Year 11 Mathematics Advanced

Topic: Probability and Data (3 of 5)

Review Questions (Worked Solutions): Conditional probability

Term 3, 2026

Multi-stage experiments and tree diagrams

1. Sample-space arrays, two-way tables and tree diagrams

Q1 Hurlstone 2023 Trial Q35 · **1 mark**

Bag A has 1 red and 5 green; Bag B has 1 red and 3 green; Bag C has 1 red and 2 green. A ball is drawn from A; if green, a ball is drawn from B; if that is green, a ball is drawn from C. Complete the probability tree.

Criteria	Marks
All branch probabilities on the tree correct	1

Solution.

Each bag gives a simple "red or green" split, so each stage of the tree carries the two probabilities for that bag:

$$\text{Bag A:}\quad P(R)=\frac{1}{6},\qquad P(G)=\frac{5}{6}.$$

$$\text{Bag B (reached only if A is green):}\quad P(R)=\frac{1}{4},\qquad P(G)=\frac{3}{4}.$$

$$\text{Bag C (reached only if B is green):}\quad P(R)=\frac{1}{3},\qquad P(G)=\frac{2}{3}.$$

The completed tree therefore reads $\frac{1}{6}, \frac{5}{6}$ on the Bag A branches; from the Bag A green branch, $\frac{1}{4}, \frac{3}{4}$ on the Bag B branches; and from the Bag B green branch, $\frac{1}{3}, \frac{2}{3}$ on the Bag C branches.

Q2 Girraween 2022 Trial Q29 · **1 mark**

A six-sided dice is rolled twice and the difference of the two results is recorded. Complete the sample-space table of differences.

Criteria	Marks
All missing cells filled in correctly	1

Solution.

Each cell is the absolute difference of the first roll (row) and second roll (column). The completed table is:

First \ Second	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

The missing entries are: row 1 column 5 is 4; row 2 column 6 is 4; row 3 columns 3 and 4 are 0 and 1; row 4 columns 1 and 6 are 3 and 2; row 6 column 5 is 1.

Q3 St George Girls 2024 Trial Q1 · **1 mark**

From the probability tree, $P(\text{Jo President}) = 0.4$, and given Jo, the Vice-President branches are Louis 0.25, Connor 0.35 and Britt (unlabelled). Find $P(\text{Jo President and VP is Britt or Louis})$.

Criteria	Marks
Correct option (B)	1

Solution.

The three Vice-President branches from Jo must sum to 1, so the Britt branch is

$$P(\text{Britt} \mid \text{Jo}) = 1 - 0.25 - 0.35 = 0.40.$$

Multiplying along the Jo branch and adding the Louis and Britt outcomes:

$$P(\text{Jo, and Britt or Louis}) = 0.4 \times (0.25 + 0.40) = 0.4 \times 0.65 = 0.26.$$

The answer is **(B)**.

Q4 2023 HSC Q2 · **1 mark**

A die (1–6) is thrown and a spinner (1–4) is spun; the score is the sum. Find $P(\text{score} \geq 7)$.

Criteria	Marks
Correct option (D)	1

Solution.

There are $6 \times 4 = 24$ equally likely (die, spinner) outcomes. A score of 7 or more needs die + spinner ≥ 7. Counting the favourable pairs:

$$(3,4),\ (4,3),\ (4,4),\ (5,2),\ (5,3),\ (5,4),\ (6,1),\ (6,2),\ (6,3),\ (6,4),$$

which is 10 outcomes. Hence

$$P(\text{score} \geq 7) = \frac{10}{24} = \frac{5}{12}.$$

The answer is **(D)**.

Q5 Ascham 2020 Trial Q12 · **3 marks**

A bag has 6 red and 12 green marbles (18 total). Two are drawn without replacement. Complete the probability tree and find $P(\text{two different colours})$.

Criteria	Marks
Correct first-draw probabilities $\frac{6}{18}, \frac{12}{18}$	1
Correct second-draw (without-replacement) probabilities on all branches	1
$P(\text{different colours}) = \frac{8}{17}$	1

Solution.

On the first draw, $P(R) = \dfrac{6}{18} = \dfrac{1}{3}$ and $P(G) = \dfrac{12}{18} = \dfrac{2}{3}$. On the second draw there is one fewer marble (17 total), so:

$$\text{after } R: \quad P(R) = \frac{5}{17},\ P(G) = \frac{12}{17}; \qquad \text{after } G: \quad P(R) = \frac{6}{17},\ P(G) = \frac{11}{17}.$$

"Two different colours" means red-then-green or green-then-red:

$$P(\text{different}) = \frac{6}{18} \cdot \frac{12}{17} + \frac{12}{18} \cdot \frac{6}{17} = \frac{72}{306} + \frac{72}{306} = \frac{144}{306} = \frac{8}{17}.$$

$$\boxed{P(\text{different colours}) = \frac{8}{17} \approx 0.471.}$$

Q6 Presbyterian 2023 Prelim Q27 · **3 marks**

A standard die (1–6) and a die numbered $3, 3, 3, 4, 4, 4$ are rolled; the number of threes is recorded. (a) Fill the six probabilities on the tree; (b) find $P(\text{at least one } 3)$.

Criteria	Marks
(a) first-die probabilities $P(3) = \frac{1}{6}$, $P(\text{not } 3) = \frac{5}{6}$	1
(a) second-die probabilities $P(3) = \frac{1}{2}$, $P(\text{not } 3) = \frac{1}{2}$ on both branches	1
(b) $P(\text{at least one } 3) = \frac{7}{12}$	1

Solution.

(a) On the standard die, $P(3) = \dfrac{1}{6}$ and $P(\text{not } 3) = \dfrac{5}{6}$. On the second die, three of the six faces are threes, so $P(3) = \dfrac{3}{6} = \dfrac{1}{2}$ and $P(\text{not } 3) = \dfrac{1}{2}$ (the same on both second-die branches).

(b) Use the complement. "At least one 3" is the opposite of "no threes at all":

$$P(\text{no } 3) = P(\text{not 3 on first}) \times P(\text{not 3 on second}) = \frac{5}{6} \times \frac{1}{2} = \frac{5}{12}.$$

$$P(\text{at least one } 3) = 1 - \frac{5}{12} = \frac{7}{12} \approx 0.583.$$

Q7 Fort St 2022 Trial Q22 · **4 marks**

Sirhind win their next game with probability 0.8 if they won the previous game, and 0.4 if they did not. They won on Monday, then played Tuesday and Wednesday. (a) Complete the tree; (b) find $P(\text{do not win Wednesday})$.

Criteria	Marks
(a) Tuesday branches correct: $W = 0.8$, $W' = 0.2$	1
(a) Wednesday branches correct: from W, $0.8/0.2$; from W', $0.4/0.6$	1
(b) identifies the two "lose Wednesday" paths and multiplies	1
(b) $P(\text{not win Wednesday}) = 0.28$	1

Solution.

(a) They won Monday, so on Tuesday $P(W) = 0.8$ and $P(W') = 0.2$. Wednesday depends on the Tuesday result: after a Tuesday win, $P(W) = 0.8$, $P(W') = 0.2$; after a Tuesday loss, $P(W) = 0.4$, $P(W') = 0.6$.

(b) "Do not win Wednesday" happens along two paths: win Tuesday then lose Wednesday, or lose Tuesday then lose Wednesday.

$$P(\text{not win Wed}) = \underbrace{0.8 \times 0.2}_{W,\,W'} + \underbrace{0.2 \times 0.6}_{W',\,W'} = 0.16 + 0.12 = 0.28.$$

$$\boxed{P(\text{do not win Wednesday}) = 0.28.}$$

Q8 Blacktown Boys 2022 Prelim Q9 · **1 mark**

After a win, Shirley wins the next game with probability 0.8; after a loss, she loses the next with probability 0.4. She has just won. Find $P(\text{exactly one win in the next two games})$.

Criteria	Marks
Correct option (A)	1

Solution.

She has just won, so game 1: $P(W) = 0.8$, $P(L) = 0.2$. The follow-on game depends on that result: after a win, $P(W) = 0.8$, $P(L) = 0.2$; after a loss, $P(W) = 0.6$ (since $P(L) = 0.4$). "Exactly one win" is win-then-lose or lose-then-win:

$$P(WL) = 0.8 \times 0.2 = 0.16, \qquad P(LW) = 0.2 \times 0.6 = 0.12.$$

$$P(\text{exactly one win}) = 0.16 + 0.12 = 0.28.$$

The answer is **(A)**.

2. Selections without replacement

Q9 St George Girls 2022 Trial Q1 · 1 mark

A jar of 30 seashells has 19 white and 11 black. Two shells are selected at random (without replacement). Find the probability both are white.

Criteria	Marks
Correct option (A)	1

Solution.

Selection is without replacement, so the probabilities multiply and the second denominator drops by one:

$$P(\text{two white}) = \frac{19}{30} \times \frac{18}{29} = \frac{57}{145} \approx 0.393.$$

This is option **A**.

Q10 Baulkham Hills 2021 Prelim Q2 · 1 mark

Two cards are drawn without replacement from a standard 52-card deck (four suits of 13). Find the probability the two cards are of different suits.

Criteria	Marks
Correct option (B)	1

Solution.

The first card can be anything. After it is removed, 51 cards remain, of which 39 belong to a different suit from the first:

$$P(\text{different suits}) = \frac{52}{52} \times \frac{39}{51} = \frac{39}{51} = \frac{13}{17}.$$

This is option **B**.

Q11 Cherrybrook Tech 2022 Prelim Q5 · **1 mark**

A bag has 4 green and 6 blue marbles. Two are selected without replacement. Find the probability at least one is green.

Criteria	Marks
Correct option (C)	1

Solution.

Use the complement "no green" (both blue):

$$P(\text{no green}) = \frac{6}{10} \times \frac{5}{9} = \frac{30}{90} = \frac{1}{3}.$$

Therefore

$$P(\text{at least one green}) = 1 - \frac{1}{3} = \frac{2}{3}.$$

This is option **C**.

Q12 Barker 2022 Trial Q18 · **2 marks**

A pencil case has 7 blue and 3 red pens. Two pens are selected without replacement. Find the probability of selecting at least one red pen.

Criteria	Marks
Computes $P(\text{no red}) = \frac{7}{10} \times \frac{6}{9}$ (both blue)	1
Correct probability by complement	1

Solution.

Use the complement "no red pen" (both blue):

$$P(\text{no red}) = \frac{7}{10} \times \frac{6}{9} = \frac{42}{90} = \frac{7}{15}.$$

Therefore

$$P(\text{at least one red}) = 1 - \frac{7}{15} = \frac{8}{15}.$$

Q13 Sydney Tech 2021 Trial Q18 · **3 marks**

Of 50 residents, 26 were willing to have the Pfizer vaccine, 22 the AstraZeneca vaccine, and 12 were NOT willing to have either. (a) Find the probability a randomly chosen resident was willing to be vaccinated by either vaccine. (b) Two residents are chosen without replacement; find the probability at least one was willing.

Criteria	Marks
(a) Identifies $50 - 12 = 38$ willing	1
(a) Correct probability $\frac{19}{25}$	1
(b) Correct probability by complement	1

Solution.

(a) The number NOT willing is 12, so the number willing (the union of the two vaccine sets) is

$$50 - 12 = 38.$$

Therefore

$$P(\text{willing by either}) = \frac{38}{50} = \frac{19}{25}.$$

(b) Use the complement "neither of the two chosen was willing." There are 12 unwilling residents, and selection is without replacement:

$$P(\text{both not willing}) = \frac{12}{50} \times \frac{11}{49} = \frac{132}{2450} = \frac{66}{1225}.$$

Therefore

$$P(\text{at least one willing}) = 1 - \frac{66}{1225} = \frac{1159}{1225} \approx 0.946.$$

Q14 2021 HSC Q6 · **1 mark**

A box has 8 chocolates: 3 peppermint (P) and 5 caramel (C). Kim then Sam each choose and eat one (without replacement). Find the probability their chocolates have different centres.

Criteria	Marks
Correct option (D)	1

Solution.

"Different centres" means one peppermint and one caramel, in either order:

$$P(PC) + P(CP) = \frac{3}{8} \times \frac{5}{7} + \frac{5}{8} \times \frac{3}{7} = \frac{15}{56} + \frac{15}{56} = \frac{30}{56} = \frac{15}{28}.$$

This is option **D**.

Q15 Merewether 2025 Trial Q19 · **4 marks**

In a survey of all Zoology students, $\frac{3}{8}$ had a pet cat, $\frac{3}{4}$ had a pet dog, and $\frac{1}{4}$ had both. 25 students had exactly one of the two pets. (a) Show that a total of 40 students were surveyed. (b) Two students are chosen at random; find the probability neither has a pet dog.

Criteria	Marks
(a) Expresses "exactly one pet" as $\frac{3}{8}N + \frac{3}{4}N - 2\times\frac{1}{4}N$	1
(a) Solves $\frac{5}{8}N = 25$ to obtain $N = 40$	1
(b) Finds $40 - 30 = 10$ students with no dog	1
(b) Correct probability $\frac{3}{52}$	1

Solution.

