Year 11 Mathematics Advanced

Topic: Trigonometry and Measure of Angles (2 of 5)

Review Questions (Worked Solutions): The sine rule, cosine rule and area of a triangle

Term 3, 2026

Non-right-angled triangles

1. The sine rule

Q1 Roseville 2022 Trial Q6 · **1 mark**

In the triangle shown, the side of length x is opposite the angle of $72°$ and the side y is opposite the angle of $65°$. From the diagram, what is the value of y? (multiple choice)

Criteria	Marks
Correct option (A)	1

Solution.

By the sine rule, each side divided by the sine of its opposite angle is equal:

$$\frac{y}{\sin 72°} = \frac{x}{\sin 65°}.$$

Making y the subject,

$$y = \frac{x \sin 72°}{\sin 65°}.$$

The answer is **(A)**.

Q2 Hunters Hill 2022 Trial Q8 · **1 mark**

In $\triangle ABC$, the side of length 29 is opposite the angle of $79°$ and the side x is opposite the angle of $61°$. What is the correct expression for the value of x? (multiple choice)

Criteria	Marks
Correct option (C)	1

Solution.

By the sine rule,

$$\frac{x}{\sin 61°} = \frac{29}{\sin 79°}.$$

Making x the subject,

$$x = \frac{29 \sin 61°}{\sin 79°}.$$

The answer is **(C)**.

Q3 Hills Grammar 2025 Trial Q26 · **2 marks**

A triangle has one side that is 5 metres in length, with the angle opposite being $72°33'$ (correct to the nearest minute). A second side is 3 metres in length. Show that the size of θ, the acute angle opposite the side of length 3 metres, is $34°55'$ correct to the nearest minute.

Criteria	**Marks**
Correct sine rule statement relating the two sides and their opposite angles	1
Solves for θ and reaches $34°55'$	1

Solution.

The 5 m side is opposite $72°33'$ and the 3 m side is opposite θ. By the sine rule,

$$\frac{\sin\theta}{3} = \frac{\sin 72°33'}{5}.$$

So

$$\sin\theta = \frac{3\sin 72°33'}{5} = 0.57239\ldots$$

Taking the acute solution,

$$\theta = \sin^{-1}(0.57239\ldots) = 34.9169°\ldots = 34°55' \text{ (nearest minute)}.$$

Q4 Hornsby Girls 2025 Trial Q22 · **2 marks**

A triangle has one side that is 8 metres in length, with an opposite angle of $82°28'$. A second angle is $41°25'$, with an opposite side of x metres. Find the length of x, correct to the nearest centimetre.

Criteria	**Marks**
Correct sine rule statement for x	1
Correct value $x = 5.34$ m	1

Solution.

The 8 m side is opposite $82°28'$ and x is opposite $41°25'$. By the sine rule,

$$\frac{x}{\sin 41°25'} = \frac{8}{\sin 82°28'}.$$

Hence

$$x = \frac{8\sin 41°25'}{\sin 82°28'} = 5.3383\ldots \text{ m} \approx 5.34 \text{ m (nearest centimetre)}.$$

Q5 Presbyterian 2021 Trial Q15 · **3 marks**

Two campsites A and B are such that campsite A is due west of campsite B, at a distance of 4 kilometres. A third campsite C bears 035° from A and 330° from B. **(a)** Show that $\angle CBA = 60°$ and $\angle BAC = 55°$. **(b)** Find the distance from campsite A to campsite C (one decimal place).

Criteria	**Marks**
(a) Establishes $\angle CBA = 60°$ and $\angle BAC = 55°$ from the bearings	1
(b) Correct sine rule statement for AC	1
(b) Correct value $AC \approx 3.8$ km	1

Solution.

(a) Since A is due west of B, the direction from A to B is due east (bearing 090°). C bears 035° from A, so

$$\angle BAC = 90° - 35° = 55°.$$

From B, campsite A is due west (bearing 270°) and C bears 330°, so

$$\angle CBA = 330° - 270° = 60°.$$

(b) The remaining angle is $\angle ACB = 180° - 55° - 60° = 65°$. Side AC is opposite $\angle CBA = 60°$, and $AB = 4$ is opposite $\angle ACB = 65°$. By the sine rule,

$$\frac{AC}{\sin 60°} = \frac{4}{\sin 65°},$$

$$AC = \frac{4 \sin 60°}{\sin 65°} = 3.8222\ldots \approx 3.8 \text{ km}.$$

Q6 Girraween 2025 Trial Q18 · **3 marks**

A ship S observes a lighthouse L on a bearing of N 26°E and a tower T on a bearing of N 70°E. The bearing of the lighthouse from the tower is N 15°E, and the distance from the lighthouse to the tower is 6.8 km. **(a)** Draw a diagram showing this information. **(b)** Find the distance of the ship from the tower (one decimal place).

