Year 11 Mathematics Advanced

Topic: Introduction to Differentiation (2 of 6)

Review Questions (Worked Solutions): The Differentiation Rules

Term 3, 2026

Chain, product and quotient rules

1. The chain rule

Q1 Barker 2021 Prelim Q12(bβ) · **1 mark**

Differentiate the following. Leave your answers with a positive, non-fractional index, where necessary. $y = (3x^2 + 5x)^4$.

Criteria	Marks
Correct derivative	1

Solution.

Apply the chain rule with outer function u^4 and inner function $u = 3x^2 + 5x$. The inner derivative is $\dfrac{du}{dx} = 6x + 5$, so

$$\frac{dy}{dx} = 4(3x^2 + 5x)^3 \cdot (6x + 5).$$

The derivative is $\dfrac{dy}{dx} = 4(6x + 5)(3x^2 + 5x)^3$.

Q2 Baulkham Hills 2021 Prelim Q11(a) · **2 marks**

Differentiate the following with respect to x. $(3x^2 + 1)^4$.

Criteria	Marks
Applies the chain rule with the correct inner derivative $6x$	1
Correct simplified derivative	1

Solution.

Apply the chain rule with outer function u^4 and inner function $u = 3x^2 + 1$, whose derivative is $6x$:

$$\frac{d}{dx}(3x^2 + 1)^4 = 4(3x^2 + 1)^3 \cdot 6x.$$

The derivative is $24x(3x^2 + 1)^3$.

Q3 Baulkham Hills 2022 Prelim Q12 · **2 marks**

Differentiate $(3x^2 + 1)^5$.

Criteria	Marks
Applies the chain rule with the correct inner derivative $6x$	1
Correct simplified derivative	1

Solution.

Apply the chain rule with outer function u^5 and inner function $u = 3x^2 + 1$, whose derivative is $6x$:

$$\frac{d}{dx}(3x^2 + 1)^5 = 5(3x^2 + 1)^4 \cdot 6x.$$

The derivative is $30x(3x^2 + 1)^4$.

Q4 Knox 2022 Prelim Q3 · **1 mark**

What is the derivative of $(1 - 2x)^5$?

(A) $5(1 - 2x)^4$

(B) $-5(1 - 2x)^4$

(C) $10(1 - 2x)^4$

(D) $-10(1 - 2x)^4$

Criteria	Marks
Correct option (D)	1

Solution.

Apply the chain rule with outer function u^5 and inner function $u = 1 - 2x$, whose derivative is -2:

$$\frac{d}{dx}(1 - 2x)^5 = 5(1 - 2x)^4 \cdot (-2) = -10(1 - 2x)^4.$$

The answer is **(D)**.

Q5 Hurlstone 2023 Prelim Q13(b(i)) · **1 mark**

Differentiate the following expression with respect to t. $2(5t+6)^8$.

Criteria	Marks
Correct derivative	1

Solution.

Apply the chain rule to $2(5t+6)^8$, with inner function $5t+6$ whose derivative is 5:

$$\frac{d}{dt}\,2(5t+6)^8 = 2\cdot 8(5t+6)^7\cdot 5.$$

The derivative is $80(5t+6)^7$.

Q6 Hornsby Girls 2022 Prelim Q17(b) · **1 mark**

Consider the semi-circle defined by function $f(x)=\sqrt{8-x^2}$. Show that $f'(x)=\dfrac{-x}{\sqrt{8-x^2}}$.

Criteria	Marks
Correctly applies the chain rule to reach the required form	1

Solution.

Write $f(x)=(8-x^2)^{1/2}$ and apply the chain rule with inner function $8-x^2$, whose derivative is $-2x$:

$$f'(x)=\tfrac{1}{2}(8-x^2)^{-1/2}\cdot(-2x)=\frac{-2x}{2\sqrt{8-x^2}}=\frac{-x}{\sqrt{8-x^2}},$$

as required.

Q7 Hurlstone 2021 Prelim Q8 · **1 mark**

The derivative of $y = 3x^2(2x^3 + 1)^3$ is given by which of the following?

