Year 11 Mathematics Advanced

Topic: Exponential and Logarithmic Functions (3 of 4)
Review Questions: Graphing exponential and logarithmic functions
Term 3, 2026

How to use this review set

These are real past-paper questions, grouped by question pattern. The grey label on each question shows where it is from and its marks.

- The questions are grouped by the pattern that keeps coming up in exams, a few of each in a row, running from the plainer patterns into the harder ones near the end. Work through them and you will meet every type an exam can throw at you.
- This set **is your diagnostic**. Working through it shows you exactly which patterns are solid and which need work. Attempt every question properly in your own book, and go back to your notes to help you work through tough questions if needed.
- Every question you get wrong, need help with or struggle with goes in your **Mistakes Log**, which you should be working on daily to get you exam-ready.
- A question marked ★ has one part that reaches into a **Year 12** topic (calculus); it is flagged, and the grey note under the question says which part.

Contents

Sketching and identifying graphs

1. Sketching exponential graphs
2. Sketching logarithmic graphs
3. Identifying a graph or finding its equation

Domain, range and intersections

4. Domain and range
5. Intersections of curves

Graphing applied models

6. Sketching an applied exponential model over its domain

Sketching and identifying graphs

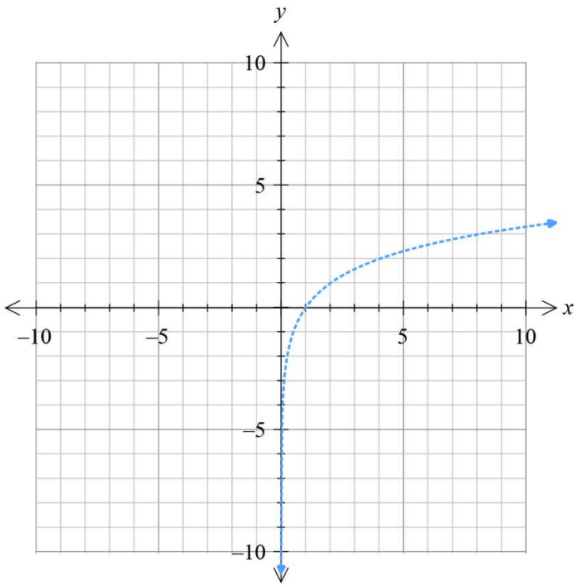
1. Sketching exponential graphs

Q1 North Sydney Boys 2024 Trial Q20(a) · **2 marks**

Sketch the curve $y = 4e^{-2x}$.

Q2 Hornsby Girls 2022 Prelim Q16(c) · **2 marks**

Sketch the graph of $y = 2^x$ on the number plane below, labelling any intercepts and asymptotes.



Q3 Sydney Grammar 2021 Prelim Q9(e)(i) · **2 marks**

Sketch the curve $y = 2^{-x}$, clearly showing all important features.

Q4 Shore 2023 Prelim Q15(a) · **2 marks**

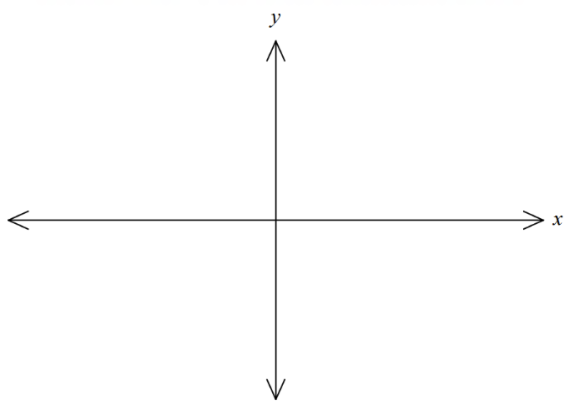
Consider the function $f(x) = 3^{-x} - 1$.

Sketch the curve $y = f(x)$, clearly showing any asymptote(s), intercept(s), and one other point.

Q5 Abbotsleigh 2024 Prelim Q16 · **2 marks**

Sketch $y = e^{-x} - 2$.

Show all intercepts and asymptotes.



Q6 Hurlstone 2024 Trial Q15 · **2 marks**

Sketch the graph of $y = e^{(1-x)} + 2$ by transforming the graph of $y = e^{x}$.

