Year 11 Mathematics Advanced

Topic: Probability and Data (2 of 5)

Review Questions (Worked Solutions): Probability

Term 3, 2026

Probability of an event

1. Basic probability from a sample space

Q1 Gosford 2024 Prelim Q11 · **1 mark**

A single card is drawn from a regular deck of 52 cards. Find the probability that the card is a red four.

Criteria	Marks
$P(\text{red four}) = \frac{1}{26}$	1

Solution.

A standard deck has 52 cards. The fours are the four of hearts, diamonds, clubs and spades. The *red* fours are the four of hearts and the four of diamonds, so there are 2 favourable cards.

$$P(\text{red four}) = \frac{2}{52} = \frac{1}{26} \approx 0.0385.$$

Q2 Blacktown Boys 2024 Trial Q2 · **1 mark**

Spinner A shows $\{1, 2, 3\}$ and Spinner B shows $\{1, 2, 2, 3\}$. Each arrow is spun once and the score is the sum of the two numbers. Find $P(\text{score} \geq 4)$.

Criteria	Marks
Correct option (C)	1

Solution.

There are $3 \times 4 = 12$ equally likely (Spinner A, Spinner B) outcomes. Completing the score table gives the sums

$$2,\ 3,\ 3,\ 4,\quad 3,\ 4,\ 4,\ 5,\quad 4,\ 5,\ 5,\ 6.$$

Counting the scores that are 4 or more: $\{4, 4, 4, 4, 5, 5, 5, 6\}$, which is 8 outcomes. Hence

$$P(\text{score} \geq 4) = \frac{8}{12} = \frac{2}{3}.$$

This is option **C**.

Q3 Abbotsleigh 2023 Trial Q3 · **1 mark**

$A = \{5, 7, 9\}$ and $B = \{11, 12, 13\}$; one number is chosen at random from each set. Find $P(\text{sum} \geq 21)$.

Criteria	Marks
Correct option (C)	1

Solution.

There are $3 \times 3 = 9$ equally likely pairs. The largest sums come from choosing 9 from A: $9 + 12 = 21$ and $9 + 13 = 22$ both satisfy sum ≥ 21. Every other pair falls short (the next largest is $9 + 11 = 20$ and $7 + 13 = 20$). So there are 2 favourable outcomes:

$$P(\text{sum} \geq 21) = \frac{2}{9}.$$

This is option **C**.

Q4 Gosford 2024 Trial Q1 · **1 mark**

A standard die (1–6) and a spinner numbered 1 to 5 are used; the score is the *product* of the two numbers. Find $P(\text{even score})$.

Criteria	Marks
Correct option (D)	1

Solution.

There are $6 \times 5 = 30$ equally likely (die, spinner) outcomes. A product is *odd* only when both factors are odd. Odd die faces are $\{1, 3, 5\}$ (3 values) and odd spinner numbers are $\{1, 3, 5\}$ (3 values), giving $3 \times 3 = 9$ odd products. The remaining $30 - 9 = 21$ products are even, so

$$P(\text{even score}) = \frac{21}{30} = \frac{7}{10}.$$

This is option **D**.

Q5 Manly 2023 Trial Q9 · **1 mark**

θ is chosen randomly and uniformly in $0 \leq \theta \leq 2\pi$. Find $P\big(-1 \leq 2\sin\theta + 1 \leq 2\big)$.

Criteria	Marks
Correct option (C)	1

Solution.

The two inequalities on $2\sin\theta + 1$ become inequalities on $\sin\theta$:

$$-1 \leq 2\sin\theta + 1 \implies \sin\theta \geq -1 \quad \text{(true for all } \theta\text{)},$$

$$2\sin\theta + 1 \leq 2 \implies \sin\theta \leq \tfrac{1}{2}.$$

So the condition is simply $\sin\theta \leq \frac{1}{2}$. On $[0, 2\pi]$ we have $\sin\theta > \frac{1}{2}$ only on the interval $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$, whose length is $\frac{5\pi}{6} - \frac{\pi}{6} = \frac{2\pi}{3}$. The favourable set therefore has length

$$2\pi - \frac{2\pi}{3} = \frac{4\pi}{3},$$

and since θ is uniform on $[0, 2\pi]$,

$$P = \frac{4\pi/3}{2\pi} = \frac{2}{3}.$$

This is option **C**.

Q6 North Sydney Girls 2022 Prelim Q33 · **4 marks**

A regular pentagon is inscribed in a circular dartboard (radius r) and divided into five identical triangles labelled 1–5. The five segments between the pentagon's edges and the circle are shaded. A dart lands uniformly at random on the board; the player wins by landing in a shaded segment next to an *odd*-numbered triangle, or inside an *even*-numbered triangle. Find $P(\text{win})$.

