Year 11 Mathematics Advanced

Topic: Introduction to Differentiation (1 of 6)

Review Questions (Worked Solutions): The Derivative and the Power Rule

Term 3, 2026

Understanding the derivative

1. Average rate of change

Q1 Hurlstone 2021 Prelim Q11 · **1 mark**

A particle's speed decreases from 34 ms^{-1} to 8 ms^{-1} in 4 seconds. What was the average rate of change of its speed?

(A) 6.5 ms^{-2}

(B) -6.5 ms^{-2}

(C) -13 ms^{-2}

(D) 13 ms^{-2}

Criteria	Marks
Correct option (B)	1

Solution.

The average rate of change is the change in speed divided by the time taken:

$$\frac{\Delta v}{\Delta t} = \frac{8-34}{4} = \frac{-26}{4} = -6.5 \text{ ms}^{-2}.$$

The speed decreases, so the rate is negative. The answer is **(B)**.

Q2 Hornsby Girls 2021 Prelim Q17(c) · **2 marks**

The population of a town is given by $P = 45\,000(3t + 2)^2$, where t is the time in years. Calculate the average rate of change of the population in the first five years.

Criteria	**Marks**
Evaluates $P(0) = 180\,000$ and $P(5) = 13\,005\,000$	1
Computes the average rate $\dfrac{P(5) - P(0)}{5} = 2\,565\,000$ per year	1

Solution.

The average rate of change over the first five years is $\dfrac{P(5) - P(0)}{5 - 0}$. Evaluating the population:

$$P(0) = 45\,000(2)^2 = 180\,000, \qquad P(5) = 45\,000(17)^2 = 13\,005\,000.$$

Therefore

$$\frac{P(5) - P(0)}{5} = \frac{13\,005\,000 - 180\,000}{5} = \frac{12\,825\,000}{5} = 2\,565\,000.$$

The average rate of change of the population is 2 565 000 people per year.

Q3 Moriah 2021 Prelim Q18(b) · **2 marks**

Sophie is breeding mice for a laboratory. The number of mice M she has after t months is given by the equation $M = 12 + 5t + 4t^2$. Find the average rate of increase in her mouse population in the first six months.

Criteria	**Marks**
Evaluates $M(0) = 12$ and $M(6) = 186$	1
Computes the average rate $\dfrac{M(6) - M(0)}{6} = 29$ mice per month	1

Solution.

The average rate of increase over the first six months is $\dfrac{M(6) - M(0)}{6 - 0}$. Evaluating:

$$M(0) = 12 + 5(0) + 4(0)^2 = 12, \qquad M(6) = 12 + 5(6) + 4(6)^2 = 12 + 30 + 144 = 186.$$

Therefore

$$\frac{M(6) - M(0)}{6} = \frac{186 - 12}{6} = \frac{174}{6} = 29.$$

The average rate of increase is 29 mice per month.

Q4 Kambala 2023 Prelim Q9 · **1 mark**

The height of a ball thrown in the air is given by the formula $h = 10t - 3t^2$ in metres, where t is measured in seconds. When the ball is thrown, what is the average rate of change of the height in the first second?

(A) 4 metres per second

(B) 7 metres

(C) $10 - 6t$

(D) 7 metres per second

Criteria	**Marks**
Correct option (D)	1

Solution.

The average rate of change of height over the first second is $\dfrac{h(1) - h(0)}{1 - 0}$. Evaluating:

$$h(0) = 10(0) - 3(0)^2 = 0, \qquad h(1) = 10(1) - 3(1)^2 = 10 - 3 = 7.$$

Therefore

$$\frac{h(1) - h(0)}{1} = \frac{7 - 0}{1} = 7 \text{ metres per second.}$$

The value is a rate, so the units are metres per second. The answer is **(D)**.

Q5 Killara 2022 Prelim Q16(a) · **2 marks**

A tank of water is being drained, and the quantity (Q) of water remaining in the tank after t minutes can be shown by the graph. It takes 7 minutes to completely drain the tank. The graph has the equation $Q(t) = 20(t-8)^2 - 20$. Show that the average rate of change is -180 L/min.

