Year 11 Mathematics Advanced

Topic: Introduction to Differentiation (5 of 6)

Review Questions (Worked Solutions): Graphical Applications of the Derivative

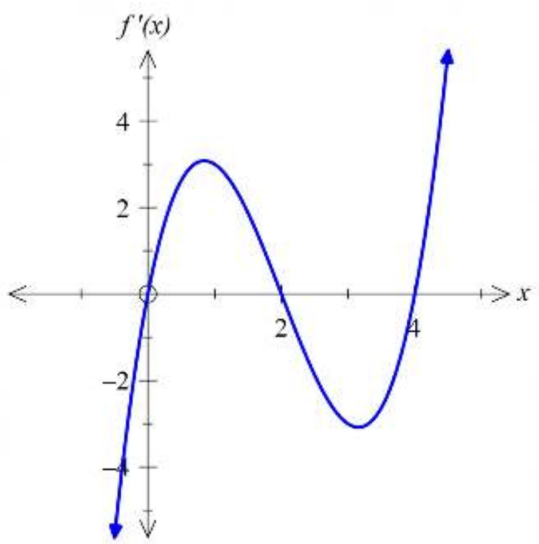
Term 3, 2026

Increasing, decreasing and stationary points

1. Where a function is increasing or decreasing

Q1 Barker 2021 Prelim Q14(cii) · 1 mark

A function $y = f(x)$ has its gradient function $y = f'(x)$ graphed below.



For what values of x is the function $f(x)$ decreasing?

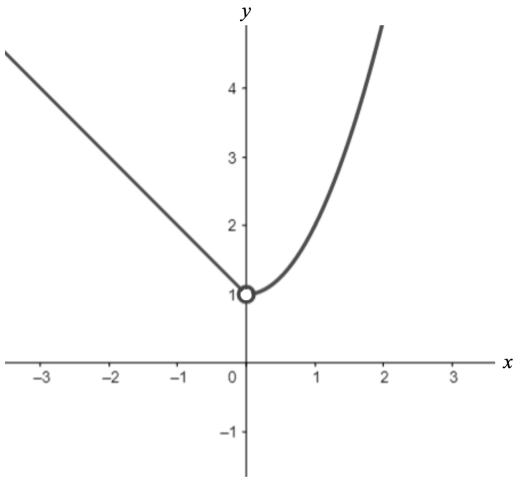
Criteria	Marks
States the interval(s) of x for which $f'(x) < 0$	1

Solution.

A function is decreasing exactly where its gradient is negative, that is, where $f'(x) < 0$. The graph shown is $y = f'(x)$, so read off the interval(s) on which the curve lies below the x-axis. Wherever $f'(x)$ is below the axis, $f(x)$ is decreasing; wherever $f'(x)$ is above the axis, $f(x)$ is increasing. Report those x-values as the answer (the endpoints are the x-intercepts of the $f'(x)$ graph, where $f'(x) = 0$).

Q2 Fort St 2021 Prelim Q6(aii) · **1 mark**

The graph of $y = f(x)$ where $x \neq 0$ is shown below.



For what values of x is $f(x)$ decreasing?

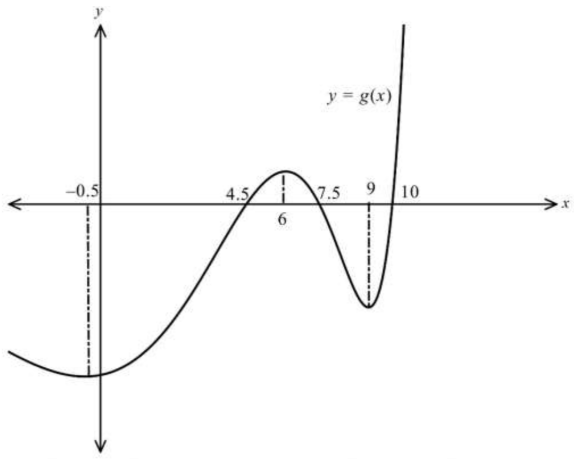
Criteria	**Marks**
Correct interval $x < 0$	1

Solution.

A function is decreasing where its gradient is negative, that is, where the curve slopes downward as x increases (left to right). Reading the given graph of $y = f(x)$, the branch to the left of the y-axis falls as x increases, so the curve is decreasing there. The function is decreasing for $x < 0$.

Q3 Moriah 2021 Prelim Q7 · **1 mark**

The graph of $y = g(x)$ is shown below.



For what values of x is $g'(x) > 0$?

(A) $(-0.5, 6) \cup (9, \infty)$

(B) $(-0.5, 4.5) \cup (7.5, \infty)$

(C) $(4.5, 7.5) \cup (10, \infty)$

(D) $(-\infty, -0.5) \cup (6, 9)$

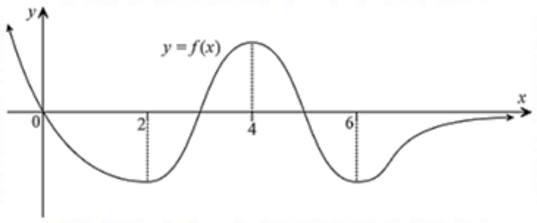
Criteria	**Marks**
Correct option	1

Solution.