(a) Let N be the total number surveyed. In terms of N the numbers are

$$\text{cat} = \tfrac{3}{8}N, \qquad \text{dog} = \tfrac{3}{4}N, \qquad \text{both} = \tfrac{1}{4}N.$$

The number with *exactly one* pet counts each single-set region once, subtracting the overlap twice:

$$\left(\tfrac{3}{8}N - \tfrac{1}{4}N\right) + \left(\tfrac{3}{4}N - \tfrac{1}{4}N\right) = \tfrac{1}{8}N + \tfrac{1}{2}N = \tfrac{5}{8}N.$$

Setting this equal to 25:

$$\tfrac{5}{8}N = 25 \quad\Longrightarrow\quad N = 25\times\tfrac{8}{5} = 40.\blacksquare$$

(b) With $N = 40$, the number with a pet dog is $\frac{3}{4}\times 40 = 30$, so the number with *no* dog is $40 - 30 = 10$. Choosing two students without replacement:

$$P(\text{neither has a dog}) = \frac{10}{40}\times\frac{9}{39} = \frac{90}{1560} = \frac{3}{52}.$$

Q16 Penrith 2025 Trial Q18 · **6 marks**

Of 40 adults, 12 pay only for sport, 6 pay for both sport and television, and 8 pay for neither. (a) Find the probability an adult pays for television shows. (b) Given an adult pays for television, find the probability they also pay for sport. (c) Two adults are chosen at random; find the probability they both pay for the same type of subscription (ignoring those who pay for neither).

Criteria	**Marks**
(a) Finds television-only = $40 - 12 - 6 - 8 = 14$	1
(a) Correct probability $\frac{20}{40} = \frac{1}{2}$	1
(b) Correct conditional probability $\frac{6}{20} = \frac{3}{10}$	1
(c) Sets up same-type pairs for sport-only, TV-only and both	1
(c) Correct numerator $14 \cdot 13 + 12 \cdot 11 + 6 \cdot 5 = 344$	1
(c) Correct probability $\frac{344}{1560} = \frac{43}{195}$	1

Solution.

The four disjoint regions (out of 40) are: sport only = 12, both = 6, neither = 8, so

$$\text{television only} = 40 - 12 - 6 - 8 = 14.$$

(a) An adult pays for television if they are in "television only" or "both":

$$P(\text{television}) = \frac{14 + 6}{40} = \frac{20}{40} = \frac{1}{2}.$$

(b) Of the 20 who pay for television, 6 also pay for sport (the "both" region):

$$P(\text{sport} \mid \text{television}) = \frac{6}{20} = \frac{3}{10}.$$

(c) Two adults are chosen without replacement. "Same type" means both from sport only (12), both from television only (14), or both from the "both" group (6):

$$P(\text{same type}) = \frac{14}{40} \cdot \frac{13}{39} + \frac{12}{40} \cdot \frac{11}{39} + \frac{6}{40} \cdot \frac{5}{39} = \frac{182 + 132 + 30}{1560} = \frac{344}{1560} = \frac{43}{195}.$$

3. 'At least one' in multi-stage experiments

Q17 Abbotsleigh 2024 Prelim Q15 · **2 marks**

The probability that Gemma gets a concert booking on any given weekend is 0.65. Find the probability that over two consecutive weekends she gets at least one booking.

Criteria	**Marks**
$P(\text{no booking either weekend}) = 0.35^2 = 0.1225$	1
Correct probability 0.8775	1

Solution.

Use the complement: "at least one booking" is the opposite of "no booking on either weekend". The two weekends are independent, so

$$P(\text{no booking either weekend}) = (1 - 0.65)^2 = 0.35^2 = 0.1225.$$

Therefore

$$P(\text{at least one booking}) = 1 - 0.1225 = \boxed{0.8775}.$$

Q18 Kings 2022 Trial Q15 · **3 marks**

James kicks a goal with probability $\frac{2}{7}$ and Kyle with probability $\frac{5}{8}$. On their next 3 attempts (each) at goal, find the probability that at least one goal is kicked.

Criteria	**Marks**
Per attempt, $P(\text{both miss}) = \frac{5}{7} \times \frac{3}{8} = \frac{15}{56}$	1
Over 3 attempts, $P(\text{no goal at all}) = \left(\frac{15}{56}\right)^3$	1
Correct probability $\frac{172241}{175616} \approx 0.98$	1

Solution.

Use the complement: "at least one goal over the three attempts" is the opposite of "no goal is ever kicked". On a single attempt James and Kyle act independently, so the probability neither kicks a goal is

$$P(\text{both miss}) = \left(1 - \frac{2}{7}\right)\left(1 - \frac{5}{8}\right) = \frac{5}{7} \times \frac{3}{8} = \frac{15}{56}.$$

The three attempts are independent, so

$$P(\text{no goal on any of the 3 attempts}) = \left(\frac{15}{56}\right)^3 = \frac{3375}{175616}.$$

Therefore

$$P(\text{at least one goal}) = 1 - \frac{3375}{175616} = \boxed{\frac{172241}{175616}} \approx 0.98.$$

Q19 Hills Grammar 2025 Trial Q11 · **3 marks**

Cody wins the first set two out of every seven matches. If he wins the first set he wins the second set $\frac{3}{5}$ of the time; if he loses the first set he wins the second set $\frac{1}{5}$ of the time. (a) Complete the probability tree and find $P(\text{Cody loses both sets})$. (b) Over 70 matches, in how many would Cody expect to win at least one set?

Criteria	Marks
(a) tree with $P(W_1)=\frac{2}{7}$, $P(L_1)=\frac{5}{7}$ and correct second-stage branches	1
(a) $P(\text{loses both})=\frac{5}{7}\times\frac{4}{5}=\frac{4}{7}$	1
(b) expected number $=70\times\frac{3}{7}=30$	1

Solution.

(a) First set: $P(W_1)=\frac{2}{7}$, so $P(L_1)=\frac{5}{7}$. If he loses the first set he wins the second $\frac{1}{5}$ of the time, so he loses the second set (given he lost the first) with probability $1-\frac{1}{5}=\frac{4}{5}$. "Loses both sets" is L_1 then loses the second:

$$P(\text{loses both sets})=\frac{5}{7}\times\frac{4}{5}=\frac{20}{35}=\boxed{\frac{4}{7}}.$$

(b) "Wins at least one set" is the complement of "loses both sets":

$$P(\text{wins at least one set})=1-\frac{4}{7}=\frac{3}{7}.$$

Over 70 matches the expected number is

$$70\times\frac{3}{7}=\boxed{30 \text{ matches}}.$$

Q20 Glenwood 2025 Trial Q30 · **4 marks**

A robot arm picks up an object on any single attempt with probability $\frac{1}{4}$. (a) Find $P(\text{first success on the third attempt, after fai}$ (b) Find $P(\text{unsuccessful in the first four attempts})$. (c) Find the minimum number of attempts so that $P(\text{at least one success}) > 0.95$.

Criteria	Marks
(a) $\left(\frac{3}{4}\right)^2 \times \frac{1}{4} = \frac{9}{64}$	1
(b) $\left(\frac{3}{4}\right)^4 = \frac{81}{256}$	1
(c) sets up $1 - \left(\frac{3}{4}\right)^n > 0.95$ (i.e. $\left(\frac{3}{4}\right)^n < 0.05$)	1
(c) solves to $n = 11$	1

Solution.

Let $p = \frac{1}{4}$ be the probability of success on one attempt, so failure has probability $\frac{3}{4}$; attempts are independent.

(a) Fail, fail, then succeed:

$$P = \frac{3}{4} \times \frac{3}{4} \times \frac{1}{4} = \boxed{\frac{9}{64}} = 0.140625.$$

(b) Four failures in a row:

$$P(\text{unsuccessful in first four}) = \left(\frac{3}{4}\right)^4 = \boxed{\frac{81}{256}} \approx 0.316.$$

(c) "At least one success in n attempts" is the complement of "all n attempts fail":

$$1 - \left(\frac{3}{4}\right)^n > 0.95 \quad \Longrightarrow \quad \left(\frac{3}{4}\right)^n < 0.05.$$

Taking logarithms (and reversing the inequality since $\log \frac{3}{4} < 0$),

$$n > \frac{\log 0.05}{\log \frac{3}{4}} \approx 10.41.$$

Since n must be a whole number, $\boxed{n = 11}$ attempts.

Q21 Cherrybrook Tech 2020 Trial Q29 · **5 marks**

15% of a large population is infected with a virus. (a) For two people chosen at random, find $P(\text{both infected})$. (b) For four people, find $P(\text{at least one infected})$ to 3 d.p. (c) Find the smallest number of people to test to have a greater than 95% chance that at least one tested person is infected.

Criteria	Marks
(a) $0.15^2 = 0.0225$	1
(b) $P(\text{none of four}) = 0.85^4$	1
(b) $P(\text{at least one}) = 0.478$	1
(c) sets up $1 - 0.85^n > 0.95$ (i.e. $0.85^n < 0.05$)	1
(c) solves to $n = 19$	1

Solution.

Each person is infected independently with probability 0.15, so not infected with probability 0.85.

(a)

$$P(\text{both infected}) = 0.15 \times 0.15 = 0.15^2 = \boxed{0.0225}.$$

(b) "At least one of four" is the complement of "none of the four":

$$P(\text{none infected}) = 0.85^4 = 0.52200625,$$

$$P(\text{at least one infected}) = 1 - 0.85^4 = 0.47799\ldots \approx \boxed{0.478}.$$

(c) Require $1 - 0.85^n > 0.95$, i.e. $0.85^n < 0.05$. Taking logarithms (reversing the sign since $\log 0.85 < 0$),

$$n > \frac{\log 0.05}{\log 0.85} \approx 18.43.$$

The smallest whole number is $\boxed{n = 19}$ people.

Q22 Hurlstone 2020 Trial Q28 · **5 marks**

Each component X fails with probability 0.05. (a)(i) A system wired in series works only if both components operate; find $P(\text{system does not fail})$ to 3 d.p. (a)(ii) A system wired in parallel works if at least one component works; find the minimum number of components so that $P(\text{system works}) > 99.999\%$. (b) A coin is tossed n times ($n > 1$); find $P(\text{at least one head and at least one tail})$.

Criteria	Marks
(a)(i) $0.95 \times 0.95 = 0.9025 \approx 0.903$	1
(a)(ii) sets up $1 - 0.05^n > 0.99999$ (i.e. $0.05^n < 0.00001$)	1
(a)(ii) solves to $n = 4$	1
(a)(ii) clear justification with logarithms / inequality sign	1
(b) $P = 1 - 2\left(\frac{1}{2}\right)^n = 1 - 2^{1-n}$	1

Solution.

A component works with probability $1 - 0.05 = 0.95$; components act independently.

(a)(i) In series the system works only if both components work:

$$P(\text{system does not fail}) = 0.95 \times 0.95 = 0.9025 \approx \boxed{0.903}.$$

(a)(ii) In parallel the system works unless all n components fail. Each fails with probability 0.05, so

$$P(\text{system works}) = 1 - (0.05)^n.$$

Require $1 - (0.05)^n > 0.99999$, i.e. $(0.05)^n < 0.00001$. Taking logarithms (reversing the sign since $\log 0.05 < 0$),

$$n > \frac{\log 0.00001}{\log 0.05} \approx 3.84.$$

The minimum whole number of components is $\boxed{n = 4}$.

(b) "At least one head and at least one tail" fails only when all tosses are the same, i.e. all heads or all tails. Each of these has probability $\left(\frac{1}{2}\right)^n$, so

$$P(\text{at least one head and one tail}) = 1 - P(\text{all heads}) - P(\text{all tails}) = 1 - \left(\frac{1}{2}\right)^n - \left(\frac{1}{2}\right)^n = \boxed{1 - 2^{1-n}}.$$

Q23 Barker 2024 Prelim Q18 · **2 marks**

Each week, Percy wins his raffle with probability $\frac{1}{9}$ and Kura wins his with probability $\frac{1}{16}$ (independently). Over the next three weeks, find $P(\text{at least one of them wins a prize})$.

Criteria	Marks
Finds $P(\text{no prize in one week}) = \dfrac{5}{6}$, then cubes and uses the complement	1
$P(\text{at least one}) = \dfrac{91}{216}$	1

Solution.

In any one week, neither wins with probability

$$\left(1-\frac{1}{9}\right)\left(1-\frac{1}{16}\right) = \frac{8}{9}\cdot\frac{15}{16} = \frac{5}{6}.$$

Over three independent weeks, no prize at all has probability $\left(\frac{5}{6}\right)^3 = \frac{125}{216}$, so by the complement

$$P(\text{at least one prize in three weeks}) = 1 - \frac{125}{216} = \frac{91}{216} \approx 0.421.$$

Q24 2024 HSC Q18 · **3 marks**

A player scores a goal on each attempt with probability 0.15. (a) Find $P(\text{no goal in the first two attempts})$. (b) Find the least number of attempts so that $P(\text{at least one goal}) > 0.8$.

Criteria	Marks
(a) $0.85^2 = 0.7225$	1
(b) sets up $1 - 0.85^n > 0.8$ (i.e. $0.85^n < 0.2$)	1
(b) solves to $n = 10$	1

Solution.

Each attempt scores with probability 0.15, so misses with probability 0.85; attempts are independent.

(a)

$$P(\text{no goal in first two attempts}) = 0.85^2 = \boxed{0.7225}.$$

(b) "At least one goal in n attempts" is the complement of "no goal in n attempts":

$$1 - 0.85^n > 0.8 \quad \Longrightarrow \quad 0.85^n < 0.2.$$

Taking logarithms (reversing the sign since $\log 0.85 < 0$),

$$n > \frac{\log 0.2}{\log 0.85} \approx 9.90.$$

The least whole number is $\boxed{n = 10}$ attempts.