Criteria	**Marks**
(a) Correct diagram with the three bearings and the 6.8 km side marked	1
(b) Finds the triangle angles ($\angle LST = 44°$, $\angle SLT = 11°$) and states the sine rule	1
(b) Correct value $ST \approx 1.9$ km	1

Solution.

(a) Mark the ship S, and from S draw SL on bearing 026° and ST on bearing 070°. From T, draw TL on bearing 015° with $LT = 6.8$ km.

(b) At S, the angle between the directions to L and T is

$$\angle LST = 70° - 26° = 44°.$$

The bearing of S from T is $070° + 180° = 250°$, and L is on bearing 015° from T, so the interior angle at T is

$$\angle LTS = 250° - 15° = 235° \implies 360° - 235° = 125°.$$

Hence

$$\angle SLT = 180° - 44° - 125° = 11°.$$

Side ST is opposite $\angle SLT = 11°$ and $LT = 6.8$ is opposite $\angle LST = 44°$. By the sine rule,

$$\frac{ST}{\sin 11°} = \frac{6.8}{\sin 44°},$$

$$ST = \frac{6.8 \sin 11°}{\sin 44°} = 1.8678\ldots \approx 1.9 \text{ km}.$$

Q7 Hurlstone 2024 Prelim Q21 · **3 marks**

In $\triangle APC$, B is the point on AC such that $\angle APB = \angle BPC = \theta$. Let $\angle PBA = x$. Using the sine rule in $\triangle APB$ and $\triangle BPC$, or otherwise, show that $AB \times PC = AP \times BC$.

Criteria	**Marks**
Applies the sine rule in each of $\triangle APB$ and $\triangle BPC$	1
Uses $\angle ABP + \angle CBP = 180°$ so $\sin \angle ABP = \sin \angle CBP$	1
Combines to reach $AB \times PC = AP \times BC$	1

Solution.

In $\triangle APB$, the sine rule gives

$$\frac{AB}{\sin\theta} = \frac{AP}{\sin \angle ABP} \quad \Rightarrow \quad \frac{AB}{AP} = \frac{\sin\theta}{\sin \angle ABP}.$$

In $\triangle BPC$, the sine rule gives

$$\frac{BC}{\sin\theta} = \frac{PC}{\sin \angle CBP} \quad \Rightarrow \quad \frac{BC}{PC} = \frac{\sin\theta}{\sin \angle CBP}.$$

Since B lies on AC, the angles $\angle ABP$ and $\angle CBP$ are supplementary:

$$\angle ABP + \angle CBP = 180° \quad \Rightarrow \quad \sin \angle ABP = \sin(180° - \angle CBP) = \sin \angle CBP.$$

Therefore the two right-hand sides are equal:

$$\frac{AB}{AP} = \frac{\sin\theta}{\sin \angle ABP} = \frac{\sin\theta}{\sin \angle CBP} = \frac{BC}{PC}.$$

Cross-multiplying,

$$AB \times PC = AP \times BC,$$

as required.

2. The cosine rule

Q8 North Sydney Boys 2022 Trial Q8 · 1 mark

In triangle ABC, $AB = 11.3$, $BC = 8.8$ and $CA = 9.9$ (not drawn to scale). Determine the size of angle A. (multiple choice)

Criteria	Marks
Correct option (A)	1

Solution.

The side opposite A is $BC = 8.8$, with adjacent sides $CA = 9.9$ and $AB = 11.3$. By the cosine rule,

$$\cos A = \frac{9.9^2 + 11.3^2 - 8.8^2}{2(9.9)(11.3)} = \frac{98.01 + 127.69 - 77.44}{223.74} = \frac{148.26}{223.74}.$$

So $A = \cos^{-1}(0.66266\ldots) = 48.5°$ (to 1 d.p.). The answer is **(A)**.

Q9 Caringbah 2020 Trial Q12 · 2 marks

In a triangle (not to scale), θ is the angle between the sides of length 16 and 20, and the side opposite θ has length 12. Find the value of θ, correct to the nearest minute.

Criteria	Marks
Correct substitution into the cosine rule	1
Correct value of θ to the nearest minute	1

Solution.