The condition $g'(x) > 0$ means the gradient of g is positive, that is, $g(x)$ is increasing. Read the given graph of $y = g(x)$ and identify every interval on which the curve rises as x increases (left to right); these are separated by the turning points of g (where $g'(x) = 0$). The correct option is the union of x-intervals over which the curve is increasing, with the turning-point x-values as the interval endpoints. Selecting the option that lists exactly those rising intervals gives the answer.

Q4 Pymble 2022 Prelim Q18 · **2 marks**

Consider the function $y = f(x)$ shown below.



For which values of x is the function increasing?

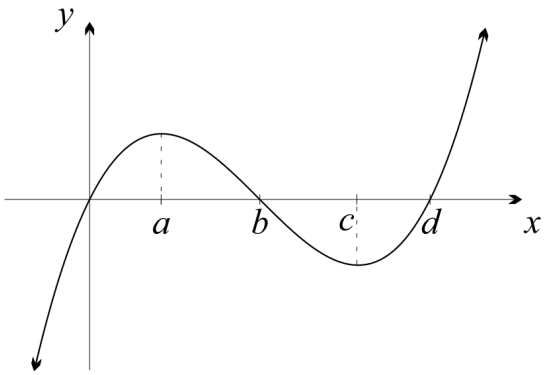
Criteria	**Marks**
Identifies the turning points of f (where $f'(x) = 0$) as the interval endpoints	1
States the correct interval(s) on which the curve is rising	1

Solution.

A function is increasing where its gradient is positive, that is, where the curve slopes upward as x increases (left to right). Locate the turning points of the given curve $y = f(x)$: at each one the tangent is horizontal, so $f'(x) = 0$, and these x-values are the endpoints of the intervals. Between the turning points, take the interval(s) on which the curve rises. The function is increasing on the union of those rising intervals, read directly from the graph.

Q5 Sydney Grammar 2023 Prelim Q4 · **1 mark**

The graph of the curve $y = f(x)$ is shown below. For what values of x will the tangent to the curve have a negative gradient?



(A) $x < a$

(B) $a < x < b$

(C) $a < x < c$

(D) $x > c$

Criteria	Marks
Correct option	1

Solution.

The tangent has a negative gradient exactly where the curve is decreasing, that is, where the curve slopes downward as x increases (left to right). Read the given graph of $y = f(x)$ and find the interval, bounded by the labelled turning points a, b and c on the x-axis, on which the curve is falling. The option matching that decreasing interval is the answer.

Q6 Sydney Girls 2020 Trial Q22 · **2 marks**

Show that $f(x) = -x^3 - 5x + 1$ is always decreasing.

Criteria	Marks
Differentiates to obtain $f'(x) = -3x^2 - 5$	1
Argues $f'(x) < 0$ for all x, so $f(x)$ is always decreasing	1

Solution.

A function is decreasing wherever its derivative is negative. Differentiating,

$$f'(x) = -3x^2 - 5.$$

For every real x, $x^2 \geq 0$, so $-3x^2 \leq 0$ and hence $f'(x) = -3x^2 - 5 \leq -5 < 0$. Thus $f'(x) < 0$ for all values of x, so $f(x)$ is always decreasing.

Q7 Girraween 2020 Trial Q31 · **2 marks**

Let $f(x) = x^3 - 3x^2 + kx + 8$, where k is a constant. Find the values of k for which $f(x)$ is an increasing function.

Criteria	Marks
Differentiates to obtain $f'(x) = 3x^2 - 6x + k$ and requires $f'(x) \geq 0$ for all x	1
Applies the discriminant condition to obtain $k \geq 3$	1

Solution.

A function is increasing when its derivative is non-negative for all x. Differentiating,

$$f'(x) = 3x^2 - 6x + k.$$

This is a quadratic in x with positive leading coefficient, so it stays ≥ 0 for all x exactly when it has no distinct real roots, that is when its discriminant is not positive:

$$\Delta = (-6)^2 - 4(3)(k) = 36 - 12k \leq 0.$$

Solving, $36 \leq 12k$, so $k \geq 3$. Therefore $f(x)$ is an increasing function for $k \geq 3$.

2. Stationary points (finding and justifying)

Q8 Knox 2021 Prelim Q12(biii) · **1 mark**

The share price P after t hours of an exchange opening was $P = -t^3 + 6t^2$ for $0 \leq t \leq 6$. When is the share price a maximum?

Criteria	Marks
States the share price is a maximum at $t = 4$ hours	1

Solution.

For a maximum, the share price is stationary, so $\dfrac{dP}{dt} = 0$. Differentiating,

$$\frac{dP}{dt} = -3t^2 + 12t = -3t(t-4).$$

Setting $\dfrac{dP}{dt} = 0$ gives $t = 0$ or $t = 4$. At $t = 0$, $P = 0$, and at $t = 4$, $P = -4^3 + 6(4)^2 = -64 + 96 = 32$. Checking the endpoints, $P(0) = 0$ and $P(6) = -216 + 216 = 0$, so the greatest value on $0 \leq t \leq 6$ occurs at $t = 4$. The share price is a maximum at $t = 4$ hours.

Q9 Baulkham Hills 2022 Prelim Q15 · **3 marks**

For what values of x is the function $y = x^3 + x^2 - x - 2$ stationary?

Criteria	Marks
Differentiates to obtain $\dfrac{dy}{dx} = 3x^2 + 2x - 1$	1
Sets $\dfrac{dy}{dx} = 0$ and factorises to $(3x-1)(x+1) = 0$	1
States $x = \dfrac{1}{3}$ and $x = -1$	1

Solution.

