Year 11 Mathematics Advanced

Topic: Exponential and Logarithmic Functions (2 of 4)

Review Questions (Worked Solutions): Logarithmic functions and laws

Term 3, 2026

Logarithmic laws and expressions

1. Evaluating logarithms

Q1 Pymble 2022 Prelim Q13 · **1 mark**

Evaluate $\log_3 27$.

Criteria	Marks
Correct value 3	1

Solution.

Ask: to what power must 3 be raised to give 27? Since $27 = 3^3$,

$$\log_3 27 = \log_3 3^3 = 3.$$

Q2 Girraween 2022 Trial Q32 · **1 mark**

Evaluate $\ln 3$ correct to three significant figures.

Criteria	Marks
Correct value 1.10	1

Solution.

$$\ln 3 = 1.0986\ldots \approx 1.10 \quad \text{(3 sig. fig.)}$$

Q3 Cherrybrook Tech 2021 Prelim Q3 · **1 mark**

What is the value of $\log_7 \dfrac{1}{49}$? (multiple choice)

Criteria	Marks
Correct option (B)	1

Solution.

Write the argument as a power of 7: $\dfrac{1}{49} = \dfrac{1}{7^2} = 7^{-2}$. Hence

$$\log_7 \frac{1}{49} = \log_7 7^{-2} = -2.$$

The answer is **(B)**.

Q4 Baulkham Hills 2021 Prelim Q3 · **1 mark**

What is the value of $\log_4\left(\frac{1}{16}\right)$? (multiple choice)

Criteria	Marks
Correct option (C)	1

Solution.

Write the argument as a power of 4: $\frac{1}{16} = \frac{1}{4^2} = 4^{-2}$. Hence

$$\log_4\left(\frac{1}{16}\right) = \log_4 4^{-2} = -2.$$

The answer is **(C)**.

Q5 Moriah 2021 Prelim Q10(a) · **1 mark**

Evaluate $\log_4(2)$.

Criteria	Marks
Correct value $\frac{1}{2}$	1

Solution.

Ask: to what power must 4 be raised to give 2? Since $2 = \sqrt{4} = 4^{1/2}$,

$$\log_4 2 = \log_4 4^{1/2} = \frac{1}{2}.$$

Q6 Sydney Grammar 2021 Prelim Q9(a) · **1 mark**

Evaluate $\log_2 16$.

Criteria	Marks
Correct value 4	1

Solution.

Ask: to what power must 2 be raised to give 16? Since $16 = 2^4$,

$$\log_2 16 = \log_2 2^4 = 4.$$

2. Simplifying expressions using the logarithmic laws

Q7 North Sydney Boys 2022 Trial Q1 · **1 mark**

Simplify $2\log_{10}(x) - \log_{10}(3x)$ (multiple choice).

Criteria	Marks
Correct option (A)	1

Solution.

Use the power law then the quotient law:

$$2\log_{10} x - \log_{10} 3x = \log_{10} x^2 - \log_{10} 3x = \log_{10}\left(\frac{x^2}{3x}\right) = \log_{10}\left(\frac{x}{3}\right).$$

The answer is **(A)**.

Q8 Hurlstone 2023 Trial Q13 · **2 marks**

Simplify $\ln 60 + \ln e^2 - \ln 15$.

Criteria	Marks
Combines the logs and uses $\ln e^2 = 2$	1
Correct simplified answer $\ln 4 + 2$	1

Solution.

Group the ln terms and evaluate $\ln e^2$:

$$\ln 60 + \ln e^2 - \ln 15 = \big(\ln 60 - \ln 15\big) + 2\ln e = \ln\left(\frac{60}{15}\right) + 2(1).$$

Since $\dfrac{60}{15} = 4$,

$$= \ln 4 + 2.$$

Q9 Hurlstone 2021 Prelim Q28 · **1 mark**

Which single logarithm equals $2\log_a x - 5\log_a y + 3\log_a z$? (multiple choice).

Criteria	Marks
Correct option (A)	1

Solution.

Apply the power law to each term, then combine with the product and quotient laws:

$$2\log_a x - 5\log_a y + 3\log_a z = \log_a x^2 - \log_a y^5 + \log_a z^3 = \log_a\left(\frac{x^2 z^3}{y^5}\right).$$

The answer is **(A)**.

Q10 Normanhurst Boys 2021 Trial Q4 · **1 mark**

Which expression equals $e^{2\log_e x}$? (multiple choice).

Criteria	Marks
Correct option (B)	1

Solution.

By the power law, $2\log_e x = \log_e x^2$, so

$$e^{2\log_e x} = e^{\log_e x^2} = x^2,$$

using the inverse relationship $e^{\log_e A} = A$. The answer is **(B)**.

Q11 Fort St 2022 Prelim Q11(a) · **2 marks**

Simplify, using the log laws, $2\log_2 25 - \log_2 16 - \log_2 20$.

