Year 11 Mathematics Advanced

Topic: Introduction to Differentiation (3 of 6)

Review Questions (Worked Solutions): Tangents, Normals and Angle of Inclination

Term 3, 2026

Tangents, normals and angle of inclination

1. Equations of tangents and normals

Q1 Sydney Grammar 2021 Prelim Q3 · **1 mark**

What is the value of the gradient of the tangent to the curve $y = 3x^2 + 2x$ at the point $(1, 5)$?

(A) 1

(B) 4

(C) 6

(D) 8

Criteria	Marks
Correct option (D)	1

Solution.

The gradient of the tangent is $\dfrac{dy}{dx} = 6x + 2$. At $x = 1$ this is $6(1) + 2 = 8$. The answer is **(D)**.

Q2 Loreto Kirribilli 2022 Prelim Q14 · **2 marks**

Find the gradient of the curve $y = \dfrac{1}{x^2}$ at the point $\left(\dfrac{1}{2}, 4\right)$.

Criteria	Marks
Correct derivative $\dfrac{dy}{dx} = -\dfrac{2}{x^3}$	1
Correct gradient at $x = \frac{1}{2}$	1

Solution.

Write $y = x^{-2}$, so $\dfrac{dy}{dx} = -2x^{-3} = -\dfrac{2}{x^3}$. At $x = \frac{1}{2}$,

$$\frac{dy}{dx} = -\frac{2}{\left(\frac{1}{2}\right)^3} = -\frac{2}{\frac{1}{8}} = -16.$$

The gradient is -16.

Q3 Gosford 2023 Prelim Q26(a) · **2 marks**

Let $y = 3x - x^2$. Show that the gradient of the tangent at $x = 3$ is equal to -3.

Criteria	**Marks**
Correct derivative $\dfrac{dy}{dx} = 3 - 2x$	1
Substitutes $x = 3$ to obtain -3	1

Solution.

Differentiating, $\dfrac{dy}{dx} = 3 - 2x$. At $x = 3$,

$$\frac{dy}{dx} = 3 - 2(3) = 3 - 6 = -3,$$

so the gradient of the tangent at $x = 3$ is -3, as required.

Q4 Sydney Grammar 2023 Prelim Q11(a) · **3 marks**

Find the equation of the normal to the curve $y = 2\sqrt{x}$, at the point on the curve where $x = 9$.

Criteria	**Marks**
Correct gradient of the tangent $\frac{1}{3}$	1
Gradient of the normal -3 and the point $(9, 6)$	1
Correct equation of the normal	1

Solution.

Write $y = 2x^{1/2}$, so $\dfrac{dy}{dx} = x^{-1/2} = \dfrac{1}{\sqrt{x}}$. At $x = 9$ the tangent gradient is $\dfrac{1}{\sqrt{9}} = \dfrac{1}{3}$, and the point on the curve is $\left(9,\ 2\sqrt{9}\right) = (9, 6)$. The normal is perpendicular to the tangent, so its gradient is $-\dfrac{1}{1/3} = -3$. Using $y - y_1 = m(x - x_1)$:

$$y - 6 = -3(x - 9) = -3x + 27,$$

so the equation of the normal is $y = -3x + 33$.

Q5 North Sydney Girls 2021 Prelim Q1(bi) · **3 marks**

Find the equation of the normal to the curve $y = (2x + 1)^4$ at the point where $x = -1$. Leave your answer in general form.

Criteria	Marks
Correct tangent gradient -8 (via the chain rule) and the point $(-1, 1)$	1
Gradient of the normal $\frac{1}{8}$	1
Correct equation in general form	1

Solution.

By the chain rule, $\dfrac{dy}{dx} = 4(2x + 1)^3 \cdot 2 = 8(2x + 1)^3$. At $x = -1$, $2x + 1 = -1$, so the tangent gradient is $8(-1)^3 = -8$, and the point is $\big(-1, (2(-1) + 1)^4\big) = (-1, 1)$. The normal gradient is $-\dfrac{1}{-8} = \dfrac{1}{8}$. Using $y - y_1 = m(x - x_1)$:

$$y - 1 = \frac{1}{8}(x + 1).$$

Multiplying by 8: $8y - 8 = x + 1$, so in general form $x - 8y + 9 = 0$.

Q6 Knox 2022 Prelim Q12(b) · **3 marks**

Find the equation of the normal to the curve $y = 3 - 4x^2 - x^3$, at the point where $x = -1$. Leave your final answer in general form.

Criteria	Marks
Correct tangent gradient 5 and the point $(-1, 0)$	1
Gradient of the normal $-\frac{1}{5}$	1
Correct equation in general form	1

Solution.