The function is stationary where $\dfrac{dy}{dx} = 0$. Differentiating,

$$\frac{dy}{dx} = 3x^2 + 2x - 1.$$

Setting this equal to zero and factorising,

$$3x^2 + 2x - 1 = 0 \quad \Longrightarrow \quad (3x-1)(x+1) = 0.$$

Hence $x = \dfrac{1}{3}$ or $x = -1$. The function is stationary at $x = \dfrac{1}{3}$ and $x = -1$.

Q10 North Sydney Girls 2021 Prelim Q2(bii) · **2 marks**

Given that $f(x) = (5x+1)^3$ and $g(x) = x^2$, let $h(x) = f(x)\,g(x)$. Find the x-coordinate(s) of the point(s) where the tangent to $y = h(x)$ is horizontal.

Criteria	Marks
Differentiates to obtain $h'(x) = x(5x+1)^2(25x+2)$	1
Solves $h'(x) = 0$ for $x = 0$, $x = -\frac{1}{5}$ and $x = -\frac{2}{25}$	1

Solution.

Using the product rule with $f(x) = (5x+1)^3$ and $g(x) = x^2$,

$$h'(x) = f'(x)g(x) + f(x)g'(x) = 15(5x+1)^2 \cdot x^2 + (5x+1)^3 \cdot 2x.$$

Factoring out $x(5x+1)^2$,

$$h'(x) = x(5x+1)^2\big[15x + 2(5x+1)\big] = x(5x+1)^2(25x+2).$$

The tangent is horizontal when $h'(x) = 0$, that is when $x = 0$, $5x+1 = 0$ or $25x+2 = 0$. Therefore the x-coordinates are $x = 0$, $x = -\frac{1}{5}$ and $x = -\frac{2}{25}$.

Q11 Loreto Kirribilli 2022 Prelim Q29(b) · **2 marks**

It is given that $y = \dfrac{2x+1}{3x^2+6}$ and that $\dfrac{dy}{dx} = \dfrac{6\left(2-x-x^2\right)}{\left(3x^2+6\right)^2}$. Hence, find the values of x where the tangent to the curve $y = \dfrac{2x+1}{3x^2+6}$ is horizontal.

Criteria	Marks
Sets $\frac{dy}{dx} = 0$, reducing to $2 - x - x^2 = 0$	1
Solves to obtain $x = -2$ and $x = 1$	1

Solution.

The tangent is horizontal when $\dfrac{dy}{dx} = 0$. Since $\left(3x^2+6\right)^2 > 0$ for all x, this requires the numerator to be zero:

$$6\left(2-x-x^2\right) = 0 \quad \Longrightarrow \quad x^2 + x - 2 = 0.$$

Factoring, $(x+2)(x-1) = 0$, so $x = -2$ or $x = 1$. The tangent to the curve is horizontal at $x = -2$ and $x = 1$.

Q12 Kambala 2023 Prelim Q23(a) · **3 marks**

Consider the curve $f(x) = x^3 - 6x^2 + 9x - 4$. Find any stationary points and determine the nature.

Criteria	Marks
Differentiates and solves $f'(x) = 0$ to obtain $x = 1$ and $x = 3$	1
Finds the stationary points $(1, 0)$ and $(3, -4)$	1
Justifies the nature: $(1, 0)$ is a maximum and $(3, -4)$ is a minimum	1

Solution.

Differentiating,

$$f'(x) = 3x^2 - 12x + 9 = 3(x^2 - 4x + 3) = 3(x - 1)(x - 3).$$

Stationary points occur where $f'(x) = 0$, that is at $x = 1$ and $x = 3$. Evaluating,

$$f(1) = 1 - 6 + 9 - 4 = 0, \qquad f(3) = 27 - 54 + 27 - 4 = -4,$$

so the stationary points are $(1, 0)$ and $(3, -4)$. To determine their nature, use the second derivative $f''(x) = 6x - 12$. At $x = 1$, $f''(1) = -6 < 0$, so $(1, 0)$ is a local maximum. At $x = 3$, $f''(3) = 6 > 0$, so $(3, -4)$ is a local minimum.

Q13 North Sydney Boys 2024 Prelim Q13 · **2 marks**

Find the coordinates of the points on the curve $y = (x^2 - 1)^3$ where the tangent to the curve is parallel to the x-axis.

Criteria	Marks
Differentiates to obtain $\dfrac{dy}{dx} = 6x(x - 1)^2(x + 1)^2$	1
Solves $\dfrac{dy}{dx} = 0$ and states the points $(-1, 0)$, $(0, -1)$ and $(1, 0)$	1

Solution.

The tangent is parallel to the x-axis when $\dfrac{dy}{dx} = 0$. Using the chain rule,

$$\frac{dy}{dx} = 3(x^2 - 1)^2 \cdot 2x = 6x(x^2 - 1)^2 = 6x(x - 1)^2(x + 1)^2.$$

Setting $\dfrac{dy}{dx} = 0$ gives $x = 0$, $x = 1$ or $x = -1$. Evaluating $y = (x^2 - 1)^3$ at each:

$$y(-1) = 0, \qquad y(0) = (-1)^3 = -1, \qquad y(1) = 0.$$

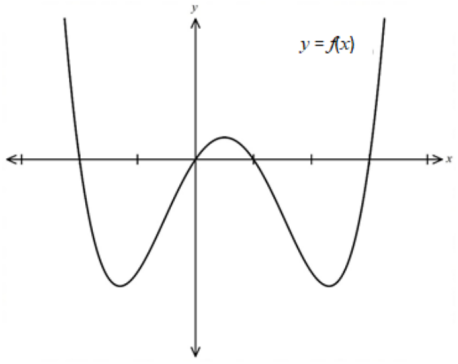
The coordinates of the points are $(-1, 0)$, $(0, -1)$ and $(1, 0)$.