Show any intercepts, asymptotes and an additional point.

2. Sketching logarithmic graphs

Q7 Barker 2021 Trial Q14(a) · **1 mark**

Sketch the curve of $y = \ln(x + 1)$ indicating intercepts and asymptotes.

Q8 Sydney Grammar 2021 Prelim Q11(a)(ii) · **2 marks**

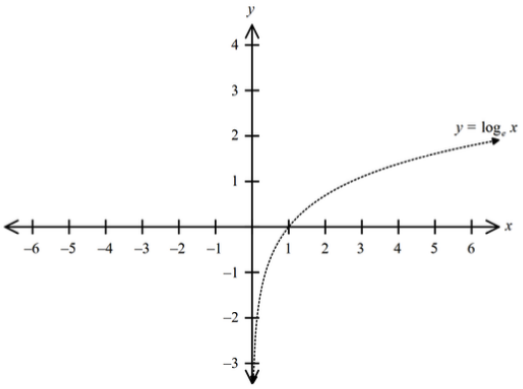
Sketch the graph of the following function on a number plane, showing all intercepts with the axes and all asymptotes. You should show at least two specific points on the graph.

$$y = \log_e x + 1$$

Q9 Presbyterian 2022 Prelim Q34(a) · **2 marks**

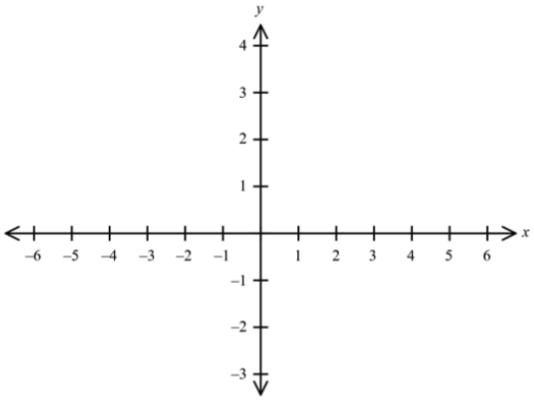
Consider the function $y = \log_e x$.

Use the sketch below of $y = \log_e x$ to sketch $y = \log_e(x + 2) - 1$ on the same axes. Show any asymptotes and the y intercept on your graph.



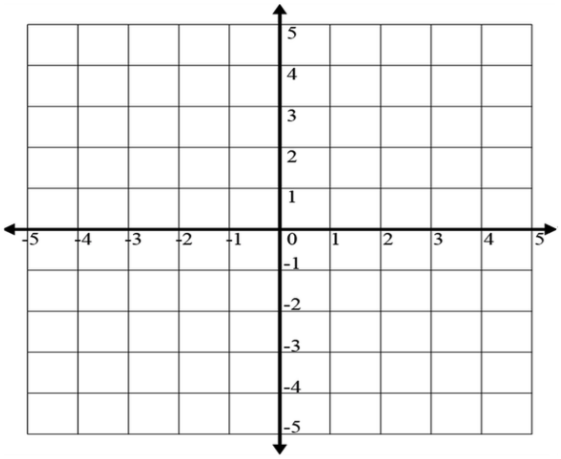
Use your sketch from part (a) to draw a sketch of $y = -\log_e(x+2)+1$.

Show **any asymptotes and the *y* intercept** on your graph.



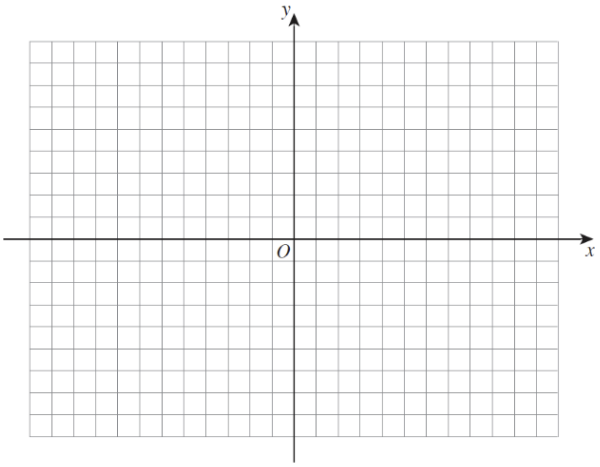
Q10 Caringbah 2023 Trial Q16 · **3 marks**

Sketch $y = \ln(x + 4) - 2$ showing all the key features and clearly labelling any intercepts in exact form.



