Year 11 Mathematics Advanced

Topic: Trigonometric Identities and Equations (2 of 3)
Review Questions (Worked Solutions): Trigonometric equations
Term 3, 2026

Basic equations

1. Solving a basic trigonometric equation on a restricted domain

Q1 Abbotsleigh 2020 Trial Q1 · **1 mark**

Solve $\sin x = \dfrac{\sqrt{3}}{2}$ for $0 \leq x \leq 2\pi$.

Criteria	Marks
Correct option (B)	1

Solution.

$\sin x = \dfrac{\sqrt{3}}{2}$ is positive, so x lies in the first and second quadrants. The related angle is $\dfrac{\pi}{3}$.

$$x = \frac{\pi}{3}, \qquad x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}.$$

The answer is **(B)**.

Q2 Barker 2024 Trial Q12 · **2 marks**

Solve $2\cos\theta = 1$ for $0 \leq \theta \leq 2\pi$.

Criteria	Marks
Reduces to $\cos\theta = \frac{1}{2}$ and finds the related angle $\frac{\pi}{3}$	1
Both correct solutions	1

Solution.

$$2\cos\theta = 1 \quad \Rightarrow \quad \cos\theta = \frac{1}{2}.$$

$\cos\theta$ is positive, so θ lies in the first and fourth quadrants. The related angle is $\dfrac{\pi}{3}$.

$$\theta = \frac{\pi}{3}, \qquad \theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}.$$

Q3 Sydney Boys 2022 Trial Q15 · **2 marks**

Solve $\cos\theta = -\dfrac{1}{2}$ for $0 \leq \theta \leq 2\pi$.

Criteria	Marks
Identifies second/third quadrants with related angle $\frac{\pi}{3}$	1
Both correct solutions	1

Solution.

$\cos\theta$ is negative, so θ lies in the second and third quadrants. The related angle (where $\cos = \frac{1}{2}$) is $\dfrac{\pi}{3}$.

$$\theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}, \qquad \theta = \pi + \frac{\pi}{3} = \frac{4\pi}{3}.$$

Q4 Baulkham Hills 2022 Prelim Q13 · **2 marks**

Solve $\sqrt{3}\tan\theta = 1$ for $0 \leq \theta \leq 2\pi$.

Criteria	Marks
Reduces to $\tan\theta = \frac{1}{\sqrt{3}}$ with related angle $\frac{\pi}{6}$	1
Both correct solutions	1

Solution.

$$\sqrt{3}\tan\theta = 1 \quad \Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}}.$$

$\tan\theta$ is positive, so θ lies in the first and third quadrants. The related angle is $\dfrac{\pi}{6}$.

$$\theta = \frac{\pi}{6}, \qquad \theta = \pi + \frac{\pi}{6} = \frac{7\pi}{6}.$$

Q5 Hurlstone 2022 Prelim Q14(a) · **2 marks**

Solve $2\cos\theta = -\sqrt{3}$ for $-\pi \leq \theta \leq \pi$.

Criteria	Marks
Reduces to $\cos\theta = -\frac{\sqrt{3}}{2}$ with related angle $\frac{\pi}{6}$	1
Both correct solutions on $[-\pi, \pi]$	1

Solution.

$$2\cos\theta = -\sqrt{3} \quad \Rightarrow \quad \cos\theta = -\frac{\sqrt{3}}{2}.$$

$\cos\theta$ is negative, so θ lies in the second and third quadrants. The related angle is $\frac{\pi}{6}$. On the domain $-\pi \leq \theta \leq \pi$:

$$\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}, \qquad \theta = -\left(\pi - \frac{\pi}{6}\right) = -\frac{5\pi}{6}.$$

Q6 Hornsby Girls 2025 Trial Q20 · **3 marks**

Find all values of θ, where $0° \leq \theta \leq 360°$, such that $\cos(\theta - 30°) = \frac{\sqrt{3}}{2}$.

Criteria	Marks
Shifts the domain: $-30° \leq \theta - 30° \leq 330°$	1
Finds all values of $\theta - 30°$ in the shifted domain	1
All three correct values of θ	1

Solution.

Let $\alpha = \theta - 30°$. As $0° \leq \theta \leq 360°$, the domain for α is

$$-30° \leq \alpha \leq 330°.$$

$\cos\alpha = \frac{\sqrt{3}}{2}$ is positive (first/fourth quadrants), related angle $30°$. Over $-30° \leq \alpha \leq 330°$:

$$\alpha = -30°, \quad 30°, \quad 330°.$$

Adding $30°$ to each:

$$\theta = 0°, \quad 60°, \quad 360°.$$

Q7 Barker 2024 Prelim Q20(a) · **2 marks**

Solve $\sin\theta - \sqrt{3}\cos\theta = 0$ for $0° \leq \theta \leq 360°$.

Criteria	Marks
Rearranges to $\tan\theta = \sqrt{3}$	1
Both correct solutions	1

Solution.

Divide through by $\cos\theta$ (valid since $\cos\theta = 0$ does not satisfy the equation):

$$\sin\theta - \sqrt{3}\cos\theta = 0 \quad \Rightarrow \quad \tan\theta = \sqrt{3}.$$

$\tan\theta$ is positive, so θ lies in the first and third quadrants. The related angle is $60°$.

$$\theta = 60°, \qquad \theta = 180° + 60° = 240°.$$

Q8 James Ruse 2024 Trial Q15(b) · **2 marks**

Solve $3\sin x + \sqrt{3}\cos x = 0$ over the domain $0 \leq x \leq 2\pi$.

Criteria	Marks
Rearranges to $\tan x = -\frac{1}{\sqrt{3}}$ with related angle $\frac{\pi}{6}$	1
Both correct solutions	1

Solution.

Divide through by $\cos x$ (valid since $\cos x = 0$ does not satisfy the equation):

$$3\sin x + \sqrt{3}\cos x = 0 \quad \Rightarrow \quad 3\tan x = -\sqrt{3} \quad \Rightarrow \quad \tan x = -\frac{1}{\sqrt{3}}.$$

$\tan x$ is negative, so x lies in the second and fourth quadrants. The related angle is $\dfrac{\pi}{6}$.

$$x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}, \qquad x = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}.$$

Q9 2023 HSC Q20 · **3 marks**

Find all the values of θ, where $0° \leq \theta \leq 360°$, such that $\sin(\theta - 60°) = -\dfrac{\sqrt{3}}{2}$.

Criteria	Marks
Shifts the domain: $-60° \leq \theta - 60° \leq 300°$	1
Finds all values of $\theta - 60°$ in the shifted domain	1
All three correct values of θ	1

Solution.

Let $\alpha = \theta - 60°$. As $0° \leq \theta \leq 360°$, the domain for α is

$$-60° \leq \alpha \leq 300°.$$

$\sin\alpha = -\dfrac{\sqrt{3}}{2}$ is negative (third/fourth quadrants), related angle $60°$. Over $-60° \leq \alpha \leq 300°$:

$$\alpha = -60°, \quad 240°, \quad 300°.$$

Adding $60°$ to each:

$$\theta = 0°, \quad 300°, \quad 360°.$$

2. Equations with a multiple or compound angle

Q10 Trinity Grammar 2022 Trial Q7 · **1 mark**

Solve $\sin 2x = 1$ in the domain $[0, 2\pi]$. (A) $\frac{\pi}{4}, \frac{5\pi}{4}$ (B) $\frac{\pi}{4}, \frac{7\pi}{4}$ (C) $\frac{\pi}{2}, \frac{5\pi}{4}$ (D) $\frac{\pi}{4}, \frac{3\pi}{4}$

Criteria	Marks
Correct option (A)	1

Solution.