Criteria	Marks
Applies power/quotient laws to reach a single logarithm	1
Correct simplified value $\log_2\left(\frac{125}{64}\right)$	1

Solution.

Use the power law, then combine into a single logarithm:

$$2\log_2 25 - \log_2 16 - \log_2 20 = \log_2 25^2 - \log_2 16 - \log_2 20 = \log_2\left(\frac{625}{16\times 20}\right).$$

Since $16\times 20 = 320$ and $\dfrac{625}{320} = \dfrac{125}{64}$,

$$= \log_2\left(\frac{125}{64}\right).$$

(Equivalently, $\log_2 125 - \log_2 64 = 3\log_2 5 - 6$.)

Q12 Ascham 2023 Trial Q12(a) · **2 marks**

Evaluate $\log_2 4x + \log_2 8x - 2\log_2 x$.

Criteria	Marks
Combines into $\log_2\left(\frac{(4x)(8x)}{x^2}\right)$	1
Correct value 5	1

Solution.

Use the power law on the last term, then the product and quotient laws:

$$\log_2 4x + \log_2 8x - 2\log_2 x = \log_2 4x + \log_2 8x - \log_2 x^2 = \log_2\left(\frac{(4x)(8x)}{x^2}\right).$$

The x^2 cancels, leaving

$$= \log_2 32 = 5,$$

since $2^5 = 32$.

Q13 Barker 2022 Trial Q7 · **1 mark**

Which expression equals $\dfrac{\ln 8^x + 3\ln 3^x}{2\ln 3 + \ln 8 - \ln 2}$? (multiple choice).

Criteria	Marks
Correct option (B)	1

Solution.

Apply the power law to the numerator:

$$\ln 8^x + 3\ln 3^x = x\ln 8 + 3x\ln 3 = x\big(\ln 8 + 3\ln 3\big) = x\ln(8\times 27) = x\ln 216.$$

Simplify the denominator with the product and quotient laws:

$$2\ln 3 + \ln 8 - \ln 2 = \ln\left(\frac{9\times 8}{2}\right) = \ln 36.$$

Since $216 = 6^3$ and $36 = 6^2$, we have $\ln 216 = 3\ln 6$ and $\ln 36 = 2\ln 6$, so

$$\frac{x\ln 216}{\ln 36} = \frac{3x\ln 6}{2\ln 6} = \frac{3x}{2}.$$

The answer is **(B)**.

3. Expressing a logarithm in terms of given logarithms

Q14 Cherrybrook Tech 2022 Prelim Q13(a) · **1 mark**

Given $\log_a b = 2.75$ and $\log_a c = 0.25$, find $\log_a\left(\dfrac{b}{c}\right)$.

Criteria	Marks
Applies the quotient law and evaluates $\log_a b - \log_a c$	1

Solution.

By the quotient law,

$$\log_a\left(\frac{b}{c}\right) = \log_a b - \log_a c = 2.75 - 0.25 = 2.5.$$

Q15 Barker 2024 Trial Q11 · **2 marks**

If $\log p = a$ and $\log q = b$, express $\log(pq^2)$ in terms of a and b.

Criteria	Marks
Applies the product law: $\log(pq^2) = \log p + \log q^2$	1
Applies the power law and states the answer $a + 2b$	1

Solution.

Using the product and power laws,

$$\log(pq^2) = \log p + \log q^2 = \log p + 2\log q = a + 2b.$$

Q16 Girraween 2022 Prelim Q11(c)(i) · **1 mark**

Given $\log_a 4 = 1.5$ and $\log_a 5 = 1.7$, find $\log_a 1.25$.

Criteria	Marks
Writes $1.25 = \frac{5}{4}$ and evaluates $\log_a 5 - \log_a 4$	1

Solution.

Since $1.25 = \dfrac{5}{4}$, the quotient law gives

$$\log_a 1.25 = \log_a\left(\frac{5}{4}\right) = \log_a 5 - \log_a 4 = 1.7 - 1.5 = 0.2.$$

Q17 Barker 2024 Prelim Q3 · **1 mark**

Given $\log_a 4 = 0.420$ and $\log_a 5 = 0.488$, find the value of $\log_a 100$.
Options: (A) 0.410 (B) 0.664 (C) 1.396 (D) 2.908.

Criteria	Marks
Correct option (C)	1

Solution.

Write $100 = 4 \times 5^2$. Then

$$\log_a 100 = \log_a 4 + \log_a 5^2 = \log_a 4 + 2\log_a 5 = 0.420 + 2(0.488) = 1.396.$$

The correct option is **(C)**.

Q18 Gosford 2024 Prelim Q19 · **2 marks**

Given $x = \log_a 3$ and $y = \log_a 7$, write an expression for $\log_a \dfrac{27}{7}$ in terms of x and y.