Criteria	**Marks**
Evaluates $Q(0) = 1260$ and $Q(7) = 0$	1
Computes the average rate $\dfrac{Q(7)-Q(0)}{7} = -180$ L/min	1

Solution.

The tank is full at $t = 0$ and empty at $t = 7$ (it takes 7 minutes to drain), so the average rate of change is $\dfrac{Q(7)-Q(0)}{7-0}$. Evaluating the quantity:

$$Q(0) = 20(0-8)^2 - 20 = 20(64) - 20 = 1260, \qquad Q(7) = 20(7-8)^2 - 20 = 20(1) - 20 = 0.$$

Therefore

$$\frac{Q(7)-Q(0)}{7} = \frac{0-1260}{7} = \frac{-1260}{7} = -180 \text{ L/min},$$

as required.

Q6 Loreto Kirribilli 2022 Prelim Q30(c) · **3 marks**

A city's population is experiencing exponential growth such that the population P at time t in years can be given by $P = 50\,000e^{at}$, where a is a positive constant. Given that the population is 52 400 after 1 year, so that $a = 0.0469$ (to 4 decimal places), find the average rate of growth of the population in the first 5 years, correct to 1 decimal place.

Criteria	**Marks**
Evaluates $P(0) = 50\,000$	1
Evaluates $P(5) = 50\,000e^{5(0.0469)} \approx 63\,213.8$	1
Computes the average rate $\dfrac{P(5)-P(0)}{5} \approx 2642.8$ per year	1

Solution.

The average rate of growth over the first five years is $\dfrac{P(5)-P(0)}{5-0}$. With $a = 0.0469$:

$$P(0) = 50\,000e^{0} = 50\,000, \qquad P(5) = 50\,000e^{5(0.0469)} = 50\,000e^{0.2345} \approx 63\,213.8.$$

Therefore

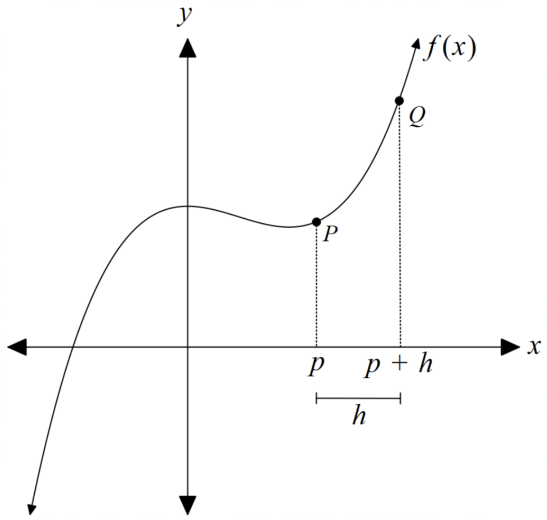
$$\frac{P(5)-P(0)}{5} = \frac{63\,213.8 - 50\,000}{5} = \frac{13\,213.8}{5} \approx 2642.8.$$

The average rate of growth of the population is approximately 2642.8 people per year.

2. Evaluating limits

Q7 International Grammar 2021 Prelim Q7 · **1 mark**

The graph of the function $f(x)$ is shown below, with points P and Q on the graph. The coordinates of P are $(p, f(p))$, and point Q's x-coordinate is h to the right of p, hence Q's coordinates are $(p+h, f(p+h))$.



Which of the following would give the gradient of the tangent at point P?

(A) $\lim\limits_{h\to 0} \dfrac{f(p)-f(p+h)}{h}$

(B) $\lim\limits_{h\to 0} \dfrac{f(p-h)+f(p)}{h}$

(C) $\lim\limits_{h\to 0} \dfrac{f(p+h)-f(p)}{h}$

(D) $\lim\limits_{h\to 0} \dfrac{f(p-h)-f(p)}{h}$

Criteria	**Marks**
Correct option (C)	1

Solution.

The gradient of the tangent at P is the limit of the gradient of the secant PQ as Q approaches P, that is, as $h \to 0$. The secant through $P(p, f(p))$ and $Q(p+h, f(p+h))$ has gradient $\dfrac{f(p+h)-f(p)}{(p+h)-p} = \dfrac{f(p+h)-f(p)}{h}$. Taking the limit gives $\lim\limits_{h\to 0} \dfrac{f(p+h)-f(p)}{h}$. The answer is **(C)**.