θ lies between the sides 16 and 20, opposite the side 12. By the cosine rule,

$$\cos\theta = \frac{16^2 + 20^2 - 12^2}{2(16)(20)} = \frac{256 + 400 - 144}{640} = \frac{512}{640} = 0.8.$$

Hence $\theta = \cos^{-1}(0.8) = 36.8699\ldots° = 36°52'$ (to the nearest minute).

Q10 Girraween 2022 Trial Q31 · **2 marks**

A circular clock face, centre O, has a minute hand $OA = 10$ cm and an hour hand $OB = 7$ cm. Calculate the length of AB when the hands show 5 o'clock, correct to 1 decimal place.

Criteria	Marks
Correct angle $\angle AOB = 150°$	1
Correct length AB to 1 d.p.	1

Solution.

At 5 o'clock the minute hand points to 12 and the hour hand points to 5, so the angle between them is $5 \times 30° = 150°$. By the cosine rule,

$$AB^2 = 10^2 + 7^2 - 2(10)(7)\cos 150° = 100 + 49 - 140\left(-\frac{\sqrt{3}}{2}\right) = 149 + 70\sqrt{3}.$$

So $AB = \sqrt{149 + 70\sqrt{3}} = 16.439\ldots \approx 16.4$ cm.

Q11 St George Girls 2022 Prelim Q30 · **3 marks**

Aria and Ben are flying kites when their kites collide mid-air. The angles of elevation of Aria's and Ben's kites are 50° and 31° respectively. Aria's kite string was 13 m and Ben's kite string was 19 m long at impact. (a) Label the information on the diagram. (b) Use the cosine rule to find the distance between Aria and Ben, to the nearest whole number.

Criteria	Marks
(a) Correctly labels the diagram (strings 13, 19 and elevations 50°, 31°)	1
(b) Correct included angle $\angle APB = 99°$	1
(b) Correct distance to the nearest whole number	1

Solution.

(a) Let P be the collision point. Mark $PA = 13$ m up to Aria's kite with the ground angle at Aria $= 50°$, and $PB = 19$ m up to Ben's kite with the ground angle at Ben $= 31°$.

(b) The angle at the collision point is $\angle APB = 180° - 50° - 31° = 99°$. By the cosine rule,

$$AB^2 = 13^2 + 19^2 - 2(13)(19)\cos 99° = 169 + 361 - 494\cos 99° = 607.28\ldots$$

So $AB = \sqrt{607.28\ldots} = 24.64\ldots \approx 25$ m.

Q12 Danebank 2021 Trial Q11 · **3 marks**

Ringo and Greg start at P. Ringo drives at 70 km/h on a bearing of 285°; Greg drives at 60 km/h on a bearing of 030°. (a) Show that $\angle GPR = 105°$. (b) Find the distance between them after 2 hours, to the nearest kilometre.

Criteria	**Marks**
(a) Shows $\angle GPR = 105°$	1
(b) Correct distances travelled (140 km and 120 km) into the cosine rule	1
(b) Correct distance to the nearest kilometre	1

Solution.

(a) The bearing of R (Ringo) is 285° and the bearing of G (Greg) is 030°. The angle between the two directions is

$$\angle GPR = 285° - 30° = 255° \quad \text{(reflex)}, \qquad 360° - 255° = 105°,$$

so $\angle GPR = 105°$.

(b) After 2 hours, $PR = 70 \times 2 = 140$ km and $PG = 60 \times 2 = 120$ km. By the cosine rule,

$$RG^2 = 140^2 + 120^2 - 2(140)(120)\cos 105° = 19600 + 14400 - 33600\cos 105° = 42695.8\ldots$$

So $RG = \sqrt{42695.8\ldots} = 206.63\ldots \approx 207$ km.

Q13 Penrith 2022 Trial Q34 · **2 marks**

Ned Kelly (N) leaves his hideout H walking at 7 km/h. At the same time the police (P), 20 km from H, walk towards H at 5 km/h along a path meeting Ned's at H at 120°. With D km the distance between them at time t hours, show that $D^2 = 39t^2 - 60t + 400$.

Criteria	**Marks**
Correct expressions $HN = 7t$ and $HP = 20 - 5t$ into the cosine rule	1
Correct expansion to $D^2 = 39t^2 - 60t + 400$	1

Solution.