Let $u = 2x$. As $0 \leq x \leq 2\pi$, the substituted domain is $0 \leq u \leq 4\pi$.

$$\sin u = 1 \quad \Rightarrow \quad u = \frac{\pi}{2}, \ \frac{\pi}{2} + 2\pi = \frac{\pi}{2}, \ \frac{5\pi}{2}.$$

Dividing by 2 (i.e. $x = \frac{u}{2}$):

$$x = \frac{\pi}{4}, \ \frac{5\pi}{4}. \qquad \textbf{(A)}$$

Q11 Shore 2023 Trial Q19 · **2 marks**

Solve $2 = 4\cos 3x$ in the interval $0 \leq x \leq \dfrac{\pi}{4}$.

Criteria	Marks
Reduces to $\cos 3x = \frac{1}{2}$ and works in the multiple-angle domain	1
Correct solution $x = \frac{\pi}{9}$	1

Solution.

$$2 = 4\cos 3x \quad \Rightarrow \quad \cos 3x = \frac{1}{2}.$$

Let $u = 3x$. As $0 \leq x \leq \dfrac{\pi}{4}$, the substituted domain is $0 \leq u \leq \dfrac{3\pi}{4}$. On this domain $\cos u = \dfrac{1}{2}$ gives only $u = \dfrac{\pi}{3}$.

$$x = \frac{u}{3} = \frac{\pi}{9}.$$

Q12 North Sydney Boys 2024 Trial Q12 · **2 marks**

Solve $\tan(2x) = \sqrt{3}$ in the domain $0 \leq x \leq \pi$.

Criteria	Marks
Expands the domain and finds the multiple-angle solutions	1
Correct solutions $x = \frac{\pi}{6}, \frac{2\pi}{3}$	1

Solution.

Let $u = 2x$. As $0 \leq x \leq \pi$, the substituted domain is $0 \leq u \leq 2\pi$. $\tan u = \sqrt{3}$ is positive, so u is in the first and third quadrants (reference angle $\dfrac{\pi}{3}$):

$$u = \frac{\pi}{3},\ \frac{\pi}{3} + \pi = \frac{\pi}{3},\ \frac{4\pi}{3}.$$

Dividing by 2:

$$x = \frac{\pi}{6},\ \frac{2\pi}{3}.$$

Q13 Barker 2022 Trial Q20(b) · **2 marks**

Given $f(x) = 3\sin\left(2x - \dfrac{\pi}{3}\right)$, solve $3\sin\left(2x - \dfrac{\pi}{3}\right) = 3$ over $0 \le x \le \pi$.

Criteria	Marks
Reduces to $\sin\left(2x - \frac{\pi}{3}\right) = 1$ and substitutes on the shifted domain	1
Correct solution $x = \frac{5\pi}{12}$	1

Solution.

$$3\sin\left(2x - \frac{\pi}{3}\right) = 3 \quad \Rightarrow \quad \sin\left(2x - \frac{\pi}{3}\right) = 1.$$

Let $u = 2x - \dfrac{\pi}{3}$. As $0 \le x \le \pi$, we have $0 \le 2x \le 2\pi$, so the substituted domain is

$$-\frac{\pi}{3} \le u \le 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}.$$

$\sin u = 1$ gives $u = \dfrac{\pi}{2} + 2k\pi$. On $\left[-\dfrac{\pi}{3}, \dfrac{5\pi}{3}\right]$ only $u = \dfrac{\pi}{2}$ lies in range.

$$2x - \frac{\pi}{3} = \frac{\pi}{2} \quad \Rightarrow \quad 2x = \frac{\pi}{2} + \frac{\pi}{3} = \frac{5\pi}{6} \quad \Rightarrow \quad x = \frac{5\pi}{12}.$$

Q14 St George Girls 2020 Trial Q17 · **3 marks**

Find the exact solutions to $1 + 2\sin 3x = 2$ in the domain $0 \le x \le \pi$.

Criteria	Marks
Reduces to $\sin 3x = \frac{1}{2}$	1
Expands the domain and finds the multiple-angle values	1
Correct solutions $x = \frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}$	1

Solution.

$$1 + 2\sin 3x = 2 \quad \Rightarrow \quad \sin 3x = \frac{1}{2}.$$

Let $u = 3x$. As $0 \le x \le \pi$, the substituted domain is $0 \le u \le 3\pi$. $\sin u = \dfrac{1}{2}$ (reference angle $\dfrac{\pi}{6}$; first and second quadrants), listing all values in $[0, 3\pi]$:

$$u = \frac{\pi}{6}, \ \frac{5\pi}{6}, \ \frac{13\pi}{6}, \ \frac{17\pi}{6}.$$

Dividing by 3:

$$x = \frac{\pi}{18}, \ \frac{5\pi}{18}, \ \frac{13\pi}{18}, \ \frac{17\pi}{18}.$$

Q15 Manly 2020 Trial Q24 · **3 marks**

Solve $2\cos(3\theta) = -1$ for $0 \leq \theta \leq \pi$.

Criteria	**Marks**
Reduces to $\cos 3\theta = -\frac{1}{2}$	1
Expands the domain and finds the multiple-angle values	1
Correct solutions $\theta = \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{8\pi}{9}$	1

Solution.

$$2\cos 3\theta = -1 \quad \Rightarrow \quad \cos 3\theta = -\frac{1}{2}.$$

Let $u = 3\theta$. As $0 \leq \theta \leq \pi$, the substituted domain is $0 \leq u \leq 3\pi$. $\cos u = -\dfrac{1}{2}$ is negative (second and third quadrants, reference angle $\dfrac{\pi}{3}$); listing all values in $[0, 3\pi]$:

$$u = \frac{2\pi}{3}, \ \frac{4\pi}{3}, \ \frac{8\pi}{3}.$$

(The next value $\dfrac{10\pi}{3} > 3\pi$ is out of range.) Dividing by 3:

$$\theta = \frac{2\pi}{9}, \ \frac{4\pi}{9}, \ \frac{8\pi}{9}.$$

Q16 Sydney Tech 2022 Trial Q26 · **3 marks**

Solve $\cos\left(\frac{x}{2}-\frac{\pi}{4}\right)=\frac{1}{2}$ for $0\le x\le 2\pi$.

Criteria	Marks
Substitutes $u=\frac{x}{2}-\frac{\pi}{4}$ and finds the shifted domain	1
Solves $\cos u=\frac{1}{2}$ on that domain	1
Correct solution $x=\frac{7\pi}{6}$	1

Solution.

Let $u=\dfrac{x}{2}-\dfrac{\pi}{4}$. As $0\le x\le 2\pi$, we have $0\le\dfrac{x}{2}\le\pi$, so the substituted domain is

$$-\frac{\pi}{4}\le u\le\pi-\frac{\pi}{4}=\frac{3\pi}{4}.$$

$\cos u=\dfrac{1}{2}$ has reference angle $\dfrac{\pi}{3}$; the candidate values are $u=\pm\dfrac{\pi}{3}$. On $\left[-\dfrac{\pi}{4},\dfrac{3\pi}{4}\right]$ only $u=\dfrac{\pi}{3}$ lies in range (since $-\dfrac{\pi}{3}<-\dfrac{\pi}{4}$).

$$\frac{x}{2}-\frac{\pi}{4}=\frac{\pi}{3}\quad\Rightarrow\quad\frac{x}{2}=\frac{\pi}{3}+\frac{\pi}{4}=\frac{7\pi}{12}\quad\Rightarrow\quad x=\frac{7\pi}{6}.$$

Q17 Sydney Girls 2024 Trial Q17 · **3 marks**

If $0 \leq \theta \leq 2\pi$, solve $\sqrt{2}\cos\left(2\theta - \dfrac{\pi}{3}\right) - 1 = 0$.

