Year 11 Mathematics Advanced

Topic: Exponential and Logarithmic Functions (1 of 4)

Review Questions (Worked Solutions): Exponential functions and Euler's number

Term 3, 2026

Exponential expressions and Euler's number

1. Evaluating exponential expressions and Euler's number e

Q1 Girraween 2021 Trial Q1 · **1 mark**

Find $8e^{-2}$ correct to 2 significant figures.

Criteria	Marks
Correct value 1.1	1

Solution.

$$8e^{-2} = \frac{8}{e^2} = 1.0826\ldots \approx 1.1 \quad \text{(2 sig. fig.)}$$

Q2 Moriah 2021 Prelim Q11(a) · **1 mark**

Find $e^{-1.75}$ correct to 3 significant figures.

Criteria	Marks
Correct value 0.174	1

Solution.

$$e^{-1.75} = 0.17377\ldots \approx 0.174 \quad \text{(3 sig. fig.)}$$

Q3 North Sydney Girls 2023 Trial Q1 · **1 mark**

Find $\dfrac{9.52}{e^{-3.8}}$ to three significant figures.

Criteria	Marks
Correct value 426	1

Solution.

Dividing by $e^{-3.8}$ is multiplying by $e^{3.8}$:

$$\frac{9.52}{e^{-3.8}} = 9.52\,e^{3.8} = 425.55\ldots \approx 426 \quad \text{(3 sig. fig.)}$$

Q4 Cherrybrook Tech 2022 Prelim Q17(a) · **1 mark**

For $f(x) = 4 - 2^{-x}$, find $f(-2)$.

Criteria	Marks
Correct value $f(-2) = 0$	1

Solution.

Substitute $x = -2$:

$$f(-2) = 4 - 2^{-(-2)} = 4 - 2^2 = 4 - 4 = 0.$$

Q5 Kambala 2024 Prelim Q17(a) · **1 mark**

For $f(x) = 4e^{3x}$, find the exact value of $f(2)$.

Criteria	Marks
Correct exact value $4e^6$	1

Solution.

Substitute $x = 2$:

$$f(2) = 4e^{3(2)} = 4e^6.$$

Q6 Sydney Girls 2020 Trial Q6 · **1 mark**

If $f(x) = b^x + b^{-x}$, find $\big(f(-x)\big)^2$ (multiple choice).

Criteria	Marks
Correct option (B)	1

Solution.

Since f is even, $f(-x) = b^{-x} + b^x = f(x)$. Squaring:

$$\big(f(-x)\big)^2 = \big(b^x + b^{-x}\big)^2 = b^{2x} + 2\,b^x b^{-x} + b^{-2x} = b^{2x} + b^{-2x} + 2.$$

Now $f(2x) = b^{2x} + b^{-2x}$, so $\big(f(-x)\big)^2 = f(2x) + 2 = 2 + f(2x)$.

The answer is **(B)**.

Q7 Gosford 2023 Prelim Q19 · **3 marks**

Let $f(x) = 1 + e^{2x}$. Show that $f(x) \times f(-x) = f(x) + f(-x)$.

Criteria	Marks
Writes $f(-x) = 1 + e^{-2x}$	1
Expands the product $f(x) \times f(-x)$	1
Shows both sides equal $2 + e^{2x} + e^{-2x}$	1

Solution.

With $f(x) = 1 + e^{2x}$, replacing x by $-x$ gives $f(-x) = 1 + e^{-2x}$.

Left-hand side:

$$f(x) \times f(-x) = \left(1 + e^{2x}\right)\left(1 + e^{-2x}\right) = 1 + e^{-2x} + e^{2x} + e^{2x}e^{-2x}.$$

Since $e^{2x}e^{-2x} = e^{0} = 1$,

$$f(x) \times f(-x) = 2 + e^{2x} + e^{-2x}.$$

Right-hand side:

$$f(x) + f(-x) = \left(1 + e^{2x}\right) + \left(1 + e^{-2x}\right) = 2 + e^{2x} + e^{-2x}.$$

Both sides equal $2 + e^{2x} + e^{-2x}$, so $f(x) \times f(-x) = f(x) + f(-x)$. ■

Solving exponential equations

2. Basic exponential equations (common base)

Q8 Sydney Boys 2022 Trial Q11 · 1 mark

Solve $2^x = \dfrac{1}{16}$.

