x-axis	1
Correct vertical translation (3 up), giving $g(x)$	1

Solution.

Starting from $y = \dfrac{1}{x^2}$: translate 1 unit **right** ($x \mapsto x - 1$) to get $\dfrac{1}{(x-1)^2}$; **reflect across the x-axis** to get $-\dfrac{1}{(x-1)^2}$; then translate 3 units **up** to get $-\dfrac{1}{(x-1)^2} + 3 = g(x)$.

Q30 Abbotsleigh 2021 Trial Q12(f) · **3 marks**

Describe the three transformations, applied in succession, taking $y = \sqrt{x}$ to $y = 4\sqrt{x-1} + 3$.

Criteria	Marks
Vertical dilation by a factor of 4	1
Horizontal translation 1 unit right	1
Vertical translation 3 units up	1

Solution.

Starting from $y = \sqrt{x}$: apply a **vertical dilation by a factor of** 4 to get $4\sqrt{x}$; translate 1 unit **right** ($x \mapsto x - 1$) to get $4\sqrt{x-1}$; then translate 3 units **up** to get $4\sqrt{x-1} + 3$. (The vertical dilation must precede the +3 translation; the horizontal shift may be applied at any point before the vertical shift.)

Q31 Caringbah 2022 Trial Q10 · **1 mark**

Let $f(x) = e^x$ and $g(x) = 6 - 8e^{-x}$. Which sequence of transformations maps $y = f(x)$ to $y = g(x)$? (multiple choice).

Criteria	Marks
Correct option: **(A)**	1

Solution.

Start from $y = e^x$ and apply the option (A) sequence in order. Vertical dilation by factor 8 gives $y = 8e^x$; reflecting in the x-axis ($y \to -y$) gives $y = -8e^x$; reflecting in the y-axis ($x \to -x$) gives $y = -8e^{-x}$; translating up 6 units gives

$$y = 6 - 8e^{-x} = g(x).$$

Only this sequence reproduces $g(x)$: option (B) uses the wrong dilation factor ($\frac{1}{8}$), and options (C) and (D) translate up *before* the reflection in the x-axis, so the +6 is itself reflected and the constant comes out wrong. The answer is **(A)**.

Q32 Glenwood 2023 Trial Q13 · **3 marks**

Describe three transformations, applied in succession, taking $y = x^2 + 5$ to $y = \left(\dfrac{x+1}{2}\right)^2$.

Criteria	Marks
Vertical translation 5 units down	1
Horizontal dilation by a scale factor of 2	1
Horizontal translation 1 unit left	1

Solution.

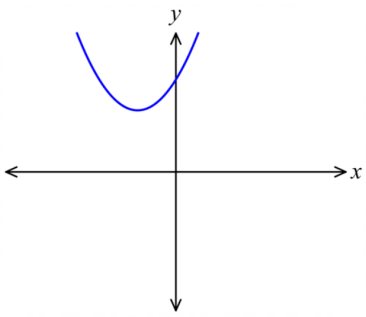
Follow the chain $x^2 + 5 \to x^2 \to \left(\dfrac{x}{2}\right)^2 \to \left(\dfrac{x+1}{2}\right)^2$: translate 5 units **down** (removing the +5) to get x^2; apply a **horizontal dilation by a scale factor of** 2 ($x \mapsto \frac{x}{2}$) to get $\left(\dfrac{x}{2}\right)^2$; then translate 1 unit **left** ($x \mapsto x + 1$) to get $\left(\dfrac{x+1}{2}\right)^2$.

5. Identify or match a transformed graph

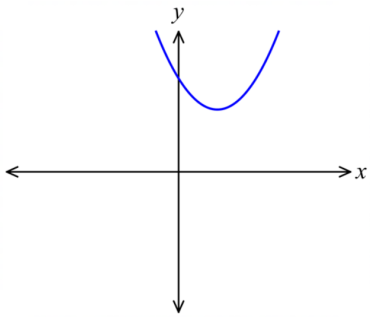
Q33 Hornsby Girls 2020 Trial Q5 · **1 mark**

Which diagram best shows the graph of the parabola $y = 2 - (x + 1)^2$?