Q25 Hurlstone 2024 Prelim Q32 · **4 marks**

A bowler scores a strike with probability 28% each bowl. (a) The bowler bowls twice; find $P(\text{no strike in both bowls})$. (b) Find the minimum number of bowls for a 90% chance of at least one strike.

Criteria	Marks
(a) $P(\text{no strike}) = 0.72$ per bowl	1
(a) $P(\text{no strike in both}) = 0.72^2 = 0.5184$	1
(b) sets up $1 - 0.72^n \geq 0.9$ (i.e. $0.72^n \leq 0.1$)	1
(b) solves to $n = 8$	1

Solution.

A strike occurs with probability 0.28, so no strike has probability 0.72; bowls are independent.

(a)

$$P(\text{no strike in both bowls}) = 0.72 \times 0.72 = 0.72^2 = \boxed{0.5184}.$$

(b) "At least one strike in n bowls" is the complement of "no strike in n bowls":

$$1 - 0.72^n \geq 0.9 \quad \Longrightarrow \quad 0.72^n \leq 0.1.$$

Taking logarithms (reversing the sign since $\log 0.72 < 0$),

$$n \geq \frac{\log 0.1}{\log 0.72} \approx 7.01.$$

The minimum whole number is $\boxed{n = 8}$ bowls.

4. Exactly k of n in repeated trials

Q26 Sydney Girls 2020 Trial Q5 · 1 mark

A fisherman catches a fish of legal length with probability $\frac{3}{5}$. Three fish are caught. Find $P(\text{exactly one is of legal length})$.

Criteria	Marks
Correct option (A)	1

Solution.

Each catch is independent with $P(\text{legal}) = \frac{3}{5}$ and $P(\text{not legal}) = \frac{2}{5}$. Exactly one of the three being legal can happen in 3 ways (which one is legal), so

$$P(\text{exactly one}) = \binom{3}{1}\left(\frac{3}{5}\right)^1\left(\frac{2}{5}\right)^2 = 3 \cdot \frac{3}{5} \cdot \frac{4}{25} = \frac{36}{125}.$$

The answer is option **A**.

Q27 Caringbah 2024 Trial Q7 · 1 mark

$P(\text{Alicia completes}) = 0.8$, $P(\text{Steven completes}) = 0.7$. Find $P(\text{only one completes})$.

Criteria	Marks
Correct option (C)	1

Solution.

The two events are independent. "Only one completes" means Alicia completes and Steven does not, *or* Steven completes and Alicia does not:

$$P = (0.8)(1-0.7) + (1-0.8)(0.7) = 0.8 \times 0.3 + 0.2 \times 0.7 = 0.24 + 0.14 = 0.38.$$

The answer is option **C**.

Q28 Barker 2021 Prelim Q12 · **2 marks**

The probability of rain on any given day is $\frac{1}{3}$. Over three days, find $P(\text{rain on exactly two days})$.

Criteria	Marks
Multiplies $\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)$ and accounts for the 3 arrangements	1
$P = \frac{2}{9}$	1

Solution.

Each day is independent with $P(\text{rain}) = \frac{1}{3}$ and $P(\text{no rain}) = \frac{2}{3}$. Exactly two of the three days having rain can occur in $\binom{3}{2} = 3$ ways, so

$$P(\text{exactly two}) = \binom{3}{2}\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^1 = 3 \cdot \frac{1}{9} \cdot \frac{2}{3} = \frac{6}{27} = \frac{2}{9} \approx 0.222.$$

Q29 Gosford 2022 Trial Q24 · **3 marks**

Greg's train runs to schedule with probability 0.96 on any day (so he is late with probability $1 - 0.96$). **(i)** Find $P(\text{late on the first day})$. **(ii)** Find $P(\text{late on exactly one of the first three days})$.

Criteria	Marks
(i) $P(\text{late}) = 0.04$	1
(ii) Accounts for the 3 arrangements of one late day among three	1
(ii) $P = 0.1106$ (to 4 d.p.)	1

Solution.

(i) Being late is the complement of running to schedule:

$$P(\text{late}) = 1 - 0.96 = 0.04.$$

(ii) Each day is independent with $P(\text{late}) = 0.04$ and $P(\text{on time}) = 0.96$. The one late day can be Day 1, Day 2 or Day 3, so there are 3 arrangements:

$$P(\text{exactly one late}) = \binom{3}{1}(0.04)(0.96)^2 = 3 \times 0.04 \times 0.9216 = \frac{1728}{15625} \approx 0.1106.$$

Q30 Sydney Grammar 2025 Trial Q17 · 3 marks

A team wins any given game with probability $\frac{1}{3}$ and plays 3 games. **(a)** Find $P(\text{at least one win})$. **(b)** Find $P(\text{exactly one win})$.

Criteria	Marks
(a) $P(\text{at least one}) = \dfrac{19}{27}$ (via the complement)	1
(b) Accounts for the 3 arrangements of the single win	1
(b) $P(\text{exactly one}) = \dfrac{4}{9}$	1

Solution.

Each game is independent with $P(\text{win}) = \frac{1}{3}$ and $P(\text{loss}) = \frac{2}{3}$.

(a) Use the complement of "no wins":

$$P(\text{at least one win}) = 1 - P(\text{no wins}) = 1 - \left(\frac{2}{3}\right)^3 = 1 - \frac{8}{27} = \frac{19}{27}.$$

(b) Exactly one win in three games can occur in $\binom{3}{1} = 3$ ways:

$$P(\text{exactly one win}) = \binom{3}{1}\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^2 = 3 \cdot \frac{1}{3} \cdot \frac{4}{9} = \frac{4}{9}.$$

Q31 Knox 2021 Trial Q12 · **5 marks**

Of 80 people: 12 had not holidayed at all, 42 had holidayed interstate (I), 36 had holidayed within NSW (N). The Venn diagram has "I only" $= x$, "both" $= y$, "N only" $= z$, and 12 outside. **(i)** Find x, y, z. **(ii)** Find $P(\text{holidayed both } I \text{ and } N)$. **(iii)** Given a person travelled, find $P(N)$. **(iv)** Two people are chosen in turn at random; find $P(\text{exactly one holidayed interstate})$.

Criteria	Marks
(i) $x = 32$, $y = 10$, $z = 26$	1
(ii) $P(I \cap N) = \dfrac{1}{8}$	1
(iii) $P(N \mid \text{travelled}) = \dfrac{9}{17}$	1
(iv) Correct two-term set-up $P(I, \bar{I}) + P(\bar{I}, I)$ without replacement	1
(iv) $P = \dfrac{399}{790}$	1

Solution.

(i) The number who travelled is $80 - 12 = 68$. Since 42 went interstate and 36 went within NSW, the overlap (both) is

$$y = 42 + 36 - 68 = 10.$$

Hence "I only" $= x = 42 - 10 = 32$ and "N only" $= z = 36 - 10 = 26$. (Check: $32 + 10 + 26 + 12 = 80$.)

(ii) 10 of the 80 people holidayed both interstate and within NSW, so

$$P(I \cap N) = \frac{10}{80} = \frac{1}{8}.$$

(iii) This is conditional on the person having travelled (68 people). Of those, 36 holidayed within NSW, so

$$P(N \mid \text{travelled}) = \frac{36}{68} = \frac{9}{17}.$$

(iv) There are 42 people who holidayed interstate and $80 - 42 = 38$ who did not. Two people are chosen in turn *without* replacement. "Exactly one interstate" happens as (interstate, then not) or (not, then interstate):

$$P = \frac{42}{80} \times \frac{38}{79} + \frac{38}{80} \times \frac{42}{79} = \frac{1596 + 1596}{6320} = \frac{3192}{6320} = \frac{399}{790} \approx 0.505.$$

5. Repeated trials and eventually winning a game

Q32 St George Girls 2022 Trial Q19 · **4 marks**

Ash and Naomi play games until one player has won two (a best-of-three series). Each game, $P(\text{Ash}) = \frac{3}{5}$ and $P(\text{Naomi}) = \frac{2}{5}$. (a) Draw a probability tree for a series of up to three games. (b) Find $P(\text{Naomi wins the series})$. (c) Find $P(\text{Naomi wins at least one game})$.

Criteria	Marks
(a) tree with correct first-game branches ($\frac{3}{5}$, $\frac{2}{5}$)	1
(a) tree extended correctly to the third game where needed	1
(b) $P(\text{Naomi series}) = \frac{44}{125}$	1
(c) $P(\text{Naomi} \geq 1 \text{ game}) = \frac{16}{25}$	1

Solution.

(a) The first game splits into Ash ($\frac{3}{5}$) or Naomi ($\frac{2}{5}$). The series stops as soon as a player reaches two wins, so AA and NN end after two games, while a split first two games (AN or NA) needs a deciding third game. The winning sequences are:

$$\text{Ash: } AA,\ ANA,\ NAA \qquad \text{Naomi: } NN,\ NAN,\ ANN.$$

(b) Naomi wins the series via NN, NAN or ANN:

$$P(\text{Naomi series}) = \underbrace{\tfrac{2}{5}\cdot\tfrac{2}{5}}_{NN} + \underbrace{\tfrac{2}{5}\cdot\tfrac{3}{5}\cdot\tfrac{2}{5}}_{NAN} + \underbrace{\tfrac{3}{5}\cdot\tfrac{2}{5}\cdot\tfrac{2}{5}}_{ANN} = \frac{4}{25} + \frac{12}{125} + \frac{12}{125} = \frac{44}{125}.$$

(c) Naomi wins *no* game only if the series is AA (Ash wins the first two). Using the complement:

$$P(\text{Naomi wins} \geq 1) = 1 - P(AA) = 1 - \tfrac{3}{5}\cdot\tfrac{3}{5} = 1 - \frac{9}{25} = \frac{16}{25}.$$

Q33 Blacktown Boys 2025 Trial Q29 · **2 marks**

Tom and Ricky play a chess match; the champion is the first to win two games. $P(\text{Tom}) = 0.7$ and $P(\text{Ricky}) = 0.3$ each game. Find the probability that Tom wins the match. (The paper writes "John"; from the given data this is the player Tom, $P = 0.7$.)

Criteria	**Marks**
Identifies the winning sequences TT, TRT, RTT (or equivalent)	1
$P(\text{Tom wins match}) = 0.784$	1

Solution.

The match ends when a player reaches two wins, so Tom wins the match via TT, TRT or RTT (where T = Tom wins a game, R = Ricky wins a game):

$$P(\text{Tom}) = \underbrace{0.7 \times 0.7}_{TT} + \underbrace{0.7 \times 0.3 \times 0.7}_{TRT} + \underbrace{0.3 \times 0.7 \times 0.7}_{RTT} = 0.49 + 0.147 + 0.147 = 0.784.$$

Equivalently $\dfrac{98}{125} = 0.784$.

Q34 Presbyterian 2021 Trial Q32 · 7 marks

Grant throws two dice (orange, green): he wins if green > orange, loses if green < orange, and re-throws if they are equal, repeating until he wins or loses. (a) Show $P(\text{win on the first roll}) = \frac{5}{12}$. (b) Find $P(\text{a second throw is needed})$. (c) Find $P(\text{win on the first, second or third throw})$, unsimplified. (d) Find $P(\text{Grant wins the game})$.

Criteria	Marks
(a) counts 15 favourable outcomes out of 36 and shows $\frac{15}{36} = \frac{5}{12}$	1
(b) recognises a second throw is needed exactly when the first is a draw	1
(b) $P = \frac{1}{6}$	1
(c) win on throw k has probability $\left(\frac{1}{6}\right)^{k-1} \frac{5}{12}$	1
(c) $\frac{5}{12} + \frac{1}{6} \cdot \frac{5}{12} + \left(\frac{1}{6}\right)^2 \frac{5}{12}$ (unsimplified)	1
(d) sums the infinite geometric series (ratio $\frac{1}{6}$)	1
(d) $P(\text{Grant wins}) = \frac{1}{2}$	1

Solution.

(a) There are $6 \times 6 = 36$ equally likely outcomes. Grant wins when green > orange. By symmetry the number of outcomes with green > orange equals the number with green < orange, and 6 outcomes are equal, so each of "win" and "lose" has $\frac{36-6}{2} = 15$ outcomes:

$$P(\text{win}) = \frac{15}{36} = \frac{5}{12}. \qquad \blacksquare$$

Similarly $P(\text{lose}) = \frac{15}{36} = \frac{5}{12}$ and $P(\text{draw, re-throw}) = \frac{6}{36} = \frac{1}{6}$.

(b) A second throw is needed exactly when the first throw is a draw:

$$P(\text{second throw needed}) = \frac{6}{36} = \frac{1}{6}.$$

(c) To win on throw k, the first $k-1$ throws are draws and throw k is a win:

$$P(\text{win 1st, 2nd or 3rd}) = \frac{5}{12} + \frac{1}{6} \cdot \frac{5}{12} + \left(\frac{1}{6}\right)^2 \cdot \frac{5}{12}.$$

(This unsimplified form equals $\dfrac{215}{432} \approx 0.498$.)