Criteria	Marks
Correct value $x = -4$	1

Solution.

Write the right-hand side as a power of 2:

$$\frac{1}{16} = \frac{1}{2^4} = 2^{-4}.$$

So $2^x = 2^{-4}$, and equating exponents gives

$$x = -4.$$

Q9 Girraween 2021 Prelim Q1(bi) · 1 mark

Solve for x: $\left(\dfrac{2}{3}\right)^x = \dfrac{27}{8}$.

Criteria	Marks
Correct value $x = -3$	1

Solution.

Note $\dfrac{27}{8} = \dfrac{3^3}{2^3} = \left(\dfrac{3}{2}\right)^3 = \left(\dfrac{2}{3}\right)^{-3}$. Hence

$$\left(\frac{2}{3}\right)^x = \left(\frac{2}{3}\right)^{-3} \quad \Longrightarrow \quad x = -3.$$

Q10 Pymble 2022 Prelim Q22(c) · **2 marks**

Solve $2^{x-3} = \dfrac{1}{4}$.

Criteria	Marks
Expresses $\frac{1}{4}$ as 2^{-2} (common base)	1
Correct value $x = 1$	1

Solution.

Write the right-hand side as a power of 2:

$$\frac{1}{4} = 2^{-2}.$$

So $2^{x-3} = 2^{-2}$. Equating exponents:

$$x - 3 = -2 \quad \Longrightarrow \quad x = 1.$$

Q11 Baulkham Hills 2022 Prelim Q22 · **2 marks**

Solve for x, $3^{1-x} = \dfrac{1}{\sqrt{27}}$.

Criteria	Marks
Expresses $\frac{1}{\sqrt{27}}$ as $3^{-3/2}$ (common base)	1
Correct value $x = \frac{5}{2}$	1

Solution.

Write the right-hand side as a power of 3. Since $27 = 3^3$,

$$\sqrt{27} = 27^{1/2} = 3^{3/2}, \qquad \frac{1}{\sqrt{27}} = 3^{-3/2}.$$

So $3^{1-x} = 3^{-3/2}$. Equating exponents:

$$1 - x = -\frac{3}{2} \quad \Longrightarrow \quad x = \frac{5}{2}.$$

Q12 Baulkham Hills 2023 Prelim Q12 · **2 marks**

Solve $5^{-3x} = 125^{x+1}$.

Criteria	Marks
Expresses 125 as 5^3 (common base)	1
Correct value $x = -\frac{1}{2}$	1

Solution.

Since $125 = 5^3$, the right-hand side is

$$125^{x+1} = \left(5^3\right)^{x+1} = 5^{3(x+1)}.$$

So $5^{-3x} = 5^{3(x+1)}$. Equating exponents:

$$-3x = 3(x+1) = 3x + 3 \quad \Longrightarrow \quad -6x = 3 \quad \Longrightarrow \quad x = -\frac{1}{2}.$$

Q13 Hurlstone 2024 Prelim Q28 · **2 marks**

Solve $\left(\sqrt{3}\right)^{x} = 9^{2x}$, leaving x in exact form.

Criteria	Marks
Writes both sides as powers of 3	1
Correct value $x = 0$	1

Solution.

Write both sides as powers of 3. Since $\sqrt{3} = 3^{1/2}$ and $9 = 3^2$,

$$\left(3^{1/2}\right)^{x} = \left(3^2\right)^{2x} \quad \Longrightarrow \quad 3^{x/2} = 3^{4x}.$$

Equating exponents:

$$\frac{x}{2} = 4x \quad \Longrightarrow \quad x = 8x \quad \Longrightarrow \quad 7x = 0 \quad \Longrightarrow \quad x = 0.$$

Q14 Moriah 2021 Prelim Q11(c) · **3 marks**

Find the value of y that solves $\left(\sqrt{8}\right)^{y-3} = (64)^{4-y}$.