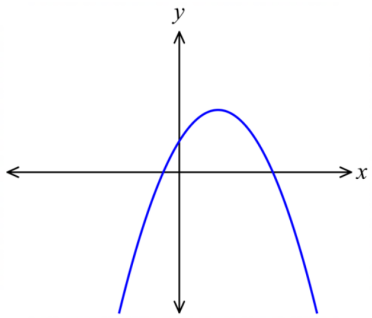
(A)



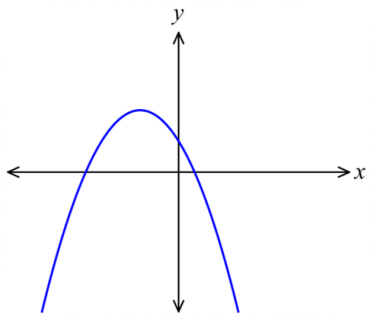
(B)



(C)



(D)



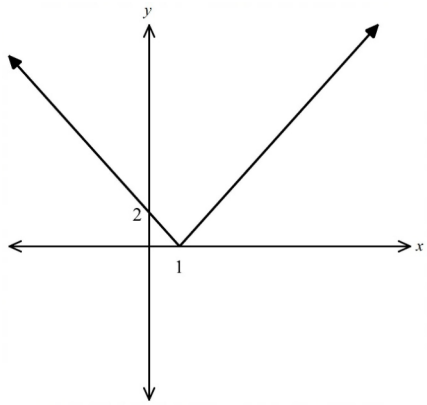
Criteria	Marks
Correct option (D)	1

Solution.

$y = 2 - (x + 1)^2$ is the parabola $y = -x^2$ translated 1 unit left and 2 units up. The leading coefficient is negative, so it is *concave down* with a maximum turning point at the vertex $(-1, 2)$, and it cuts the y-axis at $y = 2 - (1)^2 = 1$. The correct diagram is the downward parabola with vertex at $(-1, 2)$: option (D).

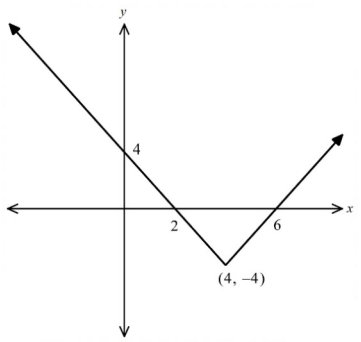
Q34 Fort St 2022 Trial Q3 · **1 mark**

The graph of $y = f(x)$ is shown below. Which of the graphs below represents $y = f(x + 3) + 4$?

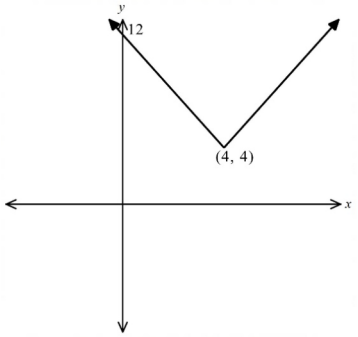


Which of the graphs below represents $y = f(x + 3) + 4$?

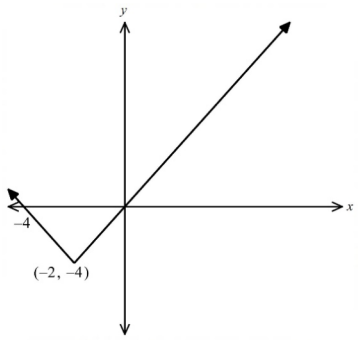
A.



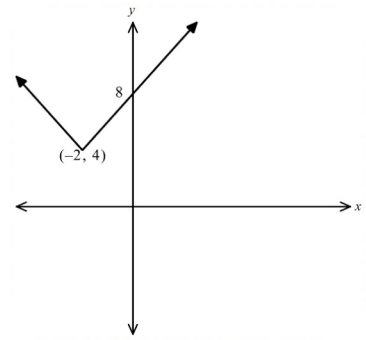
B.



C.



D.



Criteria	Marks
Correct option (translation 3 left, 4 up)	1

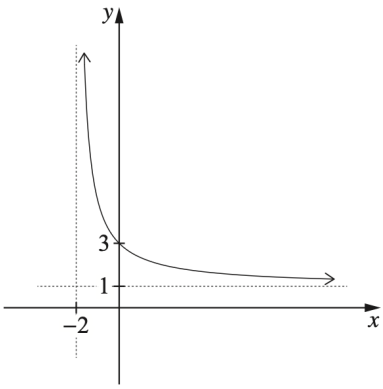
Solution.

Replacing x by $x + 3$ shifts the graph 3 units to the *left*, and the +4 shifts it 4 units *up*. So every point (x, y) on $y = f(x)$ moves to $(x - 3,\ y + 4)$. The correct option is the graph of $y = f(x)$ translated 3 units left and 4

units up.

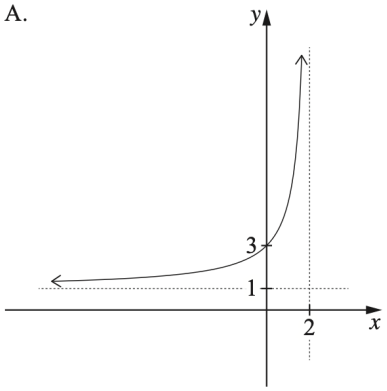
Q35 2025 HSC Q6 · **1 mark**

The graph of $y = f(x)$ is shown. Which of the following is the graph of $y = -f(-x)$?