(d) Grant wins the game if he wins on any throw. Summing the infinite geometric series with first term $\frac{5}{12}$ and ratio $\frac{1}{6}$:

$$P(\text{Grant wins}) = \frac{\frac{5}{12}}{1 - \frac{1}{6}} = \frac{\frac{5}{12}}{\frac{5}{6}} = \frac{1}{2}.$$

(By symmetry with "lose", each has probability $\frac{1}{2}$.)

Q35 Normanhurst Boys 2024 Trial Q17 · **3 marks**

A bag holds 4 red and 3 black discs. Ms Kim (needs red) and Ms Lee (needs black) draw alternately *with* replacement, Ms Kim first. **(a)** $P(\text{Kim wins on her first draw})$. **(b)** $P(\text{Kim wins in three or less of her turns})$.

Criteria	Marks
(a) $P = \dfrac{4}{7}$	1
(b) Correct multi-turn set-up (Kim misses then Lee misses between her turns)	1
(b) $P = \dfrac{12532}{16807}$	1

Solution.

Each draw is with replacement, so every draw is independent. On any draw $P(\text{red}) = \frac{4}{7}$ and $P(\text{black}) = \frac{3}{7}$.

(a) Ms Kim wins on her first draw by drawing red:

$$P = \frac{4}{7}.$$

(b) For Ms Kim to reach a later turn, the game must still be undecided: on each earlier round Ms Kim failed to draw red *and* then Ms Lee failed to draw black. Ms Kim fails with probability $\frac{3}{7}$; Ms Lee fails (draws red instead of black) with probability $\frac{4}{7}$. So the "both miss" round factor is

$$r = \frac{3}{7} \times \frac{4}{7} = \frac{12}{49}.$$

Ms Kim wins on her first, second or third turn with probability

$$P = \frac{4}{7}\left(1 + r + r^2\right) = \frac{4}{7}\left(1 + \frac{12}{49} + \frac{144}{2401}\right) = \frac{12532}{16807} \approx 0.746.$$

Q36 Sydney Girls 2023 Trial Q36 · 4 marks

Two dice are thrown: Donald wins if the sum is 6, loses if the sum is 5, and re-throws on any other sum, repeating until he wins or loses. (i) Find $P(\text{win on the first throw})$. (ii) Find $P(\text{win on the first, second or third throw})$. (iii) Find $P(\text{Donald eventually wins})$.

Criteria	Marks
(i) 5 outcomes give sum 6, so $P = \frac{5}{36}$	1
(ii) $\frac{5}{36} + \frac{3}{4} \cdot \frac{5}{36} + \left(\frac{3}{4}\right)^2 \frac{5}{36} = \frac{185}{576}$	1
(iii) sums the infinite geometric series (ratio $\frac{3}{4}$)	1
(iii) $P(\text{eventually wins}) = \frac{5}{9}$	1

Solution.

(i) A sum of 6 occurs for $(1,5), (2,4), (3,3), (4,2), (5,1)$, i.e. 5 of 36 outcomes:

$$P(\text{win first throw}) = \frac{5}{36}.$$

A sum of 5 (lose) occurs for 4 outcomes: $P(\text{lose}) = \frac{4}{36} = \frac{1}{9}$. Any other sum (re-throw) has probability $P(\text{draw}) = 1 - \frac{5}{36} - \frac{4}{36} = \frac{27}{36} = \frac{3}{4}$.

(ii) To win on throw k, the first $k-1$ throws are re-throws (each $\frac{3}{4}$) then a win ($\frac{5}{36}$):

$$P(\text{win 1st, 2nd or 3rd}) = \frac{5}{36} + \frac{3}{4} \cdot \frac{5}{36} + \left(\frac{3}{4}\right)^2 \cdot \frac{5}{36} = \frac{5}{36}\left(1 + \frac{3}{4} + \frac{9}{16}\right) = \frac{185}{576}.$$

(iii) Summing the infinite geometric series with first term $\frac{5}{36}$ and ratio $\frac{3}{4}$:

$$P(\text{Donald eventually wins}) = \frac{\frac{5}{36}}{1 - \frac{3}{4}} = \frac{\frac{5}{36}}{\frac{1}{4}} = \frac{20}{36} = \frac{5}{9}.$$

(Equivalently $\dfrac{P(\text{win})}{P(\text{win}) + P(\text{lose})} = \dfrac{5}{5+4} = \dfrac{5}{9}$.)

Q37 North Sydney Girls 2023 Trial Q33 · 4 marks

Lucas and Sophia take turns throwing two ordinary dice; the winner is the first to throw a double. Lucas throws first. (a) Show $P(\text{Sophia wins on her first or second throw}) = \frac{5}{36} + \frac{5^3}{6^4}$. (b) Find $P(\text{Sophia wins the game})$.

Criteria	Marks
(a) $P(\text{double}) = \frac{1}{6}$, $P(\text{no double}) = \frac{5}{6}$	1
(a) shows $\frac{5}{6} \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^3 \frac{1}{6} = \frac{5}{36} + \frac{5^3}{6^4}$	1
(b) sums the infinite geometric series (ratio $\left(\frac{5}{6}\right)^2$)	1
(b) $P(\text{Sophia wins}) = \frac{5}{11}$	1

Solution.

(a) A double occurs on 6 of 36 outcomes, so $P(\text{double}) = \frac{1}{6}$ and $P(\text{no double}) = \frac{5}{6}$. Lucas throws first, so Sophia's throws are the 2nd, 4th, ... throws overall.

- Sophia wins on her **first** throw: Lucas misses, then Sophia gets a double: $\frac{5}{6} \times \frac{1}{6} = \frac{5}{36}$.
- Sophia wins on her **second** throw: Lucas, Sophia, Lucas all miss, then Sophia gets a double: $\left(\frac{5}{6}\right)^3 \times \frac{1}{6} = \frac{5^3}{6^4}$.

Adding:

$$P(\text{Sophia 1st or 2nd throw}) = \frac{5}{36} + \frac{5^3}{6^4}. \qquad \blacksquare$$

(b) Sophia wins on her nth throw with probability $\left(\frac{5}{6}\right)^{2n-1} \frac{1}{6}$. Summing this infinite geometric series (first term $\frac{5}{36}$, ratio $\left(\frac{5}{6}\right)^2 = \frac{25}{36}$):

$$P(\text{Sophia wins}) = \frac{\frac{5}{36}}{1 - \frac{25}{36}} = \frac{\frac{5}{36}}{\frac{11}{36}} = \frac{5}{11}.$$

Q38 Shore 2024 Trial Q20 · 5 marks

Noreen and Larry take turns throwing two dice; the winner is the first to throw a double six or double one. Noreen starts. (a) Find $P(\text{Noreen wins on her first throw})$. (b) Complete the tree diagram. (c) Find $P(\text{Noreen wins on her first or second throw})$. (d) Find $P(\text{Noreen wins eventually})$.

Criteria	Marks
(a) $P(\text{win a throw}) = \frac{2}{36} = \frac{1}{18}$	1
(b) tree completed with win $\frac{1}{18}$ and no-win $\frac{17}{18}$ on each stage	1
(c) $P(\text{1st or 2nd throw}) = \frac{1}{18} + \left(\frac{17}{18}\right)^2 \frac{1}{18} = \frac{613}{5832}$	1
(d) sums the infinite geometric series (ratio $\left(\frac{17}{18}\right)^2$)	1
(d) $P(\text{Noreen eventually}) = \frac{18}{35}$	1

Solution.

(a) A double six or double one is 2 of the 36 outcomes, so the probability of winning on a single throw is

$$P(\text{win a throw}) = \frac{2}{36} = \frac{1}{18}, \qquad P(\text{no win}) = \frac{17}{18}.$$

So $P(\text{Noreen wins on her first throw}) = \frac{1}{18}$.

(b) On each branch, "win" has probability $\frac{1}{18}$ and "no win" has probability $\frac{17}{18}$; the game only continues down the "no win" branches, alternating Noreen and Larry.

(c) Noreen wins on her first throw ($\frac{1}{18}$), or on her second throw after Noreen and Larry both miss:

$$P(\text{1st or 2nd throw}) = \frac{1}{18} + \left(\frac{17}{18}\right)^2 \cdot \frac{1}{18} = \frac{1}{18} + \frac{289}{5832} = \frac{324 + 289}{5832} = \frac{613}{5832}.$$

(d) Noreen wins on her nth throw with probability $\left(\frac{17}{18}\right)^{2(n-1)} \frac{1}{18}$. Summing this infinite geometric series (first term $\frac{1}{18}$, ratio $\left(\frac{17}{18}\right)^2 = \frac{289}{324}$):

$$P(\text{Noreen eventually}) = \frac{\frac{1}{18}}{1 - \frac{289}{324}} = \frac{\frac{1}{18}}{\frac{35}{324}} = \frac{324}{18 \times 35} = \frac{18}{35}.$$

Q39 Fort St 2020 Trial Q13 · **5 marks**

Alice and Raoul take turns throwing darts; the winner is the first to hit the bullseye. $P(\text{Alice hit}) = \frac{1}{30}$, $P(\text{Raoul hit}) = \frac{1}{40}$, Alice first. (a) Draw a tree diagram for the first four throws (Alice, Raoul, Alice, Raoul). (b) Find $P(\text{Alice wins on her first or second throw})$. (c) Find $P(\text{Alice eventually wins})$.

Criteria	**Marks**
(a) tree with Alice $\frac{1}{30}, \frac{29}{30}$ and Raoul $\frac{1}{40}, \frac{39}{40}$ branches in order	1
(b) $P(\text{1st or 2nd}) = \frac{1}{30} + \frac{29}{30} \cdot \frac{39}{40} \cdot \frac{1}{30} = \frac{259}{4000}$	1
(c) sets up the infinite geometric series for an eventual win	1
(c) common ratio $\frac{29}{30} \cdot \frac{39}{40} = \frac{1131}{1200}$	1
(c) $P(\text{Alice eventually}) = \frac{40}{69}$	1

Solution.

(a) Alice misses with probability $\frac{29}{30}$; Raoul misses with probability $\frac{39}{40}$. The tree alternates Alice, Raoul, Alice, Raoul, each stage branching hit/miss, and play only continues down the "miss" branches.

(b) Alice wins on her first throw ($\frac{1}{30}$), or on her second throw after Alice then Raoul both miss:

$$P(\text{1st or 2nd}) = \frac{1}{30} + \frac{29}{30} \cdot \frac{39}{40} \cdot \frac{1}{30} = \frac{1}{30} + \frac{1131}{36000} = \frac{1200 + 1131}{36000} = \frac{2331}{36000} = \frac{259}{4000}.$$

(c) For Alice to win on her nth throw, every earlier throw (by both players) is a miss, then Alice hits. This is an infinite geometric series with first term $\frac{1}{30}$ and common ratio $\frac{29}{30} \cdot \frac{39}{40} = \frac{1131}{1200}$ (both players miss a full round):

$$P(\text{Alice eventually}) = \frac{\frac{1}{30}}{1 - \frac{1131}{1200}} = \frac{\frac{1}{30}}{\frac{69}{1200}} = \frac{1200}{30 \times 69} = \frac{40}{69}.$$

Q40 Hornsby Girls 2022 Trial Q31 · 5 marks

Matt and Ingrid take turns throwing two four-sided dice (faces 1, 2, 3, 4); the winner is the first to throw a double four. Matt starts. (a) Show $P(\text{Ingrid wins on her first throw}) = \frac{15}{256}$. (b) Find $P(\text{Matt wins on the first or second throw})$. (c) Find $P(\text{Matt eventually wins})$.

Criteria	Marks
(a) $P(\text{double four}) = \frac{1}{16}$, $P(\text{no double four}) = \frac{15}{16}$	1
(a) shows $\frac{15}{16} \cdot \frac{1}{16} = \frac{15}{256}$	1
(b) $\frac{1}{16} + \left(\frac{15}{16}\right)^2 \frac{1}{16} = \frac{481}{4096}$	1
(c) sums the infinite geometric series (ratio $\left(\frac{15}{16}\right)^2$)	1
(c) $P(\text{Matt eventually}) = \frac{16}{31}$	1

Solution.

(a) With two four-sided dice there are $4 \times 4 = 16$ equally likely outcomes, and a double four (both show 4) is 1 of them, so $P(\text{double four}) = \frac{1}{16}$ and $P(\text{no double four}) = \frac{15}{16}$. Matt throws first, so Ingrid wins on her first throw when Matt misses then Ingrid throws a double four:

$$P(\text{Ingrid first throw}) = \frac{15}{16} \times \frac{1}{16} = \frac{15}{256}. \qquad \blacksquare$$

(b) Matt wins on his first throw ($\frac{1}{16}$), or on his second throw after Matt then Ingrid both miss:

$$P(\text{Matt 1st or 2nd}) = \frac{1}{16} + \left(\frac{15}{16}\right)^2 \cdot \frac{1}{16} = \frac{1}{16} + \frac{225}{4096} = \frac{256 + 225}{4096} = \frac{481}{4096}.$$

(c) Matt wins on his nth throw with probability $\left(\frac{15}{16}\right)^{2(n-1)} \frac{1}{16}$. Summing this infinite geometric series (first term $\frac{1}{16}$, ratio $\left(\frac{15}{16}\right)^2 = \frac{225}{256}$):

$$P(\text{Matt eventually}) = \frac{\frac{1}{16}}{1 - \frac{225}{256}} = \frac{\frac{1}{16}}{\frac{31}{256}} = \frac{256}{16 \times 31} = \frac{16}{31}.$$

6. The product rule and multi-stage selections

Q41 Presbyterian 2023 Prelim Q32 · 1 mark

A standard 6-sided die is rolled three times. Before each roll Jane guesses the number. Find $P(\text{all three guesses correct})$.

