Year 11 Mathematics Advanced

Topic: Trigonometric Identities and Equations (1 of 3)

Review Questions (Worked Solutions): Reciprocal ratios and trigonometric identities

Term 3, 2026

Reciprocal ratios and exact values

1. Exact reciprocal ratios (sec, cosec, cot) and evaluation

Q1 Gosford 2020 Trial Q1 · 1 mark

What is the value of $\operatorname{cosec}\dfrac{\pi}{3}$ to three significant figures? (A) 1.00 (B) 1.15 (C) 1.41 (D) 2.00

Criteria	Marks
Correct option (B)	1

Solution.

$$\operatorname{cosec}\frac{\pi}{3} = \frac{1}{\sin\frac{\pi}{3}} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3} \approx 1.1547.$$

To three significant figures: 1.15. **(B)**

Q2 International Grammar 2021 Prelim Q5 · 1 mark

What is the exact value of $\sin 135° + \operatorname{cosec} 60°$? (A) $\frac{2\sqrt{2}-\sqrt{3}}{\sqrt{6}}$ (B) $\frac{2\sqrt{2}-1}{\sqrt{2}}$ (C) $\frac{2\sqrt{2}+\sqrt{3}}{\sqrt{6}}$ (D) $\frac{2\sqrt{2}+1}{\sqrt{2}}$

Criteria	Marks
Correct option (C)	1

Solution.

$$\sin 135° = \frac{\sqrt{2}}{2} \quad \text{and} \quad \operatorname{cosec} 60° = \frac{1}{\sin 60°} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}.$$

$$\sin 135° + \operatorname{cosec} 60° = \frac{\sqrt{2}}{2} + \frac{2\sqrt{3}}{3} = \frac{3\sqrt{2}+4\sqrt{3}}{6} = \frac{2\sqrt{2}+\sqrt{3}}{\sqrt{6}}. \qquad \textbf{(C)}$$

Q3 Blacktown Boys 2022 Prelim Q12 · **2 marks**

Find the exact value of $\sec\dfrac{4\pi}{3}\cot\dfrac{4\pi}{3}$.

Criteria	Marks
Correct exact value of $\sec\frac{4\pi}{3}$ and $\cot\frac{4\pi}{3}$	1
Correct simplified exact answer	1

Solution.

$\dfrac{4\pi}{3}$ lies in the third quadrant, where $\cos < 0$ and $\tan > 0$.

$$\sec\frac{4\pi}{3} = \frac{1}{\cos\frac{4\pi}{3}} = \frac{1}{-\frac{1}{2}} = -2, \qquad \cot\frac{4\pi}{3} = \frac{1}{\tan\frac{4\pi}{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}.$$

$$\therefore\ \sec\frac{4\pi}{3}\cot\frac{4\pi}{3} = (-2)\left(\frac{\sqrt{3}}{3}\right) = -\frac{2\sqrt{3}}{3}.$$

Q4 Cherrybrook Tech 2022 Prelim Q11 · **2 marks**

What is the exact value of $\cot 240^\circ \cos 120^\circ$?

Criteria	Marks
Correct exact value of $\cot 240^\circ$ and $\cos 120^\circ$	1
Correct simplified exact answer	1

Solution.

240° lies in the third quadrant (reference angle 60°), where $\tan > 0$; 120° lies in the second quadrant, where $\cos < 0$.

$$\cot 240^\circ = \frac{1}{\tan 240^\circ} = \frac{1}{\tan 60^\circ} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}, \qquad \cos 120^\circ = -\frac{1}{2}.$$

$$\therefore\ \cot 240^\circ \cos 120^\circ = \frac{\sqrt{3}}{3}\left(-\frac{1}{2}\right) = -\frac{\sqrt{3}}{6}.$$

Q5 Barker 2021 Prelim Q12(c) · **2 marks**

Find the value of $\sin\dfrac{\pi}{3}+\sec\dfrac{\pi}{4}$, leaving your answer as a single fraction in exact form.

Criteria	Marks
Correct exact value of $\sin\frac{\pi}{3}$ and $\sec\frac{\pi}{4}$	1
Combines into a single exact fraction	1

Solution.

$$\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}\quad\text{and}\quad\sec\frac{\pi}{4}=\frac{1}{\cos\frac{\pi}{4}}=\frac{1}{\frac{\sqrt{2}}{2}}=\sqrt{2}.$$

$$\sin\frac{\pi}{3}+\sec\frac{\pi}{4}=\frac{\sqrt{3}}{2}+\sqrt{2}=\frac{\sqrt{3}}{2}+\frac{2\sqrt{2}}{2}=\frac{\sqrt{3}+2\sqrt{2}}{2}.$$

2. Finding an exact ratio given another ratio and quadrant

Q6 Presbyterian 2021 Trial Q3 · **1 mark**

If $\tan\theta=-\dfrac{2}{3}$ and $\dfrac{\pi}{2}<\theta<\pi$, find $\sec\theta$. (Multiple choice.)