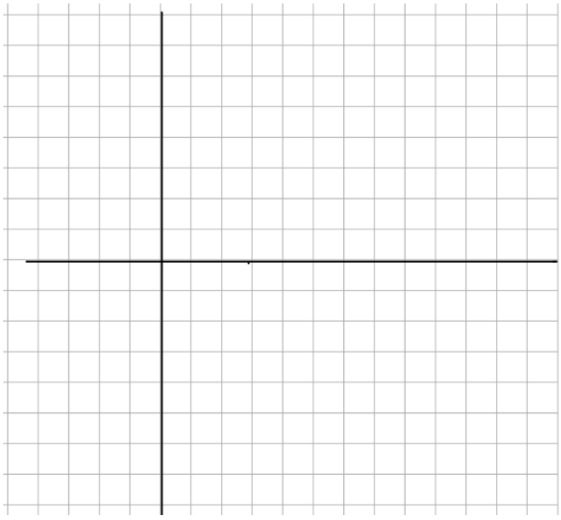
Q11 St George Girls 2022 Prelim Q31 · **3 marks**

Sketch the graph of $y = \ln x - 2$, labelling the x-intercept and vertical asymptote.



Q12 James Ruse 2023 Trial Q12 · **3 marks**

Sketch $y = 3\log_{10}(x-2)$ showing all important features.

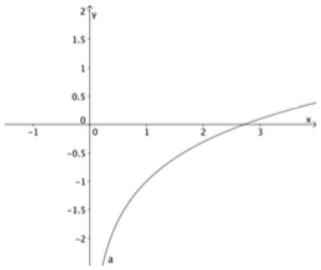


3. Identifying a graph or finding its equation

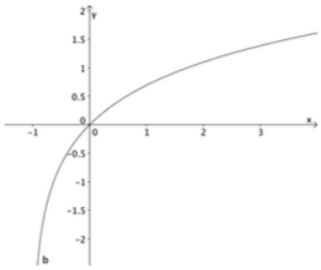
Q13 Hurlstone 2022 Trial Q9 · **1 mark**

The graph of the function $y = \ln(x + 1)$ is shown in which of the graphs below?

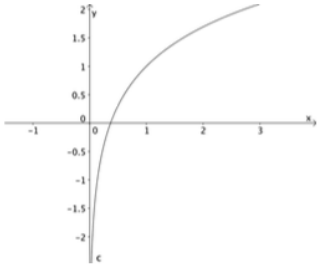
(A)



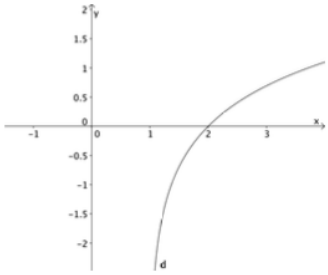
(B)



(C)



(D)



(A) (See graph A above)

(B) (See graph B above)

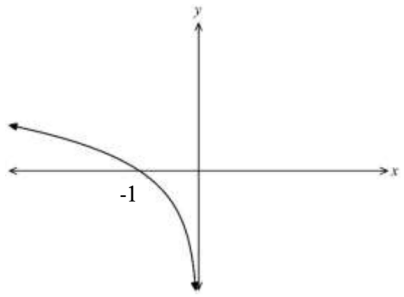
(C) (See graph C above)

(D) (See graph D above)

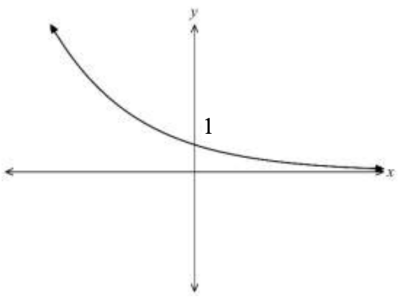
Q14 Moriah 2021 Prelim Q4 · **1 mark**

Which graph shows the curve $y = \log_2(x)$?

