Year 11 Mathematics Advanced

Topic: Introduction to Differentiation (4 of 6)

Review Questions (Worked Solutions): Rates of Change and Motion

Term 3, 2026

Rates of change

1. Rates of change from a model

Q1 Sydney Grammar 2022 Trial Q6 · **1 mark**

A ghost gum tree was planted in 2010. The height of the tree H, in metres, is a function of the tree's age, in years, after planting. Which of the following is the correct interpretation of $H'(4) = 3.2$?

(A) Between 2010 and 2014, the height increased at a rate of 3.2 m per year.

(B) Between 2010 and 2014, the height increased by 3.2 m.

(C) At the start of 2014, the height was increasing at a rate of 3.2 m per year.

(D) At the start of 2014, the height was 3.2 m.

Criteria	**Marks**
Correct option (C)	1

Solution.

The derivative $H'(4)$ is the instantaneous rate of change of the height with respect to age, evaluated at age 4 years. Since the tree was planted in 2010, an age of 4 years corresponds to the start of 2014. So $H'(4) = 3.2$ means that at the start of 2014 the height was increasing at a rate of 3.2 m per year. Options (A) and (B) describe an average change over the interval from 2010 to 2014, not an instantaneous rate, and (D) describes the height value rather than its rate of change. The answer is **(C)**.

Q2 Fort St 2021 Prelim Q5(ai) · **1 mark**

A dragster is a car that races in a straight line. Its displacement d over 400 m is modelled by the equation $d(t) = t^2 + 3t - 2$, where time t in seconds is $0 \leq t \leq 20$. The difference quotient is given by

$$\frac{d(t+h) - d(t)}{h}.$$

What is the meaning of the difference quotient in this context?

Criteria	Marks
Identifies the difference quotient as the average rate of change (average velocity/speed)	1

Solution.

The difference quotient measures the change in displacement divided by the change in time over the interval from t to $t + h$. In this context it represents the average rate of change of displacement, that is, the average velocity (or average speed) of the dragster over that period of time. It is the gradient of the secant line joining the two points on the displacement graph, not the instantaneous rate of change given by $d'(t)$.

Q3 Killara 2021 Prelim Q23(b) · **2 marks**

A petrol tank holds 35 000 litres of petrol. Petrol is draining from the tank and the volume of petrol remaining in the petrol tank after t minutes is given by $V = 35\,000\left(1 - \frac{t}{30}\right)^2$, where V is measured in litres. Find the instantaneous rate of flow at $t = 15$.

Criteria	Marks
Differentiates to obtain $\frac{dV}{dt} = \frac{700t}{9} - \frac{7000}{3}$	1
Substitutes $t = 15$ to get $\frac{dV}{dt} \approx -1166.67$ L/min	1

Solution.

The instantaneous rate of flow is $\frac{dV}{dt}$. Differentiating using the chain rule:

$$\frac{dV}{dt} = 35\,000 \cdot 2\left(1 - \frac{t}{30}\right) \cdot \left(-\frac{1}{30}\right) = -\frac{7000}{3}\left(1 - \frac{t}{30}\right).$$

Substituting $t = 15$:

$$\left.\frac{dV}{dt}\right|_{t=15} = -\frac{7000}{3}\left(1 - \frac{15}{30}\right) = -\frac{7000}{3} \cdot \frac{1}{2} = -\frac{3500}{3} \approx -1166.67.$$

The instantaneous rate of flow at $t = 15$ is $-\frac{3500}{3} \approx -1166.67$ L/min. The negative sign shows the petrol is draining at about 1166.67 litres per minute.

Q4 Hornsby Girls 2021 Prelim Q4 · **1 mark**

If the volume of water, V litres, in a tank after t minutes, is given by $V = 600t - t^3$, then after five minutes:

(A) The volume of water is decreasing at a rate of 525 litres per minute.

(B) The volume of water is increasing at a rate of 525 litres per minute.