Criteria	Marks
Writes both sides as powers of 2	1
Sets up the linear equation in the exponents	1
Correct value $y = \frac{19}{5}$	1

Solution.

Write both sides as powers of 2. Since $\sqrt{8} = 8^{1/2} = \left(2^3\right)^{1/2} = 2^{3/2}$ and $64 = 2^6$,

$$\left(2^{3/2}\right)^{y-3} = \left(2^6\right)^{4-y} \quad \Longrightarrow \quad 2^{\frac{3}{2}(y-3)} = 2^{6(4-y)}.$$

Equating exponents:

$$\frac{3}{2}(y-3) = 6(4-y).$$

Multiply through by 2:

$$3(y-3) = 12(4-y) \quad \Longrightarrow \quad 3y - 9 = 48 - 12y \quad \Longrightarrow \quad 15y = 57 \quad \Longrightarrow \quad y = \frac{57}{15} = \frac{19}{5}.$$

3. Exponential equations using logarithms

Q15 Trinity Grammar 2020 Trial Q11 · **1 mark**

Find the exact value of x if $17^x = 2$.

Criteria	Marks
Correct exact value of x	1

Solution.

Take logarithms of both sides:

$$17^x = 2 \;\Rightarrow\; x \log 17 = \log 2 \;\Rightarrow\; x = \frac{\log 2}{\log 17} = \log_{17} 2.$$

Exact value: $x = \dfrac{\log 2}{\log 17}$ (≈ 0.2447).

Q16 Normanhurst Boys 2025 Trial Q4 · **1 mark**

What is the solution of $3^x = 5$? (A) $\dfrac{\log 5}{3}$ (B) $\dfrac{3}{\log 5}$ (C) $\dfrac{\log 5}{\log 3}$ (D) $\log\left(\dfrac{5}{3}\right)$.

Criteria	Marks
Correct option (C)	1

Solution.

Take logarithms of both sides:

$$3^x = 5 \implies x \log 3 = \log 5 \implies x = \frac{\log 5}{\log 3}.$$

The answer is **(C)** $\dfrac{\log 5}{\log 3}$ (≈ 1.465).

Q17 St George Girls 2022 Prelim Q10 · **2 marks**

Solve $2^x = 5$ correct to 3 significant figures.

Criteria	Marks
Takes logarithms to isolate x	1
Correct value, 2.32 (3 sig. fig.)	1

Solution.

Take logarithms of both sides:

$$2^x = 5 \implies x \log 2 = \log 5 \implies x = \frac{\log 5}{\log 2} = 2.32192\ldots$$

Correct to 3 significant figures, $x \approx 2.32$.

Q18 St George Girls 2023 Prelim Q10 · **2 marks**

Solve for x: $2^{x-3}+2=7$, leave your answer correct to 2 decimal places.

Criteria	Marks
Reduces to $2^{x-3}=5$ and takes logarithms	1
Correct value, 5.32 (2 d.p.)	1

Solution.

Isolate the power, then take logarithms:

$$2^{x-3}+2=7 \implies 2^{x-3}=5 \implies (x-3)\log 2=\log 5.$$

$$x-3=\frac{\log 5}{\log 2}=2.3219\ldots \implies x=3+\frac{\log 5}{\log 2}=5.3219\ldots$$

Correct to 2 decimal places, $x\approx 5.32$.

Q19 Kambala 2021 Prelim Q7(a) · **2 marks**

Solve $2^x=30$, correct to two decimal places.

Criteria	Marks
Takes logarithms to isolate x	1
Correct value, 4.91 (2 d.p.)	1

Solution.

Take logarithms of both sides:

$$2^x=30 \implies x\log 2=\log 30 \implies x=\frac{\log 30}{\log 2}=4.9068\ldots$$

Correct to 2 decimal places, $x\approx 4.91$.

Q20 Girraween 2022 Trial Q11 · **3 marks**

Solve $3^{2x-1} = 5^x$, giving your answer to 2 d.p.