Differentiating, $\dfrac{dy}{dx} = -8x - 3x^2$. At $x = -1$,

$$\frac{dy}{dx} = -8(-1) - 3(-1)^2 = 8 - 3 = 5,$$

so the tangent gradient is 5. The point is $\big(-1,\ 3 - 4(-1)^2 - (-1)^3\big) = (-1,\ 3 - 4 + 1) = (-1, 0)$. The normal gradient is $-\dfrac{1}{5}$. Using $y - y_1 = m(x - x_1)$:

$$y - 0 = -\frac{1}{5}(x + 1).$$

Multiplying by 5: $5y = -(x + 1)$, so in general form $x + 5y + 1 = 0$.

Q7 Killara 2023 Prelim Q31(a) · **3 marks**

Consider the following function:

$$f(x) = \frac{1}{5}(x-2)^2(5-x).$$

Point $P(5,0)$ is on the graph of $f(x)$. Find the equation of the tangent to the graph $f(x)$ at $P(5,0)$.

Criteria	**Marks**
Correct derivative $f'(x)$	1
Correct tangent gradient $f'(5) = -\frac{9}{5}$	1
Correct equation of the tangent	1

Solution.

Expand: $f(x) = \frac{1}{5}(x-2)^2(5-x)$. Differentiating (product and chain rules):

$$f'(x) = \frac{1}{5}\Big[2(x-2)(5-x) + (x-2)^2(-1)\Big] = \frac{1}{5}(x-2)\big[2(5-x) - (x-2)\big] = \frac{1}{5}(x-2)(12-3x).$$

At $x = 5$: $f'(5) = \frac{1}{5}(5-2)(12-15) = \frac{1}{5}(3)(-3) = -\frac{9}{5}$. Using $y - y_1 = m(x - x_1)$ at $P(5,0)$:

$$y - 0 = -\frac{9}{5}(x-5) = -\frac{9}{5}x + 9,$$

so the equation of the tangent is $y = -\frac{9}{5}x + 9$ (equivalently $9x + 5y - 45 = 0$).

Q8 Baulkham Hills 2024 Prelim Q17 · **3 marks**

Find the equation of the tangent to the curve $y = 8x^3 - 6x^2 + x - 1$ at the point where $x = 1$.

Criteria	**Marks**
Correct derivative $\dfrac{dy}{dx} = 24x^2 - 12x + 1$	1
Tangent gradient 13 and the point $(1, 2)$	1
Correct equation of the tangent	1

Solution.

Differentiating, $\dfrac{dy}{dx} = 24x^2 - 12x + 1$. At $x = 1$,

$$\frac{dy}{dx} = 24(1)^2 - 12(1) + 1 = 13,$$

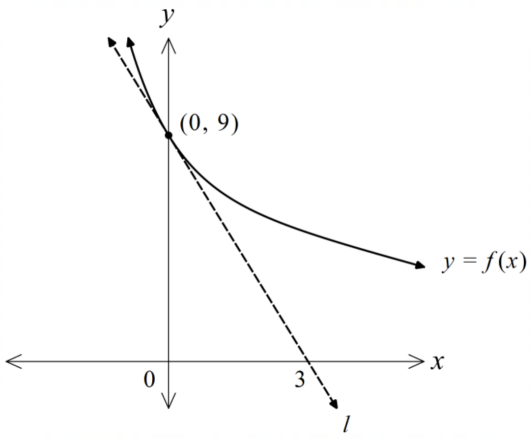
and the point is $\big(1,\ 8(1)^3 - 6(1)^2 + 1 - 1\big) = (1, 2)$. Using $y - y_1 = m(x - x_1)$:

$$y - 2 = 13(x - 1) = 13x - 13,$$

so the equation of the tangent is $y = 13x - 11$.

Q9 Killara 2022 Prelim Q36 · **2 marks**

The graph below shows a function $y = f(x)$, and its tangent at the point $(0, 9)$ represented by line l.



Given a new function $g(x) = \sqrt{f(x)}$, find the value of $g'(0)$.

Criteria	**Marks**
Differentiates $g(x) = \sqrt{f(x)}$ by the chain rule as $g'(x) = \dfrac{f'(x)}{2\sqrt{f(x)}}$	1
Evaluates using $f(0) = 9$ and the gradient of line l read from the graph	1

Solution.