Criteria	Marks
Applies the quotient law and writes $\log_a 27 = 3\log_a 3$	1
States the answer $3x - y$	1

Solution.

Using the quotient and power laws, and $27 = 3^3$,

$$\log_a \frac{27}{7} = \log_a 27 - \log_a 7 = \log_a 3^3 - \log_a 7 = 3\log_a 3 - \log_a 7 = 3x - y.$$

Q19 Hornsby Girls 2022 Prelim Q13(a) · **3 marks**

If $a = \log_{10} x$, $b = \log_{10} y$ and $c = \log_{10} z$, express $\log_{10}\left(\dfrac{x^2\sqrt{y}}{z}\right)$ in terms of a, b and c.

Criteria	Marks
Splits into $\log_{10} x^2 + \log_{10}\sqrt{y} - \log_{10} z$ via product/quotient laws	1
Applies the power law to x^2 and $\sqrt{y} = y^{1/2}$	1
States the answer $2a + \frac{1}{2}b - c$	1

Solution.

Using the product, quotient and power laws,

$$\log_{10}\left(\frac{x^2\sqrt{y}}{z}\right) = \log_{10} x^2 + \log_{10} y^{1/2} - \log_{10} z = 2\log_{10} x + \tfrac{1}{2}\log_{10} y - \log_{10} z.$$

Hence $\log_{10}\left(\dfrac{x^2\sqrt{y}}{z}\right) = 2a + \dfrac{1}{2}b - c.$

Q20 Hurlstone 2024 Prelim Q27 · **2 marks**

If $\log_3 2 = a$ and $\log_3 13 = b$, express $\log_3 52$ in terms of a and b.

Criteria	**Marks**
Factorises $52 = 2^2 \times 13$ and applies product law	1
Applies the power law and states the answer $2a + b$	1

Solution.

Factorise $52 = 4 \times 13 = 2^2 \times 13$. Then

$$\log_3 52 = \log_3 2^2 + \log_3 13 = 2\log_3 2 + \log_3 13 = 2a + b.$$

Solving logarithmic equations

4. Basic logarithmic equations

Q21 Normanhurst Boys 2024 Trial Q1 · **1 mark**

Solve $\log_3(2x-5)=2$ (multiple choice).

Criteria	Marks
Correct option (D)	1

Solution.

Rewrite in index form: $\log_3(2x-5)=2$ means $2x-5=3^2=9$, so

$$2x=14 \quad \Longrightarrow \quad x=7.$$

Check the domain: $2x-5=9>0$, valid.

The answer is **(D)**.

Q22 Baulkham Hills 2023 Prelim Q11 · **2 marks**

Solve the equation $\log_{10}(3x+1)=2$.

Criteria	Marks
Rewrites in index form $3x+1=10^2$	1
Correct solution $x=33$	1

Solution.

Rewrite in index form:

$$3x+1=10^2=100.$$

Then $3x=99$, so

$$x=33.$$

Check the domain: $3x+1=100>0$, valid.

Q23 Hurlstone 2024 Trial Q12 · **2 marks**

Solve $\log_e(x+2) = \log_e(2x)$.

Criteria	Marks
Equates arguments $x+2 = 2x$	1
Correct solution $x = 2$ (domain checked)	1

Solution.

Since $\log_e$ is one-to-one, equate the arguments:

$$x + 2 = 2x \quad \Longrightarrow \quad x = 2.$$

Check the domain: both $x + 2 = 4 > 0$ and $2x = 4 > 0$, so $x = 2$ is valid.

Q24 Kambala 2024 Prelim Q16(a) · **2 marks**

Solve the equation $\log_2(x+20) = 5$.

Criteria	Marks
Rewrites in index form $x + 20 = 2^5$	1
Correct solution $x = 12$	1

Solution.

Rewrite in index form:

$$x + 20 = 2^5 = 32.$$

Then

$$x = 12.$$

Check the domain: $x + 20 = 32 > 0$, valid.

Q25 Loreto Kirribilli 2022 Prelim Q17(b) · **2 marks**

Solve $\log_e(x) - 1 = 1$, giving the answer in exact form.

Criteria	Marks
Obtains $\log_e x = 2$	1
Correct exact solution $x = e^2$	1

Solution.

Rearrange:

$$\log_e x = 2.$$

Rewrite in index form:

$$x = e^2.$$

Check the domain: $x = e^2 > 0$, valid.

Q26 Blacktown Boys 2022 Prelim Q17(a) · **2 marks**

Solve $\log_5(7x + 1) - \log_5(x - 1) = 2$, giving the exact value of x.

Criteria	Marks
Combines logs to $\log_5 \dfrac{7x+1}{x-1} = 2$ and removes log	1
Correct solution $x = \dfrac{13}{9}$ (domain checked)	1

Solution.