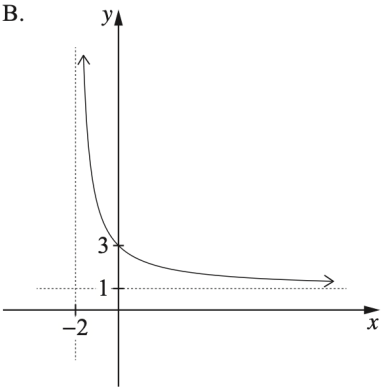


Which of the following is the graph of $y = -f(-x)$?

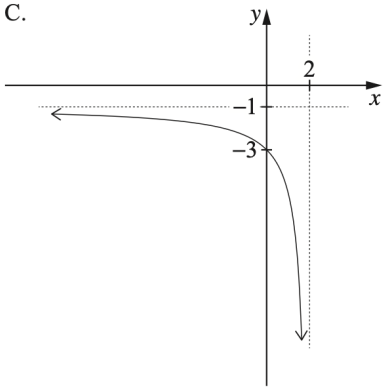
A.



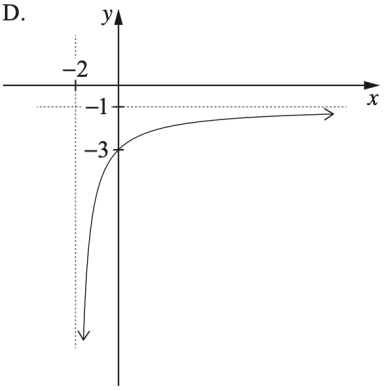
B.



C.



D.



Criteria	Marks
Correct option (C)	1

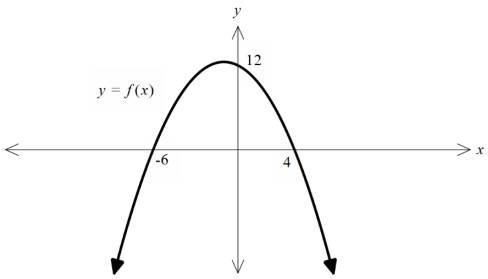
Solution.

$y = -f(-x)$ combines a reflection in the y-axis ($x \to -x$) with a reflection in the x-axis ($y \to -y$), which together is a rotation of 180° about the origin. Each point (x, y) maps to $(-x, -y)$, so the features of the

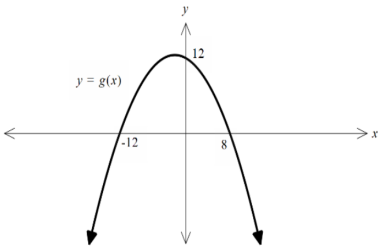
original graph in the upper-left (negative x, positive y) move to the lower-right (positive x, negative y). The correct diagram is option (C).

Q36 Roseville 2022 Trial Q9 · **1 mark**

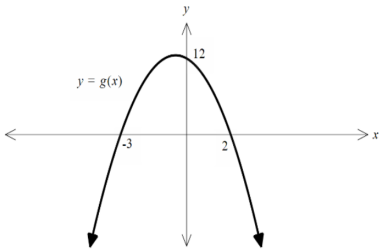
The figure below shows the graph of $y = f(x)$. If $g(x) = f\left(\frac{x}{2}\right)$, which of the following may represent the graph of $y = g(x)$?



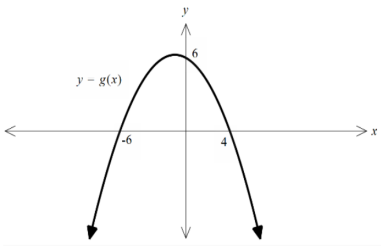
(A)



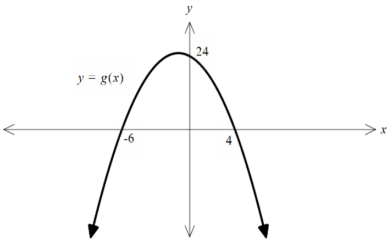
(B)



(C)



(D)



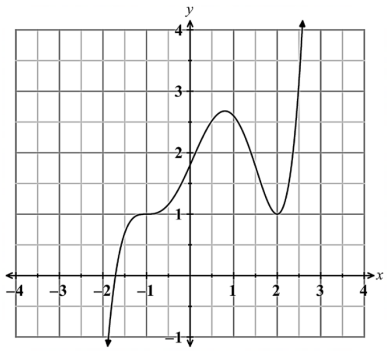
Criteria	Marks
Correct option (A)	1

Solution.