Criteria	Marks
$P = \left(\frac{1}{6}\right)^3 = \frac{1}{216}$	1

Solution.

Each roll is independent, and the probability of correctly guessing one roll of a fair die is $\dfrac{1}{6}$. By the product rule for independent stages:

$$P(\text{all three correct}) = \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{216} \approx 0.00463.$$

Q42 Riverview 2022 Trial Q14 · 3 marks

Rivendale has boys:girls = 3 : 5; South Park has boys:girls = 2 : 3. One student is chosen from each school. Find (a) $P(\text{two boys})$ and (b) $P(\text{at least one girl})$.

Criteria	Marks
(a) $P(\text{boy}) = \frac{3}{8}$ (Rivendale) and $\frac{2}{5}$ (South Park)	1
(a) $P(\text{two boys}) = \frac{3}{20}$	1
(b) $P(\text{at least one girl}) = 1 - \frac{3}{20} = \frac{17}{20}$	1

Solution.

From the ratios: at Rivendale $P(\text{boy}) = \dfrac{3}{3+5} = \dfrac{3}{8}$; at South Park $P(\text{boy}) = \dfrac{2}{2+3} = \dfrac{2}{5}$.

(a) The two choices are independent (one student from each school):

$$P(\text{two boys}) = \frac{3}{8} \times \frac{2}{5} = \frac{6}{40} = \frac{3}{20} = 0.15.$$

(b) "At least one girl" is the complement of "two boys":

$$P(\text{at least one girl}) = 1 - P(\text{two boys}) = 1 - \frac{3}{20} = \frac{17}{20} = 0.85.$$

Q43 Barker 2021 Trial Q6 · **1 mark**

A drawer has 4 red and 4 blue socks. Two are selected at random at the same time. Find $P(\text{same colour})$.

Criteria	Marks
Correct option (B)	1

Solution.

Selecting two at the same time is selection without replacement. "Same colour" means both red OR both blue:

$$P(\text{same colour}) = \underbrace{\frac{4}{8}\times\frac{3}{7}}_{\text{both red}} + \underbrace{\frac{4}{8}\times\frac{3}{7}}_{\text{both blue}} = \frac{12}{56}+\frac{12}{56} = \frac{24}{56} = \frac{3}{7}.$$

The answer is **(B)**.

Q44 Hornsby Girls 2023 Trial Q15 · **3 marks**

Three standard six-sided dice are thrown once. Find (a) $P(\text{all three show six})$, (b) $P(\text{exactly two show six})$, (c) $P(\text{three different numbers})$.

Criteria	Marks
(a) $P = \left(\frac{1}{6}\right)^3 = \frac{1}{216}$	1
(b) $P = \binom{3}{2}\left(\frac{1}{6}\right)^2\frac{5}{6} = \frac{5}{72}$ (the factor 3 for which die is not a six)	1
(c) $P = \frac{6}{6}\cdot\frac{5}{6}\cdot\frac{4}{6} = \frac{5}{9}$	1

Solution.

(a) The three dice are independent, each showing six with probability $\frac{1}{6}$:

$$P(\text{all three six}) = \left(\frac{1}{6}\right)^3 = \frac{1}{216} \approx 0.00463.$$

(b) Exactly two sixes: choose which two dice show six ($\binom{3}{2} = 3$ ways), those two show six and the third does not:

$$P(\text{exactly two six}) = 3\times\left(\frac{1}{6}\right)^2\times\frac{5}{6} = 3\times\frac{5}{216} = \frac{15}{216} = \frac{5}{72} \approx 0.0694.$$

(c) Three different numbers: the first die can be anything ($\frac{6}{6}$), the second must differ from the first ($\frac{5}{6}$), the third must differ from both ($\frac{4}{6}$):

$$P(\text{three different}) = \frac{6}{6}\times\frac{5}{6}\times\frac{4}{6} = \frac{120}{216} = \frac{5}{9} \approx 0.556.$$

Q45 Presbyterian 2021 Trial Q14 · **4 marks**

42 students study at least one of Physics or Chemistry; 27 study Physics and 33 study Chemistry. Find (a) the number studying both, (b) $P(\text{Physics} \mid \text{Chemistry})$, (c) $P(\text{two randomly chosen students both study only Physics})$.

Criteria	Marks
(a) both $= 27 + 33 - 42 = 18$	1
(b) $P(\text{Physics} \mid \text{Chemistry}) = \frac{18}{33} = \frac{6}{11}$	1
(c) only Physics $= 27 - 18 = 9$; forms $\frac{9}{42} \times \frac{8}{41}$	1
(c) $= \frac{12}{287}$	1

Solution.

(a) By the inclusion–exclusion (addition) rule, $|P \cup C| = |P| + |C| - |P \cap C|$, so

$$18 = |P \cap C| = 27 + 33 - 42.$$

So 18 students study both.

(b) Restricting to the 33 Chemistry students, 18 of them also study Physics:

$$P(\text{Physics} \mid \text{Chemistry}) = \frac{18}{33} = \frac{6}{11} \approx 0.545.$$

(c) Students studying only Physics $= 27 - 18 = 9$. Choosing two students (without replacement) from all 42, both from the "only Physics" group:

$$P = \frac{9}{42} \times \frac{8}{41} = \frac{72}{1722} = \frac{12}{287} \approx 0.0418.$$

Q46 Baulkham Hills 2023 Prelim Q23 · 3 marks

Sally hits the target $\frac{3}{4}$ of the time, Timothy $\frac{1}{3}$. (a) Both shoot one arrow: find (i) $P(\text{both hit})$ and (ii) $P(\text{at least one hits})$. (b) They take turns, Timothy first, winner is the first to hit: find $P(\text{Timothy wins with his second arrow})$.

Criteria	Marks
(a)(i) $P(\text{both hit}) = \frac{3}{4} \times \frac{1}{3} = \frac{1}{4}$	1
(a)(ii) $P(\text{at least one}) = 1 - \frac{1}{4} \times \frac{2}{3} = \frac{5}{6}$	1
(b) $P = \frac{2}{3} \times \frac{1}{4} \times \frac{1}{3} = \frac{1}{18}$	1

Solution.

$P(\text{Sally hits}) = \dfrac{3}{4}$ so $P(\text{Sally misses}) = \dfrac{1}{4}$; $P(\text{Timothy hits}) = \dfrac{1}{3}$ so $P(\text{Timothy misses}) = \dfrac{2}{3}$.

(a)(i) The two shots are independent:

$$P(\text{both hit}) = \frac{3}{4} \times \frac{1}{3} = \frac{1}{4} = 0.25.$$

(a)(ii) "At least one hits" is the complement of "both miss":

$$P(\text{at least one hits}) = 1 - P(\text{both miss}) = 1 - \frac{1}{4} \times \frac{2}{3} = 1 - \frac{1}{6} = \frac{5}{6} \approx 0.833.$$

(b) Timothy shoots first. For Timothy to win with his *second* arrow, the first three shots must be: Timothy misses, then Sally misses, then Timothy hits:

$$P = \underbrace{\frac{2}{3}}_{T \text{ miss}} \times \underbrace{\frac{1}{4}}_{S \text{ miss}} \times \underbrace{\frac{1}{3}}_{T \text{ hit}} = \frac{2}{36} = \frac{1}{18} \approx 0.0556.$$

Q47 Shore 2023 Prelim Q18 · **5 marks**

Cards 3, 4, 5, 6; draw two without replacement to form a 2-digit number (first card = tens digit). A number over 60 wins \$3 back (net +\$2), else nothing; each turn costs \$1. (a) List all 2-digit numbers with a diagram. (b) Find $P(\text{win on first turn})$. (c) A player brings \$2; find $P(\text{exactly \$3 left after two turns})$.

Criteria	Marks
(a) tree/list showing the 12 numbers: 34, 35, 36, 43, 45, 46, 53, 54, 56, 63, 64, 65	1
(a) all twelve outcomes correct and equally likely	1
(b) 3 winners (63, 64, 65) out of 12, so $P = \frac{1}{4}$	1
(c) identifies the two paths (win-then-lose, lose-then-win) each giving \$3	1
(c) $P = \frac{1}{4} \cdot \frac{3}{4} + \frac{3}{4} \cdot \frac{1}{4} = \frac{3}{8}$	1

Solution.

(a) Drawing two of $\{3, 4, 5, 6\}$ in order (without replacement) gives $4 \times 3 = 12$ equally likely 2-digit numbers:

$$34,\ 35,\ 36,\ 43,\ 45,\ 46,\ 53,\ 54,\ 56,\ 63,\ 64,\ 65.$$

(A tree diagram with each first card branching to the three remaining cards produces these twelve numbers.)

(b) A win requires a number over 60. Only 63, 64, 65 exceed 60, so there are 3 winning outcomes out of 12:

$$P(\text{win}) = \frac{3}{12} = \frac{1}{4} = 0.25.$$

(c) Each turn costs \$1; a win returns \$3 (net +\$2) and a loss returns nothing (net −\$1). Starting from \$2, to finish with exactly \$3 the net change over two turns must be +\$1. The only combinations giving +\$1 are one win and one loss (win +\$2, loss −\$1). With $P(\text{win}) = \frac{1}{4}$ and $P(\text{loss}) = \frac{3}{4}$ each turn (cards are reshuffled, so turns are independent):

$$P = P(\text{win, lose}) + P(\text{lose, win}) = \frac{1}{4} \times \frac{3}{4} + \frac{3}{4} \times \frac{1}{4} = \frac{3}{16} + \frac{3}{16} = \frac{6}{16} = \frac{3}{8} = 0.375.$$

Q48 Sydney Grammar 2021 Trial Q31 · 7 marks

A pair of dice is thrown repeatedly until the difference is 2 (Tom wins) or 3 (Tom loses); any other difference means throw again. (a) Show $P(\text{win on first throw}) = \frac{2}{9}$. (b) Find $P(\text{game continues to a second throw})$. (c) Find $P(\text{Tom wins in one of the first three throws})$ in unsimplified form. (d) Find $P(\text{Tom wins the game})$.

Criteria	Marks
(a) counts the 8 outcomes with difference 2 out of 36	1
(a) $P = \frac{8}{36} = \frac{2}{9}$	1
(b) $P(\text{diff 2 or 3}) = \frac{2}{9} + \frac{1}{6} = \frac{7}{18}$, so continue $= \frac{11}{18}$	1
(c) $P = \frac{2}{9} + \frac{11}{18} \cdot \frac{2}{9} + \left(\frac{11}{18}\right)^2 \frac{2}{9}$ (unsimplified)	1
(c) evaluates to $\frac{643}{1458}$	1
(d) recognises an infinite geometric sum with $a = \frac{2}{9}$, $r = \frac{11}{18}$ (or ratio $\frac{P(2)}{P(2 \text{ or } 3)}$)	1
(d) $P(\text{Tom wins}) = \frac{4}{7}$	1

Solution.

(a) For two six-sided dice there are 36 equally likely outcomes. A difference of 2 occurs for the pairs $(1,3), (2,4), (3,5), (4,6)$ and their reverses $(3,1), (4,2), (5,3), (6,4)$: that is 8 outcomes. Hence

$$P(\text{difference } 2) = \frac{8}{36} = \frac{2}{9}. \blacksquare$$

(b) A difference of 3 occurs for $(1,4), (2,5), (3,6)$ and their reverses: 6 outcomes, so $P(\text{difference } 3) = \frac{6}{36} = \frac{1}{6}$. The game ends on a throw only if the difference is 2 or 3:

$$P(\text{ends}) = \frac{2}{9} + \frac{1}{6} = \frac{4}{18} + \frac{3}{18} = \frac{7}{18}.$$

The game continues to a second throw exactly when the first throw does not end it:

$$P(\text{continues}) = 1 - \frac{7}{18} = \frac{11}{18} \approx 0.611.$$

(c) Tom wins on a given throw with probability $\frac{2}{9}$, and the game reaches a later throw only after each "continue" (probability $\frac{11}{18}$). Winning in one of the first three throws:

$$P = \frac{2}{9} + \frac{11}{18} \cdot \frac{2}{9} + \left(\frac{11}{18}\right)^2 \cdot \frac{2}{9} = \frac{643}{1458} \approx 0.441.$$

(d) Tom wins eventually on the first, second, third, … throw. This is an infinite geometric series with first

term $a = \frac{2}{9}$ and common ratio $r = \frac{11}{18}$:

$$P(\text{Tom wins}) = \frac{a}{1-r} = \frac{\frac{2}{9}}{1-\frac{11}{18}} = \frac{\frac{2}{9}}{\frac{7}{18}} = \frac{2}{9} \times \frac{18}{7} = \frac{4}{7} \approx 0.571.$$

(Equivalently, since each ending throw is a win with probability $\frac{2}{9}$ or a loss with probability $\frac{1}{6}$, $P(\text{win}) = \frac{2/9}{2/9 + 1/6} = \frac{4}{7}$.)