(A) $6x(2x^3 + 1)^2(9x^3 + 1)$

(B) $6x(2x^3 + 1)^2(9x^3 + 2x + 1)$

(C) $6x(2x^3 + 1)^2(11x^3 + 1)$

(D) $6x(2x^3 + 1)^2(11x^3 + 2x + 1)$

Criteria	**Marks**
Correct option (C)	1

Solution.

Use the product rule on $y = 3x^2 \cdot (2x^3+1)^3$. Differentiating $(2x^3+1)^3$ by the chain rule gives $3(2x^3+1)^2 \cdot 6x^2 = 18x^2(2x^3 + 1)^2$, so

$$\frac{dy}{dx} = 6x(2x^3 + 1)^3 + 3x^2 \cdot 18x^2(2x^3 + 1)^2.$$

Factor out $6x(2x^3 + 1)^2$:

$$\frac{dy}{dx} = 6x(2x^3 + 1)^2\left[(2x^3 + 1) + 9x^3\right] = 6x(2x^3 + 1)^2(11x^3 + 1).$$

The answer is **(C)**.

Q8 Killara 2023 Prelim Q32(b) · **3 marks**

A particle moves in a straight line so that its displacement x cm over time t seconds is given by $x = t\sqrt{25 - t^2}$. Show that the velocity as a function of t can be given by $\dfrac{dx}{dt} = \dfrac{25 - 2t^2}{\sqrt{25 - t^2}}$.

Criteria	Marks
Applies the product rule to $x = t\sqrt{25 - t^2}$	1
Differentiates $\sqrt{25 - t^2}$ correctly by the chain rule as $\dfrac{-t}{\sqrt{25 - t^2}}$	1
Combines over a common denominator to reach the required form	1

Solution.

Write $x = t(25 - t^2)^{1/2}$ and apply the product rule. Differentiating $(25 - t^2)^{1/2}$ by the chain rule gives $\frac{1}{2}(25 - t^2)^{-1/2} \cdot (-2t) = \dfrac{-t}{\sqrt{25 - t^2}}$. Therefore

$$\frac{dx}{dt} = 1 \cdot \sqrt{25 - t^2} + t \cdot \frac{-t}{\sqrt{25 - t^2}} = \sqrt{25 - t^2} - \frac{t^2}{\sqrt{25 - t^2}}.$$

Combining over the common denominator $\sqrt{25 - t^2}$:

$$\frac{dx}{dt} = \frac{(25 - t^2) - t^2}{\sqrt{25 - t^2}} = \frac{25 - 2t^2}{\sqrt{25 - t^2}},$$

as required.

Q9 Killara 2024 Prelim Q23(b) · **2 marks**

Differentiate the following function. $f(x) = \sqrt{3x - x^2}$.

Criteria	Marks
Applies the chain rule with the correct inner derivative $3 - 2x$	1
Correct simplified derivative	1

Solution.

Write $f(x) = (3x - x^2)^{1/2}$ and apply the chain rule with inner function $3x - x^2$, whose derivative is $3 - 2x$:

$$f'(x) = \tfrac{1}{2}(3x - x^2)^{-1/2} \cdot (3 - 2x) = \frac{3 - 2x}{2\sqrt{3x - x^2}}.$$

The derivative is $f'(x) = \dfrac{3 - 2x}{2\sqrt{3x - x^2}}$.

2. The product rule

Q10 Sydney Grammar 2021 Prelim Q10(aiv) · **1 mark**

Differentiate with respect to x: $y = (x-3)(x+5)$.

Criteria	Marks
Correct derivative	1

Solution.

By the product rule, $\dfrac{dy}{dx} = (1)(x+5) + (x-3)(1) = x + 5 + x - 3 = 2x + 2$. (Equivalently, expand first to $y = x^2 + 2x - 15$, giving $\dfrac{dy}{dx} = 2x + 2$.) The derivative is $\dfrac{dy}{dx} = 2x + 2$.

Q11 Moriah 2021 Prelim Q17(a) · **2 marks**

Use the product rule to find $\dfrac{dy}{dx}$ where $y = x^3(3x-2)^5$.

Criteria	Marks
Applies the product rule with correct derivatives of each factor	1
Correct simplified derivative	1

Solution.