Criteria	**Marks**
Reduces to $\cos\left(2\theta - \frac{\pi}{3}\right) = \frac{1}{\sqrt{2}}$	1
Substitutes $u = 2\theta - \frac{\pi}{3}$ and solves on the shifted domain	1
Correct solutions $\theta = \frac{\pi}{24}, \frac{7\pi}{24}, \frac{25\pi}{24}, \frac{31\pi}{24}$	1

Solution.

$$\sqrt{2}\cos\left(2\theta - \frac{\pi}{3}\right) - 1 = 0 \quad \Rightarrow \quad \cos\left(2\theta - \frac{\pi}{3}\right) = \frac{1}{\sqrt{2}}.$$

Let $u = 2\theta - \dfrac{\pi}{3}$. As $0 \leq \theta \leq 2\pi$, we have $0 \leq 2\theta \leq 4\pi$, so the substituted domain is

$$-\frac{\pi}{3} \leq u \leq 4\pi - \frac{\pi}{3} = \frac{11\pi}{3}.$$

$\cos u = \dfrac{1}{\sqrt{2}}$ has reference angle $\dfrac{\pi}{4}$; the solutions are $u = \pm\dfrac{\pi}{4} + 2k\pi$. Listing all values in $\left[-\dfrac{\pi}{3}, \dfrac{11\pi}{3}\right]$:

$$u = -\frac{\pi}{4},\ \frac{\pi}{4},\ \frac{7\pi}{4},\ \frac{9\pi}{4}.$$

Solving $2\theta = u + \dfrac{\pi}{3}$, i.e. $\theta = \dfrac{u}{2} + \dfrac{\pi}{6}$:

$$u = -\frac{\pi}{4} :\ \theta = -\frac{\pi}{8} + \frac{\pi}{6} = \frac{\pi}{24}, \qquad u = \frac{\pi}{4} :\ \theta = \frac{\pi}{8} + \frac{\pi}{6} = \frac{7\pi}{24},$$

$$u = \frac{7\pi}{4} :\ \theta = \frac{7\pi}{8} + \frac{\pi}{6} = \frac{25\pi}{24}, \qquad u = \frac{9\pi}{4} :\ \theta = \frac{9\pi}{8} + \frac{\pi}{6} = \frac{31\pi}{24}.$$

Harder equations

3. Equations involving reciprocal ratios (sec, cosec, cot)

Q18 Killara 2023 Prelim Q20(b) · **2 marks**

Solve $\operatorname{cosec}\theta = -2$ for $0 \leq \theta \leq 2\pi$.

Criteria	Marks
Converts to $\sin\theta = -\frac{1}{2}$	1
Both solutions $\theta = \dfrac{7\pi}{6},\ \dfrac{11\pi}{6}$	1

Solution.

Rewrite in terms of $\sin\theta$:

$$\operatorname{cosec}\theta = -2 \implies \frac{1}{\sin\theta} = -2 \implies \sin\theta = -\frac{1}{2}.$$

Since $\sin\theta < 0$, θ lies in the third and fourth quadrants. The related angle is $\dfrac{\pi}{6}$, so

$$\theta = \pi + \frac{\pi}{6} = \frac{7\pi}{6}, \qquad \theta = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}.$$

Both lie in $[0, 2\pi]$, so $\theta = \dfrac{7\pi}{6},\ \dfrac{11\pi}{6}$.

Q19 Baulkham Hills 2024 Trial Q12 · **3 marks**

Solve $2\sec\theta - 1 = 9$ for $0 \leq \theta \leq 2\pi$, correct to two decimal places.

Criteria	Marks
Simplifies to $\sec\theta = 5$, hence $\cos\theta = \frac{1}{5}$	1
Finds the first-quadrant solution $\theta \approx 1.37$	1
Both solutions $\theta \approx 1.37,\ 4.91$	1

Solution.

Isolate $\sec\theta$:

$$2\sec\theta - 1 = 9 \implies 2\sec\theta = 10 \implies \sec\theta = 5 \implies \cos\theta = \frac{1}{5}.$$

Since $\cos\theta > 0$, θ is in the first and fourth quadrants. The related angle is $\cos^{-1}\left(\frac{1}{5}\right) \approx 1.3694$, so

$$\theta \approx 1.3694, \qquad \theta \approx 2\pi - 1.3694 \approx 4.9137.$$

To two decimal places, $\theta \approx 1.37,\ 4.91$.

Q20 Killara 2022 Prelim Q17(b) · **2 marks**

Solve $\sec x = -\dfrac{2}{\sqrt{3}}$ for $0 \leq x \leq 2\pi$.

Criteria	Marks
Converts to $\cos x = -\frac{\sqrt{3}}{2}$	1
Both solutions $x = \dfrac{5\pi}{6}, \dfrac{7\pi}{6}$	1

Solution.

Rewrite in terms of $\cos x$:

$$\sec x = -\frac{2}{\sqrt{3}} \implies \frac{1}{\cos x} = -\frac{2}{\sqrt{3}} \implies \cos x = -\frac{\sqrt{3}}{2}.$$

Since $\cos x < 0$, x is in the second and third quadrants. The related angle is $\dfrac{\pi}{6}$, so

$$x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}, \qquad x = \pi + \frac{\pi}{6} = \frac{7\pi}{6}.$$

Both lie in $[0, 2\pi]$, so $x = \dfrac{5\pi}{6}, \dfrac{7\pi}{6}$.

Q21 North Sydney Girls 2022 Prelim Q14 · **2 marks**

Solve $\sqrt{3}\ \operatorname{cosec} x = 2$ for $-2\pi \leq x \leq \pi$.

Criteria	Marks
Converts to $\sin x = \frac{\sqrt{3}}{2}$	1
All solutions $x = -\dfrac{5\pi}{3}, -\dfrac{4\pi}{3}, \dfrac{\pi}{3}, \dfrac{2\pi}{3}$	1

Solution.

Rewrite in terms of $\sin x$:

$$\sqrt{3}\ \operatorname{cosec} x = 2 \implies \operatorname{cosec} x = \frac{2}{\sqrt{3}} \implies \sin x = \frac{\sqrt{3}}{2}.$$

Since $\sin x > 0$, the angle sits in the first and second quadrants with related angle $\dfrac{\pi}{3}$. Within $[0, \pi]$ these give $x = \dfrac{\pi}{3}$ and $x = \dfrac{2\pi}{3}$. Subtracting 2π carries each into $[-2\pi, 0]$:

$$\frac{\pi}{3} - 2\pi = -\frac{5\pi}{3}, \qquad \frac{2\pi}{3} - 2\pi = -\frac{4\pi}{3}.$$

All four lie in $[-2\pi, \pi]$, so $x = -\dfrac{5\pi}{3}, -\dfrac{4\pi}{3}, \dfrac{\pi}{3}, \dfrac{2\pi}{3}$.

Q22 Hornsby Girls 2022 Trial Q24 · **2 marks**

Find the value(s) of x satisfying $\cot^2 x + \operatorname{cosec}^2 x = 3$ for $0 \leq x \leq 2\pi$.

Criteria	**Marks**
Applies $\operatorname{cosec}^2 x = 1 + \cot^2 x$ to obtain $\cot^2 x = 1$	1
All solutions $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$	1

Solution.

Use the Pythagorean identity $\operatorname{cosec}^2 x = 1 + \cot^2 x$:

$$\cot^2 x + \left(1 + \cot^2 x\right) = 3 \implies 2\cot^2 x = 2 \implies \cot^2 x = 1.$$

Hence $\cot x = \pm 1$, i.e. $\tan x = \pm 1$. On $[0, 2\pi]$ this gives

$$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}.$$

Q23 Knox 2021 Prelim Q11(b) · **2 marks**

Solve $4\sin x = 3\operatorname{cosec} x$ for $0 \leq x \leq 2\pi$.