Combine the logarithms using the quotient law:

$$\log_5\left(\frac{7x+1}{x-1}\right) = 2.$$

Rewrite in index form:

$$\frac{7x+1}{x-1} = 5^2 = 25.$$

So $7x + 1 = 25(x - 1) = 25x - 25$, giving $18x = 26$ and

$$x = \frac{26}{18} = \frac{13}{9}.$$

Check the domain: this requires $x - 1 > 0$, i.e. $x > 1$. Since $\dfrac{13}{9} \approx 1.44 > 1$, the solution is valid.

Q27 Manly 2024 Trial Q15 · **2 marks**

Solve $\log_2(x+3) - \log_2 x = 1$.

Criteria	Marks
Combines logs to $\log_2 \dfrac{x+3}{x} = 1$ and removes log	1
Correct solution $x = 3$ (domain checked)	1

Solution.

Combine the logarithms using the quotient law:

$$\log_2\left(\frac{x+3}{x}\right) = 1.$$

Rewrite in index form:

$$\frac{x+3}{x} = 2^1 = 2.$$

So $x + 3 = 2x$, giving

$$x = 3.$$

Check the domain: both $x + 3 = 6 > 0$ and $x = 3 > 0$, valid.

5. Logarithmic equations reducing to a quadratic

Q28 Gosford 2020 Trial Q14 · **3 marks**

Solve $2\log x = \log(5x+6)$.

Criteria	Marks
Uses the power law to write $\log x^2 = \log(5x+6)$, hence $x^2 = 5x+6$	1
Solves the quadratic $x^2 - 5x - 6 = 0$, obtaining $x = 6$ or $x = -1$	1
Rejects $x = -1$ (domain $x > 0$) and states $x = 6$	1

Solution.

Apply the power law $2\log x = \log x^2$:

$$\log x^2 = \log(5x+6) \implies x^2 = 5x+6.$$

Rearranging, $x^2 - 5x - 6 = 0$, so $(x-6)(x+1) = 0$, giving $x = 6$ or $x = -1$.

The original equation contains $\log x$, so we require $x > 0$. Hence $x = -1$ is rejected. The only solution is $\boxed{x = 6}$.

Q29 SCEGGS 2022 Prelim Q25 · **3 marks**

Solve the equation $\log_{10} x + \log_{10}(x-3) = \log_{10} 18$.

Criteria	Marks
Combines the logs: $\log_{10}\big(x(x-3)\big) = \log_{10} 18$, hence $x(x-3) = 18$	1
Solves $x^2 - 3x - 18 = 0$, obtaining $x = 6$ or $x = -3$	1
Rejects $x = -3$ (need $x > 3$) and states $x = 6$	1

Solution.

Using the product law, $\log_{10} x + \log_{10}(x-3) = \log_{10}\big(x(x-3)\big)$, so

$$x(x-3) = 18 \implies x^2 - 3x - 18 = 0 \implies (x-6)(x+3) = 0.$$

This gives $x = 6$ or $x = -3$. For $\log_{10} x$ and $\log_{10}(x-3)$ to be defined we need $x > 0$ *and* $x - 3 > 0$, i.e. $x > 3$. Thus $x = -3$ is rejected, leaving $\boxed{x = 6}$.

Q30 St George Girls 2023 Trial Q20 · **3 marks**

Solve $2\ln x = \ln(2x+3)$.

Criteria	Marks
Uses the power law to write $\ln x^2 = \ln(2x+3)$, hence $x^2 = 2x + 3$	1
Solves $x^2 - 2x - 3 = 0$, obtaining $x = 3$ or $x = -1$	1
Rejects $x = -1$ (domain $x > 0$) and states $x = 3$	1

Solution.

Apply the power law $2\ln x = \ln x^2$:

$$\ln x^2 = \ln(2x+3) \implies x^2 = 2x + 3.$$

Rearranging, $x^2 - 2x - 3 = 0$, so $(x-3)(x+1) = 0$, giving $x = 3$ or $x = -1$.

Since $\ln x$ requires $x > 0$, we reject $x = -1$. The only solution is $\boxed{x = 3}$.

Q31 Sydney Tech 2023 Trial Q17 · **2 marks**

Solve $\ln(7x-12) = 2\ln x$.

Criteria	Marks
Writes $2\ln x = \ln x^2$, hence $7x-12 = x^2$, i.e. $x^2-7x+12=0$	1
Solves to obtain $x=3$ and $x=4$ (both satisfy $x > \frac{12}{7}$)	1

Solution.

Using the power law, $2\ln x = \ln x^2$, so

$$\ln(7x-12) = \ln x^2 \implies 7x-12 = x^2.$$

Rearranging, $x^2-7x+12=0$, so $(x-3)(x-4)=0$, giving $x=3$ or $x=4$.