Criteria	Marks
Takes logarithms of both sides	1
Collects the x terms and rearranges for x	1
Correct value, 1.87 (2 d.p.)	1

Solution.

Take logarithms of both sides and expand:

$$(2x-1)\log 3 = x\log 5 \;\Rightarrow\; 2x\log 3 - \log 3 = x\log 5.$$

Collect the x terms:

$$x(2\log 3 - \log 5) = \log 3 \;\Rightarrow\; x = \frac{\log 3}{2\log 3 - \log 5} = \frac{\log 3}{\log \frac{9}{5}} = 1.8690\ldots$$

Correct to 2 decimal places, $x \approx 1.87$.

Q21 Presbyterian 2022 Prelim Q14 · **2 marks**

Solve $3^{2x} = 2^{x+1}$.

Criteria	Marks
Takes logarithms and collects the x terms	1
Correct value, $x = \dfrac{\log 2}{2\log 3 - \log 2} \approx 0.46$	1

Solution.

Take logarithms of both sides and expand:

$$2x\log 3 = (x+1)\log 2 \;\Rightarrow\; 2x\log 3 - x\log 2 = \log 2.$$

Collect the x terms:

$$x(2\log 3 - \log 2) = \log 2 \;\Rightarrow\; x = \frac{\log 2}{2\log 3 - \log 2} = \frac{\log 2}{\log \frac{9}{2}} = 0.4608\ldots$$

So $x = \dfrac{\log 2}{2\log 3 - \log 2}$ (≈ 0.46).

Q22 Hornsby Girls 2022 Prelim Q24 · **2 marks**

By taking logarithms of both sides, or otherwise, solve $2 \times 3^{x-2} = 36^{x-1}$, expressing your answer correct to 2 decimal places.

Criteria	Marks
Takes logarithms of both sides and expands	1
Collects the x terms and evaluates, 0.84 (2 d.p.)	1

Solution.

Take logarithms of both sides:

$$\log 2 + (x-2)\log 3 = (x-1)\log 36.$$

Expand and collect the x terms (note $\log 36 = 2\log 6$):

$$x(\log 3 - \log 36) = -\log 36 - \log 2 + 2\log 3 = \log \frac{9}{72} = -\log 8.$$

Since $\log 3 - \log 36 = -\log 12$,

$$x = \frac{-\log 8}{-\log 12} = \frac{\log 8}{\log 12} = \frac{3\log 2}{\log 12} = 0.8368\ldots$$

Correct to 2 decimal places, $x \approx 0.84$.

4. Exponential equations reducible to a quadratic

Q23 Trinity Grammar 2020 Trial Q15 · **3 marks**

Solve the equation $4(4^x) - 5(2^x) + 1 = 0$.

Criteria	Marks
Substitutes $u = 2^x$ and forms the quadratic $4u^2 - 5u + 1 = 0$	1
Solves for u: $u = 1$ or $u = \frac{1}{4}$	1
Obtains both solutions $x = 0$ and $x = -2$	1

Solution.

Since $4^x = (2^x)^2$, let $u = 2^x$ (so $u > 0$). Then

$$4u^2 - 5u + 1 = 0 \implies (4u-1)(u-1) = 0 \implies u = \tfrac{1}{4} \text{ or } u = 1.$$

Now $2^x = 1 \implies x = 0$, and $2^x = \frac{1}{4} = 2^{-2} \implies x = -2$.

$$\boxed{x = -2 \text{ or } x = 0}$$

Q24 Normanhurst Boys 2023 Trial Q3 · **1 mark**

Which of the following represents the solutions for the equation $2^{2x} - 5(2^x) + 4 = 0$?

Criteria	Marks
Correct option (B)	1

Solution.

Let $u = 2^x$ $(u > 0)$. Then $u^2 - 5u + 4 = 0 \implies (u-1)(u-4) = 0$, so $u = 1$ or $u = 4$. Hence $2^x = 1 \implies x = 0$, and $2^x = 4 = 2^2 \implies x = 2$. The solutions are $x = 0$ or $x = 2$, which is option **(B)**.