Let $u = x^3$ and $v = (3x-2)^5$, so $u' = 3x^2$ and, by the chain rule, $v' = 5(3x-2)^4 \cdot 3 = 15(3x-2)^4$. By the product rule,

$$\frac{dy}{dx} = u'v + uv' = 3x^2(3x-2)^5 + x^3 \cdot 15(3x-2)^4.$$

Taking out the common factor $3x^2(3x-2)^4$,

$$\frac{dy}{dx} = 3x^2(3x-2)^4\big[(3x-2) + 5x\big] = 3x^2(3x-2)^4(8x-2).$$

The derivative is $\dfrac{dy}{dx} = 3x^2(3x-2)^4(8x-2) = 6x^2(3x-2)^4(4x-1)$.

Q12 Hurlstone 2022 Prelim Q11(a) · **2 marks**

Differentiate the expression $(x^2 - 5)(9x - 2x^3)$ with respect to x.

Criteria	Marks
Applies the product rule with correct derivatives of each factor	1
Correct simplified derivative	1

Solution.

Let $u = x^2 - 5$ and $v = 9x - 2x^3$, so $u' = 2x$ and $v' = 9 - 6x^2$. By the product rule,

$$\frac{dy}{dx} = u'v + uv' = (x^2 - 5)(9 - 6x^2) + (9x - 2x^3)(2x).$$

Expanding and collecting like terms,

$$\frac{dy}{dx} = (9x^2 - 6x^4 - 45 + 30x^2) + (18x^2 - 4x^4) = -10x^4 + 57x^2 - 45.$$

The derivative is $\dfrac{dy}{dx} = -10x^4 + 57x^2 - 45$.

Q13 Girraween 2022 Prelim Q8(biv) · **3 marks**

Differentiate $y = x(1 - x)^6$.

Criteria	Marks
Applies the product rule	1
Differentiates $(1 - x)^6$ correctly by the chain rule	1
Correct simplified derivative	1

Solution.

Let $u = x$ and $v = (1 - x)^6$, so $u' = 1$ and, by the chain rule, $v' = 6(1 - x)^5 \cdot (-1) = -6(1 - x)^5$. By the product rule,

$$\frac{dy}{dx} = u'v + uv' = (1 - x)^6 + x\big[-6(1 - x)^5\big] = (1 - x)^6 - 6x(1 - x)^5.$$

Taking out the common factor $(1 - x)^5$,

$$\frac{dy}{dx} = (1 - x)^5\big[(1 - x) - 6x\big] = (1 - x)^5(1 - 7x).$$

The derivative is $\dfrac{dy}{dx} = (1 - x)^5(1 - 7x)$.

Q14 Killara 2022 Prelim Q21(a) · **2 marks**

Find $\dfrac{dy}{dx}$ for $y = (7x + 4)(x + 1)^3$.

Criteria	Marks
Applies the product rule with correct derivatives of each factor	1
Correct simplified derivative	1

Solution.

Let $u = 7x + 4$ and $v = (x + 1)^3$, so $u' = 7$ and, by the chain rule, $v' = 3(x + 1)^2$. By the product rule,

$$\frac{dy}{dx} = u'v + uv' = 7(x + 1)^3 + (7x + 4) \cdot 3(x + 1)^2.$$

Taking out the common factor $(x + 1)^2$,

$$\frac{dy}{dx} = (x + 1)^2\big[7(x + 1) + 3(7x + 4)\big] = (x + 1)^2(7x + 7 + 21x + 12) = (x + 1)^2(28x + 19).$$

The derivative is $\dfrac{dy}{dx} = (x + 1)^2(28x + 19)$.

Q15 Girraween 2023 Prelim Q9(b(iv)) · **2 marks**

Differentiate $y = x^3(4 - 3x)^5$.

Criteria	Marks
Applies the product rule with correct derivatives of each factor	1
Correct simplified derivative	1

Solution.

Let $u = x^3$ and $v = (4 - 3x)^5$, so $u' = 3x^2$ and, by the chain rule, $v' = 5(4 - 3x)^4 \cdot (-3) = -15(4 - 3x)^4$. By the product rule,

$$\frac{dy}{dx} = u'v + uv' = 3x^2(4 - 3x)^5 + x^3\big[-15(4 - 3x)^4\big].$$

Taking out the common factor $3x^2(4 - 3x)^4$,

$$\frac{dy}{dx} = 3x^2(4 - 3x)^4\big[(4 - 3x) - 5x\big] = 3x^2(4 - 3x)^4(4 - 8x).$$

The derivative is $\dfrac{dy}{dx} = 3x^2(4 - 3x)^4(4 - 8x)$.