Criteria	Marks
Correct option (B)	1

Solution.

Here $\dfrac{\pi}{2}<\theta<\pi$ is the second quadrant, where $\cos\theta<0$. From $\tan\theta=-\dfrac{2}{3}$, the right triangle has opposite 2, adjacent 3, so the hypotenuse is $\sqrt{2^2+3^2}=\sqrt{13}$. Since $\cos\theta<0$,

$$\cos\theta=-\frac{3}{\sqrt{13}}\quad\Rightarrow\quad\sec\theta=\frac{1}{\cos\theta}=-\frac{\sqrt{13}}{3}.$$

The correct option is **(B)**.

Q7 Knox 2021 Prelim Q7 · **1 mark**

If $\cos\theta = -\dfrac{12}{13}$ and $180° \leq x \leq 360°$, find $\cot\theta$. (Multiple choice.)

Criteria	Marks
Correct option (D)	1

Solution.

With $\cos\theta = -\dfrac{12}{13} < 0$ and $180° \leq \theta \leq 360°$, the angle lies in the third quadrant, so $\sin\theta < 0$. Using $\sin^2\theta + \cos^2\theta = 1$,

$$\sin\theta = -\sqrt{1 - \tfrac{144}{169}} = -\frac{5}{13}.$$

Then

$$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{-\frac{12}{13}}{-\frac{5}{13}} = \frac{12}{5}.$$

The correct option is **(D)**.

Q8 Cherrybrook Tech 2021 Prelim Q14 · **1 mark**

Given $\sin\theta = \dfrac{7}{12}$ for an obtuse angle θ, find the exact value of $\cos\theta$.

Criteria	Marks
Correct exact value of $\cos\theta$	1

Solution.

An obtuse angle lies in the second quadrant, where $\cos\theta < 0$. From $\cos^2\theta = 1 - \sin^2\theta = 1 - \dfrac{49}{144} = \dfrac{95}{144}$,

$$\cos\theta = -\frac{\sqrt{95}}{12}.$$

Q9 Killara 2021 Prelim Q15 · **2 marks**

If $\sin\theta = \frac{4}{7}$, find the exact value of $\sec\theta$.

Criteria	Marks
Finds $\cos\theta = \frac{\sqrt{33}}{7}$	1
Correct exact value of $\sec\theta$	1

Solution.

Using $\cos^2\theta = 1 - \sin^2\theta = 1 - \frac{16}{49} = \frac{33}{49}$,

$$\cos\theta = \frac{\sqrt{33}}{7}.$$

Therefore

$$\sec\theta = \frac{1}{\cos\theta} = \frac{7}{\sqrt{33}} = \frac{7\sqrt{33}}{33}.$$

Q10 Sydney Girls 2021 Prelim Q3(b) · **2 marks**

If $\sin A = -\frac{1}{3}$ and $\tan A < 0$, find $\sec A$.

Criteria	Marks
Determines quadrant and finds $\cos A = \frac{2\sqrt{2}}{3}$	1
Correct exact value of $\sec A$	1

Solution.

Since $\sin A < 0$ and $\tan A < 0$, and $\tan A = \frac{\sin A}{\cos A}$, we need $\cos A > 0$ (fourth quadrant). Using $\cos^2 A = 1 - \sin^2 A = 1 - \frac{1}{9} = \frac{8}{9}$,

$$\cos A = \frac{2\sqrt{2}}{3}.$$

Therefore

$$\sec A = \frac{1}{\cos A} = \frac{3}{2\sqrt{2}} = \frac{3\sqrt{2}}{4}.$$

Q11 North Sydney Boys 2020 Trial Q22 · **2 marks**

Find the exact value of $\cot\theta$ given that $\cos\theta = 0.6$ and $\sin\theta < 0$.

Criteria	**Marks**
Finds $\sin\theta = -\frac{4}{5}$	1
Correct exact value of $\cot\theta$	1

Solution.