Criteria	**Marks**
Reduces to $\sin^2 x = \frac{3}{4}$, hence $\sin x = \pm\frac{\sqrt{3}}{2}$	1
All solutions $x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$	1

Solution.

Write $\operatorname{cosec} x = \dfrac{1}{\sin x}$:

$$4\sin x = \frac{3}{\sin x} \implies 4\sin^2 x = 3 \implies \sin^2 x = \frac{3}{4} \implies \sin x = \pm\frac{\sqrt{3}}{2}.$$

The related angle is $\dfrac{\pi}{3}$. Taking all four quadrants on $[0, 2\pi]$:

$$x = \frac{\pi}{3}, \frac{2\pi}{3} \quad (\sin x > 0), \qquad x = \frac{4\pi}{3}, \frac{5\pi}{3} \quad (\sin x < 0).$$

Q24 Sydney Girls 2022 Trial Q22 · **4 marks**

Solve $\sec\theta + 8\cos\theta = 6$ for $0 \leq \theta \leq 2\pi$, correct to 3 decimal places where necessary.

Criteria	**Marks**
Converts to the quadratic $8\cos^2\theta - 6\cos\theta + 1 = 0$	1
Solves for $\cos\theta = \frac{1}{2}$ and $\cos\theta = \frac{1}{4}$	1
Solutions from $\cos\theta = \frac{1}{2}$: $\theta = \frac{\pi}{3}, \frac{5\pi}{3}$	1
Solutions from $\cos\theta = \frac{1}{4}$: $\theta \approx 1.318,\ 4.965$	1

Solution.

Write $\sec\theta = \dfrac{1}{\cos\theta}$ and multiply through by $\cos\theta$:

$$\frac{1}{\cos\theta} + 8\cos\theta = 6 \implies 1 + 8\cos^2\theta = 6\cos\theta \implies 8\cos^2\theta - 6\cos\theta + 1 = 0.$$

Factorise: $(2\cos\theta - 1)(4\cos\theta - 1) = 0$, so $\cos\theta = \dfrac{1}{2}$ or $\cos\theta = \dfrac{1}{4}$.

For $\cos\theta = \dfrac{1}{2}$ (related angle $\dfrac{\pi}{3}$):

$$\theta = \frac{\pi}{3}, \qquad \theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}.$$

For $\cos\theta = \dfrac{1}{4}$ (related angle $\cos^{-1}\left(\frac{1}{4}\right) \approx 1.3181$):

$$\theta \approx 1.318, \qquad \theta \approx 2\pi - 1.3181 \approx 4.965.$$

Thus $\theta = \dfrac{\pi}{3}, \dfrac{5\pi}{3}, 1.318, 4.965$.

Q25 James Ruse 2020 Trial Q20 · **4 marks**

Solve $3\tan\theta - \cot\theta = 5$ for $0° \leq \theta \leq 360°$, to the nearest degree.

Criteria	**Marks**
Converts to the quadratic $3\tan^2\theta - 5\tan\theta - 1 = 0$	1
Solves for $\tan\theta = \dfrac{5 \pm \sqrt{37}}{6}$	1
Solutions from $\tan\theta \approx 1.847$: $\theta \approx 62°,\ 242°$	1
Solutions from $\tan\theta \approx -0.180$: $\theta \approx 170°,\ 350°$	1

Solution.

Write $\cot\theta = \dfrac{1}{\tan\theta}$ and multiply through by $\tan\theta$:

$$3\tan\theta - \frac{1}{\tan\theta} = 5 \implies 3\tan^2\theta - 1 = 5\tan\theta \implies 3\tan^2\theta - 5\tan\theta - 1 = 0.$$

By the quadratic formula,

$$\tan\theta = \frac{5 \pm \sqrt{25+12}}{6} = \frac{5 \pm \sqrt{37}}{6} \approx 1.8471 \text{ or } -0.1805.$$

For $\tan\theta \approx 1.8471$ (related angle $\approx 62°$), θ is in the first and third quadrants:

$$\theta \approx 62°, \qquad \theta \approx 62° + 180° = 242°.$$

For $\tan\theta \approx -0.1805$ (related angle $\approx 10°$), θ is in the second and fourth quadrants:

$$\theta \approx 180° - 10° = 170°, \qquad \theta \approx 360° - 10° = 350°.$$

Thus $\theta \approx 62°,\ 170°,\ 242°,\ 350°$.

4. Solving by factorising (common factor)

Q26 Hornsby Girls 2021 Prelim Q11 · **3 marks**

Solve $2\sin x\cos x = \sin x$ for $0° \leq x \leq 360°$.

Criteria	Marks
Moves all terms to one side and takes out the common factor $\sin x$	1
Sets each factor to zero: $\sin x = 0$ or $\cos x = \frac{1}{2}$	1
All five solutions correct	1

Solution.

Bring everything to one side (do **not** divide by $\sin x$, or solutions are lost) and take out the common factor $\sin x$:

$$2\sin x\cos x - \sin x = 0 \implies \sin x\,(2\cos x - 1) = 0.$$

So $\sin x = 0$ or $\cos x = \dfrac{1}{2}$.

For $\sin x = 0$ on $0° \leq x \leq 360°$: $x = 0°,\ 180°,\ 360°$.

For $\cos x = \dfrac{1}{2}$ (related angle $60°$, positive in the 1st and 4th quadrants): $x = 60°,\ 300°$.

$$\therefore\ x = 0°,\ 60°,\ 180°,\ 300°,\ 360°.$$

Q27 North Sydney Boys 2024 Prelim Q18 · **2 marks**

Solve $(\sqrt{3}\tan\theta - 1)(\sqrt{2}\sin\theta + 1) = 0$ for $-180° \leq \theta \leq 180°$.

Criteria	Marks
Solves $\tan\theta = \frac{1}{\sqrt{3}}$ correctly ($\theta = 30°,\ -150°$)	1
Solves $\sin\theta = -\frac{1}{\sqrt{2}}$ correctly ($\theta = -45°,\ -135°$)	1

Solution.

The equation is already a product equal to zero, so set each factor to zero:

$$\sqrt{3}\tan\theta - 1 = 0 \implies \tan\theta = \frac{1}{\sqrt{3}}, \qquad \sqrt{2}\sin\theta + 1 = 0 \implies \sin\theta = -\frac{1}{\sqrt{2}}.$$

For $\tan\theta = \dfrac{1}{\sqrt{3}}$ (related angle $30°$, positive in the 1st and 3rd quadrants) on $-180° \leq \theta \leq 180°$:

$$\theta = 30°, \qquad \theta = 30° - 180° = -150°.$$

For $\sin\theta = -\dfrac{1}{\sqrt{2}}$ (related angle $45°$, negative in the 3rd and 4th quadrants) on $-180° \leq \theta \leq 180°$:

$$\theta = -45°, \qquad \theta = -135°.$$

$$\therefore\ \theta = -150°,\ -135°,\ -45°,\ 30°.$$

Q28 Pymble 2022 Prelim Q19 · **3 marks**

Solve $2\cos^2 x - \cos x = 0$ in the domain $[0, 2\pi]$.

Criteria	**Marks**
Takes out the common factor $\cos x$	1
Sets each factor to zero: $\cos x = 0$ or $\cos x = \frac{1}{2}$	1
All four solutions correct	1

Solution.

Take out the common factor $\cos x$:

$$2\cos^2 x - \cos x = \cos x\,(2\cos x - 1) = 0.$$

So $\cos x = 0$ or $\cos x = \dfrac{1}{2}$.

For $\cos x = 0$ on $[0, 2\pi]$: $x = \dfrac{\pi}{2},\ \dfrac{3\pi}{2}$.