Graphs of the derivative

3. Interpreting a graph and its derivative

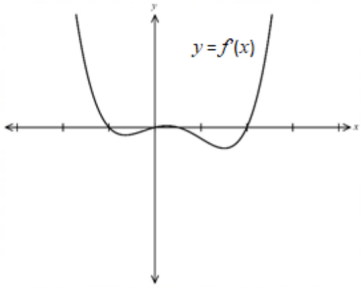
Q14 Cherrybrook Tech 2021 Prelim Q8 · 1 mark

The graph of $y = f(x)$ is shown below. Which graph could represent $y = f'(x)$?

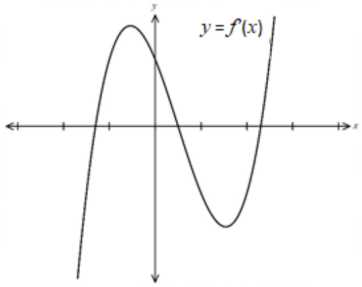


Which graph could represent $y = f'(x)$?

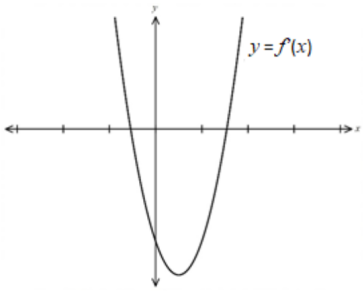
(A)



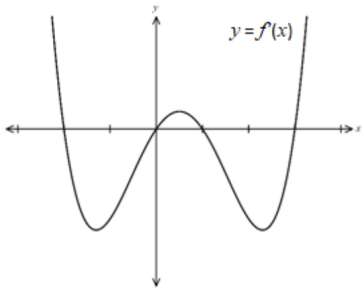
(B)



(C)



(D)



Criteria	Marks
Correct option	1

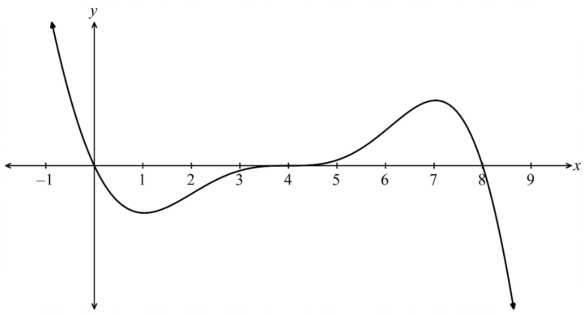
Solution.

Read the gradient of $y = f(x)$ off the given graph and match it to the correct $y = f'(x)$. At every stationary point of f (each turning point) the gradient is zero, so the graph of f' must have an x-intercept there. On every interval where f is increasing the gradient is positive, so f' lies above the x-axis; on every interval

where f is decreasing the gradient is negative, so f' lies below the x-axis. The correct option is the graph that cuts the axis at each turning point of f and takes the matching sign on each interval, and whose degree is one lower than f. Select the option whose x-intercepts sit under the turning points of f and whose sign matches where f rises and falls.

Q15 Hurlstone 2024 Prelim Q26(a) · **1 mark**

The function $y = f(x)$ is shown below. For what values of x is $f'(x) > 0$?



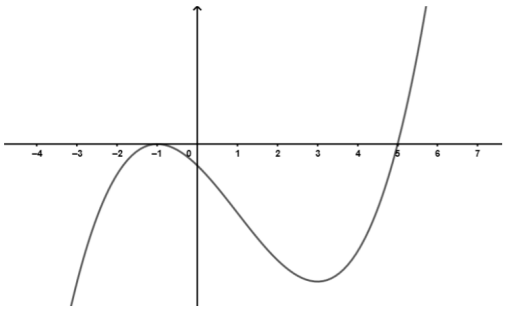
Criteria	Marks
Correct interval(s) where f is increasing	1

Solution.

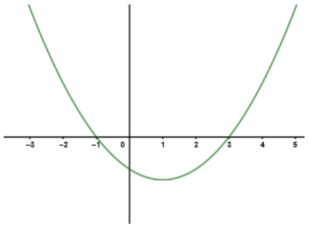
The derivative $f'(x)$ is the gradient of $y = f(x)$, so $f'(x) > 0$ exactly where the curve is increasing (sloping upward as x increases). Read off the intervals of the given graph on which f rises: between consecutive turning points where the curve goes up. From the figure the curve is increasing for $1 < x < 4$ and for $4 < x < 7$, so $f'(x) > 0$ for $1 < x < 4$ or $4 < x < 7$.

Q16 Hurlstone 2021 Prelim Q16 · **1 mark**

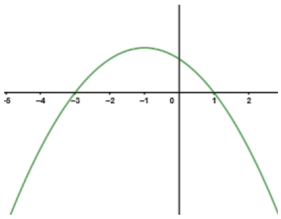
Which of the following is the graph of $\frac{dy}{dx}$ for the graph shown below?