Q16 Blacktown Boys 2022 Prelim Q23(a) · **3 marks**

Given that $f(x) = (3-2x)\sqrt{4x+1}$, show that $f'(x) = \dfrac{4(1-3x)}{\sqrt{4x+1}}$.

Criteria	Marks
Applies the product rule with the chain rule on $\sqrt{4x+1}$	1
Combines the two terms over the common denominator $\sqrt{4x+1}$	1
Simplifies the numerator to reach the required form	1

Solution.

Let $u = 3-2x$ and $v = (4x+1)^{1/2}$, so $u' = -2$ and, by the chain rule, $v' = \frac{1}{2}(4x+1)^{-1/2}\cdot 4 = \dfrac{2}{\sqrt{4x+1}}$. By the product rule,

$$f'(x) = u'v + uv' = -2\sqrt{4x+1} + (3-2x)\cdot\frac{2}{\sqrt{4x+1}}.$$

Placing both terms over $\sqrt{4x+1}$,

$$f'(x) = \frac{-2(4x+1)+2(3-2x)}{\sqrt{4x+1}} = \frac{-8x-2+6-4x}{\sqrt{4x+1}} = \frac{4-12x}{\sqrt{4x+1}}.$$

Therefore $f'(x) = \dfrac{4(1-3x)}{\sqrt{4x+1}}$, as required.

Q17 Kambala 2023 Prelim Q14(b) · **2 marks**

Differentiate the following with respect to x. $y = (x^2-2)^3(x+3)^2$.

Criteria	Marks
Applies the product rule, differentiating each factor by the chain rule	1
Factors out the common factors and simplifies	1

Solution.

Let $u = (x^2-2)^3$ and $v = (x+3)^2$. By the chain rule $u' = 3(x^2-2)^2\cdot 2x = 6x(x^2-2)^2$ and $v' = 2(x+3)$. By the product rule,

$$\frac{dy}{dx} = u'v + uv' = 6x(x^2-2)^2(x+3)^2 + (x^2-2)^3\cdot 2(x+3).$$

Take out the common factor $2(x^2-2)^2(x+3)$:

$$\frac{dy}{dx} = 2(x^2-2)^2(x+3)\big[3x(x+3)+(x^2-2)\big] = 2(x^2-2)^2(x+3)(4x^2+9x-2).$$

The derivative is $\dfrac{dy}{dx} = 2(x^2-2)^2(x+3)(4x^2+9x-2)$.

3. The quotient rule

Q18 Barker 2021 Prelim Q13(a) · **2 marks**

Differentiate $f(x) = \dfrac{x^2 - 1}{2x + 5}$.

Criteria	Marks
Applies the quotient rule with correct u, v, u', v'	1
Simplifies to the correct derivative	1

Solution.

Let $u = x^2 - 1$ and $v = 2x + 5$, so $u' = 2x$ and $v' = 2$. By the quotient rule,

$$f'(x) = \frac{vu' - uv'}{v^2} = \frac{(2x + 5)(2x) - (x^2 - 1)(2)}{(2x + 5)^2}.$$

The numerator is $4x^2 + 10x - 2x^2 + 2 = 2x^2 + 10x + 2 = 2(x^2 + 5x + 1)$, so

$$f'(x) = \frac{2(x^2 + 5x + 1)}{(2x + 5)^2}.$$

Q19 St George Girls 2021 Prelim Q1(aii) · **2 marks**

Differentiate with respect to x: $\quad y = \dfrac{4}{\sqrt{3x - 1}}$.

Criteria	Marks
Correct set-up (quotient rule, or index form with the chain rule)	1
Simplifies to the correct derivative	1

Solution.