Write $g(x) = \big(f(x)\big)^{1/2}$ and apply the chain rule:

$$g'(x) = \frac{1}{2}\big(f(x)\big)^{-1/2} \cdot f'(x) = \frac{f'(x)}{2\sqrt{f(x)}}.$$

From the graph, $f(0) = 9$ so $\sqrt{f(0)} = 3$, and $f'(0)$ equals the gradient of the tangent line l at $(0, 9)$, read from the diagram. Therefore

$$g'(0) = \frac{f'(0)}{2\sqrt{9}} = \frac{f'(0)}{6},$$

where $f'(0)$ is the gradient of line l.

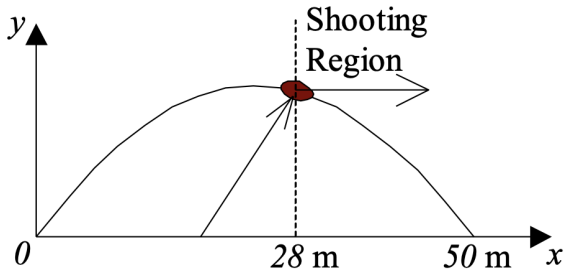
Q10 Knox 2023 Prelim Q35 · **3 marks**

In a rifle club, discs are projected and follow the path of the parabola with equation

$$y = -\frac{x^2}{25} + 2x.$$

The rules of competition include the following:

1. Competitors must only shoot at the disc when it is in the "shooting region", as shown in the figure below.
2. Competitors must shoot from a position no further than 23 m from the origin.
3. The number of points awarded for each shot is equal to the height of the disc when first struck by a bullet.
4. Bonus points are awarded if the disc is completely destroyed. (NB. To guarantee complete destruction, the direction of the bullet must be normal to the path of the disc.)
5. You may assume the direction of motion for the disc is always tangential to its parabolic path.



What advice about shooting position (relative to O) and shooting angle (relative to the x-axis) would you give a competitor wishing to achieve maximum points? Justify your answer with calculations.

Criteria	**Marks**
Recognises the bullet must be normal to the path at the entry point $x = 28$	1
Correct shooting angle	1
Correct shooting position	1

Solution.

By Rule 3, a competitor should hit the disc as soon as it enters the shooting region, at $x = 28$, where the height is greatest for a permitted shot. By Rule 4, to destroy the disc the bullet must be normal to the path there. Differentiating,

$$\frac{dy}{dx} = -\frac{2x}{25} + 2, \qquad \text{so at } x = 28, \quad \frac{dy}{dx} = -\frac{2(28)}{25} + 2 = -\frac{6}{25}.$$

This is the tangent gradient, so the gradient of the normal is $-\dfrac{1}{-6/25} = \dfrac{25}{6}$. The shooting angle to the x-axis is

$$\theta = \tan^{-1}\left(\frac{25}{6}\right) \approx 76.5^\circ.$$

The height of the disc at $x = 28$ is $y = -\dfrac{28^2}{25} + 2(28) = 24.64$ m. The bullet travels along the normal, so the horizontal distance from the shooting position to the point below the disc is $\dfrac{24.64}{\tan 76.5^\circ} \approx 5.9$ m. Hence the shooting position is about $28 - 5.9 \approx 22.1$ m from O (which satisfies the 23 m limit). Advise the competitor to shoot from approximately 22.1 m from O at an angle of about 76.5° to the x-axis.

2. Tangents from an external point

Q11 Hurlstone 2022 Trial Q11(d) · **2 marks**

The equation of a curve C is given by $y = x^2 + ax + b$, where a and b are constants. The tangent to C at $(0, 5)$ passes through $(5, 20)$. Find the values of a and b.

Criteria	**Marks**
Uses the point $(0, 5)$ on the curve to find b, and the gradient at $x = 0$	1
Uses the second point on the tangent to solve for a	1

Solution.

Since $(0, 5)$ lies on C: $5 = 0 + 0 + b$, so $b = 5$. Also $\dfrac{dy}{dx} = 2x + a$, so the gradient at $(0, 5)$ is $2(0) + a = a$. The tangent at $(0, 5)$ is $y = 5 + ax$, and it passes through $(5, 20)$:

$$20 = 5 + a(5) \implies 15 = 5a \implies a = 3.$$

Therefore $a = 3$ and $b = 5$.

Q12 Sydney Grammar 2021 Prelim Q14(c) · **4 marks**

The line ℓ is the tangent to the curve $y = 1 - \dfrac{1}{x}$ at the point where $x = a$, where a is a constant.

(i) Show that ℓ has equation $x - a^2 y = 2a - a^2$.

(ii) If the line ℓ passes through the origin, find the value of a and hence the equation of ℓ.