For $\cos x = \dfrac{1}{2}$ (related angle $\dfrac{\pi}{3}$, positive in the 1st and 4th quadrants): $x = \dfrac{\pi}{3},\ \dfrac{5\pi}{3}$.

$$\therefore\ x = \frac{\pi}{3},\ \frac{\pi}{2},\ \frac{3\pi}{2},\ \frac{5\pi}{3}.$$

Q29 Killara 2021 Prelim Q18 · **3 marks**

Solve $\sqrt{3}\sin^2\theta = \sin\theta\cos\theta$ for $0 \le \theta \le 2\pi$, giving answers in radians.

Criteria	**Marks**
Moves all terms to one side and takes out the common factor $\sin\theta$	1
Sets each factor to zero: $\sin\theta = 0$ or $\tan\theta = \frac{1}{\sqrt{3}}$	1
All five solutions correct	1

Solution.

Bring everything to one side (do **not** divide by $\sin\theta$) and take out the common factor $\sin\theta$:

$$\sqrt{3}\sin^2\theta - \sin\theta\cos\theta = 0 \;\Rightarrow\; \sin\theta\,(\sqrt{3}\sin\theta - \cos\theta) = 0.$$

So $\sin\theta = 0$, or $\sqrt{3}\sin\theta - \cos\theta = 0 \Rightarrow \tan\theta = \dfrac{1}{\sqrt{3}}$.

For $\sin\theta = 0$ on $0 \le \theta \le 2\pi$: $\theta = 0,\ \pi,\ 2\pi$.

For $\tan\theta = \dfrac{1}{\sqrt{3}}$ (related angle $\dfrac{\pi}{6}$, positive in the 1st and 3rd quadrants): $\theta = \dfrac{\pi}{6},\ \dfrac{7\pi}{6}$.

$$\therefore\ \theta = 0,\ \frac{\pi}{6},\ \pi,\ \frac{7\pi}{6},\ 2\pi.$$

Q30 Hornsby Girls 2022 Prelim Q26 · **3 marks**

Solve $3\cos^2\theta + 4\sin\theta\cos\theta = 0$ for $0° \leq \theta \leq 360°$.

Criteria	**Marks**
Takes out the common factor $\cos\theta$	1
Solves $\cos\theta = 0$ correctly ($\theta = 90°,\ 270°$)	1
Solves $\tan\theta = -\frac{3}{4}$ correctly to the nearest degree ($\theta \approx 143°,\ 323°$)	1

Solution.

Take out the common factor $\cos\theta$:

$$3\cos^2\theta + 4\sin\theta\cos\theta = \cos\theta\,(3\cos\theta + 4\sin\theta) = 0.$$

So $\cos\theta = 0$, or $3\cos\theta + 4\sin\theta = 0 \Rightarrow \tan\theta = -\dfrac{3}{4}$.

For $\cos\theta = 0$ on $0° \leq \theta \leq 360°$: $\theta = 90°,\ 270°$.

For $\tan\theta = -\dfrac{3}{4}$, the related angle is $\tan^{-1}\left(\frac{3}{4}\right) \approx 36.87°$. Since $\tan\theta$ is negative in the 2nd and 4th quadrants:

$$\theta \approx 180° - 36.87° = 143.13°, \qquad \theta \approx 360° - 36.87° = 323.13°.$$

To the nearest degree, $\theta \approx 143°,\ 323°$.

$$\therefore\ \theta = 90°,\ 143°,\ 270°,\ 323°.$$

Q31 Gosford 2025 Trial Q23 · **4 marks**

Solve $(\tan\theta - 1)(\sqrt{3}\sin\theta - \cos\theta)(\sqrt{3}\sin\theta + \cos\theta) = 0$ for $0 \le \theta \le \pi$.

Criteria	Marks
Sets each factor to zero, reducing to $\tan\theta = 1$, $\tan\theta = \frac{1}{\sqrt{3}}$, $\tan\theta = -\frac{1}{\sqrt{3}}$	1
Solves $\tan\theta = 1$ and $\tan\theta = \frac{1}{\sqrt{3}}$ on the domain	1
Solves $\tan\theta = -\frac{1}{\sqrt{3}}$ on the domain	1
Rejects the roots outside $[0, \pi]$ and gives the correct final set	1

Solution.

The equation is already a product equal to zero. Set each factor to zero and convert to a tan equation:

$$\tan\theta - 1 = 0 \Rightarrow \tan\theta = 1, \quad \sqrt{3}\sin\theta - \cos\theta = 0 \Rightarrow \tan\theta = \frac{1}{\sqrt{3}}, \quad \sqrt{3}\sin\theta + \cos\theta = 0 \Rightarrow \tan\theta = -\frac{1}{\sqrt{3}}.$$

On $0 \le \theta \le \pi$ (only the 1st and 2nd quadrants):

$\tan\theta = 1$ (related angle $\frac{\pi}{4}$): $\theta = \frac{\pi}{4}$. $\left(\text{The 3rd-quadrant root } \frac{5\pi}{4} \text{ is outside the domain.}\right)$

$\tan\theta = \frac{1}{\sqrt{3}}$ (related angle $\frac{\pi}{6}$): $\theta = \frac{\pi}{6}$. $\left(\frac{7\pi}{6} \text{ is rejected.}\right)$

$\tan\theta = -\frac{1}{\sqrt{3}}$ (related angle $\frac{\pi}{6}$, negative in the 2nd quadrant): $\theta = \frac{5\pi}{6}$. $\left(\frac{11\pi}{6} \text{ is rejected.}\right)$

$$\therefore \ \theta = \frac{\pi}{6}, \ \frac{\pi}{4}, \ \frac{5\pi}{6}.$$

5. Equations reducing to a quadratic

Q32 Manly 2024 Trial Q17 · **2 marks**

Solve $2\sin^2 x - 1 = 0$ for $-180° \leq x \leq 180°$.

Criteria	Marks
Reduces to $\sin x = \pm\frac{1}{\sqrt{2}}$	1
All four solutions correct	1

Solution.

$2\sin^2 x = 1$, so $\sin^2 x = \dfrac{1}{2}$ and $\sin x = \pm\dfrac{1}{\sqrt{2}}$.

The related angle is $45°$. On $-180° \leq x \leq 180°$:

$$\sin x = \tfrac{1}{\sqrt{2}} \Rightarrow x = 45°,\ 135°, \qquad \sin x = -\tfrac{1}{\sqrt{2}} \Rightarrow x = -45°,\ -135°.$$

$$\therefore\ x = \pm 45°,\ \pm 135°.$$

Q33 Sydney Tech 2020 Trial Q17 · **2 marks**

Solve the equation $\tan^2 x = 3$ for $0 \leq x \leq 2\pi$.

Criteria	Marks
Reduces to $\tan x = \pm\sqrt{3}$	1
All four solutions correct	1

Solution.

$\tan^2 x = 3$, so $\tan x = \pm\sqrt{3}$. The related angle is $\dfrac{\pi}{3}$.

Since tan is positive in the 1st and 3rd quadrants and negative in the 2nd and 4th, on $0 \leq x \leq 2\pi$:

$$\tan x = \sqrt{3} \Rightarrow x = \frac{\pi}{3},\ \frac{4\pi}{3}, \qquad \tan x = -\sqrt{3} \Rightarrow x = \frac{2\pi}{3},\ \frac{5\pi}{3}.$$

$$\therefore\ x = \frac{\pi}{3},\ \frac{2\pi}{3},\ \frac{4\pi}{3},\ \frac{5\pi}{3}.$$

Q34 Cherrybrook Tech 2020 Trial Q18 · **2 marks**

Solve $2\cos^2\alpha - 3\cos\alpha - 2 = 0$ for $0 \leq \alpha \leq \pi$.

Criteria	**Marks**
Factorises to $(2\cos\alpha + 1)(\cos\alpha - 2) = 0$	1
Correct solution (with $\cos\alpha = 2$ rejected)	1

Solution.