Q25 Riverview 2022 Trial Q8 · **1 mark**

The solutions to $e^{6x} - 5e^{3x} + 6 = 0$ are $x =$?

Criteria	Marks
Correct option (D)	1

Solution.

Since $e^{6x} = (e^{3x})^2$, let $u = e^{3x}$ $(u > 0)$. Then

$$u^2 - 5u + 6 = 0 \implies (u-2)(u-3) = 0 \implies u = 2 \text{ or } u = 3.$$

So $e^{3x} = 2 \implies 3x = \log_e 2 \implies x = \frac{1}{3}\log_e 2$, and $e^{3x} = 3 \implies x = \frac{1}{3}\log_e 3$. The solutions are $x = \frac{1}{3}\log_e 3$, $\frac{1}{3}\log_e 2$, which is option **(D)**.

Q26 Blacktown Boys 2022 Prelim Q17(b) · **3 marks**

Solve $3 \times 9^x - 13 \times 3^x + 4 = 0$ for the exact value of x.

Criteria	Marks
Substitutes $u = 3^x$ to reduce to the quadratic $3u^2 - 13u + 4 = 0$	1
Factorises and solves: $u = 4$ or $u = \frac{1}{3}$	1
Obtains both exact solutions $x = -1$ and $x = \log_3 4$	1

Solution.

Since $9^x = (3^x)^2$, let $u = 3^x$ $(u > 0)$. Then

$$3u^2 - 13u + 4 = 0 \implies (3u - 1)(u - 4) = 0 \implies u = \tfrac{1}{3} \text{ or } u = 4.$$

Now $3^x = \frac{1}{3} = 3^{-1} \implies x = -1$, and $3^x = 4 \implies x = \log_3 4$.

$$\boxed{x = -1 \text{ or } x = \log_3 4 \left(= \frac{2\ln 2}{\ln 3}\right)}$$

Q27 North Sydney Boys 2022 Trial Q21 · **3 marks**

Solve $5^x - \dfrac{8}{5^x} = 2$ for x.

Criteria	Marks
Multiplies through by 5^x and forms the quadratic $u^2 - 2u - 8 = 0$ with $u = 5^x$	1
Solves: $u = 4$ or $u = -2$, rejecting $u = -2$ since $5^x > 0$	1
Obtains $x = \log_5 4$	1

Solution.

Let $u = 5^x$ (so $u > 0$). Multiplying $5^x - \dfrac{8}{5^x} = 2$ by 5^x gives

$$(5^x)^2 - 2(5^x) - 8 = 0 \implies u^2 - 2u - 8 = 0 \implies (u - 4)(u + 2) = 0.$$

So $u = 4$ or $u = -2$. Since $5^x > 0$ for all x, we reject $u = -2$. Thus $5^x = 4$, giving

$$\boxed{x = \log_5 4 \left(= \frac{2\ln 2}{\ln 5}\right)}$$

Q28 Baulkham Hills 2022 Trial Q28 · **3 marks**

Solve $4^x - 2^{x+2} = 32$.

Criteria	Marks
Rewrites in terms of 2^x and forms the quadratic $u^2 - 4u - 32 = 0$	1
Factorises and solves: $u = 8$ or $u = -4$, rejecting $u = -4$ since $2^x > 0$	1
Obtains $x = 3$	1

Solution.

Note $4^x = (2^x)^2$ and $2^{x+2} = 4 \cdot 2^x$. Let $u = 2^x$ $(u > 0)$:

$$u^2 - 4u = 32 \implies u^2 - 4u - 32 = 0 \implies (u - 8)(u + 4) = 0.$$

So $u = 8$ or $u = -4$. Since $2^x > 0$, reject $u = -4$. Then $2^x = 8 = 2^3$, so

$$\boxed{x = 3}$$

Q29 Sydney Grammar 2023 Trial Q22 · **3 marks**

Solve $9^x - 3^{x+1} = -2$, leaving your answers in exact form.