Criteria	**Marks**
(i) Finds the gradient $\frac{1}{a^2}$ and the point $\left(a,\ 1 - \frac{1}{a}\right)$	1
(i) Forms the tangent and rearranges to the required form	1
(ii) Substitutes the origin to obtain $a = 2$ (rejecting $a = 0$)	1
(ii) States the equation of ℓ	1

Solution.

(i) $y = 1 - x^{-1}$, so $\dfrac{dy}{dx} = \dfrac{1}{x^2}$. At $x = a$ the gradient is $\dfrac{1}{a^2}$ and the point is $\left(a,\ 1 - \dfrac{1}{a}\right)$. The tangent is

$$y - \left(1 - \frac{1}{a}\right) = \frac{1}{a^2}(x - a).$$

Multiplying by a^2: $a^2 y - a^2 + a = x - a$, so $a^2 y = x - 2a + a^2$, giving $x - a^2 y = 2a - a^2$.
(ii) The origin $(0, 0)$ satisfies ℓ: $0 - a^2(0) = 2a - a^2$, so $a^2 - 2a = 0$, i.e. $a(a - 2) = 0$. Since $a = 0$ is not on the curve (the curve is undefined at $x = 0$), $a = 2$. With $a = 2$: $x - 4y = 4 - 4 = 0$, so $\ell :\ y = \dfrac{x}{4}$.

Q13 Abbotsleigh 2024 Prelim Q26 · **4 marks**

(a) Show that the tangent to the curve $y = x^2 + 1$ at the point $P(a,\ a^2 + 1)$ has equation $2ax - y - a^2 + 1 = 0$.

(b) Find the coordinates of any points on the curve where the tangent to the curve passes through the origin.

Criteria	Marks
(a) Finds the gradient $2a$ at P and forms the tangent	1
(a) Rearranges to the required form $2ax - y - a^2 + 1 = 0$	1
(b) Substitutes the origin to obtain $a = \pm 1$	1
(b) States the points $(1, 2)$ and $(-1, 2)$	1

Solution.

(a) $y = x^2 + 1$, so $\dfrac{dy}{dx} = 2x$. At $P(a,\ a^2 + 1)$ the gradient is $2a$. The tangent is

$$y - (a^2 + 1) = 2a(x - a), \quad \text{so} \quad y = 2ax - 2a^2 + a^2 + 1 = 2ax - a^2 + 1.$$

Rearranging gives $2ax - y - a^2 + 1 = 0$.
(b) The tangent passes through the origin $(0, 0)$:

$$2a(0) - 0 - a^2 + 1 = 0 \implies 1 - a^2 = 0 \implies a = \pm 1.$$

For $a = 1$: $P = (1, 2)$. For $a = -1$: $P = (-1, 2)$. The points are $(1, 2)$ and $(-1, 2)$.

Q14 Fort St 2022 Prelim Q9(b) · **3 marks**

Find the equation of the tangents to the curve $y = x^2 - 4x$, at the points where the curve cuts the x-axis. Hence, what is the point of intersection of these tangents?

Criteria	Marks
Finds the x-axis intercepts $x = 0, 4$ and the gradient $2x - 4$	1
Finds both tangent equations $y = -4x$ and $y = 4x - 16$	1
Correct point of intersection $(2, -8)$	1

Solution.

The curve cuts the x-axis where $x^2 - 4x = 0$, that is $x(x - 4) = 0$, so at $x = 0$ and $x = 4$. The gradient is $\dfrac{dy}{dx} = 2x - 4$. At $x = 0$ the gradient is -4, giving the tangent $y = -4x$; at $x = 4$ the gradient is 4, giving $y - 0 = 4(x - 4)$, that is $y = 4x - 16$. Setting the tangents equal, $-4x = 4x - 16$, so $8x = 16$ and $x = 2$; then $y = -4(2) = -8$. The tangents meet at $(2, -8)$.

Q15 Shore 2023 Trial Q15 · **3 marks**

The tangent to the graph of $f(x) = x^3 + ax^2 + 1$ at $x = 1$ passes through the origin. Find the value of a.

Criteria	Marks
Finds $f(1) = a + 2$ and $f'(1) = 2a + 3$	1
Forms the tangent and applies the condition that it passes through the origin	1
Solves to obtain $a = -1$	1

Solution.

$f'(x) = 3x^2 + 2ax$. At $x = 1$: the gradient is $f'(1) = 3 + 2a$, and $f(1) = 1 + a + 1 = a + 2$. The tangent at $(1,\ a + 2)$ is $y - (a + 2) = (3 + 2a)(x - 1)$. It passes through $(0, 0)$:

$$0 - (a + 2) = (3 + 2a)(0 - 1) \implies -(a + 2) = -(3 + 2a).$$

So $a + 2 = 3 + 2a$, giving $a = -1$.