Write $\cos\theta = 0.6 = \frac{3}{5}$. Using $\sin^2\theta = 1 - \cos^2\theta = 1 - \frac{9}{25} = \frac{16}{25}$, and since $\sin\theta < 0$,

$$\sin\theta = -\frac{4}{5}.$$

Therefore

$$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{\frac{3}{5}}{-\frac{4}{5}} = -\frac{3}{4}.$$

Q12 Barker 2025 Trial Q19 · **2 marks**

Given that $\cos\theta = -\frac{2}{7}$ and $\frac{\pi}{2} \leq \theta \leq \pi$, find the exact value of $\operatorname{cosec}\theta$.

Criteria	**Marks**
Finds $\sin\theta = \frac{3\sqrt{5}}{7}$	1
Correct exact value of $\operatorname{cosec}\theta$	1

Solution.

The interval $\frac{\pi}{2} \leq \theta \leq \pi$ is the second quadrant, where $\sin\theta > 0$. Using $\sin^2\theta = 1 - \cos^2\theta = 1 - \frac{4}{49} = \frac{45}{49}$,

$$\sin\theta = \frac{\sqrt{45}}{7} = \frac{3\sqrt{5}}{7}.$$

Therefore

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{7}{3\sqrt{5}} = \frac{7\sqrt{5}}{15}.$$

Q13 James Ruse 2020 Trial Q19(a) · **2 marks**

Find $\tan A$ if $\operatorname{cosec} A = -\dfrac{13}{12}$ and $\sec A > 0$.

Criteria	**Marks**
Finds $\sin A = -\dfrac{12}{13}$ and $\cos A = \dfrac{5}{13}$	1
Correct value of $\tan A$	1

Solution.

From $\operatorname{cosec} A = -\dfrac{13}{12}$ we have $\sin A = -\dfrac{12}{13} < 0$. Since $\sec A > 0$, $\cos A > 0$ (fourth quadrant). Using $\cos^2 A = 1 - \sin^2 A = 1 - \dfrac{144}{169} = \dfrac{25}{169}$,

$$\cos A = \frac{5}{13}.$$

Therefore

$$\tan A = \frac{\sin A}{\cos A} = \frac{-\frac{12}{13}}{\frac{5}{13}} = -\frac{12}{5}.$$

Q14 Sydney Girls 2023 Trial Q10 · **1 mark**

If $\tan^2\theta = 1 - e^2$, find the value of $\sec\theta + \tan^3\theta \operatorname{cosec}\theta$. (Multiple choice.)

Criteria	**Marks**
Correct option (B)	1

Solution.

Since $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$ and $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$, we have $\tan^3\theta \operatorname{cosec}\theta = \dfrac{\sin^3\theta}{\cos^3\theta} \cdot \dfrac{1}{\sin\theta} = \dfrac{\sin^2\theta}{\cos^3\theta}$. Then

$$\sec\theta + \tan^3\theta \operatorname{cosec}\theta = \frac{1}{\cos\theta} + \frac{\sin^2\theta}{\cos^3\theta} = \frac{\cos^2\theta + \sin^2\theta}{\cos^3\theta} = \frac{1}{\cos^3\theta} = \sec^3\theta.$$

Now $\sec^2\theta = 1 + \tan^2\theta = 1 + (1 - e^2) = 2 - e^2$, so

$$\sec^3\theta = \left(\sec^2\theta\right)^{\frac{3}{2}} = \left(2 - e^2\right)^{\frac{3}{2}}.$$

The correct option is **(B)**.

Working with identities

3. Simplifying expressions using identities

Q15 Abbotsleigh 2021 Trial Q11(b) · **1 mark**

Simplify $1 + \tan^2 x$.

Criteria	Marks
Correct simplification $\sec^2 x$	1

Solution.

Using the Pythagorean identity $\sec^2 x = 1 + \tan^2 x$:

$$1 + \tan^2 x = \sec^2 x.$$

Q16 Fort St 2022 Prelim Q3 · **1 mark**

Simplify the expression $\sec x \sin x$. Options: (A) $\operatorname{cosec} x$ (B) $\cos x$ (C) $\tan x$ (D) 1.

Criteria	Marks
Correct option (C)	1

Solution.

$$\sec x \sin x = \frac{1}{\cos x} \cdot \sin x = \frac{\sin x}{\cos x} = \tan x.$$

The answer is **(C)**.

Q17 Riverview 2024 Trial Q19 · **1 mark**

Simplify $\dfrac{\sin^2 x + \cos^2 x}{\cos^2 x}$.

Criteria	Marks
Correct simplification $\sec^2 x$	1

Solution.