Criteria	Marks
Area of one triangle $= \frac{1}{2}r^2 \sin\frac{2\pi}{5}$ (and/or one sector $= \frac{1}{2}r^2 \cdot \frac{2\pi}{5}$)	1
Area of one segment $= \frac{1}{2}r^2\left(\frac{2\pi}{5} - \sin\frac{2\pi}{5}\right)$	1
Winning area $= 3 \times$ (segment) $+ 2 \times$ (triangle)	1
$P(\text{win}) = \dfrac{3}{5} - \dfrac{\sin\frac{2\pi}{5}}{2\pi} \approx 0.449$	1

Solution.

Let the radius be r. Each of the five identical triangles subtends a central angle $\theta = \dfrac{2\pi}{5}$ at the centre.

Areas of the pieces. For each of the five equal sectors:

$$A_{\text{triangle}} = \tfrac{1}{2}r^2 \sin\theta = \tfrac{1}{2}r^2 \sin\tfrac{2\pi}{5}, \qquad A_{\text{sector}} = \tfrac{1}{2}r^2\theta = \tfrac{1}{2}r^2 \cdot \tfrac{2\pi}{5}.$$

The shaded segment between one pentagon edge and the arc is the sector minus the triangle:

$$A_{\text{segment}} = A_{\text{sector}} - A_{\text{triangle}} = \tfrac{1}{2}r^2\left(\tfrac{2\pi}{5} - \sin\tfrac{2\pi}{5}\right).$$

Winning region. There are 3 odd-numbered triangles $(1, 3, 5)$, so 3 winning shaded segments; and 2 even-numbered triangles $(2, 4)$, which win inside. The total winning area is

$$A_{\text{win}} = 3\,A_{\text{segment}} + 2\,A_{\text{triangle}} = 3 \cdot \tfrac{1}{2}r^2\left(\tfrac{2\pi}{5} - \sin\tfrac{2\pi}{5}\right) + 2 \cdot \tfrac{1}{2}r^2 \sin\tfrac{2\pi}{5}.$$

Simplifying,

$$A_{\text{win}} = \tfrac{1}{2}r^2\left(3 \cdot \tfrac{2\pi}{5} - 3\sin\tfrac{2\pi}{5} + 2\sin\tfrac{2\pi}{5}\right) = \tfrac{1}{2}r^2\left(\tfrac{6\pi}{5} - \sin\tfrac{2\pi}{5}\right).$$

Probability. Dividing by the whole board area πr^2:

$$P(\text{win}) = \frac{\frac{1}{2}r^2\left(\frac{6\pi}{5} - \sin\frac{2\pi}{5}\right)}{\pi r^2} = \frac{1}{2\pi}\left(\frac{6\pi}{5} - \sin\frac{2\pi}{5}\right) = \frac{3}{5} - \frac{\sin\frac{2\pi}{5}}{2\pi} \approx 0.449.$$

Q7 North Sydney Girls 2021 Prelim Q3 · **4 marks**

(a) A and B are independent with $P(A) = 0.7$, $P(B) = 0.4$; find $P(A \cup B)$.
(b) For $y = x^2 + bx + 12$ with b a randomly chosen integer in $[-10, 10]$, find the probability that $y \leq 0$ for one or more values of x.

Criteria	Marks
(a) $P(A \cap B) = 0.7 \times 0.4 = 0.28$	1
(a) $P(A \cup B) = 0.82$	1
(b) $y \leq 0$ for some $x \iff \Delta = b^2 - 48 \geq 0 \iff \lvert b\rvert \geq \sqrt{48}$	1
(b) $b \in \{\pm 7, \pm 8, \pm 9, \pm 10\}$, so $P = \frac{8}{21}$	1

Solution.

(a) Since A and B are independent, $P(A \cap B) = P(A)\,P(B) = 0.7 \times 0.4 = 0.28$. By the addition rule,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.7 + 0.4 - 0.28 = 0.82.$$

(b) The parabola $y = x^2 + bx + 12$ is concave up, so it takes a value ≤ 0 for one or more x exactly when it touches or crosses the x-axis, i.e. when the discriminant is non-negative:

$$\Delta = b^2 - 4(1)(12) = b^2 - 48 \geq 0 \implies b^2 \geq 48 \implies |b| \geq \sqrt{48} \approx 6.93.$$

Since b is an integer, this means $|b| \geq 7$. The interval $[-10, 10]$ contains 21 equally likely integers, and the favourable values are

$$b \in \{-10, -9, -8, -7, 7, 8, 9, 10\} \quad (8 \text{ values}).$$

Therefore

$$P(y \leq 0 \text{ for some } x) = \frac{8}{21} \approx 0.381.$$

Combining events

2. Probability from a Venn diagram or two-way table

Q8 Barker 2023 Trial Q4 · 1 mark

Of 15 ties, 8 are silk and 9 are striped, and 3 are neither silk nor striped. Find the probability a randomly selected tie is silk but not striped.

Criteria	Marks
Correct option (C)	1

Solution.