Q16 North Sydney Girls 2022 Trial Q32(a) · **3 marks**

The graph of $y = x^{\frac{3}{2}} - 3x$ meets the x-axis at the origin O and at the point $A(9, 0)$. Tangents to the curve at O and A meet at the point B. Show that the coordinates of B are $(3, -9)$.

Criteria	Marks
Finds $y' = \frac{3}{2}\sqrt{x} - 3$ and the tangent at O: $y = -3x$	1
Finds the tangent at $A(9, 0)$: $y = \frac{3}{2}(x - 9)$	1
Solves simultaneously to obtain $B(3, -9)$	1

Solution.

$y = x^{\frac{3}{2}} - 3x$, so $y' = \dfrac{3}{2}x^{\frac{1}{2}} - 3$. At O $(x = 0)$: $y' = -3$, so the tangent at O is $y = -3x$. At A $(x = 9)$: $y' = \dfrac{3}{2}(3) - 3 = \dfrac{3}{2}$, so the tangent at A is $y - 0 = \dfrac{3}{2}(x - 9)$, i.e. $y = \dfrac{3}{2}x - \dfrac{27}{2}$. Setting the tangents equal, $-3x = \dfrac{3}{2}x - \dfrac{27}{2}$; multiplying by 2 gives $-6x = 3x - 27$, so $9x = 27$ and $x = 3$. Then $y = -3(3) = -9$. Therefore the tangents meet at $B(3, -9)$.

Q17 Glenwood 2023 Trial Q32 · **3 marks**

Two tangents can be drawn from the origin to the graph of the curve $y = x^2 + 3x + 5$. Find the equations of these tangents.

Criteria	**Marks**
Sets up the tangent condition: gradient $2t + 3$ equals $\frac{t^2+3t+5}{t}$	1
Solves to find the points of contact $t = \pm\sqrt{5}$	1
States both tangent equations	1

Solution.

Let $(t,\ t^2 + 3t + 5)$ be a point of contact on the curve. Since $y' = 2x + 3$, the tangent gradient there is $2t + 3$. For a tangent through the origin, the gradient also equals $\dfrac{t^2 + 3t + 5}{t}$:

$$\frac{t^2 + 3t + 5}{t} = 2t + 3 \implies t^2 + 3t + 5 = 2t^2 + 3t \implies t^2 = 5,$$

so $t = \sqrt{5}$ or $t = -\sqrt{5}$. For $t = \sqrt{5}$: gradient $= 2\sqrt{5} + 3$, tangent $y = (3 + 2\sqrt{5})\,x$. For $t = -\sqrt{5}$: gradient $= 3 - 2\sqrt{5}$, tangent $y = (3 - 2\sqrt{5})\,x$.

Q18 Girraween 2022 Trial Q33 · **4 marks**

By considering the equation of the tangent to $y = x^2 - 1$ at the point $(a, a^2 - 1)$, find the equations of the two tangents to $y = x^2 - 1$ which pass through $(3, -8)$.

Criteria	**Marks**
Forms the tangent at $(a, a^2 - 1)$ with gradient $2a$	1
Substitutes $(3, -8)$ to obtain $a^2 - 6a - 7 = 0$	1
Solves for $a = -1$ and $a = 7$	1
States both tangent equations	1

Solution.

Since $\dfrac{dy}{dx} = 2x$, the gradient at the point $(a, a^2 - 1)$ is $2a$. The tangent there is

$$y - (a^2 - 1) = 2a(x - a), \qquad \text{i.e.} \qquad y = 2ax - a^2 - 1.$$

For this tangent to pass through $(3, -8)$, substitute $x = 3$, $y = -8$:

$$-8 = 2a(3) - a^2 - 1 \implies a^2 - 6a - 7 = 0.$$

Factoring, $(a + 1)(a - 7) = 0$, so $a = -1$ or $a = 7$. Substituting into $y = 2ax - a^2 - 1$:

$$a = -1 : \; y = -2x - 2, \qquad a = 7 : \; y = 14x - 50.$$

The two tangents are $y = -2x - 2$ and $y = 14x - 50$.

3. Finding a point or parameter from a gradient condition

Q19 St George Girls 2022 Prelim Q14(b) · **2 marks**

A function is given by the formula $f(x) = 2x^2 - 32ax$ where a is a constant. Find the value of a given that $f'(4) = 0$.