This is a quadratic in $\cos\alpha$. Factorising:

$$2\cos^2\alpha - 3\cos\alpha - 2 = (2\cos\alpha + 1)(\cos\alpha - 2) = 0.$$

So $\cos\alpha = -\dfrac{1}{2}$ or $\cos\alpha = 2$.

$\cos\alpha = 2$ has **no solution** (since $-1 \leq \cos\alpha \leq 1$), so it is rejected.

For $\cos\alpha = -\dfrac{1}{2}$ on $0 \leq \alpha \leq \pi$, the related angle is $\dfrac{\pi}{3}$ and cos is negative in the 2nd quadrant:

$$\therefore\ \alpha = \frac{2\pi}{3}.$$

Q35 Baulkham Hills 2021 Prelim Q15 · **2 marks**

Solve $2\sin^2 x + \sin x - 1 = 0$ for $0 \leq x \leq 2\pi$.

Criteria	**Marks**
Factorises to $(2\sin x - 1)(\sin x + 1) = 0$	1
All three solutions correct	1

Solution.

This is a quadratic in $\sin x$. Factorising:

$$2\sin^2 x + \sin x - 1 = (2\sin x - 1)(\sin x + 1) = 0.$$

So $\sin x = \dfrac{1}{2}$ or $\sin x = -1$.

For $\sin x = \dfrac{1}{2}$ on $0 \leq x \leq 2\pi$ (related angle $\dfrac{\pi}{6}$, positive in 1st and 2nd quadrants):

$$x = \frac{\pi}{6},\ \frac{5\pi}{6}.$$

For $\sin x = -1$ on $0 \leq x \leq 2\pi$:

$$x = \frac{3\pi}{2}.$$

$$\therefore\ x = \frac{\pi}{6},\ \frac{5\pi}{6},\ \frac{3\pi}{2}.$$

Q36 Knox 2022 Prelim Q13(e) · **3 marks**

Solve $2\cos^2\theta - \sin\theta - 1 = 0$ for $0 \leq \theta \leq 2\pi$.

Criteria	**Marks**
Uses $\cos^2\theta = 1 - \sin^2\theta$ to obtain a quadratic in $\sin\theta$	1
Factorises to $(2\sin\theta - 1)(\sin\theta + 1) = 0$	1
All three solutions correct	1

Solution.

Replace $\cos^2\theta = 1 - \sin^2\theta$:

$$2(1 - \sin^2\theta) - \sin\theta - 1 = 0 \implies -2\sin^2\theta - \sin\theta + 1 = 0 \implies 2\sin^2\theta + \sin\theta - 1 = 0.$$

Factorising:

$$(2\sin\theta - 1)(\sin\theta + 1) = 0,$$

so $\sin\theta = \dfrac{1}{2}$ or $\sin\theta = -1$.

For $\sin\theta = \dfrac{1}{2}$ on $0 \leq \theta \leq 2\pi$ (related angle $\dfrac{\pi}{6}$): $\theta = \dfrac{\pi}{6},\ \dfrac{5\pi}{6}$.

For $\sin\theta = -1$ on $0 \leq \theta \leq 2\pi$: $\theta = \dfrac{3\pi}{2}$.

$$\therefore\ \theta = \frac{\pi}{6},\ \frac{5\pi}{6},\ \frac{3\pi}{2}.$$

Q37 North Sydney Girls 2023 Trial Q19 · **3 marks**

Solve $2\sin^2 x - \sin x = 1$ for $x \in [0, 2\pi]$.

Criteria	**Marks**
Rearranges to $2\sin^2 x - \sin x - 1 = 0$	1
Factorises to $(2\sin x + 1)(\sin x - 1) = 0$	1
All three solutions correct	1

Solution.

Rearranging to standard form:

$$2\sin^2 x - \sin x - 1 = 0.$$

This is a quadratic in $\sin x$. Factorising:

$$(2\sin x + 1)(\sin x - 1) = 0,$$

so $\sin x = -\dfrac{1}{2}$ or $\sin x = 1$.

For $\sin x = -\dfrac{1}{2}$ on $[0, 2\pi]$ (related angle $\dfrac{\pi}{6}$, negative in 3rd and 4th quadrants):

$$x = \frac{7\pi}{6},\ \frac{11\pi}{6}.$$

For $\sin x = 1$ on $[0, 2\pi]$:

$$x = \frac{\pi}{2}.$$

$$\therefore\ x = \frac{\pi}{2},\ \frac{7\pi}{6},\ \frac{11\pi}{6}.$$

Q38 Baulkham Hills 2024 Prelim Q24 · **3 marks**

Solve $2\sin^2 t = 3\cos t$, where $0 \leq t \leq 2\pi$.

Criteria	**Marks**
Uses $\sin^2 t = 1 - \cos^2 t$ to obtain a quadratic in $\cos t$	1
Factorises to $(2\cos t - 1)(\cos t + 2) = 0$	1
Correct solution (with $\cos t = -2$ rejected)	1

Solution.

Replace $\sin^2 t = 1 - \cos^2 t$:

$$2(1 - \cos^2 t) = 3\cos t \implies 2 - 2\cos^2 t = 3\cos t \implies 2\cos^2 t + 3\cos t - 2 = 0.$$

Factorising:

$$(2\cos t - 1)(\cos t + 2) = 0,$$

so $\cos t = \dfrac{1}{2}$ or $\cos t = -2$.

$\cos t = -2$ has **no solution** (since $-1 \leq \cos t \leq 1$), so it is rejected.

For $\cos t = \dfrac{1}{2}$ on $0 \leq t \leq 2\pi$ (related angle $\dfrac{\pi}{3}$, positive in 1st and 4th quadrants):

$$\therefore\ t = \frac{\pi}{3},\ \frac{5\pi}{3}.$$

Q39 Fort St 2021 Prelim Q4(b) · 4 marks

Solve $6\sin^2\theta - \sin\theta - 1 = 0$ for $0° \leq \theta \leq 360°$, correct to the nearest degree.

Criteria	Marks
Recognises the equation is a quadratic in $\sin\theta$	1
Factorises to $(2\sin\theta - 1)(3\sin\theta + 1) = 0$, i.e. finds the two ratios	1
Solves $\sin\theta = \frac{1}{2}$ correctly ($\theta = 30°,\ 150°$)	1
Solves $\sin\theta = -\frac{1}{3}$ correctly to the nearest degree ($\theta \approx 199°,\ 341°$)	1

Solution.

This is a quadratic in $\sin\theta$. Factorising:

$$6\sin^2\theta - \sin\theta - 1 = (2\sin\theta - 1)(3\sin\theta + 1) = 0,$$

so $\sin\theta = \dfrac{1}{2}$ or $\sin\theta = -\dfrac{1}{3}$.

For $\sin\theta = \dfrac{1}{2}$ on $0° \leq \theta \leq 360°$ (related angle $30°$, positive in 1st and 2nd quadrants):

$$\theta = 30°,\ 150°.$$

For $\sin\theta = -\dfrac{1}{3}$, the related angle is $\sin^{-1}\left(\frac{1}{3}\right) \approx 19.47°$. Since $\sin\theta$ is negative in the 3rd and 4th quadrants:

$$\theta \approx 180° + 19.47° = 199.47°, \qquad \theta \approx 360° - 19.47° = 340.53°.$$

To the nearest degree, $\theta \approx 199°,\ 341°$.

$$\therefore\ \theta = 30°,\ 150°,\ 199°,\ 341°.$$

Q40 Killara 2024 Prelim Q34 · **3 marks**

Solve for y in the equation $\sin^2 y + \sqrt{3}\sin y + 1 - \cos^2 y = 0$ for $0° \leq y \leq 360°$, using trigonometric identities.