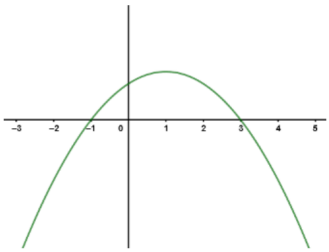
A



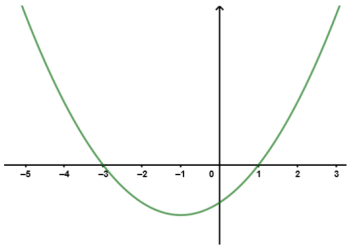
B



C



D



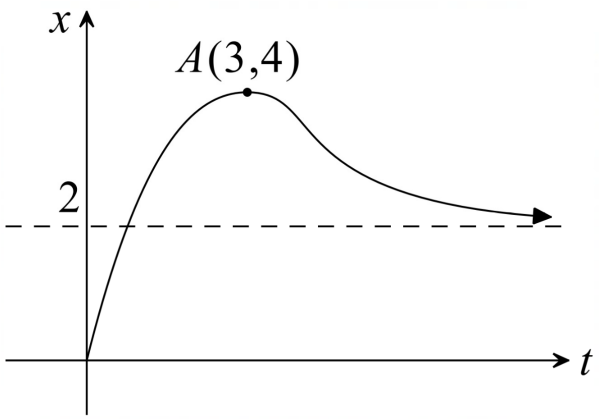
Criteria	Marks
Correct option (A)	1

Solution.

Translate the gradient of the given curve into the graph of $\frac{dy}{dx}$. The gradient of the curve is zero at its turning points, which occur at $x = -1$ and $x = 3$, so $\frac{dy}{dx} = 0$ there: the derivative graph must have x-intercepts at $x = -1$ and $x = 3$ (this narrows the choice to A or C). Next check the sign: the curve is increasing for $x < -1$ and for $x > 3$, so $\frac{dy}{dx} > 0$ (above the axis) on those intervals, and decreasing for $-1 < x < 3$, so $\frac{dy}{dx} < 0$ (below the axis) there. The graph with intercepts at $x = -1, 3$, positive outside and negative in between, is option A. The answer is **(A)**.

Q17 Baulkham Hills 2022 Prelim Q28 · **3 marks**

The graph below shows the displacement of a particle over time. Sketch a possible graph of the particle's velocity, $\frac{dx}{dt}$, on the axes above.



Criteria	Marks
Correct basic shape of the velocity graph	1
x-intercept of $\frac{dx}{dt}$ at $t = 3$ (the stationary point of the displacement graph, not a tangent to the axis)	1
Correct long-term behaviour: velocity approaching the asymptote $\frac{dx}{dt} = 0$ from below	1

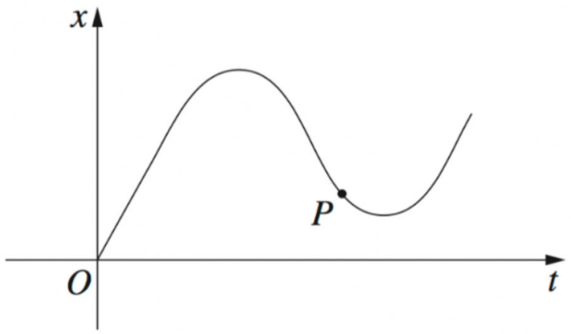
Solution.

Velocity is the derivative of displacement, so the height of the velocity graph at any time equals the gradient (slope) of the displacement graph at that time. Where the displacement graph is stationary (its turning point at $t = 3$) the gradient is zero, so the velocity graph has an x-intercept at $t = 3$, crossing the axis there rather than just touching it. Before the turning point the displacement is rising or falling according to the given curve, fixing the sign of $\frac{dx}{dt}$ on each interval; after the turning point the displacement curve flattens out, so its gradient tends to zero and the velocity graph approaches the horizontal asymptote $\frac{dx}{dt} = 0$ from below. The sketch is a smooth velocity curve with the correct overall shape, an x-intercept at $t = 3$, and a tail approaching $\frac{dx}{dt} = 0$ from below.

Q18 ★ Penrith 2022 Prelim Q10 · **1 mark**

★ *Involves acceleration, a Year 12 topic (the second derivative).*

The graph shows the displacement x of a particle moving along a straight line as a function of time t, where $t \geq 0$. Which statement describes the motion of the particle at the point P?



(A) The velocity is negative, and acceleration is positive.

(B) The velocity is negative, and acceleration is negative.

(C) The velocity is positive, and acceleration is positive.

(D) The velocity is positive, and acceleration is negative.

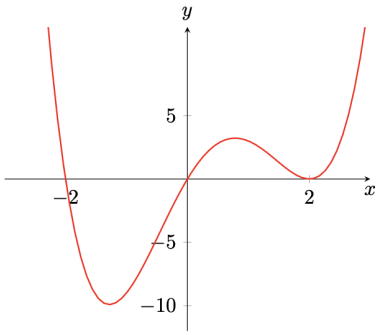
Criteria	**Marks**
Correct option	1

Solution.