Q8 Sydney Grammar 2023 Prelim Q12(a) · **1 mark**

Evaluate $\lim_{h\to 0} \dfrac{h^2-4h}{h}$.

Criteria	Marks
Correct value -4	1

Solution.

Factor the numerator and cancel the common factor of h (valid since $h \neq 0$ as $h \to 0$):

$$\frac{h^2-4h}{h} = \frac{h(h-4)}{h} = h-4.$$

Therefore $\lim_{h\to 0} \dfrac{h^2-4h}{h} = \lim_{h\to 0}(h-4) = -4$.

Q9 Hornsby Girls 2021 Prelim Q2 · **1 mark**

Find $\lim_{x\to -4} \dfrac{2x+8}{3x^2-48}$.

(A) $\dfrac{1}{12}$

(B) $-\dfrac{1}{12}$

(C) 12

(D) Undefined

Criteria	Marks
Correct option (B)	1

Solution.

Factor numerator and denominator, then cancel the common factor $(x+4)$:

$$\frac{2x+8}{3x^2-48} = \frac{2(x+4)}{3(x^2-16)} = \frac{2(x+4)}{3(x-4)(x+4)} = \frac{2}{3(x-4)}.$$

Substituting $x=-4$ gives $\dfrac{2}{3(-4-4)} = \dfrac{2}{-24} = -\dfrac{1}{12}$. The answer is **(B)**.

Q10 North Sydney Girls 2022 Prelim Q8 · **1 mark**

What is the value of $\lim_{x\to -3} \dfrac{2x+6}{x^2-9}$?

(A) $-\dfrac{1}{3}$

(B) 0

(C) $\dfrac{2}{3}$

(D) 1

Criteria	Marks
Correct option (A)	1

Solution.

Factor numerator and denominator, then cancel the common factor $(x+3)$:

$$\frac{2x+6}{x^2-9} = \frac{2(x+3)}{(x-3)(x+3)} = \frac{2}{x-3}.$$

Substituting $x=-3$ gives $\dfrac{2}{-3-3} = \dfrac{2}{-6} = -\dfrac{1}{3}$. The answer is **(A)**.

Q11 North Sydney Girls 2023 Prelim Q14 · **2 marks**

Find $\lim_{x\to 2} \dfrac{2x^2-3x-2}{x-2}$.

Criteria	Marks
Factorises the numerator as $(2x+1)(x-2)$ and cancels $(x-2)$	1
Evaluates the limit to get 5	1

Solution.

Factor the numerator, then cancel the common factor $(x-2)$:

$$\frac{2x^2-3x-2}{x-2} = \frac{(2x+1)(x-2)}{x-2} = 2x+1.$$

Therefore $\lim_{x\to 2} \dfrac{2x^2-3x-2}{x-2} = \lim_{x\to 2}(2x+1) = 2(2)+1 = 5.$

Q12 Killara 2024 Prelim Q15 · **2 marks**

Evaluate $\lim_{x\to 4} \dfrac{x^2 - x - 12}{x - 4}$.

Criteria	Marks
Factorises the numerator as $(x-4)(x+3)$ and cancels $(x-4)$	1
Evaluates the limit to get 7	1

Solution.

Factor the numerator, then cancel the common factor $(x-4)$:

$$\frac{x^2 - x - 12}{x - 4} = \frac{(x-4)(x+3)}{x-4} = x + 3.$$

Therefore $\lim_{x\to 4} \dfrac{x^2 - x - 12}{x - 4} = \lim_{x\to 4}(x+3) = 4 + 3 = 7.$

Q13 Manly 2022 Trial Q10 · **1 mark**

On a continuous function $f(x)$, h is a positive number which represents the horizontal difference between two points on the function. Which of the following would NOT give the derivative of $f(x)$?

(A) $\lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$

(B) $\lim_{h\to 0} \dfrac{f\left(x + \frac{h}{2}\right) - f\left(x - \frac{h}{2}\right)}{h}$

(C) $\lim_{h\to 0} \dfrac{f(x) - f(x-h)}{h}$

(D) $\lim_{h\to 0} \dfrac{f(x-h) - f(x)}{h}$

Criteria	Marks
Correct option (D)	1

Solution.