Since $\sin^2 x + \cos^2 x = 1$:

$$\frac{\sin^2 x + \cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x.$$

Q18 Trinity Grammar 2020 Trial Q3 · **1 mark**

Which expression is equivalent to $\tan\theta + \cot\theta$? Options: (A) $\operatorname{cosec}\theta + \sec\theta$ (B) $\sec\theta\operatorname{cosec}\theta$ (C) 1 (D) 2.

Criteria	Marks
Correct option (B)	1

Solution.

Write both terms over a common denominator:

$$\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta} = \sec\theta\operatorname{cosec}\theta.$$

The answer is **(B)**.

Q19 St George Girls 2021 Prelim Q1(b) · **2 marks**

Simplify $\dfrac{1-\cos^2\theta}{\tan\theta}$.

Criteria	Marks
Uses $1-\cos^2\theta = \sin^2\theta$ and $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$	1
Correct simplification $\sin\theta\cos\theta$	1

Solution.

Using $1-\cos^2\theta = \sin^2\theta$ and $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$:

$$\frac{1-\cos^2\theta}{\tan\theta} = \frac{\sin^2\theta}{\frac{\sin\theta}{\cos\theta}} = \sin^2\theta\cdot\frac{\cos\theta}{\sin\theta} = \sin\theta\cos\theta.$$

Q20 Moriah 2021 Prelim Q8 · **2 marks**

Simplify $\dfrac{\cos x - \cos^3 x}{\sin^3 x}$.

Criteria	Marks
Factors numerator and applies $1 - \cos^2 x = \sin^2 x$	1
Correct simplification $\cot x$	1

Solution.

Factor the numerator:

$$\frac{\cos x - \cos^3 x}{\sin^3 x} = \frac{\cos x\,(1 - \cos^2 x)}{\sin^3 x} = \frac{\cos x\ \sin^2 x}{\sin^3 x} = \frac{\cos x}{\sin x} = \cot x.$$

Q21 Riverview 2022 Trial Q19 · **2 marks**

Expand and simplify $(\operatorname{cosec} A + 1)(\operatorname{cosec} A - 1)$.

Criteria	Marks
Expands to $\operatorname{cosec}^2 A - 1$	1
Correct simplification $\cot^2 A$	1

Solution.

Expanding as a difference of two squares:

$$(\operatorname{cosec} A + 1)(\operatorname{cosec} A - 1) = \operatorname{cosec}^2 A - 1.$$

Since $\operatorname{cosec}^2 A = 1 + \cot^2 A$, we have $\operatorname{cosec}^2 A - 1 = \cot^2 A$.

Q22 Blacktown Boys 2024 Trial Q9 · **1 mark**

Which of the following is equivalent to $\dfrac{\tan^2 x - 1}{\tan^2 x + 1}$? Options: (A) $1 - 2\cos^2 x$ (B) $1 - \sec^2 x$ (C) $\dfrac{1}{\sin^2 x - \cos^2 x}$ (D) $\sec^2 x - \operatorname{cosec}^2 x$.

Criteria	**Marks**
Correct option (A)	1

Solution.

The denominator is $\tan^2 x + 1 = \sec^2 x$. Multiply numerator and denominator by $\cos^2 x$:

$$\frac{\tan^2 x - 1}{\sec^2 x} = \left(\frac{\sin^2 x}{\cos^2 x} - 1\right)\cos^2 x = \sin^2 x - \cos^2 x.$$

Then $\sin^2 x - \cos^2 x = (1 - \cos^2 x) - \cos^2 x = 1 - 2\cos^2 x$. The answer is **(A)**.

Q23 Killara 2022 Prelim Q34(b) · **3 marks**

For an angle θ with $\pi \leq \theta \leq 2\pi$, $\cos\theta = \dfrac{2}{3}$ and $\sin\theta = -\dfrac{\sqrt{5}}{3}$. Simplify $\dfrac{\cos\theta + 2\sin\theta}{\tan\theta - 3\sin\theta}$, giving the answer in the form $a\sqrt{5} + b$.

Criteria	**Marks**
Substitutes correct values into numerator and denominator	1
Simplifies numerator and denominator	1
Rationalises to reach $\dfrac{4\sqrt{5}}{15} - \dfrac{4}{3}$	1

Solution.

First, $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{-\sqrt{5}/3}{2/3} = -\dfrac{\sqrt{5}}{2}$.