Conditional probability and independence

7. Reverse conditional and total probability (Bayes-style)

Q49 Normanhurst Boys 2020 Trial Q20 · **3 marks**

A factory makes chocolate chip (C, $\frac{3}{5}$) and oatmeal (M, $\frac{2}{5}$) cookies. Of the chocolate chip, $\frac{5}{7}$ are regular (R) and $\frac{2}{7}$ sugar free (S); of the oatmeal, $\frac{2}{3}$ are sugar free and $\frac{1}{3}$ regular. (a) Label all remaining branches of the tree. (b) Given a sugar free cookie is selected, find $P(C \mid S)$.

Criteria	Marks
(a) all remaining tree branches labelled correctly	1
(b) forms the conditional-probability expression $P(C \mid S) = \dfrac{P(C \cap S)}{P(S)}$	1
(b) correct answer $\frac{9}{23}$	1

Solution.

(a) From C: $P(R \mid C) = \dfrac{5}{7}$, $P(S \mid C) = \dfrac{2}{7}$. From M: $P(S \mid M) = \dfrac{2}{3}$, $P(R \mid M) = \dfrac{1}{3}$. (The first-stage branches are $P(C) = \frac{3}{5}$ and $P(M) = \frac{2}{5}$.)

(b) A sugar free cookie can arise from either type:

$$P(C \cap S) = \frac{3}{5} \times \frac{2}{7} = \frac{6}{35}, \qquad P(S) = \frac{3}{5} \times \frac{2}{7} + \frac{2}{5} \times \frac{2}{3} = \frac{6}{35} + \frac{4}{15} = \frac{46}{105}.$$

$$P(C \mid S) = \frac{P(C \cap S)}{P(S)} = \frac{6/35}{46/105} = \frac{6}{35} \times \frac{105}{46} = \frac{9}{23} \approx 0.391.$$

$$\boxed{P(C \mid S) = \frac{9}{23}.}$$

Q50 Sydney Boys 2022 Trial Q31 · **3 marks**

Power-ups are 85% minor, 15% major. Of major, 60% are combat; of minor, 30% are combat. (a) Complete the tree. (b)(i) Find $P(\text{combat})$. (b)(ii) Find $P(\text{major} \mid \text{combat})$.

Criteria	Marks
(a) tree completed correctly (minor/major and combat/survival branches)	1
(b)(i) $P(\text{combat}) = 0.345$	1
(b)(ii) $P(\text{major} \mid \text{combat}) = \frac{6}{23}$	1

Solution.

(a) First stage: $P(\text{minor}) = 0.85$, $P(\text{major}) = 0.15$. From minor: $P(\text{combat}) = 0.30$, $P(\text{survival}) = 0.70$. From major: $P(\text{combat}) = 0.60$, $P(\text{survival}) = 0.40$.

(b)(i) By the law of total probability,

$$P(\text{combat}) = 0.85 \times 0.30 + 0.15 \times 0.60 = 0.255 + 0.09 = 0.345 = \frac{69}{200}.$$

(b)(ii)

$$P(\text{major} \mid \text{combat}) = \frac{P(\text{major} \cap \text{combat})}{P(\text{combat})} = \frac{0.15 \times 0.60}{0.345} = \frac{0.09}{0.345} = \frac{6}{23} \approx 0.261.$$

$$\boxed{P(\text{major} \mid \text{combat}) = \frac{6}{23}.}$$

Q51 Sydney Boys 2020 Trial Q28 · **4 marks**

$P(\text{wakes late}) = 0.6$; if late waking, $P(\text{late to work}) = 0.85$; if on time, $P(\text{late to work}) = 0.4$. (a) Find $P(\text{late to work})$. (b) Given she arrives late, find $P(\text{overslept})$.

Criteria	Marks
(a) sets up total probability with both paths	1
(a) $P(\text{late}) = 0.67$	1
(b) forms the conditional probability $P(\text{late waking} \mid \text{late to work})$	1
(b) $\frac{51}{67}$	1

Solution.

(a) Late to work happens whether she wakes late or on time:

$$P(\text{late}) = 0.6 \times 0.85 + 0.4 \times 0.4 = 0.51 + 0.16 = 0.67.$$

(b) "Overslept" means she woke up late; she is known to have arrived late, so this is conditional:

$$P(\text{woke late} \mid \text{late}) = \frac{P(\text{woke late} \cap \text{late})}{P(\text{late})} = \frac{0.6 \times 0.85}{0.67} = \frac{0.51}{0.67} = \frac{51}{67} \approx 0.761.$$

$$\boxed{P(\text{overslept} \mid \text{late}) = \frac{51}{67}.}$$

Q52 Baulkham Hills 2024 Trial Q18 · **5 marks**

Weather on Andrew's drive is dry $\frac{1}{2}$, rain $\frac{1}{3}$, snow $\frac{1}{6}$; $P(\text{on time})$ is $\frac{4}{5}$, $\frac{2}{5}$, $\frac{1}{10}$ respectively. (a) Complete the tree. (b) Find $P(\text{late})$. (c) Given he was late, find $P(\text{raining})$.

Criteria	Marks
(a) weather branches $\frac{1}{2}, \frac{1}{3}, \frac{1}{6}$ and on-time/late branches all correct	1
(a) late branches $\frac{1}{5}, \frac{3}{5}, \frac{9}{10}$ shown	1
(b) $P(\text{late}) = \frac{9}{20}$	1
(c) forms $P(\text{rain} \mid \text{late}) = \dfrac{P(\text{rain} \cap \text{late})}{P(\text{late})}$	1
(c) $\frac{4}{9}$	1

Solution.

(a) The "late" probabilities are the complements of "on time": dry $1-\frac{4}{5}=\frac{1}{5}$, rain $1-\frac{2}{5}=\frac{3}{5}$, snow $1-\frac{1}{10}=\frac{9}{10}$.

(b) By the law of total probability,

$$P(\text{late}) = \frac{1}{2}\cdot\frac{1}{5} + \frac{1}{3}\cdot\frac{3}{5} + \frac{1}{6}\cdot\frac{9}{10} = \frac{1}{10} + \frac{1}{5} + \frac{3}{20} = \frac{9}{20}.$$

(c)

$$P(\text{rain} \mid \text{late}) = \frac{P(\text{rain} \cap \text{late})}{P(\text{late})} = \frac{\frac{1}{3}\cdot\frac{3}{5}}{\frac{9}{20}} = \frac{\frac{1}{5}}{\frac{9}{20}} = \frac{1}{5}\times\frac{20}{9} = \frac{4}{9} \approx 0.444.$$

$$\boxed{P(\text{rain} \mid \text{late}) = \frac{4}{9}.}$$

Q53 Gosford 2024 Prelim Q20 · **2 marks**

A bag has 3 dice: two fair (1–6) and one special $(1, 5, 5, 5, 5, 6)$. One is chosen at random and tossed. (a) Find $P(\text{shows } 5)$. (b) Given a 5 shows, find $P(\text{special})$.

Criteria	Marks
(a) $P(\text{shows } 5) = \frac{1}{3}$	1
(b) $P(\text{special} \mid 5) = \frac{2}{3}$	1

Solution.

(a) A fair die gives $P(5) = \frac{1}{6}$; the special die has four 5s so $P(5) = \frac{4}{6} = \frac{2}{3}$. The die chosen is fair with probability $\frac{2}{3}$ and special with probability $\frac{1}{3}$:

$$P(\text{shows } 5) = \frac{2}{3} \cdot \frac{1}{6} + \frac{1}{3} \cdot \frac{2}{3} = \frac{2}{18} + \frac{2}{9} = \frac{1}{9} + \frac{2}{9} = \frac{1}{3}.$$

(b)

$$P(\text{special} \mid 5) = \frac{P(\text{special} \cap 5)}{P(5)} = \frac{\frac{1}{3} \cdot \frac{2}{3}}{\frac{1}{3}} = \frac{\frac{2}{9}}{\frac{1}{3}} = \frac{2}{9} \times 3 = \frac{2}{3}.$$

$$\boxed{P(\text{special} \mid 5) = \frac{2}{3}.}$$

Q54 Normanhurst Boys 2025 Trial Q17 · **4 marks**

Joe plays 80% of games. When he plays: win 77%, lose 10% (draw 13%). When he does not play: win 53%, lose 25% (draw 22%). (a) Find $P(\text{lose})$. (b) Given the team does not lose, find $P(\text{Joe played})$.

Criteria	Marks
(a) identifies both loss paths (Joe plays / Joe absent)	1
(a) $P(\text{lose}) = 0.13$	1
(b) forms $P(\text{played} \mid \text{not lose}) = \dfrac{P(\text{played} \cap \text{not lose})}{P(\text{not lose})}$	1
(b) $\frac{24}{29}$	1

Solution.

(a) Let J be "Joe plays" ($P(J) = 0.8$) and $\bar{J}$ "Joe absent" ($P(\bar{J}) = 0.2$).

$$P(\text{lose}) = P(J) \times 0.10 + P(\bar{J}) \times 0.25 = 0.8 \times 0.10 + 0.2 \times 0.25 = 0.08 + 0.05 = 0.13.$$

(b) "Not lose" has probability $1 - 0.13 = 0.87$. When Joe plays, $P(\text{not lose}) = 0.77 + 0.13 = 0.90$, so $P(J \cap \text{not lose}) = 0.8 \times 0.90 = 0.72$.

$$P(J \mid \text{not lose}) = \frac{P(J \cap \text{not lose})}{P(\text{not lose})} = \frac{0.8 \times 0.90}{0.87} = \frac{0.72}{0.87} = \frac{24}{29} \approx 0.828.$$

$$\boxed{P(\text{Joe played} \mid \text{not lose}) = \frac{24}{29}.}$$

Q55 Penrith 2022 Trial Q32 · 6 marks

If Mr Ferrarin wakes late, $P(\text{exercises}) = \frac{1}{3}$; if on time, $P(\text{exercises}) = \frac{4}{5}$. (a) He is on time Thursday and late Friday: find $P(\text{exercises on at least one day})$. (b) $P(\text{on time Monday}) = \frac{1}{4}$: (i) find $P(\text{exercises Monday})$; (ii) find $P(\text{on time} \mid \text{exercised})$.

Criteria	Marks
(a) uses the complement of "exercises on neither day"	1
(a) $P(\text{at least one}) = \frac{13}{15}$	1
(b)(i) sets up total probability with $P(\text{on time}) = \frac{1}{4}$, $P(\text{late}) = \frac{3}{4}$	1
(b)(i) $P(\text{exercises}) = \frac{9}{20}$	1
(b)(ii) forms the conditional probability	1
(b)(ii) $\frac{4}{9}$	1

Solution.

(a) Thursday (on time): $P(\text{exercises}) = \frac{4}{5}$; Friday (late): $P(\text{exercises}) = \frac{1}{3}$. Use the complement of "does not exercise on either day":

$$P(\text{at least one}) = 1 - \left(1 - \tfrac{4}{5}\right)\left(1 - \tfrac{1}{3}\right) = 1 - \frac{1}{5}\cdot\frac{2}{3} = 1 - \frac{2}{15} = \frac{13}{15} \approx 0.867.$$

(b)(i) With $P(\text{on time}) = \frac{1}{4}$ and $P(\text{late}) = \frac{3}{4}$,

$$P(\text{exercises}) = \frac{1}{4}\cdot\frac{4}{5} + \frac{3}{4}\cdot\frac{1}{3} = \frac{1}{5} + \frac{1}{4} = \frac{9}{20}.$$

(b)(ii)

$$P(\text{on time} \mid \text{exercised}) = \frac{P(\text{on time} \cap \text{exercised})}{P(\text{exercised})} = \frac{\frac{1}{4}\cdot\frac{4}{5}}{\frac{9}{20}} = \frac{\frac{1}{5}}{\frac{9}{20}} = \frac{1}{5}\times\frac{20}{9} = \frac{4}{9} \approx 0.444.$$

$$\boxed{P(\text{on time} \mid \text{exercised}) = \frac{4}{9}.}$$

Q56 Hurlstone 2023 Prelim Q14 · **6 marks**

(a) Alice/Ben/Chloe handle 48%/30%/22% of orders with mistake rates 10%/5%/2%. (i) Draw a tree diagram; (ii) find $P(\text{mistake})$. (b) A website is 98% accurate at detecting plagiarism; 5% of papers are plagiarised. Given a "Plagiarised" result, find $P(\text{actually plagiarised})$.

Criteria	Marks
(a)(i) correct tree: three employee branches, each with correct/mistake	1
(a)(i) branch probabilities correct	1
(a)(ii) $P(\text{mistake}) = 0.0674$	1
(b) sets up the conditional probability with both true and false positives	1
(b) correct denominator $0.05(0.98) + 0.95(0.02)$	1
(b) $\frac{49}{68} \approx 0.72$	1

Solution.

(a)(i) The tree has three first-stage branches Alice (0.48), Ben (0.30), Chloe (0.22); each splits into "mistake" and "correct" with $P(\text{mistake}) = 0.10,\ 0.05,\ 0.02$ respectively (so $P(\text{correct}) = 0.90,\ 0.95,\ 0.98$).