(C) The volume of water is decreasing at a rate of 2 875 litres per minute.

(D) The volume of water is increasing at a rate of 2 875 litres per minute.

Criteria	**Marks**
Correct option (B)	1

Solution.

The rate of change of volume is $\dfrac{dV}{dt} = 600 - 3t^2$. After five minutes:

$$\left.\frac{dV}{dt}\right|_{t=5} = 600 - 3(5)^2 = 600 - 75 = 525.$$

The rate is positive, so the volume is increasing at 525 litres per minute. The answer is **(B)**.

Q5 St George Girls 2023 Prelim Q25(b) · **2 marks**

The displacement x metres of a particle moving along the y-axis at a time t seconds is given by $x = 12+4t-t^2$. Find the average velocity of the particle in the first 3 seconds.

Criteria	**Marks**
Evaluates $x(0) = 12$ and $x(3) = 15$	1
Computes the average velocity $\dfrac{x(3) - x(0)}{3} = 1$ m/s	1

Solution.

The average velocity over the first 3 seconds is $\dfrac{x(3) - x(0)}{3 - 0}$. Evaluating the displacement:

$$x(0) = 12 + 4(0) - (0)^2 = 12, \qquad x(3) = 12 + 4(3) - (3)^2 = 12 + 12 - 9 = 15.$$

Therefore

$$\frac{x(3) - x(0)}{3} = \frac{15 - 12}{3} = \frac{3}{3} = 1.$$

The average velocity of the particle in the first 3 seconds is 1 m/s.

Q6 Pymble 2022 Prelim Q27(c) · **2 marks**

A swimming pool is to be emptied for maintenance. The quantity of water, Q litres, remaining in the pool at a time t minutes after it starts to drain, is given by $Q(t) = 2000(25 - t)^2$, $t \geq 0$. At what time does the rate of flow of water from the pool reach 20 000 L/minute?

Criteria	**Marks**
Differentiates to obtain $\dfrac{dQ}{dt} = 4000t - 100\,000$	1
Solves $\lvert dQ/dt \rvert = 20\,000$ for $t = 20$ minutes	1

Solution.

The rate of flow of water is $\dfrac{dQ}{dt}$. Differentiating using the chain rule:

$$\frac{dQ}{dt} = 2000 \cdot 2(25 - t) \cdot (-1) = -4000(25 - t) = 4000t - 100\,000.$$

The water is flowing out, so $\dfrac{dQ}{dt}$ is negative. Setting the rate of outflow equal to 20 000 L/minute:

$$4000t - 100\,000 = -20\,000.$$

Solving:

$$4000t = 80\,000, \qquad t = 20.$$

The rate of flow of water from the pool reaches 20 000 L/minute at $t = 20$ minutes.

Q7 North Sydney Girls 2023 Prelim Q22 · **2 marks**

While writing this exam, the teacher accidentally knocked over and spilt their coffee onto their rug creating a stain. The area (in cm^2) of the coffee stain after time t seconds is given by the equation $A = 12 - \dfrac{12}{2t+1}$. Find the instantaneous rate of change in the area of the coffee stain after 5 seconds.

Criteria	**Marks**
Differentiates to obtain $\dfrac{dA}{dt} = \dfrac{24}{(2t+1)^2}$	1
Substitutes $t = 5$ to get $\dfrac{dA}{dt} = \dfrac{24}{121}$ cm^2/s	1

Solution.

The instantaneous rate of change of the area is $\dfrac{dA}{dt}$. Writing $A = 12 - 12(2t+1)^{-1}$ and differentiating using the chain rule:

$$\frac{dA}{dt} = -12 \cdot (-1)(2t+1)^{-2} \cdot 2 = \frac{24}{(2t+1)^2}.$$

Substituting $t = 5$:

$$\left.\frac{dA}{dt}\right|_{t=5} = \frac{24}{(2(5)+1)^2} = \frac{24}{11^2} = \frac{24}{121} \approx 0.198.$$

The instantaneous rate of change in the area after 5 seconds is $\dfrac{24}{121} \approx 0.198$ cm^2/s.