Criteria	**Marks**
Differentiates to get $f'(x) = 4x - 32a$	1
Solves $f'(4) = 0$ to get $a = \frac{1}{2}$	1

Solution.

Differentiating with respect to x (treating a as a constant), $f'(x) = 4x - 32a$. Setting $f'(4) = 0$:

$$4(4) - 32a = 0 \implies 16 - 32a = 0 \implies 32a = 16 \implies a = \frac{1}{2}.$$

So $a = \frac{1}{2}$.

Q20 St George Girls 2023 Prelim Q14 · **2 marks**

Given the function $y = 10x^3 - 4$, for what value(s) of x is $\frac{dy}{dx} = 10$?

Criteria	**Marks**
Differentiates to get $\frac{dy}{dx} = 30x^2$	1
Solves $30x^2 = 10$ to get $x = \pm\frac{\sqrt{3}}{3}$	1

Solution.

Differentiating, $\frac{dy}{dx} = 30x^2$. Setting this equal to 10:

$$30x^2 = 10 \implies x^2 = \frac{1}{3} \implies x = \pm\frac{1}{\sqrt{3}} = \pm\frac{\sqrt{3}}{3}.$$

So $x = \frac{\sqrt{3}}{3}$ or $x = -\frac{\sqrt{3}}{3}$.

Q21 Cherrybrook Tech 2022 Prelim Q21(a) · **2 marks**

A tangent is drawn to the parabola $y = x^2 - 4x$ at the point P. The tangent has a gradient of 6. Find the coordinates of P.

Criteria	Marks
Differentiates and solves $2x - 4 = 6$ to get $x = 5$	1
Finds the y-coordinate to state $P = (5, 5)$	1

Solution.

The gradient of the tangent at any point is $\dfrac{dy}{dx} = 2x - 4$. Setting this equal to 6:

$$2x - 4 = 6 \implies 2x = 10 \implies x = 5.$$

The y-coordinate is $y = (5)^2 - 4(5) = 25 - 20 = 5$. So $P = (5, 5)$.

Q22 Sydney Girls 2020 Trial Q17 · **2 marks**

Find the x-values for the points on the graph of $f(x) = -x^3 + 6x^2$ at which the normal has slope $-\dfrac{1}{9}$.

Criteria	Marks
Uses the normal slope to obtain the tangent slope $f'(x) = 9$	1
Solves $-3x^2 + 12x = 9$ for $x = 1$ and $x = 3$	1

Solution.

The normal has slope $-\dfrac{1}{9}$, so the tangent has slope $-\dfrac{1}{-1/9} = 9$. Differentiating, $f'(x) = -3x^2 + 12x$. Setting this equal to 9:

$$-3x^2 + 12x = 9 \implies -3x^2 + 12x - 9 = 0 \implies x^2 - 4x + 3 = 0.$$

Factoring, $(x - 1)(x - 3) = 0$, so $x = 1$ or $x = 3$.

Q23 Sydney Grammar 2021 Prelim Q11(bii) · 3 marks

Consider the curve $y = (2x-3)^3$, for which $\dfrac{dy}{dx} = 6(2x-3)^2$. Hence, find all the points on the curve $y = (2x-3)^3$ where the gradient of the tangent is equal to 24.

Criteria	Marks
Sets $6(2x-3)^2 = 24$ and simplifies to $(2x-3)^2 = 4$	1
Solves for both x-values $x = \dfrac{1}{2}$ and $x = \dfrac{5}{2}$	1
States both points $\left(\dfrac{1}{2}, -8\right)$ and $\left(\dfrac{5}{2}, 8\right)$	1

Solution.

From part (i), $\dfrac{dy}{dx} = 6(2x-3)^2$. Setting the gradient equal to 24:

$$6(2x-3)^2 = 24 \implies (2x-3)^2 = 4 \implies 2x-3 = \pm 2.$$

So $2x-3=2$ giving $x = \dfrac{5}{2}$, or $2x-3=-2$ giving $x = \dfrac{1}{2}$. The corresponding y-values are:

$$x = \tfrac{5}{2}: \ y = (2\cdot\tfrac{5}{2}-3)^3 = (2)^3 = 8, \qquad x = \tfrac{1}{2}: \ y = (2\cdot\tfrac{1}{2}-3)^3 = (-2)^3 = -8.$$

The points are $\left(\dfrac{1}{2}, -8\right)$ and $\left(\dfrac{5}{2}, 8\right)$.

Q24 Killara 2022 Prelim Q18 · 3 marks

Consider the function $f(x) = \dfrac{a}{x} + 2x^2$ for some constant a. It is known that $f'(4) = 7$. Find the value of a.