$g(x) = f\left(\frac{x}{2}\right)$ replaces x by $\frac{x}{2}$, which is a horizontal dilation (stretch) away from the y-axis by a factor of 2. Every point (x, y) on $y = f(x)$ moves to $(2x,\ y)$: the graph keeps the same heights but is stretched to twice its width, with x-intercepts doubling. The correct diagram is option (A).

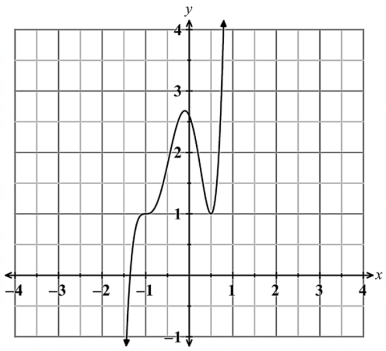
Q37 Killara 2025 Trial Q10 · **1 mark**

The graph of $y = f(x)$ is shown below. Which of the following options best represents the graph of $y = f(2x + 1)$?

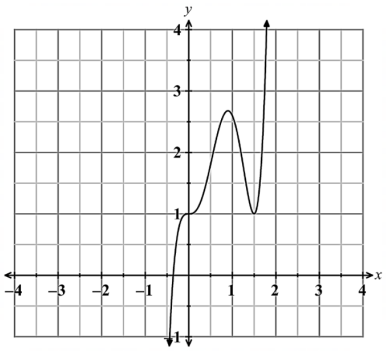


Which of the following options best represent the graph of $y = f(2x + 1)$?

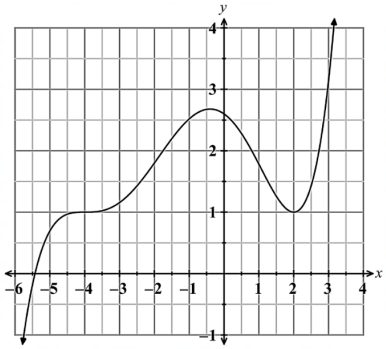
A.



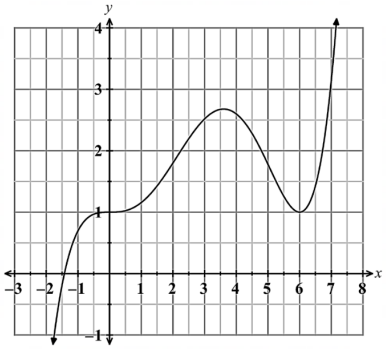
B.



C.



D.



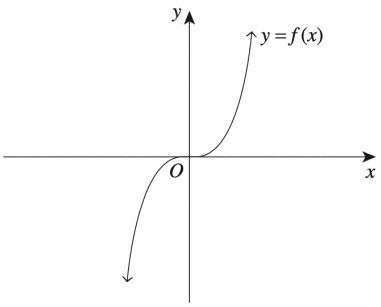
Criteria	Marks
Correct option (A)	1

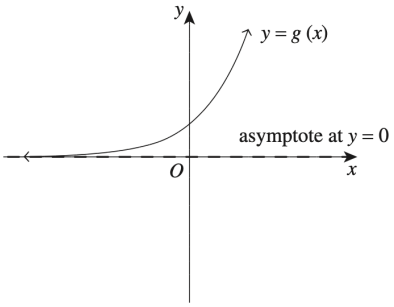
Solution.

Write $f(2x + 1) = f\left(2\left(x + \frac{1}{2}\right)\right)$. This is a horizontal compression toward the y-axis by a factor of $\frac{1}{2}$ (widths halve), followed by a translation of $\frac{1}{2}$ unit to the *left*. Each point (x, y) on $y = f(x)$ maps to $\left(\frac{x-1}{2}, y\right)$. The correct diagram is option (A).

Q38 Sydney Boys 2023 Trial Q10 · **1 mark**

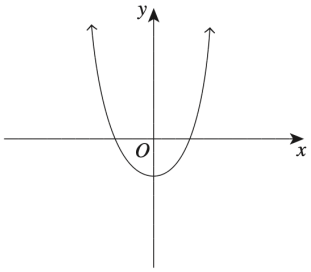
The graphs of $f(x)$ and $g(x)$ are shown (with $g(x)$ having a horizontal asymptote at $y = 0$). Which of the following best represents the graph of $y = f(g(-x))$?



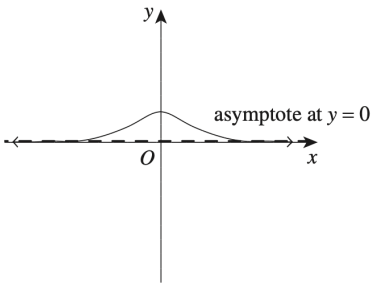


Which of the following best represents the graph of $y = f\big(g(-x)\big)$?