Q8 Killara 2024 Prelim Q29(d) · **3 marks**

Water is leaking from the bottom of a tank. It is given that V is the volume of water in litres remaining in the tank after t minutes and $0 \leq t \leq 25$, where $V(t) = 500\left(1 - \frac{1}{25}t\right)^3$. The initial rate of water leaking from the tank is 60 L/min. Hence, calculate the time at which water is leaking at half the initial rate, correct to the nearest whole number.

Criteria	**Marks**
Differentiates to obtain $\frac{dV}{dt} = -\frac{12}{125}(25-t)^2$	1
Sets $\frac{dV}{dt} = -30$ (half the initial rate) and forms $(25-t)^2 = 312.5$	1
Solves for the valid root $t = 25 - \frac{25\sqrt{2}}{2} \approx 7$ minutes	1

Solution.

The rate of leaking is $\frac{dV}{dt}$. Differentiating using the chain rule:

$$\frac{dV}{dt} = 500 \cdot 3\left(1 - \frac{t}{25}\right)^2 \cdot \left(-\frac{1}{25}\right) = -60\left(1 - \frac{t}{25}\right)^2 = -\frac{12}{125}(25-t)^2.$$

At $t = 0$ this gives $\frac{dV}{dt} = -60$, so the initial rate is 60 L/min as stated. Half the initial rate is 30 L/min, so we require $\frac{dV}{dt} = -30$:

$$-\frac{12}{125}(25-t)^2 = -30 \quad \Longrightarrow \quad (25-t)^2 = \frac{30 \cdot 125}{12} = 312.5.$$

Taking the square root, $25 - t = \pm\sqrt{312.5} = \pm\frac{25\sqrt{2}}{2}$, so

$$t = 25 \mp \frac{25\sqrt{2}}{2} \approx 7.32 \text{ or } 42.68.$$

Since $0 \leq t \leq 25$, only $t = 25 - \frac{25\sqrt{2}}{2} \approx 7.32$ is valid. To the nearest whole number, the water is leaking at half the initial rate at $t = 7$ minutes.

Motion in a straight line

2. Velocity and displacement

Q9 Killara 2021 Prelim Q20(b) · **1 mark**

The displacement in metres of a particle travelling in a straight line is given by $x = 4t^2 - 12t$, where t is the time taken in seconds. Find the velocity (v) of the particle as a function of time.

Criteria	Marks
Correct velocity function	1

Solution.

Velocity is the derivative of displacement with respect to time. Differentiating $x = 4t^2 - 12t$ gives $v = \dfrac{dx}{dt} = 8t - 12$. The velocity as a function of time is $v = 8t - 12$.

Q10 Pymble 2022 Prelim Q26(c) · **1 mark**

A particle is moving in a straight line. At time t seconds its displacement, in metres, from a fixed point O on the line is given by $x = t^2 - 2t$. What is the velocity of the particle at the time it has returned to its starting position?

Criteria	Marks
Correct velocity $v = 2$ m/s	1

Solution.

The starting position is $x(0) = 0$. The particle returns to its starting position when $x = 0$ again: $t^2 - 2t = 0$, so $t(t-2) = 0$, giving $t = 0$ or $t = 2$. It returns at $t = 2$ seconds. The velocity is $v = \dfrac{dx}{dt} = 2t - 2$, so at $t = 2$ the velocity is $v = 2(2) - 2 = 2$. The velocity is $v = 2$ m/s.

Q11 St George Girls 2021 Prelim Q5(ai) · **1 mark**

A particle is moving along the x-axis. Its displacement x centimetres at time t seconds is given by $x = 4 + 12t^2 - t^3$ for $t \geq 0$. What is the initial displacement of the particle?

Criteria	Marks
Correct initial displacement 4 cm	1

Solution.