Numerator:

$$\cos\theta + 2\sin\theta = \frac{2}{3} + 2\left(-\frac{\sqrt{5}}{3}\right) = \frac{2 - 2\sqrt{5}}{3}.$$

Denominator:

$$\tan\theta - 3\sin\theta = -\frac{\sqrt{5}}{2} - 3\left(-\frac{\sqrt{5}}{3}\right) = -\frac{\sqrt{5}}{2} + \sqrt{5} = \frac{\sqrt{5}}{2}.$$

Therefore

$$\frac{\cos\theta + 2\sin\theta}{\tan\theta - 3\sin\theta} = \frac{\frac{2-2\sqrt{5}}{3}}{\frac{\sqrt{5}}{2}} = \frac{2 - 2\sqrt{5}}{3} \cdot \frac{2}{\sqrt{5}} = \frac{4 - 4\sqrt{5}}{3\sqrt{5}}.$$

Rationalising by multiplying top and bottom by $\sqrt{5}$:

$$= \frac{(4 - 4\sqrt{5})\sqrt{5}}{15} = \frac{4\sqrt{5} - 20}{15} = \frac{4\sqrt{5}}{15} - \frac{4}{3}.$$

So $a = \dfrac{4}{15}$ and $b = -\dfrac{4}{3}$.

4. Simplifying and evaluating with related and complementary angles

Q24 Manly 2021 Trial Q3 · **1 mark**

Simplify $\operatorname{cosec}(\pi + \theta)$ (MC).

Criteria	Marks
Correct option (A)	1

Solution.

Write the reciprocal, then use the related-angle reduction in the third-quadrant form $\sin(\pi + \theta) = -\sin\theta$:

$$\operatorname{cosec}(\pi + \theta) = \frac{1}{\sin(\pi + \theta)} = \frac{1}{-\sin\theta} = -\frac{1}{\sin\theta}.$$

The answer is **(A)**.

Q25 Shore 2023 Prelim Q7 · **1 mark**

Simplify $-\sec(180^\circ - \theta)\sin\theta$ (MC).

Criteria	Marks
Correct option (B)	1

Solution.

Reduce the related angle $\sec(180^\circ - \theta) = -\sec\theta$ (second quadrant, cosine negative):

$$-\sec(180^\circ - \theta)\sin\theta = -\big(-\sec\theta\big)\sin\theta = \sec\theta\sin\theta = \frac{\sin\theta}{\cos\theta} = \tan\theta.$$

The answer is **(B)**.

Q26 Merewether 2025 Trial Q2 · **1 mark**

Simplify $\cos^2\left(\frac{\pi}{2}-\theta\right)\cot\theta$ (MC).

Criteria	Marks
Correct option (B)	1

Solution.

Use the complementary-angle identity $\cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta$, so $\cos^2\left(\frac{\pi}{2}-\theta\right)=\sin^2\theta$. Then

$$\sin^2\theta\cdot\cot\theta=\sin^2\theta\cdot\frac{\cos\theta}{\sin\theta}=\sin\theta\cos\theta.$$

The fully simplified form is $\sin\theta\cos\theta$, so the answer is **(B)**. (Note option (D) $\sin^2\theta\cot\theta$ is only a partial step and is not the simplified form.)

Q27 Killara 2021 Prelim Q8 · **2 marks**

Fully simplify $\dfrac{\sin 33^\circ+\cos 57^\circ}{\cos 57^\circ}$.

Criteria	Marks
Uses the complementary-angle identity $\cos 57^\circ=\sin 33^\circ$	1
Correct simplified value 2	1

Solution.

Since 57° and 33° are complementary, $\cos 57^\circ=\cos(90^\circ-33^\circ)=\sin 33^\circ$. Substituting:

$$\frac{\sin 33^\circ+\cos 57^\circ}{\cos 57^\circ}=\frac{\sin 33^\circ+\sin 33^\circ}{\sin 33^\circ}=\frac{2\sin 33^\circ}{\sin 33^\circ}=2.$$

Q28 Knox 2023 Prelim Q29 · **2 marks**

Simplify completely $\cos(\pi+\theta)\cos(\pi-\theta)-1$.

Criteria	Marks
Reduces both related angles: $\cos(\pi+\theta)=-\cos\theta$ and $\cos(\pi-\theta)=-\cos\theta$	1
Applies $\cos^2\theta-1=-\sin^2\theta$ for the final form	1

Solution.

Both related angles reduce to $-\cos\theta$: $\cos(\pi+\theta)=-\cos\theta$ (third quadrant) and $\cos(\pi-\theta)=-\cos\theta$ (second quadrant). So

$$\cos(\pi+\theta)\cos(\pi-\theta)-1=(-\cos\theta)(-\cos\theta)-1=\cos^2\theta-1.$$

Using the Pythagorean identity $\cos^2\theta-1=-\left(1-\cos^2\theta\right)=-\sin^2\theta$:

$$\cos(\pi+\theta)\cos(\pi-\theta)-1=-\sin^2\theta.$$

Q29 Sydney Grammar 2022 Prelim Q7 · **1 mark**

Simplify $\dfrac{\sec(180^\circ-\theta)}{\tan(360^\circ-\theta)}$ (MC).