The number that are silk or striped is $15 - 3 = 12$. By the addition rule the number that are both is

$$n(\text{silk} \cap \text{striped}) = 8 + 9 - 12 = 5.$$

So the number silk but not striped is $8 - 5 = 3$, giving

$$P(\text{silk but not striped}) = \frac{3}{15} = \frac{1}{5}.$$

This is option **C**.

Q9 Hurlstone 2021 Prelim Q21 · 1 mark

A Venn diagram splits employees by transport: B (bus) only $= 15$, both B and $T = 17$, T (train) only $= 29$, and 31 take neither. Find $P(B \cup T)$.

Criteria	Marks
Correct option (D)	1

Solution.

The total number of employees is

$$15 + 17 + 29 + 31 = 92.$$

The number who catch a bus or a train (the union) is everything inside the two circles:

$$n(B \cup T) = 15 + 17 + 29 = 61.$$

Therefore

$$P(B \cup T) = \frac{61}{92}.$$

This is option **D**.

Q10 Gosford 2024 Prelim Q5 · **1 mark**

A two-way table records 535 people by sex (Male/Female) and whether they have pets. Find the approximate probability a randomly chosen person is female or has pets.

Criteria	Marks
Correct option (a)	1

Solution.

Use the addition rule with $n(\text{Total}) = 535$, $n(\text{Female}) = 221$, $n(\text{Has pets}) = 249$, and $n(\text{Female} \cap \text{Has pets}) = 131$:

$$n(\text{Female} \cup \text{Has pets}) = 221 + 249 - 131 = 339.$$

Therefore

$$P(\text{Female or has pets}) = \frac{339}{535} = 0.6336\ldots \approx 63\%.$$

This is option **a**.

Q11 Baulkham Hills 2024 Prelim Q7 · **1 mark**

Of 100 students, 45 walked, 70 caught the bus, and 26 did neither. Find the probability a randomly chosen student walked and caught the bus.

Criteria	Marks
Correct option (D)	1

Solution.

The number who walked or caught the bus is $100 - 26 = 74$. By the addition rule the number who did both is

$$n(\text{walk} \cap \text{bus}) = 45 + 70 - 74 = 41.$$

Therefore

$$P(\text{walked and caught bus}) = \frac{41}{100} = 41\%.$$

This is option **D**.

Q12 Hurlstone 2021 Prelim Q17 · **1 mark**

Of 60 consumers, 42 used a card and 31 used cash. Find the probability a randomly chosen consumer used both cash and card.

Criteria	Marks
Correct option (A)	1

Solution.

Every consumer used at least one method, so $n(\text{Card} \cup \text{Cash}) = 60$. By the addition rule,

$$n(\text{Card} \cap \text{Cash}) = 42 + 31 - 60 = 13.$$

Therefore

$$P(\text{both cash and card}) = \frac{13}{60}.$$

This is option **A**.

Q13 Hurlstone 2023 Prelim Q7 · **1 mark**

Of 27 students, 14 swim, 11 play football, and 3 do both. Find the probability a randomly chosen student either swims or plays football, but not both.

Criteria	Marks
Correct option (B)	1

Solution.

The 3 who do both are counted in each total, so the numbers who do only one activity are

$$\text{swim only} = 14 - 3 = 11, \qquad \text{football only} = 11 - 3 = 8.$$

The number who do one activity but not both is $11 + 8 = 19$, so

$$P(\text{exactly one}) = \frac{19}{27}.$$

This is option **B**.

Probability of an event

3. Complementary events and 'at least one'

Q14 Loreto Kirribilli 2022 Prelim Q26 · **1 mark**

A coin is tossed twice. Find the probability of at least 1 head.

Criteria	Marks
Correct probability $\frac{3}{4}$	1

Solution.

Use the complement: "at least one head" is the opposite of "no heads" (that is, two tails). The two tosses are independent, so

$$P(\text{no heads}) = P(TT) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}.$$

Therefore

$$P(\text{at least 1 head}) = 1 - P(\text{no heads}) = 1 - \frac{1}{4} = \boxed{\frac{3}{4}}.$$

Q15 Sydney Boys 2020 Trial Q4 · **1 mark**

A raffle has 20 tickets and two prizes (i.e. two winning tickets). David buys 5 tickets. Find the probability that David wins at least one movie voucher.

Criteria	Marks
Correct option (D)	1

Solution.

"At least one" is easiest via the complement. There are two winning tickets among the 20, so 18 non-winning tickets. The probability that all 5 of David's tickets are non-winners is

$$P(\text{no win}) = \frac{\binom{18}{5}}{\binom{20}{5}} = \frac{15 \times 14}{20 \times 19} = \frac{21}{38}.$$

Hence

$$P(\text{at least one voucher}) = 1 - \frac{21}{38} = \frac{17}{38} \approx 0.447.$$

The answer is **(D)**.