The initial displacement occurs at $t = 0$. Substituting $t = 0$ into $x = 4+12t^2-t^3$ gives $x = 4+12(0)^2-(0)^3 = 4$. The initial displacement is 4 cm.

Q12 International Grammar 2021 Prelim Q23(a) · **1 mark**

The displacement from the origin (x metres) of a particle travelling in a straight line at a time (t seconds) is given by $x = 3t^4 - 16t^3 + 18t^2$, where $t \geq 0$. Find the velocity of the particle after 2 seconds.

Criteria	**Marks**
Correct velocity $v = -24$ m/s	1

Solution.

Velocity is the derivative of displacement: $v = \dfrac{dx}{dt} = 12t^3 - 48t^2 + 36t$. After 2 seconds, $v = 12(2)^3 - 48(2)^2 + 36(2) = 96 - 192 + 72 = -24$. The velocity after 2 seconds is $v = -24$ m/s.

Q13 Hornsby Girls 2022 Prelim Q21(c) · **2 marks**

A particle is travelling in one dimension. Its displacement in metres at time t seconds is given by $x = t^3 - 6t^2 + 9t + 3$. The particle is at rest at times t_1 and t_2 (where $t_1 < t_2$). Complete the table of values below.

t	0	$t_1 =$ _____	$t_2 =$ _____
x			

Criteria	**Marks**
Correct rest times $t_1 = 1$ and $t_2 = 3$	1
Correct displacement values $x = 3$, $x = 7$, $x = 3$	1

Solution.

The particle is at rest when $v = \dfrac{dx}{dt} = 3t^2 - 12t + 9 = 0$, that is $3(t^2 - 4t + 3) = 3(t-1)(t-3) = 0$, so $t_1 = 1$ and $t_2 = 3$. Substituting into $x = t^3 - 6t^2 + 9t + 3$: at $t = 0$, $x = 3$; at $t = 1$, $x = 1 - 6 + 9 + 3 = 7$; at $t = 3$, $x = 27 - 54 + 27 + 3 = 3$. The completed table is:

t	0	$t_1 = 1$	$t_2 = 3$
x	3	7	3

Q14 Gosford 2024 Prelim Q23(c) · **3 marks**

Gerry's displacement from home is modelled by the equation $x = -0.5t^2 + 2t + 6$, where x is measured in kilometres and t is measured in hours. Sketch the graph of velocity vs time for Gerry's journey for $0 \leq t \leq 6$.

Criteria	**Marks**
Finds the velocity function $v = 2 - t$	1
Identifies key values: $v(0) = 2$, zero at $t = 2$, $v(6) = -4$	1
Sketches the correct straight line over $0 \leq t \leq 6$	1

Solution.

Velocity is the derivative of displacement: $v = \dfrac{dx}{dt} = -t + 2 = 2 - t$. This is a straight line with v-intercept 2 and gradient -1. At $t = 0$, $v = 2$; the velocity is zero at $t = 2$; and at $t = 6$, $v = 2 - 6 = -4$. The sketch is a straight line segment starting at $(0, 2)$ on the vertical (v) axis, sloping downward, crossing the horizontal (t) axis at $(2, 0)$, and ending at $(6, -4)$.

Q15 Fort St 2022 Prelim Q12(biii) · **2 marks**

The displacement, x metres, of an object moving along a straight line over time, t seconds, is given by $x = (t^3 + 1)^4$. Show that the particle is never at the origin.

Criteria	**Marks**
Establishes $t^3 + 1 \geq 1 > 0$ for $t \geq 0$	1
Concludes $x = (t^3 + 1)^4 > 0$, so x is never 0	1

Solution.

The particle is at the origin when $x = 0$. Since $t \geq 0$, we have $t^3 \geq 0$, so $t^3 + 1 \geq 1 > 0$. Raising a positive quantity to the fourth power keeps it positive, so $x = (t^3 + 1)^4 \geq 1^4 = 1 > 0$ for all $t \geq 0$. Since x is always at least 1 and can never equal 0, the particle is never at the origin.

