Year 11 Mathematics Advanced

Topic: Exponential and Logarithmic Functions (4 of 4)
Review Questions (Worked Solutions): Mixed one-off exam questions
Term 3, 2026

Mixed one-off exam questions

Q1 Moriah 2021 Trial Q20(a) · **1 mark**

For $f(x) = 2^{-x} + 1$, complete the table of values at $x = 0, 1, 2$.

Criteria	Marks
All three values $f(0) = 2$, $f(1) = \frac{3}{2}$, $f(2) = \frac{5}{4}$ correct	1

Solution.

Substitute each x into $f(x) = 2^{-x} + 1$:

- $f(0) = 2^0 + 1 = 1 + 1 = 2$
- $f(1) = 2^{-1} + 1 = \frac{1}{2} + 1 = \dfrac{3}{2}$
- $f(2) = 2^{-2} + 1 = \frac{1}{4} + 1 = \dfrac{5}{4}$

x	0	1	2
$f(x)$	2	$\frac{3}{2}$	$\frac{5}{4}$

Q2 Blacktown Boys 2020 Trial Q24(a) · **1 mark**

For $y = \log_e(2^x - x)$, find the missing table value (at $x = 2.5$) to 3 decimal places.

Criteria	Marks
Correct value $y \approx 1.150$ (3 d.p.)	1

Solution.

The missing entry is at $x = 2.5$. Substitute into $y = \log_e(2^x - x)$:

$$y = \log_e\left(2^{2.5} - 2.5\right) = \log_e\left(4\sqrt{2} - \tfrac{5}{2}\right) = \log_e(3.1568\ldots) = 1.1495\ldots$$

So $y \approx 1.150$ (to 3 decimal places).

Q3 Knox 2021 Trial Q4 · **1 mark**

$\log_3 100$ lies between which pair of consecutive integers? (A) 0 and 1; (B) 1 and 2; (C) 4 and 5; (D) 33 and 34.

Criteria	Marks
Correct option (C)	1

Solution.

Let $\log_3 100 = x$, so $3^x = 100$. Since $3^4 = 81$ and $3^5 = 243$, we have $81 < 100 < 243$, so $3^4 < 3^x < 3^5$, giving $4 < x < 5$.

Alternatively, by the change-of-base law $\log_3 100 = \dfrac{\ln 100}{\ln 3} \approx 4.19$.

Either way, $\log_3 100$ lies between 4 and 5. The answer is **(C)**.

Q4 Gosford 2024 Prelim Q13 · **4 marks**

(a) Explain, using exponentials, between which 2 consecutive integers $\log_3 30$ lies. (b) By first expressing it in terms of $\log_{10}$, find $\log_3 30$ to 2 decimal places.

Criteria	Marks
(a) Uses $3^3 = 27$ and $3^4 = 81$ (or equivalent) to bracket 30	1
(a) Concludes $\log_3 30$ lies between 3 and 4	1
(b) Applies change of base: $\log_3 30 = \dfrac{\log_{10} 30}{\log_{10} 3}$	1
(b) Correct value 3.10 (2 d.p.)	1

Solution.

(a) Let $\log_3 30 = x$, so $3^x = 30$. Now $3^3 = 27$ and $3^4 = 81$, so

$$3^3 < 30 < 3^4 \quad \Rightarrow \quad 3^3 < 3^x < 3^4 \quad \Rightarrow \quad 3 < x < 4.$$

Hence $\log_3 30$ lies between the consecutive integers 3 and 4.

(b) By the change-of-base law,

$$\log_3 30 = \frac{\log_{10} 30}{\log_{10} 3} = \frac{1.4771\ldots}{0.4771\ldots} = 3.0959\ldots \approx 3.10 \text{ (2 d.p.)}.$$

This is consistent with part (a): 3.10 lies between 3 and 4.

Q5 Caringbah 2021 Trial Q38 · **2 marks**

Simultaneously solve $\log_{10}(2-x) - \log_{10} y = 1 - \log_{10} 2$ and $\log_{10} 2x = \log_{10} y$.

Criteria	**Marks**
Uses log laws to reduce both equations (e.g. $\dfrac{2-x}{y} = 5$ and $y = 2x$)	1
Correct solution $x = \dfrac{2}{11}$, $y = \dfrac{4}{11}$	1

Solution.