Write $y = \dfrac{4}{(3x - 1)^{1/2}}$. Using the quotient rule with $u = 4$ (so $u' = 0$) and $v = (3x - 1)^{1/2}$ (so $v' = \frac{3}{2}(3x - 1)^{-1/2}$),

$$\frac{dy}{dx} = \frac{vu' - uv'}{v^2} = \frac{-4 \cdot \frac{3}{2}(3x - 1)^{-1/2}}{3x - 1} = \frac{-6(3x - 1)^{-1/2}}{3x - 1}.$$

Therefore $\dfrac{dy}{dx} = \dfrac{-6}{(3x - 1)^{3/2}}$.

Q20 Presbyterian 2022 Prelim Q26 · **3 marks**

Differentiate $y = \dfrac{x^2+1}{4x-7}$, giving your answer in simplest form.

Criteria	Marks
Applies the quotient rule with correct u, v, u', v'	1
Expands and collects the numerator	1
States the derivative in simplest form	1

Solution.

Let $u = x^2+1$ and $v = 4x-7$, so $u' = 2x$ and $v' = 4$. By the quotient rule,

$$\frac{dy}{dx} = \frac{vu' - uv'}{v^2} = \frac{(4x-7)(2x) - (x^2+1)(4)}{(4x-7)^2}.$$

The numerator is $8x^2 - 14x - 4x^2 - 4 = 4x^2 - 14x - 4 = 2(2x^2 - 7x - 2)$, so

$$\frac{dy}{dx} = \frac{2(2x^2-7x-2)}{(4x-7)^2}.$$

Q21 Killara 2023 Prelim Q21(c) · **2 marks**

Differentiate the following function: $f(x) = \dfrac{x^2}{2x+1}$.

Criteria	Marks
Applies the quotient rule with correct u, v, u', v'	1
Simplifies to the correct derivative	1

Solution.

Let $u = x^2$ and $v = 2x+1$, so $u' = 2x$ and $v' = 2$. By the quotient rule,

$$f'(x) = \frac{vu' - uv'}{v^2} = \frac{(2x+1)(2x) - x^2(2)}{(2x+1)^2}.$$

The numerator is $4x^2 + 2x - 2x^2 = 2x^2 + 2x = 2x(x+1)$, so

$$f'(x) = \frac{2x(x+1)}{(2x+1)^2}.$$

Q22 North Sydney Girls 2021 Prelim Q3(a) · **2 marks**

Differentiate $f(x) = \dfrac{1-x}{x+1}$. Fully simplify your answer.

Criteria	Marks
Applies the quotient rule with correct u, v, u', v'	1
Simplifies to the correct derivative	1

Solution.

Let $u = 1 - x$ and $v = x + 1$, so $u' = -1$ and $v' = 1$. By the quotient rule,

$$f'(x) = \frac{vu' - uv'}{v^2} = \frac{(x+1)(-1) - (1-x)(1)}{(x+1)^2}.$$

The numerator is $-x - 1 - 1 + x = -2$, so

$$f'(x) = \frac{-2}{(x+1)^2}.$$

Q23 Barker 2024 Prelim Q15(a(iii)) · **2 marks**

Differentiate and simplify the following function with respect to x: $\quad f(x) = \dfrac{5x^2}{x^2+1}$.

Criteria	Marks
Applies the quotient rule with correct u, v, u', v'	1
Simplifies to the correct derivative	1

Solution.

Let $u = 5x^2$ and $v = x^2 + 1$, so $u' = 10x$ and $v' = 2x$. By the quotient rule,

$$f'(x) = \frac{vu' - uv'}{v^2} = \frac{(x^2+1)(10x) - 5x^2(2x)}{(x^2+1)^2}.$$

The numerator is $10x^3 + 10x - 10x^3 = 10x$, so

$$f'(x) = \frac{10x}{(x^2+1)^2}.$$

Q24 Kambala 2023 Prelim Q8 · **1 mark**

Megan tries to differentiate the function $y = \dfrac{3x}{(10-x)^2}$ with respect to x. Her working out is shown below.

$$\begin{aligned}
\frac{dy}{dx} &= \frac{3(10-x)^2 + 3x \times 2(10-x)}{(10-x)^4} && \text{Line (1)} \\
&= \frac{3(10-x)^2 - 6x(10-x)}{(10-x)^4} && \text{Line (2)} \\
&= \frac{(10-x)[3(10-x) + 6x]}{(10-x)^4} && \text{Line (3)} \\
&= \frac{30 + 3x}{(10-x)^3} && \text{Line (4)} \\
&= \frac{10 + x}{(10-x)^3} && \text{Line (5)}
\end{aligned}$$

Where did Megan make mistakes?