Q16 Barker 2023 Trial Q18 · **2 marks**

Barry passes his driving test with probability 0.7 and Doreen with probability 0.8 (independently). Find the probability that at least one of them passes.

Criteria	Marks
Finds $P(\text{both fail}) = 0.3 \times 0.2 = 0.06$	1
Correct probability 0.94	1

Solution.

Use the complement of "at least one passes", which is "both fail". The results are independent, so

$$P(\text{both fail}) = (1 - 0.7)(1 - 0.8) = 0.3 \times 0.2 = 0.06.$$

Therefore

$$P(\text{at least one passes}) = 1 - 0.06 = \boxed{0.94}.$$

Q17 Killara 2025 Trial Q12 · **3 marks**

Harry spins two fair 5-sided spinners once each. Spinner A shows 2 reds and 3 greens; Spinner B shows 4 reds and 1 green. (a) Label the probabilities on the tree diagram. (b) Find the probability that at least one spinner lands on green.

Criteria	Marks
(a) tree with Spinner A probabilities ($\frac{2}{5}$ red, $\frac{3}{5}$ green)	1
(a) tree with Spinner B probabilities ($\frac{4}{5}$ red, $\frac{1}{5}$ green) on each second-stage branch	1
(b) correct probability $\frac{17}{25}$	1

Solution.

(a) Reading the spinner faces, Spinner A lands on red with probability $\frac{2}{5}$ and on green with probability $\frac{3}{5}$; Spinner B lands on red with probability $\frac{4}{5}$ and on green with probability $\frac{1}{5}$. These label the first- and second-stage branches of the tree respectively.

(b) Use the complement: "at least one green" is the opposite of "both red". The two spins are independent, so

$$P(\text{both red}) = \frac{2}{5} \times \frac{4}{5} = \frac{8}{25}.$$

Therefore

$$P(\text{at least one green}) = 1 - \frac{8}{25} = \boxed{\frac{17}{25}} = 0.68.$$

Q18 Gosford 2020 Trial Q22 · **5 marks**

For a pet ownership survey, $P(C) = \frac{3}{7}$, $P(D \mid \overline{C}) = \frac{2}{5}$ and $P(\overline{D} \mid C) = \frac{3}{4}$, where C = "has a cat" and D = "has a dog". (a) Complete the probability tree. (b)(i) Find $P(\text{cat and dog})$; (ii) find $P(\text{at least one pet})$.

Criteria	**Marks**
(a) first-stage branches $P(C) = \frac{3}{7}$, $P(\overline{C}) = \frac{4}{7}$	1
(a) second-stage branches $P(D \mid C) = \frac{1}{4}$, $P(\overline{D} \mid C) = \frac{3}{4}$, $P(D \mid \overline{C}) = \frac{2}{5}$, $P(\overline{D} \mid \overline{C}) = \frac{3}{5}$	1
(b)(i) $P(C \cap D) = \frac{3}{28}$	1
(b)(ii) finds $P(\overline{C} \cap \overline{D}) = \frac{12}{35}$ (or equivalent)	1
(b)(ii) $P(\text{at least one pet}) = \frac{23}{35}$	1

Solution.

(a) From $P(C) = \frac{3}{7}$ we get $P(\overline{C}) = \frac{4}{7}$. From $P(\overline{D} \mid C) = \frac{3}{4}$ we get $P(D \mid C) = \frac{1}{4}$, and from $P(D \mid \overline{C}) = \frac{2}{5}$ we get $P(\overline{D} \mid \overline{C}) = \frac{3}{5}$. These are the branch probabilities on the tree.

(b)(i) "A cat and a dog" is $C \cap D$:

$$P(C \cap D) = P(C)\,P(D \mid C) = \frac{3}{7} \times \frac{1}{4} = \boxed{\frac{3}{28}}.$$

(b)(ii) "At least one pet" is the complement of "no pet" ($\overline{C} \cap \overline{D}$):

$$P(\overline{C} \cap \overline{D}) = P(\overline{C})\,P(\overline{D} \mid \overline{C}) = \frac{4}{7} \times \frac{3}{5} = \frac{12}{35}.$$

Therefore

$$P(\text{at least one pet}) = 1 - \frac{12}{35} = \boxed{\frac{23}{35}} \approx 0.657.$$

Q19 North Sydney Boys 2024 Trial Q25 · **3 marks**

Isis gets a seat each morning with probability $\frac{1}{8}$, Layla with probability $\frac{1}{21}$. Find the probability that, over the next three mornings, at least one girl gets a seat.

Criteria	**Marks**
Per morning, $P(\text{neither gets a seat}) = \frac{7}{8} \times \frac{20}{21} = \frac{5}{6}$	1
Over 3 mornings, $P(\text{no seat at all}) = \left(\frac{5}{6}\right)^3 = \frac{125}{216}$	1
$P(\text{at least one seat}) = \frac{91}{216}$	1

Solution.