Second equation: $\log_{10} 2x = \log_{10} y \Rightarrow y = 2x$.

First equation: $\log_{10}(2-x) - \log_{10} y = 1 - \log_{10} 2$. Rewrite the right side using $1 = \log_{10} 10$:

$$\log_{10} \frac{2-x}{y} = \log_{10} 10 - \log_{10} 2 = \log_{10} \frac{10}{2} = \log_{10} 5 \quad \Rightarrow \quad \frac{2-x}{y} = 5.$$

Substitute $y = 2x$:

$$\frac{2-x}{2x} = 5 \;\Rightarrow\; 2 - x = 10x \;\Rightarrow\; 2 = 11x \;\Rightarrow\; x = \frac{2}{11}, \quad y = 2x = \frac{4}{11}.$$

Check domain: $2 - x = \frac{20}{11} > 0$, $x > 0$ and $y > 0$, so all logarithms are defined. The solution is $x = \dfrac{2}{11}$, $y = \dfrac{4}{11}$.

Q6 International Grammar 2021 Prelim Q22 · **3 marks**

For $f(x) = a + \log_b x$ with x-intercept 9 and passing through $(243, 3)$, find a and b.

Criteria	Marks
Forms the two equations $a + \log_b 9 = 0$ and $a + \log_b 243 = 3$	1
Subtracts to obtain $\log_b 27 = 3$, hence $b = 3$	1
Solves back for $a = -2$	1

Solution.

The x-intercept is 9, so $f(9) = 0$; and $f(243) = 3$. This gives two equations:

$$a + \log_b 9 = 0 \quad \ldots(1) \qquad a + \log_b 243 = 3 \quad \ldots(2)$$

Subtract (1) from (2):

$$\log_b 243 - \log_b 9 = 3 \;\Rightarrow\; \log_b \frac{243}{9} = 3 \;\Rightarrow\; \log_b 27 = 3 \;\Rightarrow\; b^3 = 27 \;\Rightarrow\; b = 3.$$

Substitute $b = 3$ into (1):

$$a + \log_3 9 = 0 \;\Rightarrow\; a + 2 = 0 \;\Rightarrow\; a = -2.$$

So $a = -2$ and $b = 3$ (i.e. $f(x) = -2 + \log_3 x$). Check: $f(243) = -2 + \log_3 243 = -2 + 5 = 3$. ✓

Q7 Riverview 2023 Trial Q18 · **3 marks**

Solve simultaneously $\log_2 x + \log_2 y = 3$ and $\log_x y = 2$.

Criteria	Marks
From (1): $\log_2(xy) = 3 \Rightarrow xy = 8$	1
From (2): $\log_x y = 2 \Rightarrow y = x^2$	1
Solves the pair to get $x = 2$, $y = 4$	1

Solution.

Equation (1): $\log_2 x + \log_2 y = \log_2(xy) = 3 \Rightarrow xy = 2^3 = 8$.

Equation (2): $\log_x y = 2 \Rightarrow y = x^2$.

Substitute $y = x^2$ into $xy = 8$:

$$x \cdot x^2 = 8 \;\Rightarrow\; x^3 = 8 \;\Rightarrow\; x = 2, \qquad y = x^2 = 4.$$

Check: $x = 2 > 0$ and $x \neq 1$, so it is a valid logarithm base; $\log_2(2 \cdot 4) = \log_2 8 = 3$ ✓ and $\log_2 4 = 2$ ✓. The solution is $x = 2$, $y = 4$.

Q8 Cherrybrook Tech 2021 Trial Q26(b) · **3 marks**

The probability that a planted seed yields a red rose is 0.02. How many seeds must be planted to be at least 98% certain of obtaining at least one red rose?

Criteria	**Marks**
Sets up $P(\text{at least one}) = 1 - 0.98^n \geq 0.98$	1
Takes logs correctly: $n \geq \dfrac{\log 0.02}{\log 0.98}$ (with inequality reversal)	1
Concludes $n = 194$	1

Solution.

For n seeds, $P(\text{no red rose}) = (1 - 0.02)^n = 0.98^n$, so

$$P(\text{at least one red rose}) = 1 - 0.98^n.$$

We require this to be at least 98% = 0.98:

$$1 - 0.98^n \geq 0.98 \implies 0.98^n \leq 0.02.$$

Take logarithms of both sides. Since $\log 0.98 < 0$, the inequality *reverses*:

$$n \log 0.98 \leq \log 0.02 \implies n \geq \frac{\log 0.02}{\log 0.98} = 193.6\ldots$$

As n must be a whole number, $n = 194$. (Check: $1 - 0.98^{193} = 0.9797 < 0.98$, but $1 - 0.98^{194} = 0.9801 \geq 0.98$.)