Velocity is the gradient of the displacement-time graph and acceleration is the rate of change of that gradient (the concavity of the graph). At P, read the slope of the curve: if the displacement is increasing there the velocity is positive, and if it is decreasing the velocity is negative. Then read the concavity: if the curve is concave up at P the gradient is increasing, so the acceleration is positive, and if the curve is concave down the gradient is decreasing, so the acceleration is negative. Match the sign of the slope and the sign of the concavity at P from the figure to select the correct statement.

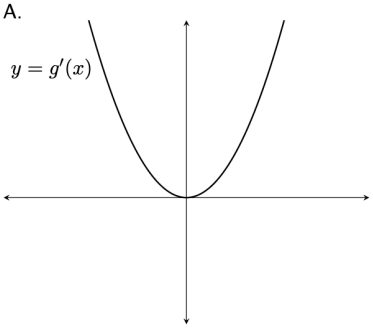
Q19 Kambala 2024 Prelim Q10 · **1 mark**

The graph below shows the curve $y = g(x)$. Which of the graphs below shows the curve $y = g'(x)$?

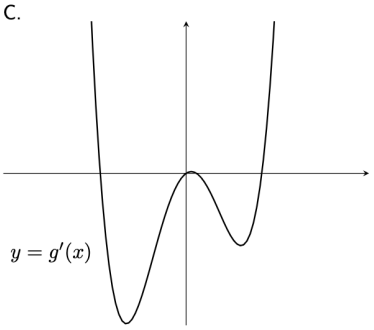


Which of the graphs below shows the curve $y = g'(x)$?

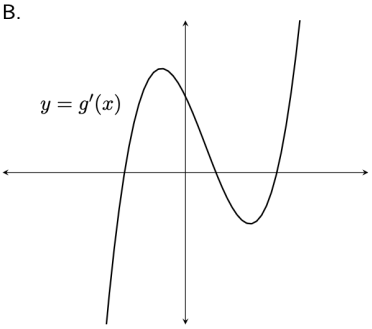
A.



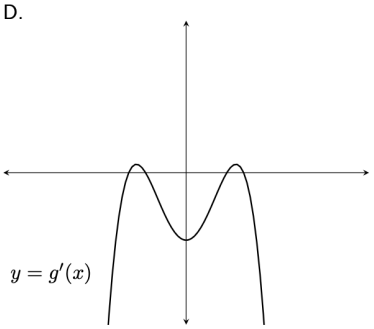
C.



B.



D.



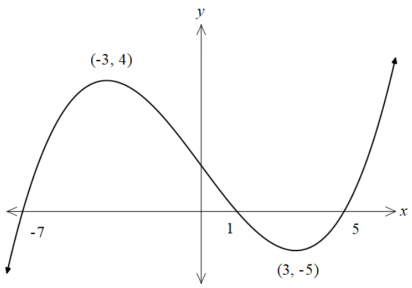
Criteria	Marks
Correct option	1

Solution.

The graph of $y = g'(x)$ records the gradient of $y = g(x)$. Wherever g has a stationary point (a turning point) the gradient is zero, so g' has an x-intercept at that x-value. On intervals where g is increasing the gradient is positive, so g' lies above the x-axis; on intervals where g is decreasing the gradient is negative, so g' lies below the x-axis. The correct option is the graph whose x-intercepts line up with the turning points of g, whose sign on each interval matches where g rises and falls, and whose degree is one lower than g.

Q20 North Sydney Boys 2024 Prelim Q4 · **1 mark**

The graph of $y = f(x)$ is shown below. For what values of x is $f(x) < 0$ and $f'(x) < 0$?



(A) $(-\infty, -7) \cup (1, 5)$

(B) $(-\infty, -3) \cup (3, \infty)$

(C) $(-3, 3)$

(D) $(1, 3)$

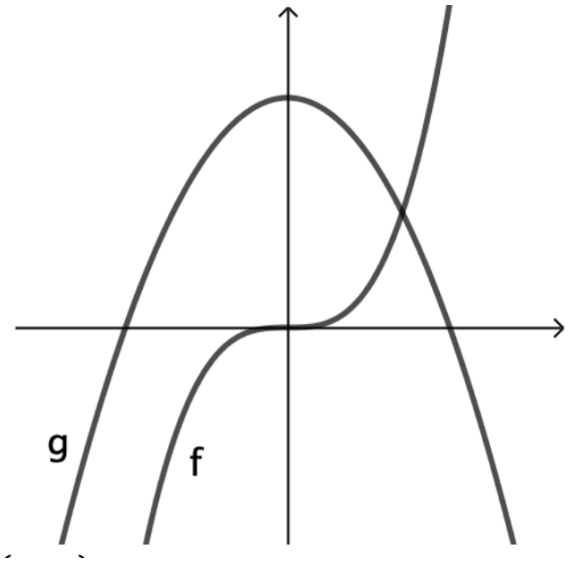
Criteria	Marks
Correct option	1

Solution.

Two conditions must hold at the same time. First, $f(x) < 0$ means the curve lies below the x-axis, so identify the x-values where the graph is beneath the axis (between the relevant x-intercepts). Second, $f'(x) < 0$ means the gradient is negative, so the curve must also be decreasing (sloping downward) there. The required set of x-values is the overlap of these two regions: where the curve is both below the x-axis and falling. Read both features off the figure and choose the interval where they hold simultaneously.

Q21 Knox 2021 Prelim Q10 · **1 mark**

The graph shows two functions $y = f(x)$ and $y = g(x)$.



Define $h(x) = f(g(x))$. How many stationary points does $y = h(x)$ have?