(A) Line 1 & Line 5

(B) Line 3 & Line 4

(C) Line 1 & Line 3

(D) Line 3 & Line 5

Criteria	**Marks**
Correct option (D)	1

Solution.

The correct derivative is $\dfrac{dy}{dx} = \dfrac{3(x+10)}{(x-10)^3} = \dfrac{-3(x+10)}{(10-x)^3}$. Checking each line against this value, Line 1, Line 2 and Line 4 are correct, while Line 3 and Line 5 are wrong. At Line 3 the factoring is incorrect: Line 2 has numerator $3(10-x)^2 - 6x(10-x) = (10-x)[3(10-x) - 6x]$, but Megan writes $[3(10-x) + 6x]$, flipping the sign of the $6x$ term. At Line 5 she divides the numerator $30 + 3x$ by 3 to get $10 + x$ without dividing the denominator, which is invalid. The answer is **(D)**.

Q25 Killara 2024 Prelim Q26(a) · **3 marks**

Given that $f(x) = \dfrac{\sqrt{x}}{x+1}$, show that $f'(x) = -\dfrac{x-1}{2\sqrt{x}\,(x+1)^2}$.

Criteria	**Marks**
Applies the quotient rule with $u = \sqrt{x}$, $v = x+1$ and correct u', v'	1
Simplifies the numerator over the common denominator $2\sqrt{x}$	1
Reaches the required form	1

Solution.

Let $u = \sqrt{x} = x^{1/2}$ and $v = x+1$, so $u' = \dfrac{1}{2\sqrt{x}}$ and $v' = 1$. By the quotient rule,

$$f'(x) = \frac{vu' - uv'}{v^2} = \frac{(x+1)\cdot\dfrac{1}{2\sqrt{x}} - \sqrt{x}\cdot 1}{(x+1)^2}.$$

Writing the numerator over $2\sqrt{x}$ (using $\sqrt{x} = \dfrac{2x}{2\sqrt{x}}$),

$$\text{numerator} = \frac{(x+1) - 2x}{2\sqrt{x}} = \frac{1-x}{2\sqrt{x}}.$$

Therefore

$$f'(x) = \frac{1-x}{2\sqrt{x}\,(x+1)^2} = -\frac{x-1}{2\sqrt{x}\,(x+1)^2},$$

as required.

Q26 Presbyterian 2023 Prelim Q24 · 3 marks

Show that $\dfrac{d}{dx}\left(\dfrac{x}{\sqrt{x-1}}\right) = \dfrac{x-2}{2\sqrt{(x-1)^3}}$.

Criteria	Marks
Applies the quotient rule with $u = x$, $v = (x-1)^{1/2}$ and correct u', v'	1
Combines the numerator over the common denominator $2\sqrt{x-1}$	1
Reaches the required form	1

Solution.

Let $u = x$ and $v = (x-1)^{1/2}$, so $u' = 1$ and, by the chain rule, $v' = \frac{1}{2}(x-1)^{-1/2} = \dfrac{1}{2\sqrt{x-1}}$. By the quotient rule,

$$\frac{d}{dx}\left(\frac{x}{\sqrt{x-1}}\right) = \frac{vu' - uv'}{v^2} = \frac{\sqrt{x-1}\cdot 1 - x\cdot\dfrac{1}{2\sqrt{x-1}}}{x-1}.$$

Writing the numerator over $2\sqrt{x-1}$ (using $\sqrt{x-1} = \dfrac{2(x-1)}{2\sqrt{x-1}}$),

$$\text{numerator} = \frac{2(x-1) - x}{2\sqrt{x-1}} = \frac{x-2}{2\sqrt{x-1}}.$$

Therefore

$$\frac{d}{dx}\left(\frac{x}{\sqrt{x-1}}\right) = \frac{x-2}{2\sqrt{x-1}}\cdot\frac{1}{x-1} = \frac{x-2}{2\sqrt{(x-1)^3}},$$

as required.