Criteria	Marks
Rewrites in terms of 3^x and forms the quadratic $u^2 - 3u + 2 = 0$	1
Factorises and solves: $u = 1$ or $u = 2$	1
Obtains both exact solutions $x = 0$ and $x = \log_3 2$	1

Solution.

Note $9^x = (3^x)^2$ and $3^{x+1} = 3 \cdot 3^x$. Let $u = 3^x$ $(u > 0)$:

$$u^2 - 3u = -2 \implies u^2 - 3u + 2 = 0 \implies (u - 1)(u - 2) = 0.$$

So $u = 1$ or $u = 2$. Then $3^x = 1 \implies x = 0$, and $3^x = 2 \implies x = \log_3 2$.

$$\boxed{x = 0 \text{ or } x = \log_3 2 \left(= \frac{\ln 2}{\ln 3}\right)}$$

Q30 Killara 2024 Prelim Q32(a) · **3 marks**

The graph of $f(x) = 3^{2x} - 10(3^x) + 9$ is shown. Find the x-intercepts using an algebraic method.

Criteria	**Marks**
Sets $f(x) = 0$, substitutes $u = 3^x$ to form $u^2 - 10u + 9 = 0$	1
Factorises and solves: $u = 1$ or $u = 9$	1
Obtains both x-intercepts $x = 0$ and $x = 2$	1

Solution.

The x-intercepts occur where $f(x) = 0$. Since $3^{2x} = (3^x)^2$, let $u = 3^x$ $(u > 0)$:

$$u^2 - 10u + 9 = 0 \implies (u-1)(u-9) = 0 \implies u = 1 \text{ or } u = 9.$$

Then $3^x = 1 \implies x = 0$, and $3^x = 9 = 3^2 \implies x = 2$.

$$\boxed{x\text{-intercepts at } x = 0 \text{ and } x = 2}$$

Exponential growth and decay models

5. Exponential growth and decay models

Q31 Presbyterian 2022 Prelim Q31 · **3 marks**

An ant colony is modelled by $P = 1000e^{0.34t}$, where t is in weeks. (a) Find the population after 10 weeks (nearest 10). (b) When will the population exceed 80 000 ants?

Criteria	Marks
(a) Evaluates $P(10) \approx 29\,960$	1
(b) Sets $1000e^{0.34t} = 80\,000$ and takes logs	1
(b) Solves $t = \dfrac{\ln 80}{0.34} \approx 12.9$ weeks (exceeds 80 000 after 13 weeks)	1

Solution.

(a) Substitute $t = 10$:

$$P = 1000e^{0.34\times 10} = 1000e^{3.4} = 29\,964.1\ldots$$

To the nearest 10, the population is $\boxed{29\,960}$ ants.

(b) Set $P = 80\,000$:

$$1000e^{0.34t} = 80\,000 \implies e^{0.34t} = 80.$$

Take natural logs of both sides:

$$0.34t = \ln 80 \implies t = \frac{\ln 80}{0.34} = 12.888\ldots$$

The population reaches 80 000 at $t \approx 12.9$ weeks, so it exceeds 80 000 ants after about $\boxed{13 \text{ weeks}}$.

Q32 Hornsby Girls 2022 Prelim Q19 · **4 marks**

Yeast cell numbers are given by $N = 100 \times (2.7)^{1.03t}$, t in hours. (a) Initial number of cells. (b) Cells after 6 hours (nearest whole). (c) Time to reach one million cells (nearest hour).

Criteria	Marks
(a) States $N(0) = 100$	1
(b) Evaluates $N(6) \approx 46\,326$	1
(c) Sets $N = 1\,000\,000$ and applies logs	1
(c) Solves $t \approx 9$ hours	1

Solution.

(a) At $t = 0$: $N = 100 \times (2.7)^0 = 100 \times 1 = \boxed{100}$ cells.

(b) At $t = 6$:

$$N = 100 \times (2.7)^{1.03\times 6} = 100 \times (2.7)^{6.18} = 46\,326.3\ldots$$

To the nearest whole number, $\boxed{46\,326}$ cells.