After t hours, Ned is $HN = 7t$ km from H and the police are $HP = 20 - 5t$ km from H. The angle at H between the paths is 120°, so by the cosine rule,

$$\begin{aligned} D^2 &= (7t)^2 + (20 - 5t)^2 - 2(7t)(20 - 5t)\cos 120^\circ \\ &= 49t^2 + (400 - 200t + 25t^2) - 14t(20 - 5t)\left(-\tfrac{1}{2}\right) \\ &= 49t^2 + 25t^2 - 200t + 400 + 7t(20 - 5t) \\ &= 74t^2 - 200t + 400 + 140t - 35t^2 \\ &= 39t^2 - 60t + 400. \qquad \blacksquare \end{aligned}$$

Q14 North Sydney Boys 2023 Trial Q18 · 5 marks

A ship sails 225 km from Adhiban Island (A) on a bearing of 147° to Port Bologan (B), then 318 km on a bearing of 071° to Port Cramling (C). (i) Show $\angle ABC = 104°$. (ii) Show $AC \approx 431.7$ km. (iii) Find the bearing from C straight back to A.

Criteria	Marks
(i) Shows $\angle ABC = 104°$	1
(ii) Correct cosine-rule substitution	1
(ii) Shows $AC \approx 431.7$ km	1
(iii) Correct angle $\angle BCA$ by the sine rule	1
(iii) Correct return bearing	1

Solution.

(i) The reverse bearing of A from B is $147° + 180° = 327°$. The bearing of C from B is 071°. Hence

$$\angle ABC = 327° - 71° = 256° \text{ (reflex)}, \qquad 360° - 256° = 104°.$$

(ii) By the cosine rule in $\triangle ABC$ with $AB = 225$, $BC = 318$, $\angle ABC = 104°$,

$$AC^2 = 225^2 + 318^2 - 2(225)(318)\cos 104° = 50625 + 101124 - 143100\cos 104° = 186368.6\ldots$$

So $AC = \sqrt{186368.6\ldots} = 431.70\ldots \approx 431.7$ km.

(iii) By the sine rule, $\dfrac{\sin \angle BCA}{AB} = \dfrac{\sin \angle ABC}{AC}$, so

$$\sin \angle BCA = \frac{225 \sin 104°}{431.70\ldots} = 0.50568\ldots, \qquad \angle BCA = 30.38\ldots°.$$

The bearing of B from C is $071° + 180° = 251°$. Rotating towards A,

$$\text{bearing of } A \text{ from } C = 251° + 30.38° = 281.4° \approx 281°.$$

Q15 Hills Grammar 2025 Trial Q35 · 6 marks

Buoys A, B, C; H is vertically above C. From H, the angles of depression of A and B are $60°$ and $30°$; the bearings of A and B from C are $250°$ and $130°$; $AB = 100\sqrt{13}$ km. (a) Show $AC = 100$ km. (b) Destroyers leave A (along AC, 20 km/h) and B (along BC, 50 km/h); show their separation after t hours is $10\sqrt{1300 - 450t + 39t^2}$ km.

Criteria	Marks
(a) Correct $CA = h/\tan 60°$ and $CB = h/\tan 30°$	1
(a) Correct $\angle ACB = 120°$ and cosine rule for AB	1
(a) Solves for h and shows $AC = 100$ km	1
(b) Correct distances from C: $100 - 20t$ and $300 - 50t$	1
(b) Correct cosine-rule set-up with $\angle ACB = 120°$	1
(b) Correct simplification to $10\sqrt{1300 - 450t + 39t^2}$	1

Solution.

(a) Let the helicopter height be $h = HC$. Since HC is vertical, the angles of depression give

$$\tan 60° = \frac{h}{CA} \Rightarrow CA = \frac{h}{\tan 60°}, \qquad \tan 30° = \frac{h}{CB} \Rightarrow CB = \frac{h}{\tan 30°}.$$

The bearings of A and B from C are $250°$ and $130°$, so $\angle ACB = 250° - 130° = 120°$. By the cosine rule in $\triangle ACB$,

$$AB^2 = CA^2 + CB^2 - 2\,CA \cdot CB \cos 120° = \frac{h^2}{3} + 3h^2 - 2 \cdot \frac{h}{\sqrt{3}} \cdot h\sqrt{3}\left(-\tfrac{1}{2}\right) = \frac{h^2}{3} + 3h^2 + h^2 = \frac{13h^2}{3}.$$

Given $AB = 100\sqrt{13}$, $AB^2 = 130000$, so $\dfrac{13h^2}{3} = 130000 \Rightarrow h^2 = 30000 \Rightarrow h = 100\sqrt{3}$. Then

$$AC = \frac{h}{\tan 60°} = \frac{100\sqrt{3}}{\sqrt{3}} = 100 \text{ km}. \qquad \blacksquare$$