Q9 Presbyterian 2023 Prelim Q37 · **3 marks**

Solve $e^{2\ln x} = 8 - 2x$.

Criteria	Marks
Simplifies $e^{2\ln x} = x^2$ (noting the domain $x > 0$)	1
Forms and solves $x^2 + 2x - 8 = 0 \Rightarrow x = 2$ or $x = -4$	1
Rejects $x = -4$ (not in domain $x > 0$), so $x = 2$	1

Solution.

The expression $e^{2\ln x}$ requires $x > 0$. Simplify the left-hand side:

$$e^{2\ln x} = e^{\ln x^2} = x^2.$$

So the equation becomes

$$x^2 = 8 - 2x \;\Rightarrow\; x^2 + 2x - 8 = 0 \;\Rightarrow\; (x+4)(x-2) = 0 \;\Rightarrow\; x = -4 \text{ or } x = 2.$$

Since $\ln x$ requires $x > 0$, we **reject** $x = -4$. Therefore $x = 2$.

Q10 Caringbah 2025 Trial Q28(b) · **2 marks**

The graph of $y = \dfrac{-\sin(\ln x)}{kx}$ for $0 < x \leq 1$ is shown. Show that one of the x-intercepts is $x = e^{-\pi}$.

Criteria	Marks
Sets $y = 0$, giving $\sin(\ln x) = 0$ (numerator zero, $kx \neq 0$)	1
Solves $\ln x = -\pi$ within $0 < x \leq 1$ to obtain $x = e^{-\pi}$	1

Solution.

An x-intercept occurs where $y = 0$. Since the denominator $kx \neq 0$ for $0 < x \leq 1$, this needs the numerator to be zero:

$$-\sin(\ln x) = 0 \;\Rightarrow\; \sin(\ln x) = 0 \;\Rightarrow\; \ln x = n\pi, \quad n \in \mathbb{Z}.$$

For $0 < x \leq 1$ we have $\ln x \leq 0$, so we need $n \leq 0$. Taking $n = -1$:

$$\ln x = -\pi \;\Rightarrow\; x = e^{-\pi}.$$

Since $e^{-\pi} \approx 0.043$ lies in $(0, 1]$, it is a valid x-intercept. (The intercept shown at $x = 1$ corresponds to $n = 0$; $e^{-\pi}$ is the next one, near the y-axis.) ■

Q11 Gosford 2024 Trial Q26 · **3 marks**

Show that $\log_e(1+\sin x) - \log_e(1-\sin x) = 2\log_e(\sec x + \tan x)$.

Criteria	**Marks**
Combines the LHS into $\log_e \dfrac{1+\sin x}{1-\sin x}$	1
Multiplies by $\dfrac{1+\sin x}{1+\sin x}$ and uses $1-\sin^2 x = \cos^2 x$	1
Reaches $2\log_e \dfrac{1+\sin x}{\cos x} = 2\log_e(\sec x + \tan x)$	1

Solution.

Start with the left-hand side and combine using the log-quotient law:

$$\text{LHS} = \log_e(1+\sin x) - \log_e(1-\sin x) = \log_e \frac{1+\sin x}{1-\sin x}.$$

Multiply the numerator and denominator inside the logarithm by $(1+\sin x)$:

$$= \log_e \frac{(1+\sin x)^2}{(1-\sin x)(1+\sin x)} = \log_e \frac{(1+\sin x)^2}{1-\sin^2 x}.$$

Since $1-\sin^2 x = \cos^2 x$,

$$= \log_e \frac{(1+\sin x)^2}{\cos^2 x} = \log_e \left(\frac{1+\sin x}{\cos x}\right)^2 = 2\log_e \frac{1+\sin x}{\cos x}.$$

Splitting the fraction:

$$\frac{1+\sin x}{\cos x} = \frac{1}{\cos x} + \frac{\sin x}{\cos x} = \sec x + \tan x.$$

Therefore

$$\text{LHS} = 2\log_e(\sec x + \tan x) = \text{RHS}.\blacksquare$$