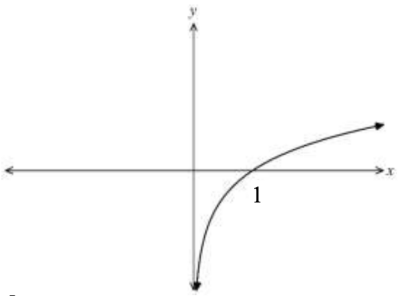
A.



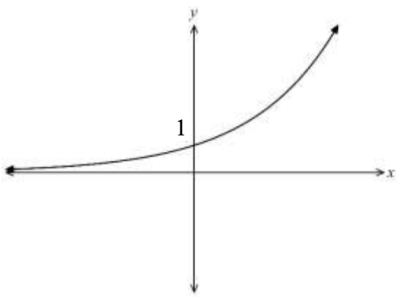
B.



C.



D.



(A) (See graph A above)

(B) (See graph B above)

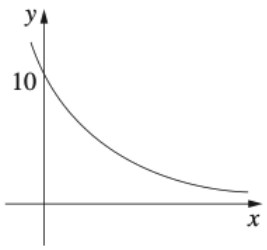
(C) (See graph C above)

(D) (See graph D above)

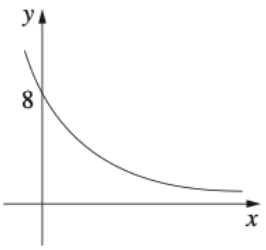
Q15 2021 HSC Q5 · **1 mark**

Which of the following best represents the graph of $y = 10(0.8)^x$?

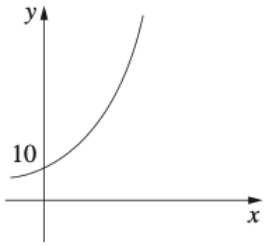
A.



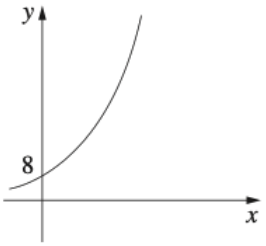
B.



C.



D.



(A) (See graph A above)

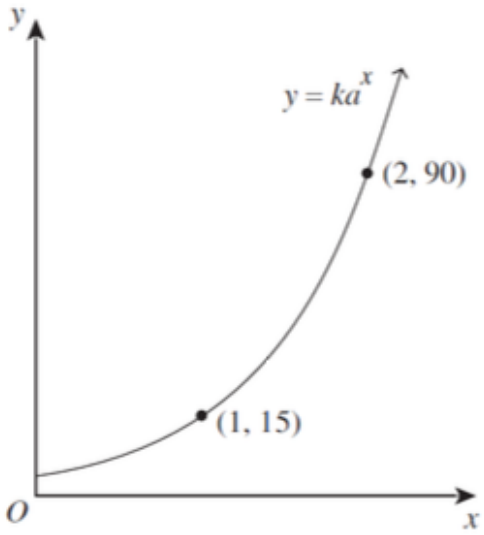
(B) (See graph B above)

(C) (See graph C above)

(D) (See graph D above)

Q16 SCEGGS 2022 Prelim Q7 · **1 mark**

The graph shows $y = ka^x$, passing through the points $(1, 15)$ and $(2, 90)$.



What are the values of a and k?

(A) $a = 15, k = 6$

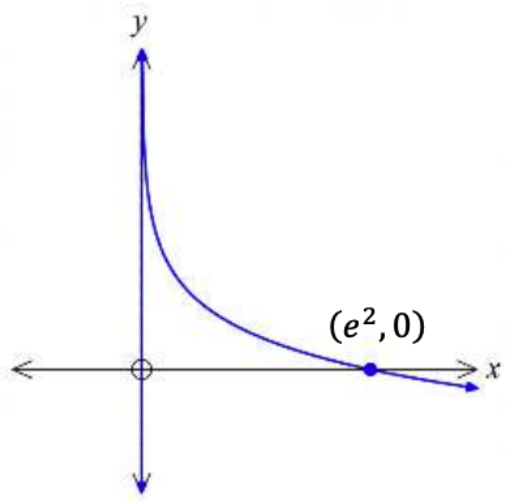
(B) $a = 6, k = 15$

(C) $a = 2.5, k = 6$

(D) $a = 6, k = 2.5$

Q17 Barker 2021 Prelim Q10 · **1 mark**

The graph of the function $y = a \ln x + b$ is shown below, passing through the point $(e^2, 0)$.