Criteria	Marks
Correct option (D)	1

Solution.

Reduce each related angle: $\sec(180^\circ-\theta)=-\sec\theta$ (second quadrant) and $\tan(360^\circ-\theta)=-\tan\theta$ (fourth quadrant). Then

$$\frac{\sec(180^\circ-\theta)}{\tan(360^\circ-\theta)}=\frac{-\sec\theta}{-\tan\theta}=\frac{\sec\theta}{\tan\theta}=\frac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}=\frac{1}{\sin\theta}=\operatorname{cosec}\theta.$$

The answer is **(D)**.

Q30 Killara 2023 Prelim Q24 · **3 marks**

Fully simplify $\dfrac{\sin\theta}{\cos 60^\circ} + \dfrac{\cos(270^\circ - \theta)}{\tan 45^\circ}$.

Criteria	**Marks**
Evaluates exact values $\cos 60^\circ = \frac{1}{2}$ and $\tan 45^\circ = 1$	1
Reduces the related angle $\cos(270^\circ - \theta) = -\sin\theta$	1
Combines to the fully simplified answer $\sin\theta$	1

Solution.

Evaluate the exact values in the denominators: $\cos 60^\circ = \dfrac{1}{2}$ and $\tan 45^\circ = 1$. So

$$\frac{\sin\theta}{\cos 60^\circ} = \frac{\sin\theta}{\frac{1}{2}} = 2\sin\theta.$$

For the related angle, $\cos(270^\circ - \theta) = \cos 270^\circ \cos\theta + \sin 270^\circ \sin\theta = (0)\cos\theta + (-1)\sin\theta = -\sin\theta$. Hence

$$\frac{\cos(270^\circ - \theta)}{\tan 45^\circ} = \frac{-\sin\theta}{1} = -\sin\theta.$$

Adding the two terms:

$$2\sin\theta + (-\sin\theta) = \sin\theta.$$

5. Proving identities

Q31 Hurlstone 2020 Trial Q34 · **2 marks**

By expressing $\sec\theta$ and $\tan\theta$ in terms of $\sin\theta$ and $\cos\theta$, show that $\sec^2\theta - \tan^2\theta = 1$.

Criteria	Marks
Writes $\sec\theta = \dfrac{1}{\cos\theta}$ and $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$	1
Combines over a common denominator and uses $\sin^2\theta + \cos^2\theta = 1$ to reach 1	1

Solution.

Start from the left-hand side and rewrite in terms of $\sin\theta$ and $\cos\theta$:

$$\text{LHS} = \sec^2\theta - \tan^2\theta = \frac{1}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta} = \frac{1-\sin^2\theta}{\cos^2\theta}.$$

Using the Pythagorean identity $1 - \sin^2\theta = \cos^2\theta$,

$$= \frac{\cos^2\theta}{\cos^2\theta} = 1 = \text{RHS}.$$

Q32 Hornsby Girls 2021 Prelim Q12 · **2 marks**

Prove: $\tan x + \cot x = \sec x \operatorname{cosec} x$.

Criteria	Marks
Writes $\tan x + \cot x = \dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x}$ over a common denominator	1
Applies $\sin^2 x + \cos^2 x = 1$ and identifies $\sec x \operatorname{cosec} x$	1

Solution.

Working on the left-hand side,

$$\text{LHS} = \tan x + \cot x = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{\sin^2 x + \cos^2 x}{\sin x \cos x}.$$

Since $\sin^2 x + \cos^2 x = 1$,

$$= \frac{1}{\sin x \cos x} = \frac{1}{\cos x} \cdot \frac{1}{\sin x} = \sec x \operatorname{cosec} x = \text{RHS}.$$

Q33 Hunters Hill 2023 Trial Q17 · **2 marks**

Prove that $(\sec x + \tan x)(\sec x - \tan x) = 1$.

Criteria	Marks
Expands the difference of two squares to $\sec^2 x - \tan^2 x$	1
Uses $\sec^2 x - \tan^2 x = 1$ (or reduces via sin/cos) to reach 1	1

Solution.