Criteria	Marks
Rewrites as $f(x) = ax^{-1} + 2x^2$ and differentiates	1
Correct derivative $f'(x) = -\dfrac{a}{x^2} + 4x$	1
Substitutes $x = 4$, solves $f'(4) = 7$ to get $a = 144$	1

Solution.

Write $f(x) = ax^{-1} + 2x^2$. Then

$$f'(x) = -ax^{-2} + 4x = -\frac{a}{x^2} + 4x.$$

Substituting $x = 4$ and setting $f'(4) = 7$:

$$-\frac{a}{16} + 16 = 7 \implies -\frac{a}{16} = -9 \implies a = 144.$$

So $a = 144$.

Q25 Blacktown Boys 2022 Prelim Q21 · **3 marks**

The curve $y = ax^3 + bx$ passes through the point $(1, 7)$. The tangent at this point is parallel to the line $y = 3x - 8$. Find the values of a and b.

Criteria	**Marks**
Uses the point $(1, 7)$ to get $a + b = 7$	1
Uses the parallel condition $\dfrac{dy}{dx} = 3$ at $x = 1$ to get $3a + b = 3$	1
Solves the system for $a = -2$ and $b = 9$	1

Solution.

Since $(1, 7)$ lies on the curve, substitute $x = 1$, $y = 7$:

$$a(1)^3 + b(1) = 7 \implies a + b = 7.$$

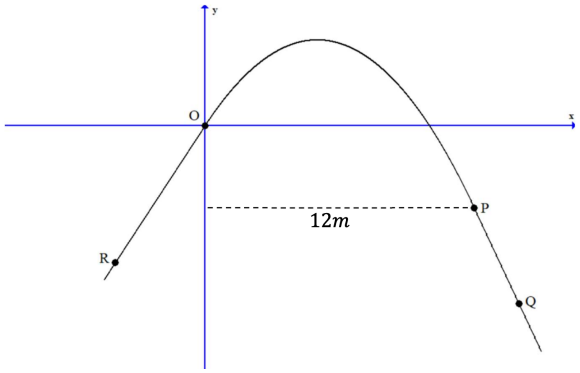
The gradient is $\dfrac{dy}{dx} = 3ax^2 + b$. The line $y = 3x - 8$ has gradient 3, and the tangent at $x = 1$ is parallel to it, so

$$3a(1)^2 + b = 3 \implies 3a + b = 3.$$

Subtracting the first equation from the second: $2a = -4$, so $a = -2$. Then $b = 7 - a = 7 - (-2) = 9$. So $a = -2$ and $b = 9$.

Q26 Gosford 2024 Prelim Q10 · **1 mark**

Luna Park is designing a new roller coaster, and the diagram below illustrates part of its design. The section RO is a straight line with $m = 2.5$ and the section PQ is a straight line with $m = -3.5$. The section OP is a parabola of the form $y = ax^2 + bx$. The horizontal distance from the y-axis to P is 12 m.



For the ride to be smooth, the straight-line sections must be tangents to the parabola at O and P. What are the values of a and b that make the ride smooth?

(A) $a = -0.0625$ and $b = 2.5$

(B) $a = -0.25$ and $b = 2.5$

(C) $a = -2.5$ and $b = -3.5$

(D) $a = -2$ and $b = 1$

Criteria	Marks
Correct option (B)	1

Solution.

The parabola $y = ax^2 + bx$ passes through the origin O, so the gradient there is $\dfrac{dy}{dx} = 2ax + b$. For the ride to be smooth, the parabola's gradient must match the adjoining straight line at each junction. At O (where $x = 0$) the gradient equals the gradient of RO: $2a(0) + b = 2.5$, so $b = 2.5$. At P (where $x = 12$) the gradient equals the gradient of PQ: $2a(12) + b = -3.5$, so $24a + 2.5 = -3.5$, giving $24a = -6$ and $a = -0.25$. The answer is **(B)**.

4. Angle of inclination ($\tan\theta = m$)

Q27 Hurlstone 2021 Prelim Q13 · 1 mark

For the equation $y = 3x^2 - 4$, what is the angle of inclination of the tangent to the curve at the point where $x = 1$?

(A) $8°$

(B) $6°$

(C) $61°$

(D) $81°$

Criteria	Marks
Correct option (D)	1

Solution.

Differentiate to get the gradient function: $\dfrac{dy}{dx} = 6x$. At $x = 1$ the tangent gradient is $m = 6$. The angle of inclination θ satisfies $\tan\theta = m$, so $\theta = \tan^{-1}(6) \approx 80.54°$, which is $81°$ to the nearest degree. The answer is **(D)**.