(c) Set $N = 1\,000\,000$:

$$100 \times (2.7)^{1.03t} = 1\,000\,000 \implies (2.7)^{1.03t} = 10\,000.$$

Take logs:

$$1.03t \ln 2.7 = \ln 10\,000 \implies t = \frac{\ln 10\,000}{1.03 \ln 2.7} = 9.002\ldots$$

To the nearest hour, it takes $\boxed{9 \text{ hours}}$.

Q33 Girraween 2022 Prelim Q12(a) · **5 marks**

A tank starts with 3000 litres and leaks as $V = V_0\,e^{-0.135t}$, t in hours. (a) Find V_0. (b) Find V after 5 hours (1 dp). (c) Time for V to fall to 250 litres (nearest minute).

Criteria	Marks
(a) States $V_0 = 3000$ (initial volume)	1
(b) Evaluates $V(5) \approx 1527.5$ litres	1
(c) Sets $3000e^{-0.135t} = 250$ and takes logs	1
(c) Solves $t = \dfrac{\ln 12}{0.135} \approx 18.41$ hours	1
(c) Converts to nearest minute: 18 h 24 min	1

Solution.

(a) At $t = 0$ the volume is the initial 3000 litres, so $V_0 = V(0) = \boxed{3000}$ litres.

(b) At $t = 5$:

$$V = 3000e^{-0.135\times 5} = 3000e^{-0.675} = 1527.469\ldots$$

To 1 decimal place, $\boxed{1527.5 \text{ litres}}$.

(c) Set $V = 250$:

$$3000e^{-0.135t} = 250 \;\Rightarrow\; e^{-0.135t} = \frac{250}{3000} = \frac{1}{12}.$$

Take natural logs:

$$-0.135t = \ln\frac{1}{12} = -\ln 12 \;\Rightarrow\; t = \frac{\ln 12}{0.135} = 18.406\ldots \text{ hours}.$$

Now $0.406\ldots \times 60 = 24.4\ldots$ minutes, so $t \approx 18$ hours 24 minutes, i.e. $\boxed{1104 \text{ minutes}}$ (to the nearest minute).

Q34 Girraween 2021 Prelim Q1(d) · **5 marks**

Bacteria triple every two hours; initially 50. The model is $P = 50 \times 3^{\frac{n}{x}}$ after n hours. (a) Find x. (b) Bacteria after 13 hours. (c) Time to reach 1 000 000 (nearest hour).

Criteria	Marks
(a) States $x = 2$ (tripling period is 2 hours)	1
(b) Evaluates $P(13) \approx 63\,133$	1
(c) Sets $50 \times 3^{n/2} = 1\,000\,000$ and takes logs	1
(c) Solves $\dfrac{n}{2} = \dfrac{\ln 20\,000}{\ln 3}$	1
(c) States $n \approx 18$ hours	1

Solution.

(a) The population triples every 2 hours, so when $n = 2$ the factor $3^{n/x}$ must equal 3^1. Hence $\dfrac{2}{x} = 1$, giving $\boxed{x = 2}$ and $P = 50 \times 3^{n/2}$.

(b) At $n = 13$:

$$P = 50 \times 3^{13/2} = 50 \times 3^{6.5} = 63\,133.2\ldots$$

To the nearest whole number, $\boxed{63\,133}$ bacteria.

(c) Set $P = 1\,000\,000$:

$$50 \times 3^{n/2} = 1\,000\,000 \;\Rightarrow\; 3^{n/2} = 20\,000.$$

Take natural logs:

$$\frac{n}{2}\ln 3 = \ln 20\,000 \;\Rightarrow\; n = \frac{2\ln 20\,000}{\ln 3} = 18.029\ldots$$

To the nearest hour, it takes $\boxed{18 \text{ hours}}$.

Q35 Girraween 2023 Prelim Q10(d) · **4 marks**

A city's population is $P = 35\,000e^{0.024t}$, t in years. (a) Population after 10 years. (b) When will P reach 200 000?