Use the complement: "at least one girl gets a seat (over the three mornings)" is the opposite of "no girl ever gets a seat". On a single morning the two girls act independently, so

$$P(\text{neither gets a seat}) = \left(1 - \frac{1}{8}\right)\left(1 - \frac{1}{21}\right) = \frac{7}{8} \times \frac{20}{21} = \frac{5}{6}.$$

The three mornings are independent, so

$$P(\text{no seat on any of the 3 mornings}) = \left(\frac{5}{6}\right)^3 = \frac{125}{216}.$$

Therefore

$$P(\text{at least one girl gets a seat}) = 1 - \frac{125}{216} = \boxed{\frac{91}{216}} \approx 0.421.$$

Q20 Blacktown Boys 2024 Trial Q31 · **4 marks**

Each of 5 people adopts a dog with probability $P(D) = \frac{9}{14}$ and a cat with probability $P(C)$, where $P(D \mid C) = \frac{6}{13}$ and $P(C \mid D) = \frac{1}{3}$. (a) Show that $P(C) = \frac{13}{28}$. (b) Find the probability that at least one of the 5 people will NOT adopt a dog (to 2 d.p.).

Criteria	Marks
(a) finds $P(C \cap D) = P(C \mid D)\,P(D) = \frac{3}{14}$	1
(a) rearranges $P(D \mid C) = \frac{P(C \cap D)}{P(C)}$ to obtain $P(C) = \frac{13}{28}$	1
(b) $P(\text{all 5 adopt a dog}) = \left(\frac{9}{14}\right)^5$	1
(b) $P(\text{at least one does NOT}) \approx 0.89$	1

Solution.

(a) First find $P(C \cap D)$ from the conditional $P(C \mid D)$:

$$P(C \cap D) = P(C \mid D)\,P(D) = \frac{1}{3} \times \frac{9}{14} = \frac{3}{14}.$$

Now use $P(D \mid C) = \dfrac{P(C \cap D)}{P(C)}$ and solve for $P(C)$:

$$P(C) = \frac{P(C \cap D)}{P(D \mid C)} = \frac{\frac{3}{14}}{\frac{6}{13}} = \frac{3}{14} \times \frac{13}{6} = \frac{39}{84} = \frac{13}{28}. \qquad \blacksquare$$

(b) Use the complement: "at least one of the 5 does NOT adopt a dog" is the opposite of "all 5 adopt a dog". Each person adopts a dog independently with probability $\frac{9}{14}$, so

$$P(\text{all 5 adopt a dog}) = \left(\frac{9}{14}\right)^5 = \frac{59049}{537824} \approx 0.1098.$$

Therefore

$$P(\text{at least one does NOT adopt a dog}) = 1 - \left(\frac{9}{14}\right)^5 \approx 0.8902 \approx \boxed{0.89}.$$

Combining events

4. The addition rule and mutually exclusive events

Q21 Barker 2022 Trial Q6 · **1 mark**

16 T-shirts: 10 are blue, 8 have stripes, 4 are neither blue nor striped. Find $P(\text{blue and striped})$.

Criteria	Marks
Correct option (C)	1

Solution.

The number that are blue *or* striped is $16 - 4 = 12$. By the addition rule,

$$P(\text{blue} \cup \text{striped}) = P(\text{blue}) + P(\text{striped}) - P(\text{blue} \cap \text{striped}),$$

so, counting T-shirts,

$$12 = 10 + 8 - n(\text{blue} \cap \text{striped}) \implies n(\text{blue} \cap \text{striped}) = 6.$$

Hence

$$P(\text{blue and striped}) = \frac{6}{16} = \frac{3}{8}.$$

The answer is option **C**.

Q22 St George Girls 2020 Trial Q3 · **1 mark**

An ordinary die is thrown twice. Find the probability the second number is more than the first.

Criteria	Marks
Correct option (C)	1

Solution.

There are $6 \times 6 = 36$ equally likely ordered outcomes. The number of outcomes with (second > first) equals the number of ways to choose two *different* values and assign the larger to the second throw, i.e. the number of pairs $\{a, b\}$ with $a \neq b$:

$$\binom{6}{2} = 15.$$

(Equivalently, of the $36 - 6 = 30$ outcomes with the two numbers different, exactly half have the second larger.) Therefore

$$P(\text{second} > \text{first}) = \frac{15}{36} = \frac{5}{12}.$$

The answer is option **C**.

Q23 Ascham 2022 Trial Q3 · **1 mark**

One card is drawn from cards labelled 1 to 50. Find $P(\text{odd or greater than } 40)$.

Criteria	Marks
Correct option (C)	1

Solution.