Q28 North Sydney Girls 2022 Prelim Q32(b) · 1 mark

Consider the function $f(x) = -\sqrt{x-4}$.

(a) Determine the equation of the normal to the curve at the point where $x = 8$. Write your answer in gradient-intercept form.

(b) Hence, find the angle of inclination of the normal with the positive x-axis. Give your answer to the nearest degree.

Criteria	Marks
Correct angle of inclination $76°$ from the normal gradient	1

Solution.

From part (a), $f'(x) = -\dfrac{1}{2\sqrt{x-4}}$, so at $x = 8$ the tangent gradient is $f'(8) = -\dfrac{1}{2\sqrt{4}} = -\dfrac{1}{4}$. The normal gradient is the negative reciprocal, $m = 4$, giving the normal $y = 4x - 34$. The angle of inclination θ satisfies $\tan\theta = m = 4$, so $\theta = \tan^{-1}(4) \approx 75.96°$, which is $76°$ to the nearest degree.

Q29 Hornsby Girls 2022 Prelim Q2 · **1 mark**

The tangent to the curve $y = x^2$ at the point $(2, 4)$ makes the angle θ with the positive direction of the x-axis. What is the value of θ, correct to the nearest minute?

(A) $1°19'$

(B) $1°20'$

(C) $75°57'$

(D) $75°58'$

Criteria	Marks
Correct option (D)	1

Solution.

Differentiate to get the gradient function: $\dfrac{dy}{dx} = 2x$. At $x = 2$ the tangent gradient is $m = 4$. The angle of inclination θ satisfies $\tan\theta = m = 4$, so $\theta = \tan^{-1}(4) \approx 75.9638° = 75°57.8'$, which is $75°58'$ to the nearest minute. The answer is **(D)**.

Q30 Hurlstone 2022 Prelim Q8 · **1 mark**

What is the angle of inclination to the positive x-axis, to the nearest degree, of the tangent to the curve $y = \dfrac{x^3}{6} + 2x$ at the point where $x = 1$?

(A) $65°$

(B) $68°$

(C) $79°$

(D) $112°$

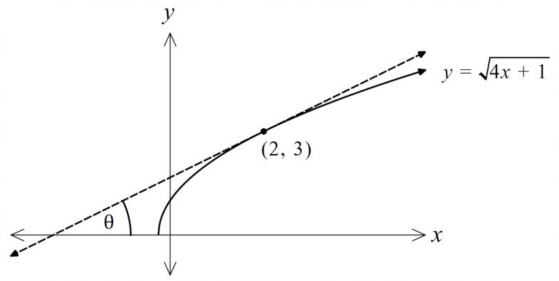
Criteria	Marks
Correct option (B)	1

Solution.

Differentiate to get the gradient function: $\dfrac{dy}{dx} = \dfrac{x^2}{2} + 2$. At $x = 1$ the tangent gradient is $m = \dfrac{1}{2} + 2 = \dfrac{5}{2}$. The angle of inclination θ satisfies $\tan\theta = m = 2.5$, so $\theta = \tan^{-1}(2.5) \approx 68.20°$, which is $68°$ to the nearest degree. The answer is **(B)**.

Q31 Killara 2022 Prelim Q30 · **3 marks**

The function $y = \sqrt{4x+1}$ has been graphed below. The dotted line shows the tangent to the curve at the point $(2, 3)$.



Find the angle the tangent line makes to the x-axis from the positive side (θ), correct to the nearest degree.

Criteria	**Marks**
Differentiates to $\dfrac{dy}{dx} = \dfrac{2}{\sqrt{4x+1}}$	1
Evaluates the tangent gradient at $(2, 3)$ as $\dfrac{2}{3}$	1
Finds $\theta = \tan^{-1}\left(\dfrac{2}{3}\right) \approx 34°$	1

Solution.

Write $y = (4x+1)^{1/2}$ and differentiate using the chain rule: $\dfrac{dy}{dx} = \dfrac{1}{2}(4x+1)^{-1/2} \cdot 4 = \dfrac{2}{\sqrt{4x+1}}$. At the point $(2, 3)$, where $x = 2$, the tangent gradient is $m = \dfrac{2}{\sqrt{4(2)+1}} = \dfrac{2}{\sqrt{9}} = \dfrac{2}{3}$. The angle of inclination θ satisfies $\tan\theta = m = \dfrac{2}{3}$, so $\theta = \tan^{-1}\left(\dfrac{2}{3}\right) \approx 33.69°$, which is $34°$ to the nearest degree.