(a)(ii) By the law of total probability,

$$P(\text{mistake}) = 0.48 \times 0.10 + 0.30 \times 0.05 + 0.22 \times 0.02 = 0.048 + 0.015 + 0.0044 = 0.0674\ (6.74\%).$$

(b) Let $P(\text{plag}) = 0.05$, $P(\overline{\text{plag}}) = 0.95$. The website is correct 98% of the time, so $P(\text{"Plag"} \mid \text{plag}) = 0.98$ (true positive) and $P(\text{"Plag"} \mid \overline{\text{plag}}) = 0.02$ (false positive):

$$P(\text{"Plag"}) = 0.05 \times 0.98 + 0.95 \times 0.02 = 0.049 + 0.019 = 0.068.$$

$$P(\text{plag} \mid \text{"Plag"}) = \frac{0.05 \times 0.98}{0.068} = \frac{0.049}{0.068} = \frac{49}{68} \approx 0.721\ (\approx 72\%).$$

$$\boxed{P(\text{plagiarised} \mid \text{"Plagiarised"}) = \frac{49}{68} \approx 72\%.}$$

8. Conditional probability from a two-way table or Venn

Q57 Trinity Grammar 2020 Trial Q6 · 1 mark

A Venn diagram of A and B shows "A only" = 0.1, the intersection = p, "B only" = q, and 0.4 outside both. Given $P(A) = 0.25$, find $P(B)$.

Criteria	Marks
Correct option (C)	1

Solution.

Since $P(A) = 0.25$ and the "A only" region is 0.1, the intersection is

$$p = P(A \cap B) = 0.25 - 0.1 = 0.15.$$

All four regions sum to 1, so "B only" is

$$q = 1 - 0.1 - 0.15 - 0.4 = 0.35.$$

Hence

$$P(B) = p + q = 0.15 + 0.35 = 0.50.$$

The answer is option **C**.

Q58 Kings 2022 Trial Q7 · 1 mark

A three-circle Venn diagram (Surf Club, AFL, Soccer) shows: Surf only = 5, Surf∩AFL only = 6, Surf∩Soccer only = 4, all three = 3, AFL only = 4, AFL∩Soccer only = 5, Soccer only = 6. A soccer-club member is chosen at random. Find $P(\text{surf} \mid \text{soccer})$.

Criteria	Marks
Correct option (C)	1

Solution.

The number of soccer-club members is the whole Soccer circle:

$$n(\text{Soccer}) = 6 + 4 + 5 + 3 = 18.$$

Of these, the ones who are also in the Surf circle are the two overlap regions with Soccer that lie inside Surf: Surf∩Soccer only (4) and the centre (3), giving $4 + 3 = 7$. Hence

$$P(\text{surf} \mid \text{soccer}) = \frac{n(\text{surf} \cap \text{soccer})}{n(\text{soccer})} = \frac{7}{18}.$$

The answer is option **C**.

Q59 Normanhurst Boys 2020 Trial Q10 · **1 mark**

A Venn diagram for 100 readers shows "Movie only" = 46, "both" = 17, "Novel only" = 34, and 3 outside. Given the reader watched the movie, find $P(\text{read the novel})$.

Criteria	Marks
Correct option (B)	1

Solution.

The number who watched the movie is

$$n(\text{Movie}) = 46 + 17 = 63.$$

Of those, 17 also read the novel, so

$$P(\text{Novel} \mid \text{Movie}) = \frac{n(\text{Movie} \cap \text{Novel})}{n(\text{Movie})} = \frac{17}{63}.$$

The answer is option **B**.

Q60 Hurlstone 2021 Prelim Q22 · **1 mark**

From the two-way table of P-plate drivers, with S = lost license for speeding and M = male: males who lost their license = 13, total males = 64. Find $P(S \mid M)$.

Criteria	Marks
Correct option (B)	1

Solution.

Conditioning on the driver being male restricts attention to the 64 males, of whom 13 lost their license for speeding:

$$P(S \mid M) = \frac{n(S \cap M)}{n(M)} = \frac{13}{64}.$$

The answer is option **B**.

Q61 Killara 2022 Prelim Q7 · **1 mark**

A Venn diagram of sets F and G shows "F only" = 6, $F \cap G = 14$, "G only" = x, and 9 outside both. Given $P(\bar{G}) = \frac{1}{2}$, find x.

Criteria	Marks
Correct option (A)	1

Solution.

The total number of scores is $6 + 14 + x + 9 = 29 + x$. The complement $\bar{G}$ consists of everything outside G, that is "F only" and the region outside both:

$$n(\bar{G}) = 6 + 9 = 15.$$

Setting $P(\bar{G}) = \frac{1}{2}$ gives

$$\frac{15}{29 + x} = \frac{1}{2} \quad \Longrightarrow \quad 29 + x = 30 \quad \Longrightarrow \quad x = 1.$$

The answer is option **A**.

Q62 Hurlstone 2024 Trial Q33 · **2 marks**

Of 60 students: 26 study Mathematics Advanced, 29 study Biology, 14 study both. **(a)** Draw a Venn diagram. **(b)** Find $P(\text{Biology} \mid \text{Mathematics Advanced})$.

Criteria	Marks
(a) Correct Venn diagram (Maths Adv only 12, both 14, Biology only 15, neither 19)	1
(b) $P(\text{Bio} \mid \text{MA}) = \frac{7}{13}$	1

Solution.

(a) The "both" region is 14. Then "Mathematics Advanced only" = $26 - 14 = 12$ and "Biology only" = $29 - 14 = 15$. The number studying neither is

$$60 - (12 + 14 + 15) = 19.$$

So the Venn diagram has 12 in Mathematics Advanced only, 14 in the overlap, 15 in Biology only, and 19 outside both circles.

(b) Conditioning on studying Mathematics Advanced restricts to those 26 students, of whom 14 also study Biology:

$$P(\text{Biology} \mid \text{Mathematics Advanced}) = \frac{14}{26} = \frac{7}{13}.$$

Q63 Normanhurst Boys 2022 Trial Q27 · **2 marks**

120 soccer fans: 74 support Manchester United, 55 support Tottenham, 8 support neither. **(a)** Represent the data in a Venn diagram. **(b)** Given a fan supports Manchester United, find $P(\text{Tottenham})$.

Criteria	Marks
(a) Correct Venn diagram (Man Utd only 57, both 17, Tottenham only 38, neither 8)	1
(b) $P(\text{Tot} \mid \text{Man Utd}) = \dfrac{17}{74}$	1

Solution.

(a) The number who support at least one team is $120 - 8 = 112$. By the addition rule, the number supporting both is

$$n(\text{both}) = 74 + 55 - 112 = 17.$$

Hence "Manchester United only" $= 74 - 17 = 57$ and "Tottenham only" $= 55 - 17 = 38$. The Venn diagram has 57, 17, 38 in the three regions and 8 outside. (Check: $57 + 17 + 38 + 8 = 120$.)

(b) Conditioning on being a Manchester United fan restricts to those 74 fans, of whom 17 also support Tottenham:

$$P(\text{Tottenham} \mid \text{Manchester United}) = \frac{17}{74}.$$

Q64 Danebank 2020 Trial Q17 · **3 marks**

A lie detector test on 200 people gives a two-way table: lying/positive 96, lying/negative 24, not-lying/positive 8, not-lying/negative 72. (a) Find the probability that a person who was lying tested positive. (b) Find the probability that the test was inaccurate.

Criteria	Marks
(a) $P(\text{positive} \mid \text{lying}) = \frac{96}{120} = \frac{4}{5}$	1
(b) identifies the inaccurate outcomes ($24 + 8 = 32$)	1
(b) $P(\text{inaccurate}) = \frac{32}{200} = \frac{4}{25}$	1

Solution.

(a) "A person who was lying" restricts us to the $96 + 24 = 120$ people who were lying, of whom 96 tested positive:

$$P(\text{positive} \mid \text{lying}) = \frac{96}{120} = \frac{4}{5} = 0.8.$$

(b) The test is inaccurate when its result disagrees with the truth: a lying person who tests negative (24) or a truthful person who tests positive (8). Out of all 200 people:

$$P(\text{inaccurate}) = \frac{24 + 8}{200} = \frac{32}{200} = \frac{4}{25} = 0.16.$$

Q65 Cranbrook 2023 Trial Q19 · **3 marks**

An incomplete two-way table has Dog: (Cat = A, No Cat = 32) and No Dog: (Cat = 45, No Cat = 35). Given that a student who owns a dog owns a cat with probability 60%, find $P(\text{owns both a cat and a dog})$.

Criteria	Marks
Sets up $\dfrac{A}{A+32} = 0.6$ from the conditional probability	1
Solves $A = 48$ and finds the total = 160	1
$P(\text{cat and dog}) = \dfrac{3}{10}$	1

Solution.

The dog owners number $A + 32$. Being told a dog owner owns a cat with probability 60% means

$$P(\text{cat} \mid \text{dog}) = \frac{A}{A+32} = 0.6.$$

Solving:

$$A = 0.6(A+32) \implies 0.4A = 19.2 \implies A = 48.$$

The total number of students is

$$48 + 32 + 45 + 35 = 160,$$

so

$$P(\text{owns both a cat and a dog}) = \frac{48}{160} = \frac{3}{10} = 0.3.$$

9. Conditional probability from a tree or the formula

Q66 Sydney Girls 2021 Prelim Q2 · **1 mark**

Given $P(X \cap Y) = 0.2$, $P(X) = 0.3$ and $P(Y) = 0.4$, find $P(X \mid Y)$.

Criteria	Marks
$P(X \mid Y) = \dfrac{1}{2}$	1

Solution.

By the conditional-probability formula,

$$P(X \mid Y) = \frac{P(X \cap Y)}{P(Y)} = \frac{0.2}{0.4} = \frac{1}{2} = 0.5.$$

(The value $P(X) = 0.3$ is not needed.)

Q67 St George Girls 2022 Trial Q5 · **1 mark**

Failures occur in the hard drive (40%), monitor (25%), disk drive (20%) or microprocessor (15%). Given the problem is not in the monitor, find $P(\text{hard drive})$.

Criteria	Marks
Correct option (D)	1

Solution.

Condition on "not the monitor", which has probability $1 - 0.25 = 0.75$. The hard drive accounts for 0.40, so

$$P(\text{hard drive} \mid \text{not monitor}) = \frac{P(\text{hard drive})}{P(\text{not monitor})} = \frac{0.40}{0.75} = \frac{8}{15} \approx 0.533.$$

The answer is option **D**.

Q68 Killara 2020 Trial Q17 · **4 marks**

200 raffle tickets are sold; the two prize tickets are drawn without replacement, the first drawn winning first prize. A student holds 5 tickets. **(a)** Complete the two-stage tree (Win/Lose on each draw) and list the sample space. **(b)** Find $P(\text{wins at least one prize})$. **(c)** Find $P(\text{wins second prize} \mid \text{did not win first prize})$.

Criteria	Marks
(a) Correct first-draw probabilities $\frac{5}{200}, \frac{195}{200}$	1
(a) Correct (dependent) second-draw probabilities and sample space listed	1
(b) $P(\text{at least one}) = \dfrac{197}{3980}$	1
(c) $P(\text{second} \mid \text{not first}) = \dfrac{5}{199}$	1

Solution.

Let W and L denote winning and losing a prize on a draw. The student holds 5 of the 200 tickets. Since the draw is without replacement, the second-draw probabilities depend on the first:

$$P(W_1) = \frac{5}{200}, \qquad P(L_1) = \frac{195}{200},$$

$$P(WW) = \frac{5}{200}\cdot\frac{4}{199}, \quad P(WL) = \frac{5}{200}\cdot\frac{195}{199}, \quad P(LW) = \frac{195}{200}\cdot\frac{5}{199}, \quad P(LL) = \frac{195}{200}\cdot\frac{194}{199}.$$

(b) "At least one prize" is the complement of winning neither, i.e. of LL:

$$P(\text{at least one}) = 1 - P(LL) = 1 - \frac{195}{200}\cdot\frac{194}{199} = 1 - \frac{3783}{3980} = \frac{197}{3980} \approx 0.0495.$$

(c) Given the first prize was not won, only the second-draw branch from L matters. Of the 199 remaining tickets the student still holds all 5:

$$P(\text{second} \mid \text{not first}) = \frac{5}{199} \approx 0.0251.$$

Q69 SCEGGS 2022 Prelim Q24 · **3 marks**

For dependent events A and B: $P(A) = 0.22$, $P(B) = 0.54$, $P(A \cup B) = 0.72$. Find $P(A \mid B)$ to 3 decimal places.

Criteria	**Marks**
Finds $P(A \cap B) = 0.04$ via the addition rule	1
Applies $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$	1
$P(A \mid B) \approx 0.074$	1

Solution.

First find the intersection from the addition rule:

$$P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.22 + 0.54 - 0.72 = 0.04.$$

Then

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.04}{0.54} = \frac{2}{27} \approx 0.074.$$

Q70 Sydney Grammar 2023 Trial Q29 · **5 marks**

A bowl holds 18 apples: 11 red, 7 green. Matthew eats one at random, then Evelyn eats one at random (without replacement). **(a)** Find $P(\text{same colour})$. **(b)** Given the two apples are the same colour, find $P(\text{both red})$.

Criteria	Marks
(a) $P(RR) = \frac{11}{18}\cdot\frac{10}{17}$ and $P(GG) = \frac{7}{18}\cdot\frac{6}{17}$	1
(a) $P(\text{same}) = \frac{76}{153}$	1
(b) Sets up the conditional $\frac{P(RR)}{P(\text{same})}$	1
(b) $P(\text{both red} \mid \text{same}) = \frac{55}{76}$	1
Correct without-replacement probabilities used throughout	1

Solution.

The two apples are eaten without replacement, so the second probability depends on the first.