What are the values of a and b?

(A) $a = 1, b = e^2$

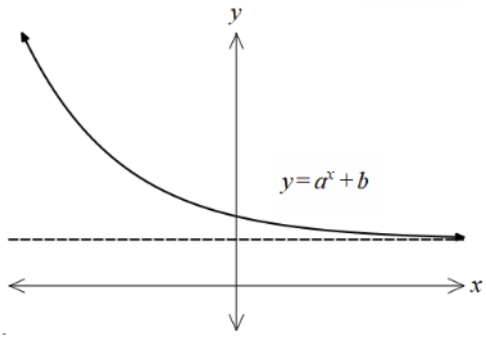
(B) $a = 1, b = 0$

(C) $a = -1, b = 2$

(D) $a = -1, b = e$

Q18 North Sydney Boys 2024 Prelim Q2 · **1 mark**

The figure shows the graph of $y = a^x + b$, where a and b are constants.



Which of the following must be true?

(A) $0 < a < 1$ and $b > 0$

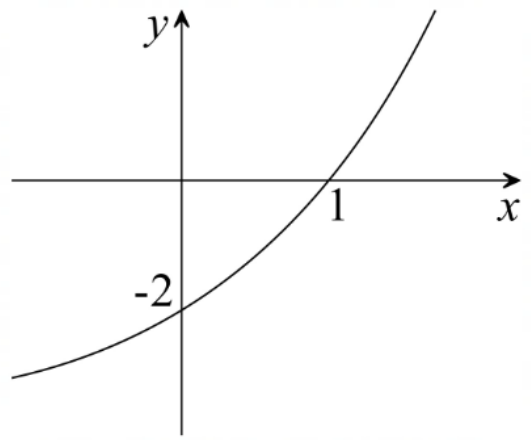
(B) $0 < a < 1$ and $b < 0$

(C) $a > 1$ and $b > 0$

(D) $a > 1$ and $b < 0$

Q19 Baulkham Hills 2022 Prelim Q6 · **1 mark**

The graph of $y = 2^{x+a} + b$ is shown.



What are the values of a and b?

(A) $a = 0$ and $b = -2$

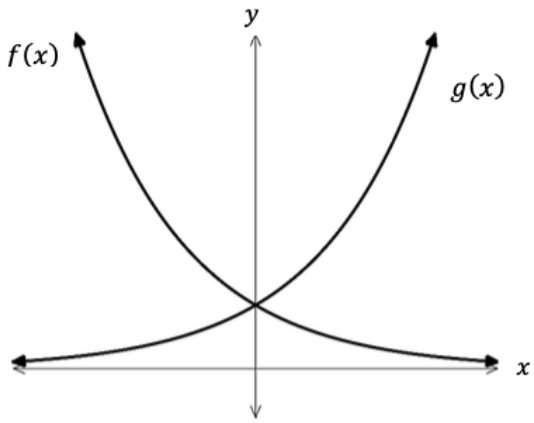
(B) $a = 0$ and $b = -3$

(C) $a = 1$ and $b = -2$

(D) $a = 1$ and $b = -4$

Q20 St George Girls 2023 Prelim Q5 · **1 mark**

The diagram shows the graphs of $f(x) = a^x$ and $g(x) = b^x$, and $f(x)$ is a reflection of $g(x)$ along the y-axis.



Which of the following are true?

I. $0 < a < 1$

II. $b > 1$

III. $ab = 1$

(A) I and II

(B) I and III

(C) II and III

(D) I, II and III

Domain, range and intersections

4. Domain and range

Q21 Sydney Tech 2022 Trial Q2 · **1 mark**

What is the domain of $y = \ln(x - 1)$?

(A) $x > 0$

(B) $x \geq 1$

(C) $x \geq -1$

(D) $x > 1$

Q22 Barker 2023 Trial Q2 · **1 mark**

What is the range of the function $y = 3^{-x} + 2$?