The derivative is the limit of a gradient of a secant as $h \to 0$. Option (A) is the standard forward-difference definition; option (C) is the backward difference $\dfrac{f(x) - f(x-h)}{h}$; and option (B) is the symmetric difference $\dfrac{f(x + \frac{h}{2}) - f(x - \frac{h}{2})}{h}$. All three approach $f'(x)$. Option (D) is $\dfrac{f(x-h) - f(x)}{h} = -\dfrac{f(x) - f(x-h)}{h}$, which approaches $-f'(x)$, not $f'(x)$. The answer is **(D)**.

3. Differentiation from first principles

Q14 Sydney Grammar 2021 Prelim Q13(ai) · **1 mark**

Consider the function $f(x) = x^2 - 3x + 7$. Write down an expression for $f(x + h)$.

Criteria	Marks
Correct expression for $f(x + h)$	1

Solution.

Replace every x in $f(x) = x^2 - 3x + 7$ with $x + h$:

$$f(x + h) = (x + h)^2 - 3(x + h) + 7 = x^2 + 2xh + h^2 - 3x - 3h + 7.$$

So $f(x + h) = x^2 + 2xh + h^2 - 3x - 3h + 7$.

Q15 St George Girls 2021 Prelim Q3(a) · **3 marks**

Differentiate $f(x) = 4x^2 - 5x$, using the method of first principles.

Criteria	Marks
Writes the first-principles limit with $f(x + h)$ substituted	1
Expands and simplifies the difference quotient	1
Evaluates the limit to get $f'(x) = 8x - 5$	1

Solution.

$f'(x) = \lim_{h \to 0} \dfrac{f(x + h) - f(x)}{h}$. Here

$$f(x + h) = 4(x + h)^2 - 5(x + h) = 4x^2 + 8xh + 4h^2 - 5x - 5h,$$

so

$$f(x + h) - f(x) = 8xh + 4h^2 - 5h = h(8x + 4h - 5).$$

Therefore $f'(x) = \lim_{h \to 0} \dfrac{h(8x + 4h - 5)}{h} = \lim_{h \to 0} (8x + 4h - 5) = 8x - 5.$

Q16 Pymble 2022 Prelim Q15 · **3 marks**

Find the derivative of $f(x) = 5x^2 - 2x$ by first principles.

Criteria	Marks
Writes the first-principles limit with $f(x+h)$ substituted	1
Expands and simplifies the difference quotient	1
Evaluates the limit to get $f'(x) = 10x - 2$	1

Solution.

$f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$. Here

$$f(x+h) = 5(x+h)^2 - 2(x+h) = 5x^2 + 10xh + 5h^2 - 2x - 2h,$$

so

$$f(x+h) - f(x) = 10xh + 5h^2 - 2h = h(10x + 5h - 2).$$

Therefore $f'(x) = \lim_{h\to 0} \dfrac{h(10x+5h-2)}{h} = \lim_{h\to 0}(10x + 5h - 2) = 10x - 2.$

Q17 Baulkham Hills 2024 Prelim Q20 · **2 marks**

Differentiate $y = x^2 + 6x$ from first principles.

Criteria	Marks
Sets up the first-principles limit and expands the difference quotient	1
Evaluates the limit to get $\dfrac{dy}{dx} = 2x + 6$	1

Solution.

With $f(x) = x^2 + 6x$, $\dfrac{dy}{dx} = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$. Here

$$f(x+h) = (x+h)^2 + 6(x+h) = x^2 + 2xh + h^2 + 6x + 6h,$$

so

$$f(x+h) - f(x) = 2xh + h^2 + 6h = h(2x + h + 6).$$

Therefore $\dfrac{dy}{dx} = \lim_{h\to 0} \dfrac{h(2x+h+6)}{h} = \lim_{h\to 0}(2x + h + 6) = 2x + 6.$

Q18 Fort St 2023 Trial Q9 · **1 mark**

Which of the following is the correct expression for differentiating $f(x) = x^2 - 2x$ with first principles?

