Year 11 Mathematics Advanced

Topic: Graph Transformations (2 of 2)

Review Questions (Worked Solutions): Mixed one-off exam questions

Term 3, 2026

Mixed one-off exam questions

Q1 Hornsby Girls 2022 Prelim Q9 · 1 mark

The graph of $f(x) = \dfrac{1}{x}$ is shown. What are the domain and range of $g(x) = 2f(x) + 1$? (multiple choice).

Criteria	Marks
Correct option: domain $x \neq 0$, range $y \neq 1 \rightarrow$ (C)	1

Solution.

Here $g(x) = 2 \cdot \dfrac{1}{x} + 1 = \dfrac{2}{x} + 1$. Division by x is undefined only at $x = 0$, so the domain is all real x with $x \neq 0$ (a vertical dilation and vertical shift do not change the domain). As x varies, $\dfrac{2}{x}$ takes every real value except 0, so $g(x) = \dfrac{2}{x} + 1$ takes every real value except 1. Hence the range is all real y with $y \neq 1$. The answer is **(C)**.

Q2 Presbyterian Ladies' College 2021 Trial Q4 · 1 mark

Given $y = f(x)$ has asymptote $x = 3$ and is defined only for $x > 3$, which inequality gives the domain of $y = -f(-x)$? (multiple choice).

Criteria	Marks
Correct option: $D: \ x < -3 \rightarrow$ (D)	1

Solution.

From the graph, f is defined only for $x > 3$. In $y = -f(-x)$, the outer negative sign is a vertical reflection and does not affect the domain; only the inner $-x$ matters. We need $-x$ to lie in the domain of f, i.e.

$$-x > 3 \implies x < -3.$$

Hence the domain of $y = -f(-x)$ is $x < -3$. The answer is **(D)**.

Q3 North Sydney Girls 2022 Prelim Q28 · **3 marks**

Consider the circle $x^2 + y^2 - 4y + 6x = 12$; find its centre and radius, and show $(2, 4)$ lies outside it.

Criteria	Marks
(a) Completes the square to reach $(x + 3)^2 + (y - 2)^2 = 25$	1
(a) Correct centre $(-3, 2)$ and radius 5	1
(b) Shows the distance from the centre to $(2, 4)$ is $\sqrt{29} > 5$, so the point lies outside	1

Solution.

(a) Group and complete the square in both variables:

$$(x^2 + 6x + 9) + (y^2 - 4y + 4) = 12 + 9 + 4$$

$$(x + 3)^2 + (y - 2)^2 = 25.$$

Hence the centre is $(-3, 2)$ and the radius is $\sqrt{25} = 5$.

(b) The distance from the centre $(-3, 2)$ to the point $(2, 4)$ is

$$d = \sqrt{(2 - (-3))^2 + (4 - 2)^2} = \sqrt{5^2 + 2^2} = \sqrt{29} \approx 5.39.$$

Since $d = \sqrt{29} > 5 = r$, the point $(2, 4)$ lies further from the centre than the radius, so it lies outside the circle.

Q4 Manly 2021 Trial Q6 · **1 mark**

The tangent to $y = f(x)$ at $(2, 7)$ is $y = 5x - 3$; find the tangent to $y = f(x + 2) + 1$ at $(0, 8)$. (multiple choice).

Criteria	Marks
Correct option: $y = 5x + 8 \rightarrow$ (C)	1

Solution.

The curve $y = f(x + 2) + 1$ is the graph of $y = f(x)$ translated 2 units left and 1 unit up. A translation moves every feature of the graph rigidly, so it moves the tangent line the same way. The point of tangency $(2, 7)$ moves to $(2 - 2,\ 7 + 1) = (0, 8)$, which matches the point in the question. Applying the same translation to the tangent line $y = 5x - 3$ (replace x with $x + 2$, then add 1):

$$y = 5(x + 2) - 3 + 1 = 5x + 10 - 3 + 1 = 5x + 8.$$

The gradient is unchanged at 5 because a translation does not rotate the line. Hence the tangent is $y = 5x + 8$. The answer is **(C)**.

Q5 2023 HSC Q10 · **1 mark**

$y = x^2$ meets $y = k$ at P, Q with $|PQ| = L$; $y = \dfrac{x^2}{a^2}$ meets $y = k$ at S, T. Find $|ST|$. (multiple choice).

Criteria	Marks
Correct option: $\lvert ST\rvert = aL \rightarrow$ (C)	1

Solution.

For $y = x^2$: solving $x^2 = k$ gives $x = \pm\sqrt{k}$, so P and Q are $(-\sqrt{k}, k)$ and $(\sqrt{k}, k)$, giving

$$L = 2\sqrt{k}.$$

For $y = \dfrac{x^2}{a^2}$: solving $\dfrac{x^2}{a^2} = k$ gives $x^2 = a^2k$, so $x = \pm a\sqrt{k}$, and

$$|ST| = 2a\sqrt{k} = a \cdot 2\sqrt{k} = aL.$$

(The curve $y = x^2/a^2$ is a horizontal dilation of $y = x^2$ by factor a, which stretches the chord at height k by the same factor a.) The answer is **(C)**.