(A) 0

(B) 1

(C) 2

(D) 3

Criteria	Marks
Correct option	1

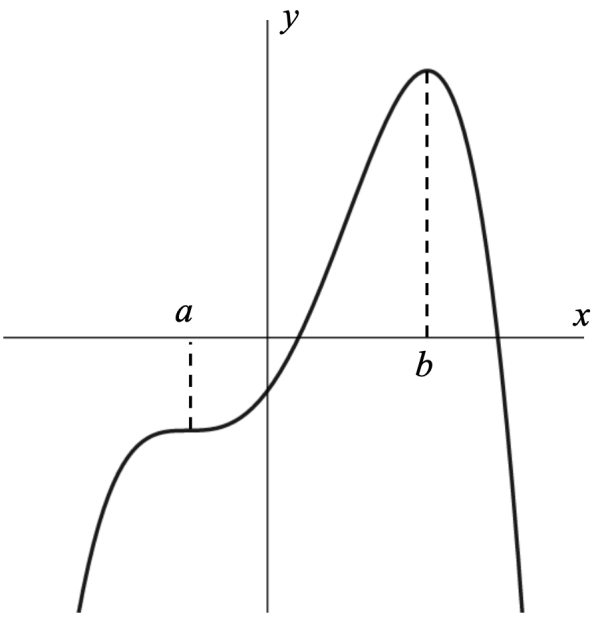
Solution.

By the chain rule, $h'(x) = f'(g(x)) \cdot g'(x)$, so $y = h(x)$ is stationary wherever $h'(x) = 0$, that is wherever either factor is zero: where $g'(x) = 0$ (the stationary points of g), or where $f'(g(x)) = 0$ (the values of x at which $g(x)$ equals an input where f is stationary). Read the two given graphs: locate the x-values where g has a horizontal tangent, and the x-values where $g(x)$ takes a value at which f has a horizontal tangent. Count the distinct x-values satisfying either condition; that count is the number of stationary points of $y = h(x)$, and the matching option is the answer.

4. Sketching $y = f'(x)$ from $y = f(x)$

Q22 SCEGGS 2022 Prelim Q19 · **2 marks**

The graph of $y = f(x)$ is shown below. On the number plane below sketch $y = f'(x)$.



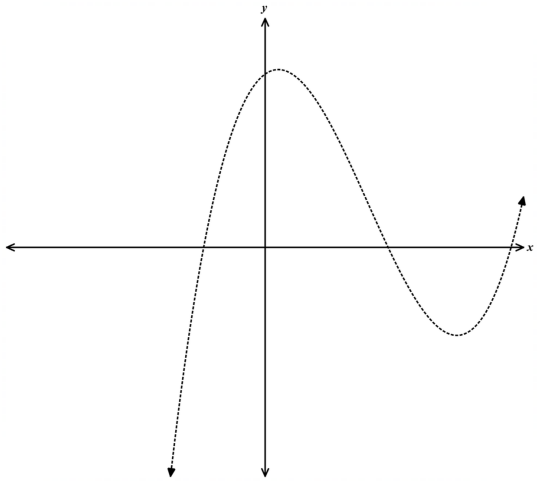
Criteria	Marks
$f'(x)$ has x-intercepts at the stationary points of f, with correct sign of f' each side	1
Correct overall shape of f'	1

Solution.

Read the gradient of $y = f(x)$ off the given graph and translate it into the height of $y = f'(x)$. Wherever f has a stationary point (a turning point), the gradient is zero, so $f'(x) = 0$ there: these give the x-intercepts of f' (they occur at the labelled turning-point x-values on the figure). On any interval where f is increasing the gradient is positive, so f' lies above the x-axis; on any interval where f is decreasing the gradient is negative, so f' lies below the x-axis. The sketch of f' therefore cuts the x-axis at each turning point of f and switches sign there, sitting above the axis wherever f rises and below wherever f falls, with a smooth curve of one degree lower than f connecting these features.

Q23 Killara 2023 Prelim Q26 · **2 marks**

The function $f(x)$ is given on the axis below. Hence sketch the gradient function on the same axis.



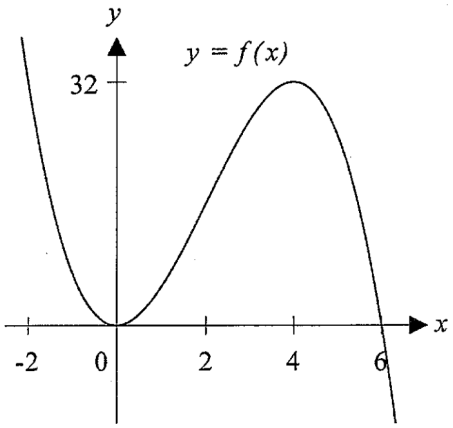
Criteria	**Marks**
Shows the stationary points explicitly ($f' = 0$ at each turning point of f)	1
Correct shape of the gradient function	1

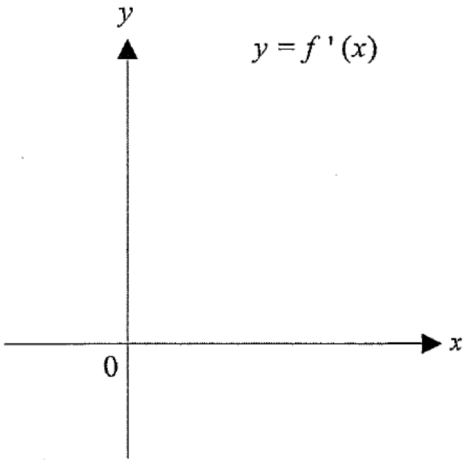
Solution.