(A) $f'(x) = \lim\limits_{h\to 0} \dfrac{(x-h)^2 - 2(x-h) - (x^2 - 2x)}{h}$

(B) $f'(x) = \lim\limits_{h\to 0} \dfrac{(x-h)^2 - (x-h) - (x^2 - 2x)}{h}$

(C) $f'(x) = \lim\limits_{h\to 0} \dfrac{(x+h)^2 - 2(x+h) - (x^2 - 2x)}{h}$

(D) $f'(x) = \lim\limits_{h\to 0} \dfrac{(x+h)^2 - 2(x+h) + (x^2 - 2x)}{h}$

Criteria	**Marks**
Correct option (C)	1

Solution.

The first-principles definition is $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$. Substituting $f(x+h) = (x+h)^2 - 2(x+h)$ and subtracting $f(x) = x^2 - 2x$ gives $\lim\limits_{h\to 0} \dfrac{(x+h)^2 - 2(x+h) - (x^2 - 2x)}{h}$. Option (A) uses $x - h$ instead of $x + h$; option (B) drops the coefficient 2; option (D) adds $f(x)$ instead of subtracting it. The answer is **(C)**.

Q19 Penrith 2023 Trial Q11 · **3 marks**

A student was asked to differentiate $f(x) = 2x^2 + x$ using the first Principles of Differential Calculus. The student began the solution as shown below.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Complete the solution.

Criteria	**Marks**
Substitutes $f(x+h)$ and forms the difference quotient	1
Expands and simplifies the difference quotient	1
Evaluates the limit to get $f'(x) = 4x + 1$	1

Solution.

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$. Here

$$f(x+h) = 2(x+h)^2 + (x+h) = 2x^2 + 4xh + 2h^2 + x + h,$$

so

$$f(x+h) - f(x) = 4xh + 2h^2 + h = h(4x + 2h + 1).$$

Therefore $f'(x) = \lim_{h \to 0} \frac{h(4x+2h+1)}{h} = \lim_{h \to 0} (4x + 2h + 1) = 4x + 1.$

Q20 North Sydney Girls 2022 Prelim Q19 · **2 marks**

Differentiate $f(x) = 3x - 4x^2$ using first principles, given $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$.

Criteria	**Marks**
Substitutes $f(x+h)$ and expands the difference quotient	1
Evaluates the limit to get $f'(x) = 3 - 8x$	1

Solution.

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$. Here

$$f(x+h) = 3(x+h) - 4(x+h)^2 = 3x + 3h - 4x^2 - 8xh - 4h^2,$$

so

$$f(x+h) - f(x) = 3h - 8xh - 4h^2 = h(3 - 8x - 4h).$$

Therefore $f'(x) = \lim_{h \to 0} \frac{h(3-8x-4h)}{h} = \lim_{h \to 0} (3 - 8x - 4h) = 3 - 8x.$

Q21 North Sydney Girls 2023 Prelim Q21 · **3 marks**

Differentiate $f(x) = \dfrac{1}{x}$ from first principles, given that $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$.

Criteria	Marks
Writes the first-principles limit with $f(x+h) = \dfrac{1}{x+h}$ substituted	1
Combines the fractions over a common denominator and simplifies the difference quotient	1
Evaluates the limit to get $f'(x) = -\dfrac{1}{x^2}$	1

Solution.

$f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$. Here $f(x+h) = \dfrac{1}{x+h}$, so

$$f(x+h) - f(x) = \frac{1}{x+h} - \frac{1}{x} = \frac{x - (x+h)}{x(x+h)} = \frac{-h}{x(x+h)}.$$

Dividing by h:

$$\frac{f(x+h) - f(x)}{h} = \frac{-h}{h\,x(x+h)} = \frac{-1}{x(x+h)}.$$

Therefore $f'(x) = \lim\limits_{h\to 0} \dfrac{-1}{x(x+h)} = \dfrac{-1}{x \cdot x} = -\dfrac{1}{x^2}$.

Q22 Kambala 2024 Prelim Q25 · **3 marks**

Differentiation from first principles is defined by $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$. Use differentiation from first principles to find the derivative of $x^2 + 3x - 4$.

Criteria	Marks
Correct substitution of $f(x+h)$ into the difference quotient	1
Expands and simplifies the difference quotient	1
Evaluates the limit to get $f'(x) = 2x + 3$	1

Solution.