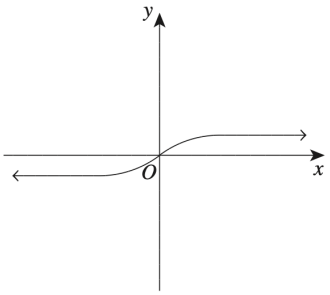
A.



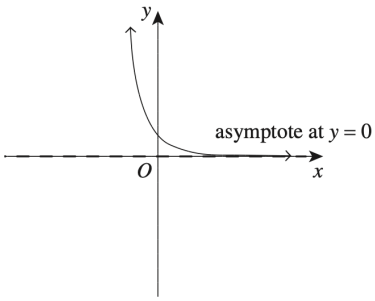
B.



C.



D.



Criteria	Marks
Correct option (D)	1

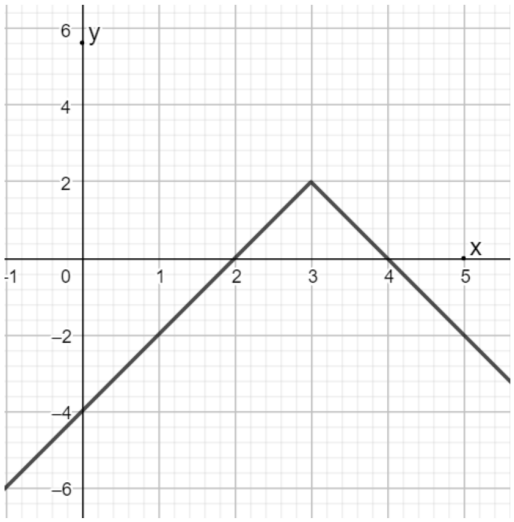
Solution.

The inner $g(-x)$ reflects g in the y-axis, so $g(-x) \to 0$ as $x \to \infty$; and since $g(x) \to 0$, the input to f approaches 0, where $f(x) \to 0$. This narrows the candidates to (B) or (D). The composite $y = f(g(-x))$ is neither odd nor even, and only option (D) has that (non-symmetric) shape. The correct diagram is option (D).

6. Find unknown transformation parameters

Q39 Girraween 2020 Trial Q27 · **2 marks**

The function $f(x) = |x|$ is transformed to $y = kf(x + b) + c$. From the graph shown, find k, b and c.



Criteria	Marks
Reads the vertex $(2, 1)$, giving $b = -2$ and $c = 1$	1
Reads the slope and orientation to get $k = -3$	1

Solution.

The graph is $y = k|x + b| + c$, an absolute-value "V" with vertex where $x + b = 0$, i.e. at $x = -b$ with height c. The vertex is at $(2, 1)$, so $-b = 2$ giving $b = -2$, and $c = 1$. The V opens *downward*, so $k < 0$, and each arm falls 3 units for every 1 unit across, so $|k| = 3$. Hence

$$k = -3, \quad b = -2, \quad c = 1,$$

i.e. $y = -3|x - 2| + 1$.

Q40 Blacktown Boys 2023 Trial Q27 · **2 marks**

The graph of $y = x^2$ maps to $y = 3x^2 + 24x + 33$ under a vertical stretch k, a horizontal translation p and a vertical translation q. Write down k, p and q.

Criteria	**Marks**
Completes the square to $y = 3(x + 4)^2 - 15$	1
Correct $k = 3$, $p = -4$, $q = -15$	1

Solution.

Complete the square on the target:

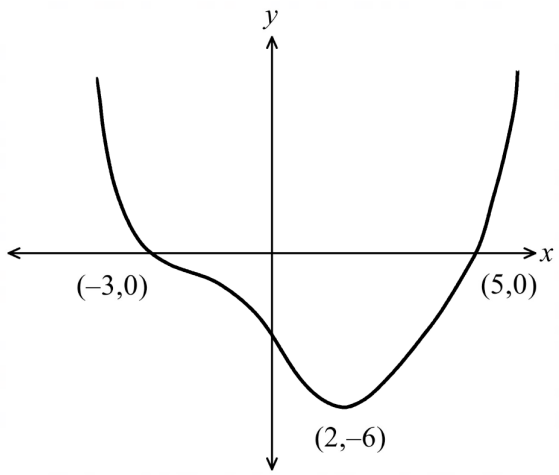
$$3x^2 + 24x + 33 = 3(x^2 + 8x) + 33 = 3(x + 4)^2 - 48 + 33 = 3(x + 4)^2 - 15.$$

Comparing with $y = k(x + p)^2 + q$ applied to $y = x^2$ (stretch k, shift p left=negative, shift q):

$$k = 3, \quad p = -4, \quad q = -15.$$

Q41 North Sydney Girls 2020 Trial Q27 · 3 marks

$y = f(x)$ passes through $(-3, 0)$ and $(5, 0)$ with a minimum turning point at $(2, -6)$. (a) Find the possible values of a so that $y = 4f(x + a)$ passes through the origin. (b) Find b so that the stationary point of $y = bf(x + 2)$ has y-coordinate 4.