We need $x>0$ (for $\ln x$) and $7x-12>0$, i.e. $x>\frac{12}{7}$. Both $x=3$ and $x=4$ satisfy this, so both are valid: $\boxed{x=3 \text{ or } x=4}$.

Q32 North Sydney Girls 2023 Prelim Q26 · **3 marks**

Solve $\log_{10}(x-3)+\log_{10}(x+2) = \log_{10} 6$.

Criteria	Marks
Combines the logs: $\log_{10}\big((x-3)(x+2)\big) = \log_{10} 6$, hence $(x-3)(x+2)=6$	1
Solves $x^2-x-12=0$, obtaining $x=4$ or $x=-3$	1
Rejects $x=-3$ (need $x>3$) and states $x=4$	1

Solution.

Using the product law,

$$\log_{10}\big((x-3)(x+2)\big) = \log_{10} 6 \implies (x-3)(x+2)=6.$$

Expanding, $x^2-x-6=6$, so $x^2-x-12=0$, giving $(x-4)(x+3)=0$ and hence $x=4$ or $x=-3$.

For both logarithms to be defined we require $x-3>0$ and $x+2>0$, i.e. $x>3$. Thus $x=-3$ is rejected, leaving $\boxed{x=4}$.

Q33 North Sydney Boys 2024 Prelim Q22 · **3 marks**

Solve the equation $\log_2(x+4) + \log_2(x-2) = 4$.

Criteria	Marks
Combines the logs and converts to index form: $(x+4)(x-2) = 2^4 = 16$	1
Solves $x^2 + 2x - 24 = 0$, obtaining $x = 4$ or $x = -6$	1
Rejects $x = -6$ (need $x > 2$) and states $x = 4$	1

Solution.

Using the product law, $\log_2\big((x+4)(x-2)\big) = 4$. Converting to index form,

$$(x+4)(x-2) = 2^4 = 16.$$

Expanding, $x^2 + 2x - 8 = 16$, so $x^2 + 2x - 24 = 0$, giving $(x-4)(x+6) = 0$ and hence $x = 4$ or $x = -6$.

For $\log_2(x+4)$ and $\log_2(x-2)$ to be defined we need $x + 4 > 0$ and $x - 2 > 0$, i.e. $x > 2$. Thus $x = -6$ is rejected, leaving $\boxed{x = 4}$.

Q34 Sydney Tech 2024 Trial Q17 · **2 marks**

Solve $\ln x - \dfrac{3}{\ln x} = 2$ giving your answers in exact form.

Criteria	Marks
Lets $u = \ln x$ and forms the quadratic $u^2 - 2u - 3 = 0$, solving for $u = 3$ or $u = -1$	1
Back-substitutes to give the exact solutions $x = e^3$ and $x = e^{-1}$	1

Solution.

Let $u = \ln x$ (defined for $x > 0$). Multiplying through by u:

$$u - \frac{3}{u} = 2 \;\Rightarrow\; u^2 - 2u - 3 = 0 \;\Rightarrow\; (u-3)(u+1) = 0,$$

so $u = 3$ or $u = -1$. (Note $u = 0$, i.e. $x = 1$, is excluded since we divided by $\ln x$; neither solution is $x = 1$.)

Back-substituting $u = \ln x$:

$$\ln x = 3 \Rightarrow x = e^3, \qquad \ln x = -1 \Rightarrow x = e^{-1}.$$

Both give $x > 0$, so both are valid. The exact solutions are $\boxed{x = e^3 \text{ or } x = e^{-1}}$.

Q35 James Ruse 2023 Trial Q26 · **2 marks**

Solve $\ln x - \dfrac{6}{\ln x} = 5$.

Criteria	Marks
Lets $u = \ln x$ and forms the quadratic $u^2 - 5u - 6 = 0$, solving for $u = 6$ or $u = -1$	1
Back-substitutes to give the solutions $x = e^6$ and $x = e^{-1}$	1

Solution.

Let $u = \ln x$ (defined for $x > 0$). Multiplying through by u:

$$u - \frac{6}{u} = 5 \implies u^2 - 5u - 6 = 0 \implies (u-6)(u+1) = 0,$$

so $u = 6$ or $u = -1$. (Here $u = 0$, i.e. $x = 1$, is excluded as we divided by $\ln x$; neither solution is $x = 1$.)

Back-substituting $u = \ln x$:

$$\ln x = 6 \Rightarrow x = e^6, \qquad \ln x = -1 \Rightarrow x = e^{-1}.$$

Both give $x > 0$, so both are valid. The solutions are $\boxed{x = e^6 \text{ or } x = e^{-1}}$.

Q36 Abbotsleigh 2024 Prelim Q30 · **3 marks**

Solve the equation $\log_2 x - \dfrac{3}{\log_2 x} = 2$.