Sketch $y = f'(x)$ from the given graph of f. Identify every stationary point (turning point) of f on the figure: at each one the tangent is horizontal, so the gradient is zero and f' has an x-intercept directly below/above that x-value. Between these points, decide the sign of the gradient: where f slopes upward $f'(x) > 0$ (the gradient graph is above the x-axis) and where f slopes downward $f'(x) < 0$ (below the axis). Draw a smooth curve that passes through the x-intercepts at the stationary points and takes the correct sign on each interval, with the gradient function being one degree lower than f. The final sketch shows f' crossing the axis at each turning point of f and lying above/below the axis to match where f is rising/falling.

Q24 St George Girls 2023 Prelim Q12 · **2 marks**

The diagram shows the graph of the curve $y = f(x)$. Sketch the graph of the curve $y = f'(x)$ showing the shape of the curve and the x-intercepts.





Criteria	Marks
Correct x-intercepts of f' (at the stationary points of f)	1
Correct shape of f'	1

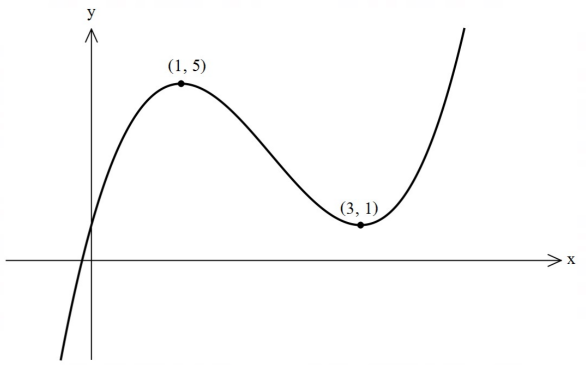
Solution.

Convert the gradient of the given curve $y = f(x)$ into the graph $y = f'(x)$. The x-intercepts of f' occur exactly at the stationary points of f: wherever the given curve has a turning point the gradient is zero, so f' crosses the x-axis there. Determine the sign of f' between the intercepts from the slope of f: $f'(x) > 0$ (above the axis) on intervals where f is increasing and $f'(x) < 0$ (below the axis) where f is decreasing. Join these with a smooth curve one degree lower than f. The final sketch must clearly show the x-intercepts (at

the turning points of f) and the correct overall shape, sitting above the axis where f rises and below where f falls.

Q25 North Sydney Girls 2022 Prelim Q23(b) · **2 marks**

The graph of a function $y = f(x)$ passes through the points $(1, 5)$ and $(3, 1)$ as shown below. On the number plane above, sketch the gradient function $y = f'(x)$, indicating all necessary features.



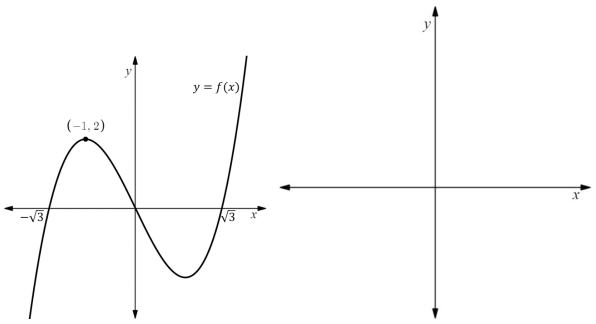
Criteria	Marks
$f'(x)$ has x-intercepts at the stationary points of f (the turning points shown)	1
Correct shape of f', with correct sign on each interval and all necessary features shown	1

Solution.

Sketch $y = f'(x)$ by reading the gradient of the given curve. The turning points of f (the maximum and minimum shown on the figure, near the marked points) are where the gradient is zero, so f' has an x-intercept at each of those x-values. Between and beyond the turning points, take the sign of f' from the slope of f: where f is increasing $f'(x) > 0$ (above the x-axis) and where f is decreasing $f'(x) < 0$ (below the axis). Draw a smooth gradient curve one degree lower than f that cuts the axis at each stationary point and has the correct sign on each interval. Indicate all necessary features: the x-intercepts (at the stationary points) and the intervals where f' is positive or negative, matching where the original curve rises and falls.

Q26 North Sydney Boys 2024 Prelim Q16 · **2 marks**

The diagram above on the left shows the graph of the curve $y = f(x)$. On the set of axes above on the right, sketch the graph of the curve $y = f'(x)$ showing the shape of the curve and its x intercepts.



Criteria	Marks
Correct x-intercepts of f' (at the stationary points of f)	1
Correct shape of f' (correct sign on each interval)	1

Solution.

Draw $y = f'(x)$ on the right-hand axes from the given graph of f on the left. The x-intercepts of f' are located at the stationary points of f: at each turning point the tangent is horizontal, so the gradient is zero and f' meets the x-axis there. Between the intercepts, set the sign of f' from the slope of f: $f'(x) > 0$ (curve above the axis) wherever f is increasing and $f'(x) < 0$ (curve below the axis) wherever f is decreasing. Connect with a smooth curve one degree lower than f. The completed sketch shows f' crossing the x-axis at each turning point of f and taking the correct sign on every interval, matching where the original curve rises and falls.