Among 1–50: there are 25 odd numbers, and 10 numbers greater than 40 (i.e. 41–50). The overlap "odd *and* greater than 40" is $\{41, 43, 45, 47, 49\}$, which is 5 numbers. By the addition rule,

$$n(\text{odd} \cup >40) = 25 + 10 - 5 = 30.$$

Hence

$$P(\text{odd or} > 40) = \frac{30}{50} = 0.6.$$

The answer is option **C**.

Q24 Barker 2025 Trial Q4 · **1 mark**

73 teachers: 11 teach Mathematics, 10 teach Science, 54 teach neither. Find $P(\text{Mathematics only or Science only})$.

Criteria	Marks
Correct option (A)	1

Solution.

The number teaching at least one of the two subjects is $73 - 54 = 19$. By the addition rule the number teaching *both* is

$$n(M \cap S) = 11 + 10 - 19 = 2.$$

So Mathematics only $= 11 - 2 = 9$ and Science only $= 10 - 2 = 8$, giving

$$P(\text{Maths only or Science only}) = \frac{9 + 8}{73} = \frac{17}{73}.$$

The answer is option **A**.

Q25 Presbyterian 2022 Prelim Q28 · **4 marks**

A and B are independent, $P(A) = 0.3$, $P(B) = 0.6$. **(a)** Find $P(A \cap B)$. **(b)** Find $P(A \cup B)$. **(c)** Shade $A \cap B'$ on the Venn diagram. **(d)** Find $P(A \cap B')$.

Criteria	Marks
(a) $P(A \cap B) = 0.18$	1
(b) $P(A \cup B) = 0.72$	1
(c) Shades $A \cap B'$ (the "A only" region)	1
(d) $P(A \cap B') = 0.12$	1

Solution.

(a) Since A and B are independent,

$$P(A \cap B) = P(A)\,P(B) = 0.3 \times 0.6 = 0.18.$$

(b) By the addition rule,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.3 + 0.6 - 0.18 = 0.72.$$

(c) $A \cap B'$ is the set of outcomes in A but *not* in B: shade only the part of circle A that lies outside circle B (the "A only" crescent). Leave the overlap, the "B only" region, and the outside region unshaded.

(d) A splits into "A only" and the overlap, so

$$P(A \cap B') = P(A) - P(A \cap B) = 0.3 - 0.18 = 0.12.$$

Q26 Knox 2021 Prelim Q13 · **2 marks**

Two six-sided dice are rolled, then a third. You win if the third die's number lies strictly between the first two. Find $P(\text{win})$.

Criteria	**Marks**
Sets up the count of winning outcomes (or equivalent per-case reasoning)	1
$P(\text{win}) = \dfrac{5}{27}$	1

Solution.

There are $6^3 = 216$ equally likely ordered outcomes. A roll wins when the third number lies strictly between the first two. Group by the pair {first, second}: if the two rolls differ by d (so the values are k apart), the number of integers strictly between them is $d - 1$, and each ordered arrangement of the first two dice is counted.

Counting directly, the number of ordered triples (a, b, c) with c strictly between a and b is

$$\sum_{a=1}^{6}\sum_{b=1}^{6}\big(\max(a,b) - \min(a,b) - 1\big)_+ = 40.$$

Therefore

$$P(\text{win}) = \frac{40}{216} = \frac{5}{27} \approx 0.185.$$

Harder applications

5. Setting up equations to find an unknown probability

Q27 Cranbrook 2023 Trial Q7 · 1 mark

A bag has n discs, exactly 7 red. Two are drawn without replacement and $P(\text{both red}) = \frac{21}{55}$. Choose the quadratic that finds n.

Criteria	Marks
Correct option (B)	1

Solution.

Drawing two red discs without replacement:

$$P(\text{both red}) = \frac{7}{n} \times \frac{6}{n-1} = \frac{42}{n(n-1)} = \frac{21}{55}.$$

Cross-multiplying:

$$42 \times 55 = 21\, n(n-1) \;\Rightarrow\; 2310 = 21n^2 - 21n.$$

Dividing by 21:

$$110 = n^2 - n \;\Rightarrow\; n^2 - n - 110 = 0.$$

(This factors as $(n-11)(n+10) = 0$, giving $n = 11$.) The answer is **(B)**.

Q28 Fort St 2022 Trial Q10 · 1 mark

A bag has 10 green and W white marbles with $P(\text{white}) = \frac{4}{9}$. Find how many more white marbles must be added so that $P(\text{white}) = \frac{3}{5}$.

Criteria	Marks
Correct option (A)	1

Solution.

First find W from the initial probability:

$$\frac{W}{10+W} = \frac{4}{9} \;\Rightarrow\; 9W = 4(10+W) \;\Rightarrow\; 5W = 40 \;\Rightarrow\; W = 8.$$

So there are 8 white and 18 marbles in total. Let x more white marbles be added:

$$\frac{8+x}{18+x} = \frac{3}{5} \;\Rightarrow\; 5(8+x) = 3(18+x) \;\Rightarrow\; 40 + 5x = 54 + 3x \;\Rightarrow\; 2x = 14 \;\Rightarrow\; x = 7.$$

The answer is **(A)**.