Criteria	Marks
(a) Substitutes $t = 10$ into P	1
(a) Evaluates $P(10) \approx 44\,494$	1
(b) Sets $35\,000e^{0.024t} = 200\,000$ and takes logs	1
(b) Solves $t = \dfrac{1}{0.024}\ln\dfrac{200\,000}{35\,000} \approx 72.6$ years	1

Solution.

(a) At $t = 10$:

$$P = 35\,000e^{0.024\times 10} = 35\,000e^{0.24} = 44\,493.7\ldots$$

To the nearest whole number, $\boxed{44\,494}$ people.

(b) Set $P = 200\,000$:

$$35\,000e^{0.024t} = 200\,000 \;\Rightarrow\; e^{0.024t} = \frac{200\,000}{35\,000} = \frac{40}{7}.$$

Take natural logs:

$$0.024t = \ln\frac{40}{7} \;\Rightarrow\; t = \frac{1}{0.024}\ln\frac{40}{7} = 72.62\ldots$$

The population reaches 200 000 after approximately $\boxed{72.6 \text{ years}}$ (about 73 years).

Q36 Loreto Kirribilli 2022 Prelim Q30 · **6 marks**

A city grows as $P = 50\,000e^{at}$ (t in years, $a > 0$). (a) Initial population. (b) Find a (4 dp) given $P = 52\,400$ after 1 year. (c) Average rate of growth over the first 5 years (1 dp).

Criteria	Marks
(a) States $P(0) = 50\,000$	1
(b) Sets $50\,000e^{a} = 52\,400$ and takes logs	1
(b) States $a = \ln\dfrac{52\,400}{50\,000} \approx 0.0469$	1
(c) Uses average rate $= \dfrac{P(5) - P(0)}{5}$	1
(c) Evaluates $P(5)$	1
(c) States average rate ≈ 2642.8 people per year	1

Solution.

(a) At $t = 0$: $P = 50\,000e^{0} = \boxed{50\,000}$ people.

(b) Set $P = 52\,400$ at $t = 1$:

$$50\,000e^{a} = 52\,400 \implies e^{a} = \frac{52\,400}{50\,000} = 1.048.$$

Take natural logs:

$$a = \ln 1.048 = 0.046883\ldots \approx \boxed{0.0469}.$$

(c) The average rate of growth over the first 5 years is the change in population divided by the time:

$$\text{average rate} = \frac{P(5) - P(0)}{5}.$$

Using $a = 0.0469$: $P(5) = 50\,000e^{0.0469\times 5} = 50\,000e^{0.2345} = 63\,213.8\ldots$, so

$$\text{average rate} = \frac{63\,213.8\ldots - 50\,000}{5} = \frac{13\,213.8\ldots}{5} = 2642.76\ldots$$

The average rate of growth is approximately $\boxed{2642.8 \text{ people per year}}$. (Using the unrounded a gives ≈ 2641.7 people per year.)

Q37 Abbotsleigh 2024 Prelim Q19 · **3 marks**

A car's value is $V = 40\,000e^{-0.4t}$ ($\$V$, t years after purchase). Find, in years and months (nearest month), the time for the value to fall to \$1000.

Criteria	**Marks**
Sets $40\,000e^{-0.4t} = 1000$ and simplifies to $e^{-0.4t} = \frac{1}{40}$	1
Takes logs and solves $t = \dfrac{\ln 40}{0.4} \approx 9.22$ years	1
Converts to 9 years 3 months (nearest month)	1

Solution.

Set $V = 1000$:

$$40\,000e^{-0.4t} = 1000 \implies e^{-0.4t} = \frac{1000}{40\,000} = \frac{1}{40}.$$

Take natural logs of both sides:

$$-0.4t = \ln\frac{1}{40} = -\ln 40 \implies t = \frac{\ln 40}{0.4} = 9.2222\ldots \text{ years.}$$

The fractional part is $0.2222\ldots \times 12 = 2.67$ months, which rounds to 3 months. So the value falls to \$1000 after about $\boxed{\text{9 years and 3 months}}$.