Criteria	Marks
(a) Sets $f(a) = 0$ using the x-intercepts	1
(a) Both values $a = -3$ and $a = 5$	1
(b) Correct $b = -\frac{2}{3}$	1

Solution.

(a) At the origin $x = 0,\ y = 0$: $4f(0 + a) = 0$, so $f(a) = 0$. The zeros of f are its x-intercepts $x = -3$ and $x = 5$, so

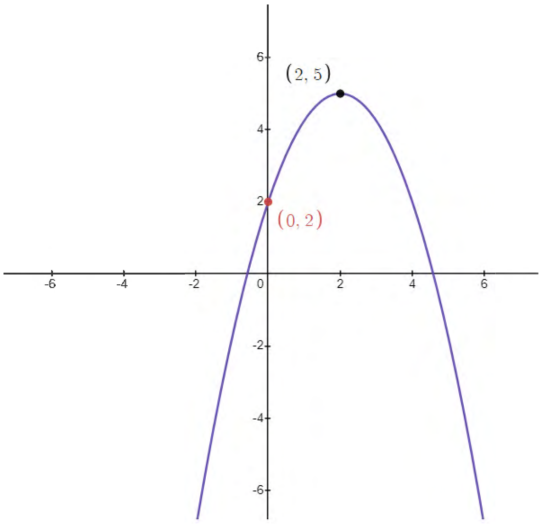
$$a = -3 \quad \text{or} \quad a = 5.$$

(b) $y = bf(x + 2)$ shifts the graph 2 left, moving the turning point to $x = 0$ with y-value $b \times (-6)$. Setting this equal to 4:

$$-6b = 4 \implies b = -\frac{2}{3}.$$

Q42 Sydney Tech 2023 Trial Q20 · **2 marks**

$f(x) = x^2$ is transformed to the parabola shown. Find $g(x) = k(x+b)^2 + c$.



Criteria	Marks
Reads the vertex $(2,5)$, giving $b = -2$ and $c = 5$	1
Uses the point $(0,2)$ to get $k = -\frac{3}{4}$	1

Solution.

The vertex is at $(2,5)$, so in $g(x) = k(x+b)^2 + c$ we have $b = -2$ and $c = 5$, i.e. $g(x) = k(x-2)^2 + 5$. The curve passes through $(0,2)$:

$$2 = k(0-2)^2 + 5 = 4k + 5 \implies 4k = -3 \implies k = -\tfrac{3}{4}.$$

Hence

$$g(x) = -\tfrac{3}{4}(x-2)^2 + 5, \qquad k = -\tfrac{3}{4},\ b = -2,\ c = 5.$$

Q43 Barker 2023 Trial Q23 · 3 marks

$f(x) = x^2$ is translated a left, dilated vertically by b and translated 3 up to give $g(x) = 2x^2 + 16x + 35$. Find a and b.

Criteria	Marks
Writes the transformed form $b(x + a)^2 + 3$	1
Matches coefficients / completes the square	1
Correct $a = 4$ and $b = 2$	1

Solution.

Applying the transformations to x^2: translate a left gives $(x + a)^2$, dilate vertically by b gives $b(x + a)^2$, translate 3 up gives

$$g(x) = b(x + a)^2 + 3.$$

Complete the square on the target:

$$2x^2 + 16x + 35 = 2(x^2 + 8x) + 35 = 2(x + 4)^2 - 32 + 35 = 2(x + 4)^2 + 3.$$

Matching $b(x + a)^2 + 3 = 2(x + 4)^2 + 3$ gives

$$a = 4, \quad b = 2.$$

Q44 2022 HSC Q19 · **3 marks**

$f(x) = x^2$ is translated m right, dilated vertically by k and translated 5 down to give $g(x) = 3x^2 - 12x + 7$. Find m and k.

Criteria	Marks
Writes the transformed form $k(x-m)^2 - 5$	1
Matches coefficients / completes the square	1
Correct $m = 2$ and $k = 3$	1

Solution.

Applying the transformations to x^2: translate m right gives $(x-m)^2$, dilate vertically by k gives $k(x-m)^2$, translate 5 down gives

$$g(x) = k(x-m)^2 - 5.$$

Complete the square on the target:

$$3x^2 - 12x + 7 = 3(x^2 - 4x) + 7 = 3(x-2)^2 - 12 + 7 = 3(x-2)^2 - 5.$$

Matching $k(x-m)^2 - 5 = 3(x-2)^2 - 5$ gives

$$m = 2, \quad k = 3.$$

Circles

7. Find the centre and radius by completing the square

Q45 Killara 2020 Trial Q4 · **1 mark**

The radius and centre of the circle with equation $x^2 - 6x + y^2 + 10y + 18 = 0$ is (multiple choice).