Let $f(x) = x^2 + 3x - 4$, so $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$. Here

$$f(x+h) = (x+h)^2 + 3(x+h) - 4 = x^2 + 2xh + h^2 + 3x + 3h - 4,$$

so

$$f(x+h) - f(x) = 2xh + h^2 + 3h = h(2x + h + 3).$$

Therefore $f'(x) = \lim\limits_{h\to 0} \dfrac{h(2x+h+3)}{h} = \lim\limits_{h\to 0} (2x + h + 3) = 2x + 3.$

Q23 Hornsby Girls 2021 Prelim Q14 · **2 marks**

Find the derivative of $f(x) = \dfrac{6}{x}$ using the first principles method.

Criteria	**Marks**
Writes the first-principles limit with $f(x+h) = \dfrac{6}{x+h}$ substituted	1
Combines the fractions, simplifies and evaluates the limit to get $f'(x) = -\dfrac{6}{x^2}$	1

Solution.

$f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$. Here $f(x+h) = \dfrac{6}{x+h}$, so

$$f(x+h) - f(x) = \frac{6}{x+h} - \frac{6}{x} = \frac{6x - 6(x+h)}{x(x+h)} = \frac{-6h}{x(x+h)}.$$

Dividing by h:

$$\frac{f(x+h) - f(x)}{h} = \frac{-6h}{h\,x(x+h)} = \frac{-6}{x(x+h)}.$$

Therefore $f'(x) = \lim\limits_{h\to 0} \dfrac{-6}{x(x+h)} = \dfrac{-6}{x \cdot x} = -\dfrac{6}{x^2}$.

The power rule

4. The power rule: polynomials, sums and multiples

Q24 Hornsby Girls 2021 Prelim Q13(a) · 1 mark

Differentiate $y = 3x^3 + 4x^5$.

Criteria	Marks
Correct derivative	1

Solution.

Apply the power rule term by term, multiplying by the index and reducing it by one:

$$\frac{dy}{dx} = 3(3x^2) + 4(5x^4) = 9x^2 + 20x^4.$$

So $\dfrac{dy}{dx} = 20x^4 + 9x^2$.

Q25 Cherrybrook Tech 2022 Prelim Q18(a) · 1 mark

Differentiate the following with respect to x. Leave your answer in simplified form with positive indices: $y = 7x^3 - 5x + 2$.

Criteria	Marks
Correct derivative	1

Solution.

Differentiate term by term. The constant 2 differentiates to 0:

$$\frac{dy}{dx} = 7(3x^2) - 5(1) + 0 = 21x^2 - 5.$$

So $\dfrac{dy}{dx} = 21x^2 - 5$.

Q26 Hurlstone 2021 Prelim Q7 · **1 mark**

Given the function $y = 4x^5$, which one of the following is correct?

(A) $\dfrac{dy}{dx} = 4x^4$

(B) $\dfrac{dy}{dx} = 5x^4$

(C) $\dfrac{dy}{dx} = 1024x^4$

(D) $\dfrac{dy}{dx} = 20x^4$

Criteria	Marks
Correct option (D)	1

Solution.

Apply the power rule: multiply by the index 5 and reduce it by one, $\dfrac{dy}{dx} = 4 \times 5x^4 = 20x^4$. The answer is **(D)**.

Q27 Shore 2023 Prelim Q16(a) · **1 mark**

Differentiate the following with respect to x: $y = 15x^3 - x^2 - 2$.

Criteria	Marks
Correct derivative	1

Solution.

Differentiate term by term. The constant -2 differentiates to 0:

$$\frac{dy}{dx} = 15(3x^2) - 2x - 0 = 45x^2 - 2x.$$

So $\dfrac{dy}{dx} = 45x^2 - 2x$.

Q28 St George Girls 2023 Prelim Q8 · **1 mark**

Find $\dfrac{dy}{dx}$, if $y = 6x^3 - 5x - 1$.

Criteria	Marks
Correct derivative	1

Solution.

Differentiate term by term with the power rule. The constant -1 differentiates to 0:

$$\frac{dy}{dx} = 6(3x^2) - 5(1) - 0 = 18x^2 - 5.$$

So $\dfrac{dy}{dx} = 18x^2 - 5$.