The left-hand side is a difference of two squares:

$$\text{LHS} = (\sec x + \tan x)(\sec x - \tan x) = \sec^2 x - \tan^2 x.$$

Writing in terms of $\sin x$ and $\cos x$,

$$= \frac{1}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x} = \frac{1 - \sin^2 x}{\cos^2 x} = \frac{\cos^2 x}{\cos^2 x} = 1 = \text{RHS}.$$

Q34 North Sydney Boys 2022 Trial Q17 · **2 marks**

Show that $\cot A + \dfrac{\sin A}{1 + \cos A} = \operatorname{cosec} A$.

Criteria	Marks
Writes $\cot A = \dfrac{\cos A}{\sin A}$ and combines the two terms over $\sin A(1 + \cos A)$	1
Uses $\cos^2 A + \sin^2 A = 1$, cancels $(1 + \cos A)$ and reaches $\operatorname{cosec} A$	1

Solution.

Working on the left-hand side, write $\cot A = \dfrac{\cos A}{\sin A}$ and put over a common denominator $\sin A(1 + \cos A)$:

$$\text{LHS} = \frac{\cos A}{\sin A} + \frac{\sin A}{1 + \cos A} = \frac{\cos A(1 + \cos A) + \sin^2 A}{\sin A(1 + \cos A)}.$$

Expanding the numerator and using $\cos^2 A + \sin^2 A = 1$:

$$= \frac{\cos A + \cos^2 A + \sin^2 A}{\sin A(1 + \cos A)} = \frac{\cos A + 1}{\sin A(1 + \cos A)}.$$

Cancelling the factor $(1 + \cos A)$,

$$= \frac{1}{\sin A} = \operatorname{cosec} A = \text{RHS}.$$

Q35 2020 HSC Q19 · **2 marks**

Prove that $\sec\theta - \cos\theta = \sin\theta\tan\theta$.

Criteria	Marks
Writes $\sec\theta = \dfrac{1}{\cos\theta}$ and combines $\dfrac{1-\cos^2\theta}{\cos\theta}$	1
Uses $1-\cos^2\theta = \sin^2\theta$ and rewrites as $\sin\theta\tan\theta$	1

Solution.

Starting from the left-hand side and writing $\sec\theta = \dfrac{1}{\cos\theta}$:

$$\text{LHS} = \sec\theta - \cos\theta = \frac{1}{\cos\theta} - \cos\theta = \frac{1-\cos^2\theta}{\cos\theta}.$$

Using $1-\cos^2\theta = \sin^2\theta$,

$$= \frac{\sin^2\theta}{\cos\theta} = \sin\theta\cdot\frac{\sin\theta}{\cos\theta} = \sin\theta\tan\theta = \text{RHS}.$$

Q36 Baulkham Hills 2024 Trial Q17 · **2 marks**

Prove $\dfrac{1}{\sec x - \tan x} + \dfrac{1}{\sec x + \tan x} = 2\sec x$.

Criteria	Marks
Adds the two fractions over the common denominator $(\sec x - \tan x)(\sec x + \tan x)$	1
Uses $\sec^2 x - \tan^2 x = 1$ to reduce the result to $2\sec x$	1

Solution.

Add the two fractions on the left over their common denominator:

$$\text{LHS} = \frac{(\sec x + \tan x) + (\sec x - \tan x)}{(\sec x - \tan x)(\sec x + \tan x)} = \frac{2\sec x}{\sec^2 x - \tan^2 x}.$$

Since $\sec^2 x - \tan^2 x = 1$,

$$= \frac{2\sec x}{1} = 2\sec x = \text{RHS}.$$

Q37 Sydney Tech 2023 Trial Q15 · **3 marks**

Prove that $\sin^4\theta - \cos^4\theta = 2\sin^2\theta - 1$.

Criteria	Marks
Factors the LHS as the difference of two squares $(\sin^2\theta - \cos^2\theta)(\sin^2\theta + \cos^2\theta)$	1
Uses $\sin^2\theta + \cos^2\theta = 1$ to reach $\sin^2\theta - \cos^2\theta$	1
Replaces $\cos^2\theta = 1 - \sin^2\theta$ to obtain $2\sin^2\theta - 1$	1

Solution.

Factor the left-hand side as a difference of two squares:

$$\text{LHS} = \sin^4\theta - \cos^4\theta = (\sin^2\theta - \cos^2\theta)(\sin^2\theta + \cos^2\theta).$$

Since $\sin^2\theta + \cos^2\theta = 1$, the second factor is 1:

$$= \sin^2\theta - \cos^2\theta.$$

Now replace $\cos^2\theta = 1 - \sin^2\theta$:

$$= \sin^2\theta - (1 - \sin^2\theta) = 2\sin^2\theta - 1 = \text{RHS}.$$

Q38 Penrith 2023 Prelim Q29 · **3 marks**

Prove that $\dfrac{1+\cos\varphi}{1-\cos\varphi} - \dfrac{1-\cos\varphi}{1+\cos\varphi} = 4\cot\varphi\ \operatorname{cosec}\varphi$.