Criteria	Marks
Lets $u = \log_2 x$ and forms the quadratic $u^2 - 2u - 3 = 0$	1
Solves for $u = 3$ or $u = -1$	1
Back-substitutes to give the solutions $x = 8$ and $x = \frac{1}{2}$	1

Solution.

Let $u = \log_2 x$ (defined for $x > 0$). Multiplying through by u:

$$u - \frac{3}{u} = 2 \implies u^2 - 2u - 3 = 0 \implies (u-3)(u+1) = 0,$$

so $u = 3$ or $u = -1$. (The value $u = 0$, i.e. $x = 1$, is excluded since we divided by $\log_2 x$; neither solution is $x = 1$.)

Back-substituting $u = \log_2 x$:

$$\log_2 x = 3 \Rightarrow x = 2^3 = 8, \qquad \log_2 x = -1 \Rightarrow x = 2^{-1} = \tfrac{1}{2}.$$

Both give $x > 0$, so both are valid. The solutions are $\boxed{x = 8 \text{ or } x = \frac{1}{2}}$.

6. Making a variable the subject (inverse form)

Q37 Hornsby Girls 2020 Trial Q32(a) · **1 mark**

Given $y = \ln(x + r)$ (where $r > 1$), show that $x = e^y - r$.

Criteria	Marks
Exponentiates correctly to reach $x = e^y - r$	1

Solution.

Rewrite the logarithmic relation in exponential form. Since $y = \ln(x + r) = \log_e(x + r)$,

$$e^y = x + r.$$

Making x the subject,

$$x = e^y - r. \blacksquare$$

Q38 St George Girls 2020 Trial Q6 · **1 mark**

$y = \log_a x^{-3}$ is equivalent to which index form (multiple choice)?

Criteria	Marks
Correct option (B)	1

Solution.

Apply the power law of logarithms:

$$y = \log_a x^{-3} = -3\log_a x \implies \log_a x = -\frac{y}{3}.$$

Rewriting in index form,

$$x = a^{-\frac{y}{3}}.$$

The answer is **(B)**.

Q39 Loreto Kirribilli 2022 Prelim Q9 · **1 mark**

If $y = \dfrac{e^x}{e^x + 1}$, make x the subject (multiple choice).

Criteria	Marks
Correct option (B)	1

Solution.

Clear the denominator:

$$y\,(e^x + 1) = e^x \implies y\,e^x + y = e^x.$$

Collect the e^x terms:

$$y = e^x - y\,e^x = e^x(1 - y) \implies e^x = \frac{y}{1 - y}.$$

Taking natural logarithms,

$$x = \ln\left(\frac{y}{1 - y}\right).$$

The answer is **(B)**.

Q40 Shore 2023 Prelim Q8 · **1 mark**

If $y = \dfrac{e^x}{2 - e^x}$, make x the subject (multiple choice).

Criteria	Marks
Correct option (A)	1

Solution.

Clear the denominator:

$$y\,(2 - e^x) = e^x \implies 2y - y\,e^x = e^x.$$

Collect the e^x terms:

$$2y = e^x + y\,e^x = e^x(1 + y) \implies e^x = \frac{2y}{1 + y}.$$

Taking natural logarithms,

$$x = \ln\left(\frac{2y}{1 + y}\right).$$

The answer is **(A)**.

Q41 Abbotsleigh 2024 Prelim Q9 · **1 mark**

If $x = \log_e(1+y) - \log_e(1-y)$, where $-1 < y < 1$, make y the subject (multiple choice).

Criteria	**Marks**
Correct option (C)	1

Solution.

Combine the logarithms and rewrite in exponential form:

$$x = \log_e\left(\frac{1+y}{1-y}\right) \implies e^x = \frac{1+y}{1-y}.$$

Clear the denominator and collect the y terms:

$$e^x(1-y) = 1+y \implies e^x - 1 = y + y\,e^x = y\,(1+e^x)\,.$$

Therefore

$$y = \frac{e^x - 1}{e^x + 1}.$$

(Note $1 - y > 0$ throughout since $-1 < y < 1$, so the division is valid.)

The answer is **(C)**.

Change of base and applications

7. Change of base and applications

Q42 Danebank 2020 Trial Q19(a) · 1 mark

Show that $\log_x 2 = \dfrac{1}{\log_2 x}$.

Criteria	Marks
Applies the change-of-base rule to obtain the result	1

Solution.

Change the base of $\log_x 2$ to base 2:

$$\log_x 2 = \frac{\log_2 2}{\log_2 x} = \frac{1}{\log_2 x},$$

since $\log_2 2 = 1$. Hence $\log_x 2 = \dfrac{1}{\log_2 x}$, as required.

Q43 North Sydney Boys 2024 Prelim Q1 · 1 mark

Find the value of x such that $\log_x 2 = \dfrac{1}{2}$. (Options: (A) $\sqrt{2}$, (B) 2, (C) 4, (D) $2\sqrt{2}$.)