Criteria	**Marks**
Uses $1 - \cos^2 y = \sin^2 y$ to reduce to $2\sin^2 y + \sqrt{3}\sin y = 0$	1
Factorises to $\sin y\,(2\sin y + \sqrt{3}) = 0$	1
All five solutions correct	1

Solution.

Since $1 - \cos^2 y = \sin^2 y$, the equation becomes

$$\sin^2 y + \sqrt{3}\sin y + \sin^2 y = 0 \implies 2\sin^2 y + \sqrt{3}\sin y = 0.$$

Factorising out $\sin y$:

$$\sin y\,(2\sin y + \sqrt{3}) = 0,$$

so $\sin y = 0$ or $\sin y = -\dfrac{\sqrt{3}}{2}$.

For $\sin y = 0$ on $0° \leq y \leq 360°$: $y = 0°,\ 180°,\ 360°$.

For $\sin y = -\dfrac{\sqrt{3}}{2}$ (related angle $60°$, negative in 3rd and 4th quadrants):

$$y = 180° + 60° = 240°, \qquad y = 360° - 60° = 300°.$$

$$\therefore\ y = 0°,\ 180°,\ 240°,\ 300°,\ 360°.$$

6. Proving an identity, then solving an equation with it

Q41 Fort St 2022 Trial Q16 · **5 marks**

(a) Show that $(\operatorname{cosec}^2\theta - 1)\sin^2\theta = \cos^2\theta$. **(b)** Hence solve $(\operatorname{cosec}^2\theta - 1)\sin^2\theta = \dfrac{3}{4}$ for $-\pi \le x \le \pi$.

Criteria	Marks
(a) Uses $\operatorname{cosec}^2\theta - 1 = \cot^2\theta$ (or expands) to reach $\cos^2\theta$	2
(b) Reduces to $\cos^2\theta = \frac{3}{4}$, i.e. $\cos\theta = \pm\frac{\sqrt{3}}{2}$	1
(b) Obtains $\theta = \pm\frac{\pi}{6}$	1
(b) Obtains $\theta = \pm\frac{5\pi}{6}$	1

Solution.

(a) Since $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$,

$$(\operatorname{cosec}^2\theta - 1)\sin^2\theta = \left(\frac{1}{\sin^2\theta} - 1\right)\sin^2\theta = 1 - \sin^2\theta = \cos^2\theta,$$

as required.

(b) By part (a) the equation becomes $\cos^2\theta = \dfrac{3}{4}$, so $\cos\theta = \pm\dfrac{\sqrt{3}}{2}$. On $-\pi \le \theta \le \pi$:

$$\cos\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \pm\frac{\pi}{6}, \qquad \cos\theta = -\frac{\sqrt{3}}{2} \Rightarrow \theta = \pm\frac{5\pi}{6}.$$

Hence $\theta = -\dfrac{5\pi}{6}, -\dfrac{\pi}{6}, \dfrac{\pi}{6}, \dfrac{5\pi}{6}$.

Q42 St George Girls 2023 Prelim Q23 · **4 marks**

(a) Show that $2\cos^2\theta+\sin\theta = 2+\sin\theta-2\sin^2\theta$. **(b)** Hence solve $2\cos^2\theta+\sin\theta = 1$ given that $0 \le \theta \le \pi$.

Criteria	**Marks**
(a) Substitutes $\cos^2\theta = 1-\sin^2\theta$ to reach the RHS	1
(b) Forms and factorises $2\sin^2\theta-\sin\theta-1=0$	1
(b) Solves $\sin\theta = 1$ or $\sin\theta = -\frac{1}{2}$ and rejects the invalid root	1
(b) States $\theta = \frac{\pi}{2}$	1

Solution.

(a) Using $\cos^2\theta = 1-\sin^2\theta$,

$$2\cos^2\theta+\sin\theta = 2(1-\sin^2\theta)+\sin\theta = 2-2\sin^2\theta+\sin\theta = 2+\sin\theta-2\sin^2\theta,$$

as required.

(b) Using part (a), $2+\sin\theta-2\sin^2\theta = 1$, i.e.

$$2\sin^2\theta-\sin\theta-1=0 \implies (2\sin\theta+1)(\sin\theta-1)=0,$$

so $\sin\theta = -\dfrac{1}{2}$ or $\sin\theta = 1$. On $0 \le \theta \le \pi$ we have $\sin\theta \ge 0$, so $\sin\theta = -\frac{1}{2}$ is rejected. Thus $\sin\theta = 1$, giving $\theta = \dfrac{\pi}{2}$.

Q43 Killara 2022 Prelim Q26 · **5 marks**

(a) Show that $2\cos^2 x - 3\sin^2 x + 1 = 5\cos^2 x - 2$. **(b)** Hence solve $2\cos^2 x - 3\sin^2 x + 1 = 0$ for $0° \leq x \leq 360°$, to the nearest degree.

Criteria	Marks
(a) Substitutes $\sin^2 x = 1 - \cos^2 x$ and simplifies to $5\cos^2 x - 2$	2
(b) Reduces to $\cos^2 x = \frac{2}{5}$, i.e. $\cos x = \pm\sqrt{\frac{2}{5}}$	1
(b) Two correct solutions (51°, 309°)	1
(b) Two correct solutions (129°, 231°)	1

Solution.

(a) Using $\sin^2 x = 1 - \cos^2 x$,

$$2\cos^2 x - 3\sin^2 x + 1 = 2\cos^2 x - 3(1 - \cos^2 x) + 1 = 2\cos^2 x - 3 + 3\cos^2 x + 1 = 5\cos^2 x - 2,$$

as required.

(b) By part (a) the equation becomes $5\cos^2 x - 2 = 0$, so $\cos^2 x = \dfrac{2}{5}$ and $\cos x = \pm\sqrt{\dfrac{2}{5}}$. The related angle is $\cos^{-1}\sqrt{\frac{2}{5}} \approx 50.77°$. On $0° \leq x \leq 360°$:

$$\cos x = +\sqrt{\tfrac{2}{5}} \Rightarrow x \approx 51°,\ 309°, \qquad \cos x = -\sqrt{\tfrac{2}{5}} \Rightarrow x \approx 129°,\ 231°.$$

Hence $x \approx 51°,\ 129°,\ 231°,\ 309°$.

Q44 Abbotsleigh 2024 Prelim Q28 · **4 marks**

(a) Show that $\tan^2 A\,(1-\sin^2 A) = \sin^2 A$. **(b)** Hence solve $4\tan^2 A\,(1-\sin^2 A) = 1$ for $0 \le A \le 2\pi$.

Criteria	Marks
(a) Uses $\tan^2 A = \dfrac{\sin^2 A}{\cos^2 A}$ and $1-\sin^2 A = \cos^2 A$	1
(b) Reduces to $4\sin^2 A = 1$, i.e. $\sin A = \pm\frac{1}{2}$	1
(b) Two correct solutions ($\frac{\pi}{6}$, $\frac{5\pi}{6}$)	1
(b) Two correct solutions ($\frac{7\pi}{6}$, $\frac{11\pi}{6}$)	1

Solution.

(a) Writing $\tan^2 A = \dfrac{\sin^2 A}{\cos^2 A}$ and $1-\sin^2 A = \cos^2 A$,

$$\tan^2 A\,(1-\sin^2 A) = \frac{\sin^2 A}{\cos^2 A}\cdot\cos^2 A = \sin^2 A,$$

as required.

(b) By part (a), $4\sin^2 A = 1$, so $\sin^2 A = \dfrac{1}{4}$ and $\sin A = \pm\dfrac{1}{2}$. On $0 \le A \le 2\pi$:

$$\sin A = \tfrac{1}{2} \;\Rightarrow\; A = \frac{\pi}{6}, \frac{5\pi}{6}, \qquad \sin A = -\tfrac{1}{2} \;\Rightarrow\; A = \frac{7\pi}{6}, \frac{11\pi}{6}.$$

Hence $A = \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$.