Q16 Penrith 2023 Prelim Q22(d) · **2 marks**

A particle is moving in a straight line. Its displacement, x m, from a fixed point, O, after t seconds is given by $x = 3t^2 - 6t$, where $t \geq 0$. Is the particle speeding up or slowing down when $t = 0.5$ seconds? Explain how you determined this.

Criteria	**Marks**
Finds $v(0.5) = -3$ and $a(0.5) = 6$	1
Concludes slowing down because velocity and acceleration have opposite signs	1

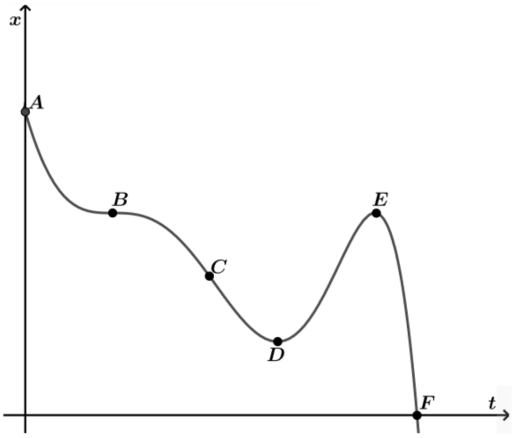
Solution.

The velocity is $v = \dfrac{dx}{dt} = 6t - 6$ and the acceleration is $a = \dfrac{dv}{dt} = 6$. At $t = 0.5$: $v = 6(0.5) - 6 = -3$ m/s and $a = 6$ m/s^2. The velocity is negative and the acceleration is positive, so velocity and acceleration act in opposite directions. When velocity and acceleration have opposite signs the particle is slowing down. Therefore the particle is slowing down at $t = 0.5$ seconds.

3. At rest, changing direction and distance travelled

Q17 Hurlstone 2021 Prelim Q12 · **1 mark**

For the displacement graph below, at which of the points will the particle be at rest?



(A) B, C, D and E only

(B) A and F only

(C) D and E only

(D) B, D and E only

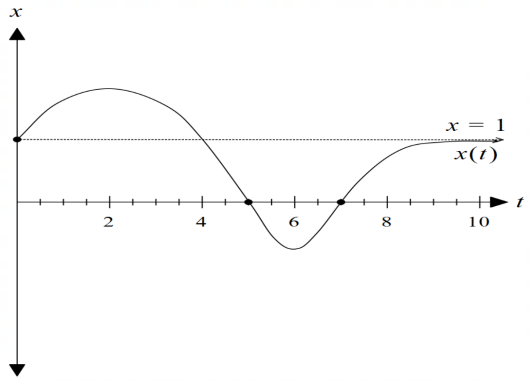
Criteria	Marks
Correct option (D)	1

Solution.

A particle is at rest when its velocity is zero, that is, where the displacement graph has a horizontal tangent. This occurs at the stationary points and the horizontal point of inflection: B (point of inflection), D (minimum turning point) and E (maximum turning point). The answer is **(D)**.

Q18 Manly 2022 Prelim Q29(a) · **1 mark**

A particle is moving along the x-axis, and its displacement from the origin is given by $x(t)$, where t is the time in seconds. $x(t)$ has a horizontal asymptote along the line $x = 1$. The graph of $x(t)$ is shown below. When is the particle at rest?



Criteria	Marks
States the particle is at rest at $t = 2$ and $t = 6$	1

Solution.

The particle is at rest when its velocity is zero, that is, where the graph of $x(t)$ has a horizontal tangent. Reading the graph, this occurs at $t = 2$ and $t = 6$ seconds.

Q19 St George Girls 2022 Prelim Q29(a) · **2 marks**

A particle moves from rest at a point A in a straight line, so that t seconds after leaving A its displacement, x metres, is given by $x = 16t^2 - 2t^4$. Find what time the particle next comes to rest.