Q29 Killara 2022 Prelim Q12(a) · **1 mark**

Differentiate the following function: $f(x) = x^{15}$.

Criteria	Marks
Correct derivative	1

Solution.

Apply the power rule: multiply by the index 15 and reduce it by one:

$$f'(x) = 15x^{14}.$$

So $f'(x) = 15x^{14}$.

Q30 Glenwood 2020 Trial Q1 · **1 mark**

If $f(x) = x^2 - 2x$, what is the value of $f'(3)$?

(A) 0

(B) 6

(C) 3

(D) 4

Criteria	Marks
Correct option (D)	1

Solution.

Differentiate using the power rule: $f'(x) = 2x - 2$. Substituting $x = 3$ gives $f'(3) = 2(3) - 2 = 4$. The answer is **(D)**.

Q31 Gosford 2024 Prelim Q14(b) · **3 marks**

Differentiate the following, simplifying fully where possible: $f(x) = 3x(4x + 5)$.

Criteria	Marks
Expands the product into a polynomial	1
Differentiates using the power rule	1
Correct simplified answer	1

Solution.

Expand the brackets first so the power rule applies term by term:

$$f(x) = 3x(4x + 5) = 12x^2 + 15x.$$

Differentiating:

$$f'(x) = 12(2x) + 15 = 24x + 15.$$

So $f'(x) = 24x + 15$.

5. The power rule: negative and fractional indices

Q32 Kambala 2021 Prelim Q3(a) · **1 mark**

Find $f'(x)$ where $f(x) = x^2 + \dfrac{3}{x}$.

Criteria	Marks
Correct derivative $f'(x) = 2x - \dfrac{3}{x^2}$	1

Solution.

Rewrite with a negative index: $f(x) = x^2 + 3x^{-1}$. Applying the power rule term by term,

$$f'(x) = 2x + 3(-1)x^{-2} = 2x - 3x^{-2}.$$

The derivative is $f'(x) = 2x - \dfrac{3}{x^2}$.

Q33 Sydney Grammar 2022 Prelim Q10(bi) · **1 mark**

Differentiate $y = \dfrac{3}{x}$ with respect to x.

Criteria	Marks
Correct derivative $\dfrac{dy}{dx} = -\dfrac{3}{x^2}$	1

Solution.

Write y with a negative index: $y = 3x^{-1}$. By the power rule,

$$\frac{dy}{dx} = 3(-1)x^{-2} = -3x^{-2}.$$

The derivative is $\dfrac{dy}{dx} = -\dfrac{3}{x^2}$.

Q34 Killara 2021 Prelim Q11(a) · **2 marks**

Differentiate $y = \dfrac{x^3 - 3x}{x^2}$.

Criteria	Marks
Simplifies to $y = x - 3x^{-1}$ (or equivalent split into powers of x)	1
Correct derivative $\dfrac{dy}{dx} = 1 + \dfrac{3}{x^2}$	1

Solution.

Split the fraction and simplify each term using index laws:

$$y = \frac{x^3 - 3x}{x^2} = \frac{x^3}{x^2} - \frac{3x}{x^2} = x - 3x^{-1}.$$

Differentiating term by term,

$$\frac{dy}{dx} = 1 - 3(-1)x^{-2} = 1 + 3x^{-2}.$$

The derivative is $\dfrac{dy}{dx} = 1 + \dfrac{3}{x^2}$.

Q35 Presbyterian 2022 Prelim Q7 · **1 mark**

What is the derivative of $-\dfrac{3}{x^2}$?

(A) $-\dfrac{3}{x}$

(B) $-\dfrac{6}{x^3}$

(C) $-\dfrac{3}{x^3}$

(D) $\dfrac{6}{x^3}$

Criteria	Marks
Correct option (D)	1

Solution.

Write the function with a negative index: $-\dfrac{3}{x^2} = -3x^{-2}$. By the power rule,

$$\frac{d}{dx}\left(-3x^{-2}\right) = -3(-2)x^{-3} = 6x^{-3} = \frac{6}{x^3}.$$

The answer is **(D)**.