(A) $(3, \infty)$

(B) $(-\infty, 2)$

(C) $(2, \infty)$

(D) $(-\infty, 1)$

Q23 Riverview 2022 Trial Q1 · **1 mark**

The domain of $y = \log_e(x + 2)$ is:

(A) $x > 0$

(B) $x > 1$

(C) $x > -3$

(D) $x > -2$

Q24 Girraween 2023 Prelim Q5 · **1 mark**

The domain of the graph of $y = \log_2(2x - 3)$ is:

(A) All real x, $x > \frac{3}{2}$

(B) All real x, $x \neq \frac{3}{2}$

(C) $x > 0$

(D) All real x.

Q25 Blacktown Boys 2022 Prelim Q6 · **1 mark**

The range of $y = e^{-x} + 1$ is

(A) $(-\infty, 1)$

(B) $(-\infty, \infty)$

(C) $(-1, \infty)$

(D) $(1, \infty)$

Q26 Fort St 2025 Trial Q6 · **1 mark**

What is the domain of $f(x) = \log_{10}\big[(x - 3)(x - 5)\big]$?

(A) $\{3 \leq x \leq 5\}$

(B) $\{3 < x < 5\}$

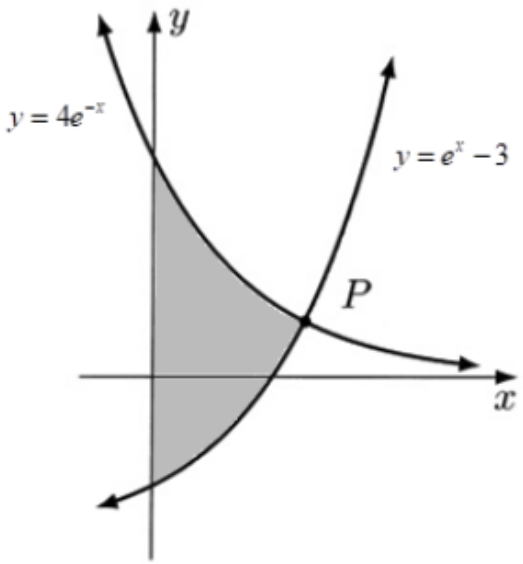
(C) $\{x \leq 3 \text{ or } x \geq 5\}$

(D) $\{x < 3 \text{ or } x > 5\}$

5. Intersections of curves

Q27 Fort St 2020 Trial Q14 · **3 marks**

The diagram shows the graphs of $y = 4e^{-x}$ and $y = e^x - 3$.



(a) Show that the curves intersect when $e^{2x} - 3e^x - 4 = 0$.

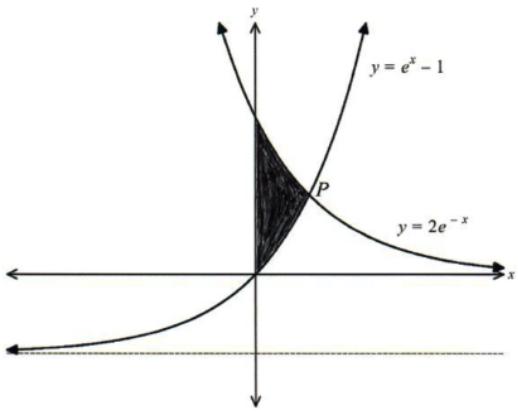
(b) Hence, show the x coordinate of the point P is $x = \ln 4$.

Q28 Barker 2022 Trial Q25 · **2 marks**

Find the point(s) of intersection of the graphs of $y = 9^x + 4$ and $y = 13 - 8 \times 3^x$.

Q29 Cherrybrook Tech 2021 Trial Q28(a) · **3 marks**

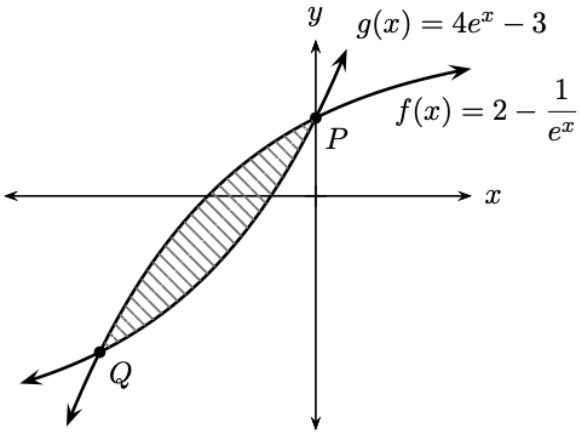
Two curves $y = 2e^{-x}$ and $y = e^x - 1$ intersect at point P.