Criteria	Marks
Differentiates to get $v = \dfrac{dx}{dt} = 32t - 8t^3$	1
Solves $v = 0$ and selects $t = 2$ as the time it next comes to rest	1

Solution.

The velocity is $v = \dfrac{dx}{dt} = 32t - 8t^3 = 8t(4 - t^2) = 8t(2 - t)(2 + t)$. The particle is at rest when $v = 0$:

$$8t(2 - t)(2 + t) = 0 \implies t = 0,\ t = 2 \text{ or } t = -2.$$

Since the particle starts from rest at $t = 0$ and $t \geq 0$, the time it next comes to rest is $t = 2$ seconds.

Q20 Gosford 2023 Prelim Q20(b) · **1 mark**

A particle moves in a straight line described by the function $x(t) = -t^2 + 5t + 6$. When does the particle change directions?

Criteria	Marks
Differentiates and solves $v = 5 - 2t = 0$ to get $t = \dfrac{5}{2}$	1

Solution.

The velocity is $v = \dfrac{dx}{dt} = -2t + 5$. The particle changes direction when its velocity is zero:

$$-2t + 5 = 0 \implies t = \frac{5}{2}.$$

So the particle changes direction at $t = \dfrac{5}{2} = 2.5$ seconds.

Q21 Moriah 2021 Prelim Q27(b) · **2 marks**

The displacement from the origin (x metres) of a particle travelling in a straight line at a time (t seconds) is given by $x = 3t^4 - 16t^3 + 18t^2$, where $t \geq 0$. It can be shown that the velocity $\dfrac{dx}{dt} = 0$ at time $t = 1$ second. Find any other time(s) when the particle is stationary.

Criteria	Marks
Differentiates to get $v = 12t^3 - 48t^2 + 36t$ and factorises	1
Solves $v = 0$ to give the other stationary times $t = 0$ and $t = 3$	1

Solution.

The velocity is $v = \dfrac{dx}{dt} = 12t^3 - 48t^2 + 36t = 12t(t^2 - 4t + 3) = 12t(t-1)(t-3)$. The particle is stationary when $v = 0$:

$$12t(t-1)(t-3) = 0 \implies t = 0,\ t = 1 \text{ or } t = 3.$$

Apart from $t = 1$, the particle is also stationary at $t = 0$ and $t = 3$ seconds.

Q22 Hornsby Girls 2021 Prelim Q19 · **3 marks**

A particle moves in a straight line so that its displacement $x(t)$ from a fixed point on the line at time $t \geq 0$ is given by $x(t) = 6 + 8t - 4\sqrt{t^2+16}$. Show that the particle never comes to rest.

Criteria	**Marks**
Differentiates using the chain rule to get $x'(t) = 8 - \dfrac{4t}{\sqrt{t^2+16}}$	1
States the particle is at rest when $x'(t) = 0$ and forms the equation	1
Shows there is no real solution, so the particle never comes to rest	1

Solution.

Write $x(t) = 6 + 8t - 4(t^2+16)^{1/2}$. Differentiating with the chain rule:

$$x'(t) = 8 - 4 \cdot \tfrac{1}{2}(t^2+16)^{-1/2}(2t) = 8 - \frac{4t}{\sqrt{t^2+16}}.$$

The particle comes to rest when $x'(t) = 0$:

$$8 - \frac{4t}{\sqrt{t^2+16}} = 0 \implies \frac{4t}{\sqrt{t^2+16}} = 8 \implies 4t = 8\sqrt{t^2+16} \implies t = 2\sqrt{t^2+16}.$$

Squaring both sides, $t^2 = 4(t^2+16) = 4t^2 + 64$, so $3t^2 = -64$. There is no real value of t satisfying this, so $x'(t)$ is never zero. Therefore the particle never comes to rest.