Q36 St George Girls 2023 Prelim Q17(a(ii)) · 2 marks

Differentiate the following with respect to x: $f(x) = \sqrt{x} + \dfrac{3}{x}$.

Criteria	Marks
Rewrites as $f(x) = x^{1/2} + 3x^{-1}$ (both terms in index form)	1
Correct derivative $f'(x) = \dfrac{1}{2\sqrt{x}} - \dfrac{3}{x^2}$	1

Solution.

Rewrite each term with an index: $f(x) = x^{1/2} + 3x^{-1}$. By the power rule,

$$f'(x) = \tfrac{1}{2}x^{-1/2} + 3(-1)x^{-2} = \tfrac{1}{2}x^{-1/2} - 3x^{-2}.$$

The derivative is $f'(x) = \dfrac{1}{2\sqrt{x}} - \dfrac{3}{x^2}$.

Q37 Barker 2021 Prelim Q12(bγ) · 2 marks

Differentiate the following. Leave your answer with a positive, non-fractional index, where necessary: $f(x) = 5x^{-2} - 6x^2$.

Criteria	Marks
Differentiates using the power rule	1
Correct answer with a positive index	1

Solution.

Apply the power rule to each term, keeping the negative index for now:

$$f'(x) = 5(-2x^{-3}) - 6(2x) = -10x^{-3} - 12x.$$

Rewriting the negative index as a positive-index (non-fractional) form:

$$f'(x) = -\frac{10}{x^3} - 12x.$$

Q38 Presbyterian 2023 Prelim Q23(a) · **2 marks**

Differentiate the following with respect to x: $y = \sqrt[3]{x^2}$.

Criteria	Marks
Rewrites in index form $y = x^{2/3}$	1
Correct derivative $\dfrac{dy}{dx} = \dfrac{2}{3\sqrt[3]{x}}$	1

Solution.

Rewrite the cube root in index form: $y = \sqrt[3]{x^2} = x^{2/3}$. By the power rule,

$$\frac{dy}{dx} = \tfrac{2}{3}x^{2/3-1} = \tfrac{2}{3}x^{-1/3}.$$

The derivative is $\dfrac{dy}{dx} = \dfrac{2}{3\sqrt[3]{x}}$.

Q39 North Sydney Girls 2023 Prelim Q19(b) · **2 marks**

Differentiate $y = \dfrac{-x^2 + 10x - 16}{x^2}$.

Criteria	Marks
Simplifies to $y = -1 + 10x^{-1} - 16x^{-2}$ (each term as a power of x)	1
Correct derivative $\dfrac{dy}{dx} = -\dfrac{10}{x^2} + \dfrac{32}{x^3}$	1

Solution.

Split the fraction and simplify each term using index laws:

$$y = \frac{-x^2 + 10x - 16}{x^2} = \frac{-x^2}{x^2} + \frac{10x}{x^2} - \frac{16}{x^2} = -1 + 10x^{-1} - 16x^{-2}.$$

Differentiating term by term,

$$\frac{dy}{dx} = 0 + 10(-1)x^{-2} - 16(-2)x^{-3} = -10x^{-2} + 32x^{-3}.$$

The derivative is $\dfrac{dy}{dx} = -\dfrac{10}{x^2} + \dfrac{32}{x^3}$.

Q40 North Sydney Girls 2022 Prelim Q10 · **1 mark**

Let $g(x) = x^3 + 3$ and $f(x) = \sqrt{x}$. What is the simplified expression for $f'(g(x))$?

(A) $x\sqrt{3}$

(B) $\dfrac{1}{2\sqrt{x^3+3}}$

(C) $\sqrt{x^3+3}$

(D) $x\sqrt{x+3}$

Criteria	**Marks**
Correct option (B)	1

Solution.

First differentiate $f(x) = \sqrt{x} = x^{1/2}$ using the power rule: $f'(x) = \frac{1}{2}x^{-1/2} = \dfrac{1}{2\sqrt{x}}$. Then substitute $g(x) = x^3 + 3$ in place of x:

$$f'(g(x)) = \frac{1}{2\sqrt{g(x)}} = \frac{1}{2\sqrt{x^3+3}}.$$

The answer is **(B)**.