Q29 Glenwood 2023 Trial Q16 · **5 marks**

In a large population 25% of people are left-handed. (a) For three people chosen at random, find $P(\text{at least one left-handed})$. (b) Find the smallest number of people needed so that $P(\text{at least one left-handed}) > 99\%$.

Criteria	Marks
(a) uses the complement $1 - P(\text{none left-handed})$	1
(a) $\frac{37}{64}$	1
(b) sets up $1 - (0.75)^n > 0.99$	1
(b) solves $n > \frac{\ln 0.01}{\ln 0.75} \approx 16.0$	1
(b) smallest $n = 17$	1

Solution.

(a) $P(\text{left-handed}) = 0.25$, so $P(\text{not left-handed}) = 0.75$. Using the complement of "none of the three is left-handed":

$$P(\text{at least one}) = 1 - (0.75)^3 = 1 - \frac{27}{64} = \frac{37}{64} \approx 0.578.$$

(b) For n people we need

$$1 - (0.75)^n > 0.99 \implies (0.75)^n < 0.01.$$

Taking natural logs (note $\ln 0.75 < 0$ reverses the inequality):

$$n \ln(0.75) < \ln(0.01) \implies n > \frac{\ln 0.01}{\ln 0.75} \approx 16.008.$$

Since n must be a whole number, the smallest value is $n = 17$. (Check: $1 - (0.75)^{16} \approx 0.9900$ which just fails > 0.99, while $1 - (0.75)^{17} \approx 0.9925$.)

Q30 Killara 2022 Prelim Q35 · **4 marks**

A bag has n balls, k black and the rest white. $P(\text{white}) = \frac{13}{15}$ and $P(\text{two black, without replacement}) = \frac{1}{66}$. Find n and k (with $k \neq 0$).

Criteria	Marks
Uses $P(\text{white}) = \frac{13}{15}$ to get $\frac{k}{n} = \frac{2}{15}$ (so $n = \frac{15k}{2}$)	1
Sets up $\frac{k}{n} \cdot \frac{k-1}{n-1} = \frac{1}{66}$	1
Solves for $k = 6$	1
$n = 45$	1

Solution.

Since k balls are black and $n - k$ are white, the first condition gives

$$P(\text{white}) = \frac{n-k}{n} = \frac{13}{15} \Rightarrow \frac{k}{n} = 1 - \frac{13}{15} = \frac{2}{15} \Rightarrow n = \frac{15k}{2}. \qquad (1)$$

The second condition (two black without replacement) is

$$P(\text{two black}) = \frac{k}{n} \times \frac{k-1}{n-1} = \frac{1}{66}. \qquad (2)$$

Substituting $\frac{k}{n} = \frac{2}{15}$ into (2):

$$\frac{2}{15} \times \frac{k-1}{n-1} = \frac{1}{66} \Rightarrow \frac{k-1}{n-1} = \frac{15}{2 \times 66} = \frac{15}{132} = \frac{5}{44}.$$

Now use $n = \frac{15k}{2}$ so $n - 1 = \frac{15k-2}{2}$:

$$\frac{k-1}{\frac{15k-2}{2}} = \frac{5}{44} \Rightarrow \frac{2(k-1)}{15k-2} = \frac{5}{44}.$$

Cross-multiplying:

$$88(k-1) = 5(15k-2) \Rightarrow 88k - 88 = 75k - 10 \Rightarrow 13k = 78 \Rightarrow k = 6.$$

Then from (1), $n = \frac{15 \times 6}{2} = 45$.

$$\boxed{n = 45, \quad k = 6.}$$

(Check: $P(\text{white}) = \frac{39}{45} = \frac{13}{15}$ and $P(\text{two black}) = \frac{6}{45} \times \frac{5}{44} = \frac{30}{1980} = \frac{1}{66}$.)

Q31 Gosford 2024 Trial Q20 · **5 marks**

Sixty people visited a pool: 23 wore goggles (13 of them adults) and 19 were children. (a) Complete the goggles-vs-adult/child two-way table. (b) Find $P(\text{wore goggles} \mid \text{adult})$. (c) After an extra group of adults with no goggles arrives, $P(\text{adult, no goggles}) = 0.6$; find how many additional adults arrived.

Criteria	**Marks**
(a) fills the goggles/adults & children entries	1
(a) completes all totals correctly	1
(b) $P(\text{goggles} \mid \text{adult}) = \frac{13}{41}$	1
(c) forms $\dfrac{28+x}{60+x} = 0.6$	1
(c) $x = 20$ additional adults	1

Solution.

(a) There are 60 people and 19 children, so $60 - 19 = 41$ adults. Of the 23 who wore goggles, 13 were adults, so $23 - 13 = 10$ children wore goggles. Adults with no goggles: $41 - 13 = 28$; children with no goggles: $19 - 10 = 9$; total with no goggles: $28 + 9 = 37$.