Q45 Fort St 2023 Trial Q33 · **3 marks**

(a) Show that $\dfrac{\cos^2\theta}{1-\sin\theta} - \dfrac{\cos^2\theta}{1+\sin\theta} = 2\sin\theta$. **(b)** Hence solve $\dfrac{\cos^2\theta}{1-\sin\theta} - \dfrac{\cos^2\theta}{1+\sin\theta} = 1$ for $0 \le \theta \le \dfrac{\pi}{2}$.

Criteria	Marks
(a) Combines over the common denominator $(1-\sin\theta)(1+\sin\theta) = \cos^2\theta$	1
(a) Simplifies the numerator to $2\sin\theta\cos^2\theta$ and cancels to $2\sin\theta$	1
(b) Reduces to $2\sin\theta = 1$ and states $\theta = \frac{\pi}{6}$	1

Solution.

(a) Over a common denominator, with $(1-\sin\theta)(1+\sin\theta) = 1-\sin^2\theta = \cos^2\theta$,

$$\frac{\cos^2\theta}{1-\sin\theta} - \frac{\cos^2\theta}{1+\sin\theta} = \frac{\cos^2\theta(1+\sin\theta) - \cos^2\theta(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)} = \frac{\cos^2\theta\big[(1+\sin\theta)-(1-\sin\theta)\big]}{\cos^2\theta}.$$

The numerator bracket is $2\sin\theta$, and $\cos^2\theta$ cancels, giving $2\sin\theta$, as required.

(b) By part (a) the equation becomes $2\sin\theta = 1$, so $\sin\theta = \dfrac{1}{2}$. On $0 \le \theta \le \dfrac{\pi}{2}$ the only solution is $\theta = \dfrac{\pi}{6}$.

Q46 North Sydney Girls 2020 Trial Q17 · **5 marks**

(a) Show that $\dfrac{4\cos\theta}{\tan\theta}+15=0$ can be written as $4\sin^2\theta-15\sin\theta-4=0$. **(b)** Hence solve $\dfrac{4\cos\theta}{\tan\theta}+15=0$ for $0\le\theta\le 2\pi$.

Criteria	Marks
(a) Writes $\dfrac{\cos\theta}{\tan\theta}=\dfrac{\cos^2\theta}{\sin\theta}$ and clears the denominator	1
(a) Uses $\cos^2\theta=1-\sin^2\theta$ and rearranges to the required quadratic	1
(b) Factorises and rejects $\sin\theta=4$, keeping $\sin\theta=-\frac{1}{4}$	1
(b) One correct solution	1
(b) Second correct solution	1

Solution.

(a) Since $\tan\theta=\dfrac{\sin\theta}{\cos\theta}$, we have $\dfrac{\cos\theta}{\tan\theta}=\dfrac{\cos\theta\cdot\cos\theta}{\sin\theta}=\dfrac{\cos^2\theta}{\sin\theta}$. Thus

$$\frac{4\cos^2\theta}{\sin\theta}+15=0.$$

Multiplying by $\sin\theta$ gives $4\cos^2\theta+15\sin\theta=0$. Using $\cos^2\theta=1-\sin^2\theta$:

$$4(1-\sin^2\theta)+15\sin\theta=0\implies 4-4\sin^2\theta+15\sin\theta=0\implies 4\sin^2\theta-15\sin\theta-4=0,$$

as required.

(b) Factorising, $4\sin^2\theta-15\sin\theta-4=(4\sin\theta+1)(\sin\theta-4)=0$, so $\sin\theta=-\dfrac{1}{4}$ or $\sin\theta=4$. Since $-1\le\sin\theta\le 1$, reject $\sin\theta=4$. With $\sin\theta=-\dfrac{1}{4}$, the related angle is $\sin^{-1}\frac{1}{4}\approx 0.2527$, and $\sin\theta<0$ places θ in the third and fourth quadrants:

$$\theta=\pi+\sin^{-1}\tfrac{1}{4}\approx 3.39,\qquad \theta=2\pi-\sin^{-1}\tfrac{1}{4}\approx 6.03.$$

Hence $\theta\approx 3.39$ and $\theta\approx 6.03$ (radians).

Q47 Loreto Kirribilli 2022 Prelim Q33 · **5 marks**

(a) Show that $\dfrac{\cos\theta}{1+\sin\theta}+\dfrac{1+\sin\theta}{\cos\theta}=2\sec\theta$. **(b)** Hence solve $\dfrac{\cos\theta}{1+\sin\theta}+\dfrac{1+\sin\theta}{\cos\theta}=2$ for $0^\circ\le\theta\le 360^\circ$.

Criteria	Marks
(a) Combines over the common denominator $\cos\theta(1+\sin\theta)$	1
(a) Expands the numerator to $2+2\sin\theta$ using $\cos^2\theta+\sin^2\theta=1$	1
(a) Cancels $(1+\sin\theta)$ to reach $\dfrac{2}{\cos\theta}=2\sec\theta$	1
(b) Reduces to $\cos\theta=1$	1
(b) States $\theta=0^\circ, 360^\circ$	1

Solution.

(a) Over a common denominator,

$$\frac{\cos\theta}{1+\sin\theta}+\frac{1+\sin\theta}{\cos\theta}=\frac{\cos^2\theta+(1+\sin\theta)^2}{\cos\theta(1+\sin\theta)}.$$

Expanding the numerator: $\cos^2\theta+1+2\sin\theta+\sin^2\theta=1+1+2\sin\theta=2+2\sin\theta=2(1+\sin\theta)$. Hence

$$\frac{2(1+\sin\theta)}{\cos\theta(1+\sin\theta)}=\frac{2}{\cos\theta}=2\sec\theta,$$

as required.

(b) By part (a), $2\sec\theta=2$, so $\sec\theta=1$, i.e. $\cos\theta=1$. On $0^\circ\le\theta\le 360^\circ$ this gives $\theta=0^\circ, 360^\circ$.

Q48 Penrith 2025 Trial Q12 · **5 marks**

(a) Show that $\dfrac{1}{1-\sin x}+\dfrac{1}{1+\sin x}=2\sec^2 x$. **(b)** Hence solve $\dfrac{1}{1-\sin x}+\dfrac{1}{1+\sin x}=8$ for $0^\circ \le x \le 360^\circ$.

Criteria	Marks
(a) Combines the fractions over $(1-\sin x)(1+\sin x)=\cos^2 x$	1
(a) Simplifies the numerator to 2	1
(a) Reaches $\dfrac{2}{\cos^2 x}=2\sec^2 x$	1
(b) Reduces to $\cos^2 x=\frac{1}{4}$ and solves $\cos x=\pm\frac{1}{2}$	1
(b) All four solutions $60^\circ, 120^\circ, 240^\circ, 300^\circ$	1

Solution.

(a) Over a common denominator,

$$\frac{1}{1-\sin x}+\frac{1}{1+\sin x}=\frac{(1+\sin x)+(1-\sin x)}{(1-\sin x)(1+\sin x)}=\frac{2}{1-\sin^2 x}=\frac{2}{\cos^2 x}=2\sec^2 x,$$

as required.

(b) By part (a), $2\sec^2 x=8$, so $\sec^2 x=4$ and $\cos^2 x=\dfrac{1}{4}$, giving $\cos x=\pm\dfrac{1}{2}$. On $0^\circ \le x \le 360^\circ$:

$$\cos x=\tfrac{1}{2} \Rightarrow x=60^\circ, 300^\circ, \qquad \cos x=-\tfrac{1}{2} \Rightarrow x=120^\circ, 240^\circ.$$

Hence $x=60^\circ, 120^\circ, 240^\circ, 300^\circ$.