Criteria	Marks
Correct option (C)	1

Solution.

Rewrite $\log_x 2 = \dfrac{1}{2}$ in index form:

$$x^{1/2} = 2 \quad \Longrightarrow \quad x = 2^2 = 4.$$

The answer is **(C)** 4.

Q44 Cherrybrook Tech 2022 Prelim Q13(b) · **1 mark**

Given that $\log_a b = 2.75$ and $\log_a c = 0.25$, find the value of $\log_c b$.

Criteria	Marks
Uses change of base to evaluate $\log_c b = 11$	1

Solution.

Using the change-of-base rule (to base a),

$$\log_c b = \frac{\log_a b}{\log_a c} = \frac{2.75}{0.25} = 11.$$

Hence $\log_c b = 11$.

Q45 North Sydney Girls 2022 Prelim Q27(a) · **1 mark**

The Richter scale relates the magnitude M and energy E (in Joules) of an earthquake by $\log_{10}(E) = a + 1.5M$, where a is a real constant. The 1906 San Francisco earthquake registered $M = 8.2$ and released $E = 5.011 \times 10^{16}$ J. Find a to 1 decimal place.

Criteria	Marks
Substitutes the given values and evaluates a correct to 1 decimal place	1

Solution.

Substitute $E = 5.011 \times 10^{16}$ and $M = 8.2$ into $\log_{10}(E) = a + 1.5M$:

$$\log_{10}\left(5.011 \times 10^{16}\right) = a + 1.5(8.2).$$

Now $\log_{10}(5.011 \times 10^{16}) \approx 16.6999$ and $1.5 \times 8.2 = 12.3$, so

$$a = 16.6999 - 12.3 \approx 4.3999.$$

Correct to 1 decimal place, $a \approx \boxed{4.4}$.

Q46 Killara 2021 Trial Q29(a) · **1 mark**

Show that $\log_a b = \dfrac{1}{\log_b a}$.

Criteria	Marks
Applies the change-of-base rule to obtain the result	1

Solution.

Change the base of $\log_a b$ to base b:

$$\log_a b = \frac{\log_b b}{\log_b a} = \frac{1}{\log_b a},$$

since $\log_b b = 1$. Hence $\log_a b = \dfrac{1}{\log_b a}$, as required.

Q47 Caringbah 2020 Trial Q21(b) · **2 marks**

Solve the equation $\log_2 x = 4 \log_x 2$.

Criteria	Marks
Uses change of base to form a quadratic in $\log_2 x$	1
Solves and states both solutions $x = 4$ and $x = \frac{1}{4}$	1

Solution.

Let $t = \log_2 x$. By change of base, $\log_x 2 = \dfrac{1}{\log_2 x} = \dfrac{1}{t}$, so the equation becomes

$$t = \frac{4}{t} \quad \Longrightarrow \quad t^2 = 4 \quad \Longrightarrow \quad t = \pm 2.$$

Then $\log_2 x = 2 \Rightarrow x = 4$, or $\log_2 x = -2 \Rightarrow x = 2^{-2} = \frac{1}{4}$. Both values are positive and not equal to 1, so both are valid:

$$x = 4 \quad \text{or} \quad x = \tfrac{1}{4}.$$

Q48 Hurlstone 2023 Prelim Q15(e)(i) · **2 marks**

On the Richter scale, an earthquake of intensity I has magnitude $M(I) = \log_{10}\left(\dfrac{I}{I_0}\right)$, where I_0 is a fixed reference intensity. The Newcastle earthquake had magnitude 5.9 and the Turkey earthquake magnitude 7.8. Find how many times more intense the Turkey earthquake was than the Newcastle earthquake.

Criteria	**Marks**
Expresses each intensity as $I = I_0 \times 10^M$ (or forms the ratio $10^{M_T - M_N}$)	1
Evaluates the ratio $10^{1.9} \approx 79.4$	1

Solution.

Let I_N and I_T be the intensities of the Newcastle and Turkey earthquakes. From $M(I) = \log_{10}(I/I_0)$,

$$\log_{10}\left(\frac{I_N}{I_0}\right) = 5.9 \;\Rightarrow\; I_N = I_0\,10^{5.9}, \qquad \log_{10}\left(\frac{I_T}{I_0}\right) = 7.8 \;\Rightarrow\; I_T = I_0\,10^{7.8}.$$

The ratio of intensities is therefore

$$\frac{I_T}{I_N} = \frac{I_0\,10^{7.8}}{I_0\,10^{5.9}} = 10^{\,7.8-5.9} = 10^{1.9} \approx 79.4.$$

The Turkey earthquake was approximately $\boxed{79.4}$ times more intense than the Newcastle earthquake.