Criteria	Marks
Correct option: radius 4, centre $(3, -5)$ $\rightarrow$ (D)	1

Solution.

Group and complete the square in both variables:

$$(x^2 - 6x + 9) + (y^2 + 10y + 25) = -18 + 9 + 25$$

$$(x - 3)^2 + (y + 5)^2 = 16.$$

Hence centre $(3, -5)$ and radius $\sqrt{16} = 4$. The answer is **(D)**.

Q46 Cherrybrook Tech 2022 Prelim Q2 · **1 mark**

What is the centre and radius of the circle with equation $(x + 3)^2 + (y - 1)^2 = 25$? (multiple choice).

Criteria	Marks
Correct option: centre $(-3, 1)$, radius 5 $\rightarrow$ (B)	1

Solution.

The equation is already in standard form $(x - a)^2 + (y - b)^2 = r^2$, so read off $a = -3$, $b = 1$ and $r^2 = 25$. Hence centre $(-3, 1)$ and radius $\sqrt{25} = 5$. The answer is **(B)**.

Q47 Shore 2023 Trial Q11 · **2 marks**

Find the centre and radius of the circle with the equation $x^2 - 4x + y^2 - 8y - 5 = 0$.

Criteria	Marks
Completes the square in both variables to reach $(x-2)^2 + (y-4)^2 = 25$	1
Correct centre $(2, 4)$ and radius 5	1

Solution.

Group and complete the square:

$$(x^2 - 4x + 4) + (y^2 - 8y + 16) = 5 + 4 + 16$$

$$(x-2)^2 + (y-4)^2 = 25.$$

Hence centre $(2, 4)$ and radius $\sqrt{25} = 5$.

Q48 Killara 2022 Prelim Q29(a) · **1 mark**

The equation $x^2 - 12x + y^2 + 4y + 27 = 0$ represents a circle. Show that the radius of the circle is $\sqrt{13}$ units.

Criteria	Marks
Completes the square to reach $(x-6)^2 + (y+2)^2 = 13$ and states $r = \sqrt{13}$	1

Solution.

Group and complete the square:

$$(x^2 - 12x + 36) + (y^2 + 4y + 4) = -27 + 36 + 4$$

$$(x-6)^2 + (y+2)^2 = 13.$$

Hence $r^2 = 13$, so the radius is $\sqrt{13}$ units, as required.

Q49 Loreto Kirribilli 2022 Prelim Q4 · **1 mark**

What is the radius of the circle $x(x-4)+y(y-2)=4$? (multiple choice).

Criteria	Marks
Correct option: radius $3 \rightarrow$ (C)	1

Solution.

Expand and rearrange, then complete the square:

$$x^2-4x+y^2-2y=4$$

$$(x^2-4x+4)+(y^2-2y+1)=4+4+1$$

$$(x-2)^2+(y-1)^2=9.$$

Hence $r^2=9$, so the radius is $\sqrt{9}=3$. The answer is **(C)**.

Q50 International Grammar 2021 Prelim Q17 · **2 marks**

Draw a sketch of the circle with equation $x^2-2x+y^2+6y+6=0$.

Criteria	Marks
Completes the square to reach $(x-1)^2+(y+3)^2=4$ (centre $(1,-3)$, radius 2)	1
Correct sketch: circle centred at $(1,-3)$ with radius 2	1

Solution.

Group and complete the square:

$$(x^2-2x+1)+(y^2+6y+9)=-6+1+9$$

$$(x-1)^2+(y+3)^2=4.$$

Hence the circle has centre $(1,-3)$ and radius $\sqrt{4}=2$. Sketch a circle of radius 2 centred at $(1,-3)$: it meets the x-axis nowhere (lowest point $(1,-5)$, highest $(1,-1)$) and spans $x=-1$ to $x=3$.

Q51 Sydney Grammar 2021 Prelim Q8 · **1 mark**

A point lies on the circle with equation $x^2 + y^2 - 24x - 10y + 48 = 0$. What is the furthest distance, in units, the point could be from the origin? (multiple choice).

Criteria	Marks
Correct option: furthest distance 24 $\rightarrow$ (B)	1

Solution.

Complete the square to find the centre and radius:

$$(x^2 - 24x + 144) + (y^2 - 10y + 25) = -48 + 144 + 25$$

$$(x - 12)^2 + (y - 5)^2 = 121.$$

So the centre is $(12, 5)$ and the radius is $\sqrt{121} = 11$. The distance from the origin to the centre is

$$\sqrt{12^2 + 5^2} = \sqrt{169} = 13.$$

The furthest point on the circle from the origin lies along this line beyond the centre, at distance $13 + 11 = 24$ units. The answer is **(B)**.