(a) "Same colour" means both red or both green:

$$P(RR) = \frac{11}{18}\cdot\frac{10}{17} = \frac{110}{306} = \frac{55}{153}, \qquad P(GG) = \frac{7}{18}\cdot\frac{6}{17} = \frac{42}{306} = \frac{7}{51} = \frac{21}{153},$$

$$P(\text{same}) = \frac{55}{153} + \frac{21}{153} = \frac{76}{153} \approx 0.497.$$

(b) Condition on the same-colour event:

$$P(\text{both red} \mid \text{same}) = \frac{P(RR)}{P(\text{same})} = \frac{55/153}{76/153} = \frac{55}{76} \approx 0.724.$$

Q71 St George Girls 2024 Trial Q30 · **3 marks**

Lucio plays with probability 0.9. His team wins with probability 0.85 when he plays and 0.40 when he does not. **(a)** Find $P(\text{Lucio plays and the team wins})$. **(b)** Find $P(\text{Lucio played} \mid \text{team won})$.

Criteria	Marks
(a) $P(\text{plays and wins}) = 0.765$	1
(b) Finds total $P(\text{win}) = 0.805$	1
(b) $P(\text{played} \mid \text{won}) = \dfrac{153}{161} \approx 0.950$	1

Solution.

(a) Multiply along the "plays then wins" branch:

$$P(\text{plays and wins}) = 0.9 \times 0.85 = 0.765.$$

(b) The total probability of a win adds the two winning branches:

$$P(\text{win}) = \underbrace{0.9 \times 0.85}_{0.765} + \underbrace{0.1 \times 0.40}_{0.04} = 0.805.$$

Then

$$P(\text{played} \mid \text{won}) = \frac{P(\text{plays and wins})}{P(\text{win})} = \frac{0.765}{0.805} = \frac{153}{161} \approx 0.950.$$

10. Independence and the multiplication rule

Q72 Cherrybrook Tech 2020 Trial Q3 · **1 mark**

A and B are independent with $P(A), P(B)$ non-zero. Which expression gives the probability that A occurs and then B also occurs?

Criteria	Marks
Correct option (D)	1

Solution.

"A occurs and then B also occurs" is the event $A \cap B$. For independent events the multiplication rule gives

$$P(A \cap B) = P(A)\,P(B).$$

Option (A) $P(B \mid A)$ and option (B) $P(A \mid B)$ are conditional probabilities (single events), and option (C) $P(A \cup B)$ is "A or B". The answer is **(D)**.

Q73 North Sydney Boys 2024 Trial Q8 · **1 mark**

X and Y are independent with $P(X) = 0.2$, $P(Y) = 0.52$. Find $P(\overline{X} \cap \overline{Y})$.

Criteria	Marks
Correct option (C)	1

Solution.

If X and Y are independent then so are their complements $\overline{X}$ and $\overline{Y}$, so

$$P(\overline{X} \cap \overline{Y}) = P(\overline{X})\,P(\overline{Y}) = (1 - 0.2)(1 - 0.52) = 0.8 \times 0.48 = 0.384.$$

The answer is **(C)**.

Q74 Ravenswood 2021 Trial Q10 · **4 marks**

30 students take chemistry only, 20 physics only, 10 both, 60 neither. Find (a) $P(\text{physics})$, (b) $P(\text{physics} \mid \text{chemistry})$, (c) whether taking chemistry and taking physics are independent.

Criteria	Marks
(a) total = 30 + 20 + 10 + 60 = 120; physics = 20 + 10 = 30	1
(a) $P(\text{physics}) = \frac{30}{120} = \frac{1}{4}$	1
(b) $P(\text{physics} \mid \text{chemistry}) = \frac{10}{40} = \frac{1}{4}$	1
(c) $P(\text{physics} \mid \text{chemistry}) = P(\text{physics}) = \frac{1}{4}$, so the events *are* independent	1

Solution.

The total number of students is $30 + 20 + 10 + 60 = 120$.

(a) Students taking physics = 20 (physics only) + 10 (both) = 30, so

$$P(\text{physics}) = \frac{30}{120} = \frac{1}{4} = 0.25.$$

(b) Restricting to the $30 + 10 = 40$ chemistry students, 10 also take physics:

$$P(\text{physics} \mid \text{chemistry}) = \frac{10}{40} = \frac{1}{4} = 0.25.$$

(c) The events are independent iff $P(\text{physics} \mid \text{chemistry}) = P(\text{physics})$. Here both equal $\dfrac{1}{4}$, so

$$P(\text{physics} \mid \text{chemistry}) = P(\text{physics}) = \frac{1}{4},$$

and the events **are independent**. (Equivalently $P(\text{P} \cap \text{C}) = \frac{10}{120} = \frac{1}{12}$ and $P(\text{P})P(\text{C}) = \frac{1}{4} \times \frac{40}{120} = \frac{1}{12}$.)

Q75 Cherrybrook Tech 2022 Prelim Q10 · **1 mark**

In which case are A and B independent? (A) $P(A|B) = 0.5, P(A) = 0.4, P(B) = 0.5$; (B) $P(A) = 0.9, P(B) = 0.4, P(A \cap B) = 0.36$; (C) $P(A|B) = \frac{3}{4}, P(A) = \frac{3}{5}, P(B) = \frac{7}{10}$; (D) $P(A) = \frac{1}{5}, P(B) = \frac{1}{3}, P(A \cap B) = \frac{1}{5}$.

Criteria	**Marks**
Correct option (B)	1

Solution.

A and B are independent iff $P(A \cap B) = P(A)\,P(B)$ (equivalently $P(A \mid B) = P(A)$). Testing each option:

$$\text{(A) } P(A|B) = 0.5 \neq 0.4 = P(A); \qquad \text{(C) } P(A|B) = \tfrac{3}{4} \neq \tfrac{3}{5} = P(A);$$

$$\text{(D) } P(A)P(B) = \tfrac{1}{5} \times \tfrac{1}{3} = \tfrac{1}{15} \neq \tfrac{1}{5} = P(A \cap B).$$

$$\text{(B) } P(A)P(B) = 0.9 \times 0.4 = 0.36 = P(A \cap B). \checkmark$$

The answer is **(B)**.

Q76 Shore 2023 Trial Q24 · 3 marks

600 students; two-way table of Rugby/Soccer/AFL preference by sex (Female 52, 130, 88; Male 104, 155, 71). (a) Find $P(\text{Rugby})$. (b) Determine whether "Female" and "prefers Rugby" are independent, with reasoning.

Criteria	Marks
(a) $P(\text{Rugby}) = \frac{52+104}{600} = \frac{156}{600} = \frac{13}{50}$	1
(b) computes both $P(\text{Female})P(\text{Rugby})$ and $P(\text{Female} \cap \text{Rugby})$	1
(b) $\frac{13}{150} \neq \frac{117}{1000}$, so the events are *not* independent	1

Solution.

(a) The number preferring Rugby is 52 + 104 = 156 out of 600:

$$P(\text{Rugby}) = \frac{156}{600} = \frac{13}{50} = 0.26.$$

(b) There are 52 + 130 + 88 = 270 females, so $P(\text{Female}) = \dfrac{270}{600} = \dfrac{9}{20}$. If the events were independent we would need $P(\text{Female} \cap \text{Rugby}) = P(\text{Female})\,P(\text{Rugby})$:

$$P(\text{Female})\,P(\text{Rugby}) = \frac{9}{20} \times \frac{13}{50} = \frac{117}{1000} = 0.117.$$

But the actual joint probability is

$$P(\text{Female} \cap \text{Rugby}) = \frac{52}{600} = \frac{13}{150} \approx 0.0867.$$

Since $\dfrac{13}{150} \neq \dfrac{117}{1000}$, the events are **not independent**.

Q77 Riverview 2023 Trial Q27 · **4 marks**

Class of 24: 8 study neither, 8 study Chemistry, 12 study Physics (Venn diagram given). (a) Determine whether "studies Chemistry" and "studies Physics" are independent. (b) For the whole year group, $P(C) = \frac{1}{3}$, $P(P) = \frac{2}{5}$, $P(P \mid C) = \frac{3}{7}$; find $P(C \text{ or } P)$.

Criteria	Marks
(a) from the Venn: both $= 8 + 12 - (24 - 8) = 4$, so $P(C \cap P) = \frac{4}{24} = \frac{1}{6}$	1
(a) $P(C)P(P) = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6} = P(C \cap P)$, so the events *are* independent	1
(b) $P(C \cap P) = P(P \mid C)\, P(C) = \frac{3}{7} \times \frac{1}{3} = \frac{1}{7}$	1
(b) $P(C \cup P) = \frac{1}{3} + \frac{2}{5} - \frac{1}{7} = \frac{62}{105}$	1

Solution.

(a) Of the 24 students, 8 study neither, so $24 - 8 = 16$ study at least one subject. By the addition rule for counts, $|C \cap P| = |C| + |P| - |C \cup P| = 8 + 12 - 16 = 4$. Hence

$$P(C) = \frac{8}{24} = \frac{1}{3}, \quad P(P) = \frac{12}{24} = \frac{1}{2}, \quad P(C \cap P) = \frac{4}{24} = \frac{1}{6}.$$

The events are independent iff $P(C \cap P) = P(C)P(P)$:

$$P(C)\,P(P) = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6} = P(C \cap P),$$

so the events **are independent**.

(b) From the conditional probability, $P(C \cap P) = P(P \mid C)\, P(C) = \frac{3}{7} \times \frac{1}{3} = \frac{1}{7}$. By the addition rule,

$$P(C \cup P) = P(C) + P(P) - P(C \cap P) = \frac{1}{3} + \frac{2}{5} - \frac{1}{7} = \frac{35 + 42 - 15}{105} = \frac{62}{105} \approx 0.590.$$

Q78 Killara 2022 Prelim Q27 · **5 marks**

100 students; Venn diagram gives Mathematics only 45, both 25, Chemistry only 20, neither 10. (a) Find $|\overline{M} \cup C|$. (b) Explain why M and C are not mutually exclusive. (c) Find $P(C \mid M)$. (d) Determine whether M and C are independent.

Criteria	Marks
(a) $\overline{M} \cup C = 100 - (M \text{ only}) = 100 - 45 = 55$	1
(b) $M \cap C = 25 \neq 0$, so both events can occur together	1
(c) $P(C \mid M) = \frac{25}{70} = \frac{5}{14}$	1
(d) $P(C) = \frac{45}{100} = \frac{9}{20}$ (or $P(M)P(C) = \frac{63}{200}$)	1
(d) $P(C \mid M) = \frac{5}{14} \neq \frac{9}{20} = P(C)$, so *not* independent	1

Solution.

From the Venn diagram: Mathematics only = 45, both = 25, Chemistry only = 20, neither = 10 (total 100). So $|M| = 45 + 25 = 70$, $|C| = 25 + 20 = 45$, $|M \cap C| = 25$.

(a) $\overline{M} \cup C$ contains everyone except those in M but not C (i.e. "Mathematics only"). There are 45 students in "Mathematics only", so

$$|\overline{M} \cup C| = 100 - 45 = 55.$$

(b) Two events are mutually exclusive only if they cannot occur together, i.e. $M \cap C = \varnothing$. Here 25 students study both, so $M \cap C \neq \varnothing$ and the events are **not mutually exclusive**.

(c) Restricting to the 70 Mathematics students, 25 also study Chemistry:

$$P(C \mid M) = \frac{25}{70} = \frac{5}{14} \approx 0.357.$$

(d) The events are independent iff $P(C \mid M) = P(C)$. Here $P(C) = \dfrac{45}{100} = \dfrac{9}{20} = 0.45$, but $P(C \mid M) = \dfrac{5}{14} \approx 0.357$:

$$P(C \mid M) = \frac{5}{14} \neq \frac{9}{20} = P(C),$$

so M and C are **not independent**. (Equivalently $P(M \cap C) = \frac{25}{100} = 0.25$ but $P(M)P(C) = \frac{70}{100} \times \frac{45}{100} = 0.315$.)

Q79 Fort St 2025 Trial Q25 · **4 marks**

$P(A) = \frac{3k}{4}$, $P(B) = k$, $P(\overline{A} \cap B) = k^2$ with $0 < k < 1$. (a) Find $P(A \cap B)$ in terms of k. (b) Given A, B independent, find k.

Criteria	Marks
(a) $B = (A \cap B) \cup (\overline{A} \cap B)$, so $P(A \cap B) = P(B) - P(\overline{A} \cap B) = k - k^2$	1
(b) independence gives $P(A \cap B) = P(A)P(B)$, i.e. $k - k^2 = \frac{3k}{4} \cdot k$	1
(b) simplifies to $1 = \frac{7k}{4}$ (dividing by $k \neq 0$)	1
(b) $k = \frac{4}{7}$	1

Solution.

(a) The event B splits into the part inside A and the part outside A: $B = (A \cap B) \cup (\overline{A} \cap B)$, and these are mutually exclusive. Hence

$$P(A \cap B) = P(B) - P(\overline{A} \cap B) = k - k^2.$$

(b) If A and B are independent then $P(A \cap B) = P(A)\,P(B)$:

$$k - k^2 = \frac{3k}{4} \times k = \frac{3k^2}{4}.$$

Since $0 < k < 1$ we may divide by k:

$$1 - k = \frac{3k}{4} \implies 1 = k + \frac{3k}{4} = \frac{7k}{4} \implies k = \frac{4}{7}.$$