Q49 Sydney Girls 2023 Trial Q35 · **3 marks**

Find the exact value of x given $\log_3 x = \log_x \sqrt[9]{3}$.

Criteria	**Marks**
Writes $\log_x \sqrt[9]{3}$ using change of base to form an equation in $\log_3 x$	1
Reduces to $(\log_3 x)^2 = \frac{1}{9}$ and solves for $\log_3 x = \pm\frac{1}{3}$	1
States both exact values $x = 3^{1/3}$ and $x = 3^{-1/3}$	1

Solution.

Let $t = \log_3 x$. For the right-hand side, first note $\sqrt[9]{3} = 3^{1/9}$, so

$$\log_x \sqrt[9]{3} = \log_x 3^{1/9} = \tfrac{1}{9}\log_x 3 = \frac{1}{9}\cdot\frac{1}{\log_3 x} = \frac{1}{9t},$$

using change of base ($\log_x 3 = 1/\log_3 x$). The equation becomes

$$t = \frac{1}{9t} \quad\Longrightarrow\quad 9t^2 = 1 \quad\Longrightarrow\quad t^2 = \tfrac{1}{9} \quad\Longrightarrow\quad t = \pm\tfrac{1}{3}.$$

Hence $\log_3 x = \frac{1}{3} \Rightarrow x = 3^{1/3} = \sqrt[3]{3}$, or $\log_3 x = -\frac{1}{3} \Rightarrow x = 3^{-1/3} = \dfrac{1}{\sqrt[3]{3}}$. Both are positive and not equal to 1, so both are valid:

$$x = 3^{1/3} \quad \text{or} \quad x = 3^{-1/3}.$$

Q50 James Ruse 2022 Trial Q35 · **3 marks**

Solve $2\log_2 x - 9\log_x 2 = 3$.

Criteria	Marks
Uses change of base to write $\log_x 2 = 1/\log_2 x$	1
Forms and solves the quadratic $2t^2 - 3t - 9 = 0$ giving $t = 3,\ -\frac{3}{2}$	1
States both solutions $x = 8$ and $x = \frac{\sqrt{2}}{4}$	1

Solution.

Let $t = \log_2 x$. By change of base, $\log_x 2 = \dfrac{1}{\log_2 x} = \dfrac{1}{t}$, so

$$2t - \frac{9}{t} = 3.$$

Multiply through by t:

$$2t^2 - 9 = 3t \quad \Longrightarrow \quad 2t^2 - 3t - 9 = 0 \quad \Longrightarrow \quad (2t+3)(t-3) = 0,$$

so $t = 3$ or $t = -\frac{3}{2}$. Then:

$$\log_2 x = 3 \Rightarrow x = 2^3 = 8, \qquad \log_2 x = -\tfrac{3}{2} \Rightarrow x = 2^{-3/2} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}.$$

Both are positive and not equal to 1, so both are valid:

$$x = 8 \quad \text{or} \quad x = \frac{\sqrt{2}}{4}.$$

Q51 North Sydney Boys 2024 Prelim Q20 · **3 marks**

The apparent stellar magnitude is $m = -\sqrt[5]{100}\,\log\left(\dfrac{f}{f_V}\right)$, where f is a star's energy flux and f_V is the reference energy flux. Betelgeuse has magnitude 0.45 and Deneb has magnitude 1.26. Determine which star is brighter, and by how much.

Criteria	Marks
Recognises that a smaller magnitude means greater brightness, so Betelgeuse is brighter	1
Rearranges the definition to $\dfrac{f}{f_V} = 10^{-m/\sqrt[5]{100}}$ (or forms the brightness ratio)	1
Evaluates the ratio $10^{(1.26-0.45)/\sqrt[5]{100}} \approx 2.1$	1

Solution.

A smaller apparent magnitude corresponds to a larger energy flux (a brighter star). Since $0.45 < 1.26$, **Betelgeuse is the brighter star**.

Note that $\sqrt[5]{100} = 100^{1/5} \approx 2.5119$. Rearranging the definition for the flux ratio,

$$m = -\sqrt[5]{100}\,\log\left(\frac{f}{f_V}\right) \implies \log\left(\frac{f}{f_V}\right) = -\frac{m}{\sqrt[5]{100}} \implies \frac{f}{f_V} = 10^{-m/\sqrt[5]{100}}.$$

Writing f_B and f_D for the fluxes of Betelgeuse and Deneb, the ratio of their brightnesses is

$$\frac{f_B}{f_D} = \frac{10^{-0.45/\sqrt[5]{100}}}{10^{-1.26/\sqrt[5]{100}}} = 10^{(1.26-0.45)/\sqrt[5]{100}} = 10^{0.81/2.5119} \approx 10^{0.3225} \approx 2.1.$$

Betelgeuse is brighter than Deneb by a factor of approximately $\boxed{2.1}$ (its energy flux is about 2.1 times that of Deneb).