8. Transform a circle or find its equation

Q52 Hornsby Girls 2021 Trial Q7 · **1 mark**

The relation $(x - 1)^2 + y^2 = 4$ is transformed to $(x - 3)^2 + (y + 1)^2 = 4$ by a horizontal translation followed by a vertical translation. Which row shows the correct direction and distance of the translation?

Criteria	Marks
Correct answer (A)	1

Solution.

The centre moves from $(1, 0)$ to $(3, -1)$. Horizontally $1 \rightarrow 3$ is a shift right 2 units; vertically $0 \rightarrow -1$ is a shift down 1 unit. Answer $\boxed{\text{(A)}}$.

Q53 Hornsby Girls 2022 Trial Q2 · **1 mark**

The circle $(x-2)^2+(y+4)^2=9$ is enlarged by a factor of 2 with centre the origin. What is its resulting equation?

Criteria	Marks
Correct answer (C)	1

Solution.

An enlargement of factor 2 about the origin doubles every coordinate: the centre $(2,-4)$ maps to $(4,-8)$ and the radius 3 becomes 6. Hence $(x-4)^2+(y+8)^2=36$. Answer $\boxed{\text{(C)}}$.

Q54 Kings 2022 Trial Q20 · **2 marks**

The circle $x^2-4x+y^2+2y-5=0$ is reflected in the y-axis. Determine the coordinates of the centre and radius of the reflected circle.

Criteria	Marks
Completes the square to reach $(x-2)^2+(y+1)^2=10$ (centre $(2,-1)$, radius $\sqrt{10}$)	1
Correct reflected centre $(-2,-1)$ and radius $\sqrt{10}$	1

Solution.

Completing the square,

$$x^2-4x+y^2+2y-5=0 \implies (x-2)^2+(y+1)^2=10,$$

so the original circle has centre $(2,-1)$ and radius $\sqrt{10}$. Reflecting in the y-axis sends $x \to -x$, which negates the x-coordinate of the centre and leaves the radius unchanged. The reflected circle has centre $(-2,-1)$ and radius $\sqrt{10}$.

Q55 St George Girls 2023 Trial Q19 · **3 marks**

The circle $x^2 + y^2 - 6x + 8y - 11 = 0$ is translated to the left by 4 units and up by 3 units. What is the centre and radius of the new circle?

Criteria	Marks
Completes the square to reach $(x-3)^2 + (y+4)^2 = 36$	1
Correct original centre $(3, -4)$ and radius 6	1
Correct translated centre $(-1, -1)$ and radius 6	1

Solution.

Completing the square,

$$x^2 + y^2 - 6x + 8y - 11 = 0 \implies (x-3)^2 + (y+4)^2 = 36,$$

so the original circle has centre $(3, -4)$ and radius 6. Translating left 4 (subtract 4 from the x-coordinate) and up 3 (add 3 to the y-coordinate) moves the centre to $(3-4,\ -4+3) = (-1, -1)$; the radius is unchanged. The new circle has centre $(-1, -1)$ and radius 6.

Q56 Caringbah 2022 Trial Q12 · **2 marks**

A circle has the equation $x^2 - 6x + y^2 + 4y + k = 0$ for some constant k. Find the set of possible values for k.

Criteria	Marks
Completes the square to reach $(x-3)^2 + (y+2)^2 = 13 - k$	1
Applies $r^2 > 0$ to conclude $k < 13$	1

Solution.

Completing the square,

$$x^2 - 6x + y^2 + 4y + k = 0 \implies (x-3)^2 + (y+2)^2 = 13 - k.$$

For this to be a genuine circle the radius squared must be positive, so $13 - k > 0$, giving $k < 13$.

Q57 2020 HSC Q24 · **3 marks**

The circle $x^2 - 6x + y^2 + 4y - 3 = 0$ is reflected in the x-axis. Sketch the reflected circle, showing the coordinates of the centre and the radius.

Criteria	**Marks**
Completes the square to reach $(x-3)^2 + (y+2)^2 = 16$ (centre $(3,-2)$, radius 4)	1
Correct reflected centre $(3,2)$ and radius 4	1
Correct sketch of the reflected circle with centre and radius labelled	1

Solution.

Completing the square,

$$x^2 - 6x + y^2 + 4y - 3 = 0 \implies (x-3)^2 + (y+2)^2 = 16,$$

so the original circle has centre $(3,-2)$ and radius 4. Reflecting in the x-axis sends $y \to -y$, negating the y-coordinate of the centre and leaving the radius unchanged. The reflected circle has centre $(3,2)$ and radius 4, with equation $(x-3)^2 + (y-2)^2 = 16$. Sketch a circle of radius 4 centred at $(3,2)$, marking the centre and radius.