(a) Show that the coordinates of P are $(\ln 2,\ 1)$.

Q30 Normanhurst Boys 2020 Trial Q29 · **3 marks**

The functions $f(x) = 2 - \dfrac{1}{e^x}$ and $g(x) = 4e^x - 3$ intersect at points P and Q as shown.

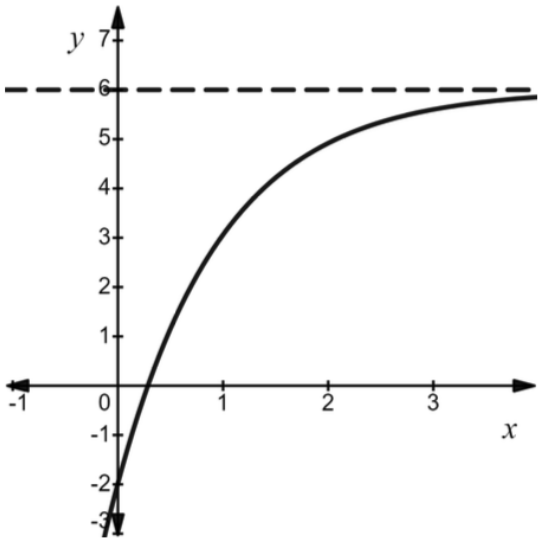


(a) Show that the curves intersect when $4e^{2x} - 5e^x + 1 = 0$.

(b) Show that the points of intersection occur at $x = 0$ and $x = \ln \dfrac{1}{4}$.

Q31 Sydney Girls 2022 Trial Q29(a) · **4 marks**

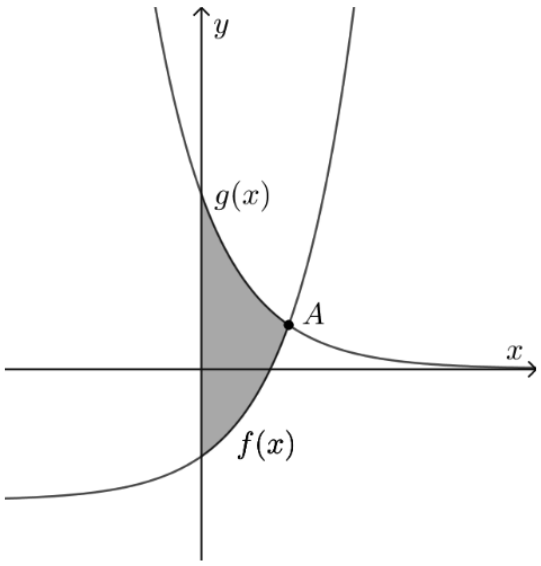
The diagram shows the graph of $y = 6 - 8e^{-x}$.



On the diagram, sketch the graph of $y = e^x$, which passes through the point $(1, e)$. Show that the x coordinates of the two points of intersection, (x_1 and x_2, where $x_1 < x_2$), are the solutions of the equation $e^{2x} - 6e^x + 8 = 0$ and solve this equation to find the exact values of x_1 and x_2.

Q32 Barker 2021 Trial Q24(a) · **3 marks**

The graph below shows the functions $f(x) = 2^x - 3$ and $g(x) = 4 \times 2^{-x}$.



(a) By creating a quadratic equation show that the coordinates of A are $(2, 1)$.

Graphing applied models

6. Sketching an applied exponential model over its domain

Q33 ★ Blacktown Boys 2022 Prelim Q20 · **6 marks**

★ Part (b) uses Year-12 calculus (rate of change). An experimental vaccine was injected into a rat. The amount, M millilitres, of vaccine present in the bloodstream of the rat, t hours later, was given by

$$M = e^{-\frac{t}{4}} + 2.$$

(a) How much vaccine was initially injected into the rat?