	Wore goggles	No goggles	Total
Adults	13	28	41
Children	10	9	19
Total	23	37	60

(b) Restricting to the 41 adults, 13 wore goggles:

$$P(\text{wore goggles} \mid \text{adult}) = \frac{13}{41} \approx 0.317.$$

(c) Let x be the number of additional adults (all with no goggles). The number of adults with no goggles becomes $28 + x$, and the total number of people becomes $60 + x$:

$$\frac{28+x}{60+x} = 0.6 = \frac{3}{5}.$$

Cross-multiplying:

$$5(28+x) = 3(60+x) \;\Rightarrow\; 140 + 5x = 180 + 3x \;\Rightarrow\; 2x = 40 \;\Rightarrow\; x = 20.$$

So $\boxed{20}$ additional adults arrived.

Q32 Normanhurst Boys 2020 Trial Q22 · **6 marks**

Each purchase wins a prize with probability 0.07; Benjamin buys one item per day until he wins. (a) Find $P(\text{he wins on the first or second day})$ to two decimal places. (b) Find the smallest number of days for which $P(\text{he has won}) > 80\%$.

Criteria	Marks
(a) forms $P = 0.07 + 0.93 \times 0.07$ (or equivalent)	1
(a) $= 0.14$ (2 d.p.)	1
(b) forms $1 - (0.93)^n > 0.8$ (or the equivalent geometric-sum inequality)	1
(b) reduces to $(0.93)^n < 0.2$	1
(b) solves $n > \frac{\ln 0.2}{\ln 0.93} \approx 22.2$	1
(b) smallest $n = 23$	1

Solution.

(a) With $P(\text{win on a purchase}) = 0.07$ and $P(\text{lose}) = 0.93$, winning on the first or second day means winning on day 1, OR losing on day 1 then winning on day 2:

$$P = 0.07 + 0.93 \times 0.07 = 0.07 + 0.0651 = 0.1351 \approx 0.14.$$

(b) Benjamin has won within n days unless he has lost every one of the n days. Using the complement:

$$P(\text{won within } n \text{ days}) = 1 - (0.93)^n.$$

(Equivalently, summing the geometric series of daily win probabilities $0.07,\ 0.93{\times}0.07, \ldots$ gives $\dfrac{0.07(1-0.93^n)}{1-0.93} = 1 - 0.93^n$.) We require

$$1 - (0.93)^n > 0.8 \implies (0.93)^n < 0.2.$$

Taking natural logs ($\ln 0.93 < 0$ reverses the inequality):

$$n > \frac{\ln 0.2}{\ln 0.93} \approx 22.178.$$

The smallest whole number is $n = 23$. (Check: $1 - (0.93)^{22} \approx 0.7974 < 0.8$, while $1 - (0.93)^{23} \approx 0.8116 > 0.8$.)

Q33 Sydney Tech 2025 Trial Q34 · **3 marks**

A loaded die has $P(1) = \frac{x}{4}$, $P(2) = \frac{1}{4}$, $P(6) = \frac{1}{4}(1-x)$, and $P(3) = P(4) = P(5) = \frac{1}{6}$. Two such dice are rolled and totalled. Given $P(\text{total} = 7) = \frac{5}{36}$, find $P(\text{rolling a } 1)$.

Criteria	Marks
Writes $P(\text{total } 7)$ as a sum over the pairs $(1,6), (2,5), (3,4)$ and their reverses	1
Forms and solves the equation $f(x) = \frac{5}{36}$	1
$P(1) = \dfrac{1}{4}$	1

Solution.

A total of 7 occurs via the ordered pairs $(1,6), (6,1), (2,5), (5,2), (3,4), (4,3)$. With $P(1) = \frac{x}{4}$, $P(6) = \frac{1}{4}(1-x)$, $P(2) = \frac{1}{4}$, and $P(3) = P(4) = P(5) = \frac{1}{6}$,

$$f(x) = P(\text{total } 7) = 2 \cdot \frac{x}{4} \cdot \frac{1-x}{4} + 2 \cdot \frac{1}{4} \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} \cdot \frac{1}{6}.$$

Simplifying,

$$f(x) = \frac{x(1-x)}{8} + \frac{1}{12} + \frac{1}{18} = -\frac{x(x-1)}{8} + \frac{5}{36}.$$

Setting $f(x) = \frac{5}{36}$ gives

$$-\frac{x(x-1)}{8} = 0 \implies x(x-1) = 0 \implies x = 0 \text{ or } x = 1.$$

Since the die is described as one that *can* roll a 1, take $x = 1$, giving

$$P(1) = \frac{x}{4} = \frac{1}{4}.$$

($x = 0$ also satisfies the equation, but then $P(1) = 0$: the die never shows a 1.)