Criteria	**Marks**
Combines the two fractions over the common denominator $(1-\cos\varphi)(1+\cos\varphi)$	1
Simplifies the numerator to $4\cos\varphi$ and the denominator to $\sin^2\varphi$	1
Rewrites $\dfrac{4\cos\varphi}{\sin^2\varphi}$ as $4\cot\varphi\ \operatorname{cosec}\varphi$	1

Solution.

Combine the two fractions on the left over the common denominator $(1-\cos\varphi)(1+\cos\varphi) = 1-\cos^2\varphi = \sin^2\varphi$:

$$\text{LHS} = \frac{(1+\cos\varphi)^2 - (1-\cos\varphi)^2}{(1-\cos\varphi)(1+\cos\varphi)}.$$

Expanding the numerator,

$$(1+\cos\varphi)^2 - (1-\cos\varphi)^2 = (1+2\cos\varphi+\cos^2\varphi) - (1-2\cos\varphi+\cos^2\varphi) = 4\cos\varphi,$$

so, using the denominator $\sin^2\varphi$,

$$\text{LHS} = \frac{4\cos\varphi}{\sin^2\varphi} = 4\cdot\frac{\cos\varphi}{\sin\varphi}\cdot\frac{1}{\sin\varphi} = 4\cot\varphi\ \operatorname{cosec}\varphi = \text{RHS}.$$

Q39 2025 HSC Q22 · **2 marks**

Prove that $\dfrac{\sin^4\theta+\cos^4\theta}{\sin^2\theta\cos^2\theta}+2=\sec^2\theta\ \operatorname{cosec}^2\theta$.

Criteria	**Marks**
Writes $\sin^4\theta+\cos^4\theta=1-2\sin^2\theta\cos^2\theta$ (from $(\sin^2\theta+\cos^2\theta)^2$)	1
Simplifies the full LHS to $\dfrac{1}{\sin^2\theta\cos^2\theta}=\sec^2\theta\ \operatorname{cosec}^2\theta$	1

Solution.

Use $(\sin^2\theta+\cos^2\theta)^2=\sin^4\theta+2\sin^2\theta\cos^2\theta+\cos^4\theta=1$, which rearranges to

$$\sin^4\theta+\cos^4\theta=1-2\sin^2\theta\cos^2\theta.$$

Substitute this into the left-hand side:

$$\text{LHS}=\frac{1-2\sin^2\theta\cos^2\theta}{\sin^2\theta\cos^2\theta}+2=\frac{1}{\sin^2\theta\cos^2\theta}-2+2=\frac{1}{\sin^2\theta\cos^2\theta}.$$

Finally,

$$=\frac{1}{\cos^2\theta}\cdot\frac{1}{\sin^2\theta}=\sec^2\theta\ \operatorname{cosec}^2\theta=\text{RHS}.$$

Q40 Sydney Girls 2024 Trial Q28 · **3 marks**

Prove that $(\cot\theta + \operatorname{cosec}\theta)^2 = \dfrac{1+\cos\theta}{1-\cos\theta}$.

Criteria	**Marks**
Writes $\cot\theta + \operatorname{cosec}\theta = \dfrac{\cos\theta+1}{\sin\theta}$ and squares to $\dfrac{(1+\cos\theta)^2}{\sin^2\theta}$	1
Replaces $\sin^2\theta = 1-\cos^2\theta = (1-\cos\theta)(1+\cos\theta)$	1
Cancels a factor of $(1+\cos\theta)$ to reach $\dfrac{1+\cos\theta}{1-\cos\theta}$	1

Solution.

Working on the left-hand side, combine over $\sin\theta$ and square:

$$\text{LHS} = (\cot\theta + \operatorname{cosec}\theta)^2 = \left(\frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta}\right)^2 = \frac{(\cos\theta+1)^2}{\sin^2\theta}.$$

Replace $\sin^2\theta = 1-\cos^2\theta = (1-\cos\theta)(1+\cos\theta)$:

$$= \frac{(1+\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)}.$$

Cancelling a factor of $(1+\cos\theta)$,

$$= \frac{1+\cos\theta}{1-\cos\theta} = \text{RHS}.$$