(b) At what rate was the amount of vaccine decreasing at the end of 3 hours?

(c) Show that there will always be more than 2 millilitres of vaccine present in the rat's bloodstream.

(d) Sketch the curve of $M = e^{-\frac{t}{4}} + 2$ to show how the amount of vaccine present in the rat's bloodstream changes over time.

Q34 Gosford 2023 Prelim Q25 · **6 marks**

A metal bar is heated and then allowed to cool. Its temperature T in °C is found to be

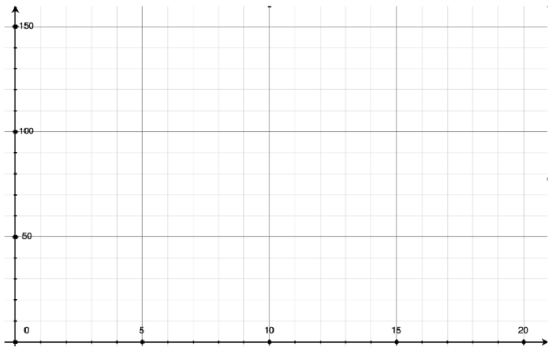
$$T = 15 + 125e^{-0.3t},$$

where t (in minutes) is the time of cooling.

(a) Find the initial temperature of the metal bar.

(b) How cool is the metal bar after 10 minutes?

(c) Graph the temperature vs time for $0 \leq t \leq 20$.



(d) Using the graph or otherwise, find the time when the temperature reaches 30°C.

Q35 Hurlstone 2024 Prelim Q30 · **5 marks**

The amount of electrical energy $E(t)$, stored in a battery after charging for t minutes, is given by the equation

$$E(t) = 100 - Ae^{-0.1t}.$$

(a) When $t = 0$, there is 10 units of electrical energy stored in the battery. Show that $A = 90$.

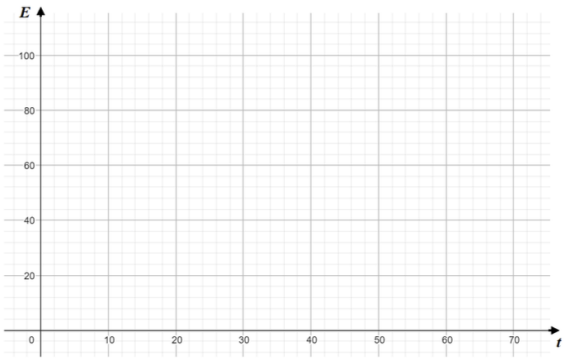
(b) Fill in the missing value of $E(t)$ in the table below to 1 decimal place.

t	0	10	20	30	40
$E(t)$	10		87.8	95.5	98.4

(c) Hence, draw the graph of $E(t)$ below for $0 \leq t \leq 4$.

t	0	10	20	30	40
$E(t)$	10		87.8	95.5	98.4

Hence, draw the graph of $E(t)$ below for $0 \leq t \leq 4$.



(d) What value will the electrical energy approach as t increases?

Q36 Barker 2021 Trial Q26 · **5 marks**

A rare species of bird lives on a remote island. In 2020 a model predicts that the bird population, P, will be

$$P = 150 + 300e^{-0.05t},$$

where t is the number of years after observations began at the start of 2020.

(a) According to the model, how many birds were there when observations began in 2020?

(b) Sketch a graph of P against t for $t \geq 0$, clearly indicating the limiting value of the bird population.

(c) The species will become eligible for inclusion in the endangered species list when the population falls below 200. In what year does the model predict this will occur?

Q37 ★ 2024 HSC Q17 · **6 marks**

★ Part (c) uses Year-12 calculus (rate of change). In a particular electrical circuit, the voltage V (volts) across a capacitor is given by

$$V(t) = 6.5\left(1 - e^{-kt}\right),$$

where k is a positive constant and t is the number of seconds after the circuit is switched on.

(a) Draw a sketch of the graph of $V(t)$, showing its behaviour as t increases.

(b) When $t = 1$, the voltage across the capacitor is 2.6 volts. Find the value of k, correct to 3 decimal places.

(c) Find the rate at which the voltage is increasing when $t = 2$, correct to 3 decimal places.