(b) From part (a), $AC = 100$ km and $CB = \dfrac{100\sqrt{3}}{\tan 30°} = 300$ km. After t hours the destroyers are at distances from C

$$100 - 20t \quad \text{(from } A\text{)}, \qquad 300 - 50t \quad \text{(from } B\text{)},$$

with $\angle ACB = 120°$ between them. By the cosine rule, if d is their separation,

$$\begin{aligned} d^2 &= (100 - 20t)^2 + (300 - 50t)^2 - 2(100 - 20t)(300 - 50t)\cos 120° \\ &= (100 - 20t)^2 + (300 - 50t)^2 + (100 - 20t)(300 - 50t) \\ &= 3900t^2 - 45000t + 130000 = 100\left(39t^2 - 450t + 1300\right). \end{aligned}$$

Hence $d = \sqrt{100(39t^2 - 450t + 1300)} = 10\sqrt{1300 - 450t + 39t^2}$ km. $\blacksquare$

3. The ambiguous case of the sine rule

Q16 Sydney Boys 2025 Trial Q12 · **2 marks**

Triangle ABC has $\angle A = 38°$, $BC = 16$ cm and $AC = 18$ cm. Find $\angle B$, where $\angle B$ is obtuse, to the nearest minute.

Criteria	**Marks**
Applies the sine rule $\dfrac{\sin B}{18} = \dfrac{\sin 38°}{16}$	1
Takes the obtuse solution, $\angle B \approx 136°10'$	1

Solution.

The side $AC = 18$ is opposite $\angle B$ and $BC = 16$ is opposite $\angle A = 38°$, so by the sine rule

$$\frac{\sin B}{18} = \frac{\sin 38°}{16} \implies \sin B = \frac{18 \sin 38°}{16} = 0.69262.$$

The acute solution is $\sin^{-1}(0.69262) = 43°51'$, so the obtuse solution is

$$\angle B = 180° - 43°51' = 136°9'44'' \approx 136°10'.$$

This is valid since then $\angle C = 180° - 38° - 136°10' \approx 5°50' > 0$.

Q17 Gosford 2023 Trial Q18 · **3 marks**

In triangle ABC, $\angle BAC = 26°$, $BC = 7$ (the side opposite $\angle A$) and $AC = 15$ (the side opposite $\angle B$). Find the size of the obtuse angle ABC, correct to the nearest degree.

Criteria	**Marks**
Writes the sine rule $\dfrac{\sin B}{15} = \dfrac{\sin 26°}{7}$	1
Finds the acute angle $\sin^{-1}\left(\dfrac{15 \sin 26°}{7}\right) \approx 70°$	1
Takes the obtuse angle $\angle ABC \approx 110°$	1

Solution.

Since $AC = 15$ is opposite $\angle B$ and $BC = 7$ is opposite $\angle A = 26°$, the sine rule gives

$$\frac{\sin B}{15} = \frac{\sin 26°}{7} \implies \sin B = \frac{15 \sin 26°}{7} = 0.93937.$$

The acute solution is $\sin^{-1}(0.93937) \approx 69.9°$, so the obtuse angle is

$$\angle ABC = 180° - 69.9° \approx 110°.$$

Q18 Barker 2021 Prelim Q11 · **2 marks**

In the triangle shown, an angle of 18° is opposite the side of length 15 cm, and the obtuse angle θ is opposite the side of length 36 cm. Find the value of the obtuse angle θ, rounding your answer to the nearest minute.

Criteria	Marks
Applies the sine rule $\dfrac{\sin\theta}{36} = \dfrac{\sin 18^\circ}{15}$	1
Takes the obtuse solution, $\theta \approx 132^\circ 8'$	1

Solution.

The angle 18° is opposite 15 cm and θ is opposite 36 cm, so

$$\frac{\sin\theta}{36} = \frac{\sin 18^\circ}{15} \implies \sin\theta = \frac{36\sin 18^\circ}{15} = 0.74164.$$

The acute solution is $\sin^{-1}(0.74164) = 47^\circ 52'$, so the obtuse solution is

$$\theta = 180^\circ - 47^\circ 52' = 132^\circ 7' 43'' \approx 132^\circ 8'.$$

Q19 Hurlstone 2021 Prelim Q4 · **1 mark**

In the triangle shown, θ is obtuse. The side of length 9.6 is opposite a 23° angle, and the side of length 12.2 is opposite θ. To the nearest degree, what is the size of θ? (multiple choice)

(A) 147°

(B) 150°

(C) 153°

(D) 156°

Criteria	Marks
Correct option (B)	1

Solution.

By the sine rule, $\dfrac{\sin\theta}{12.2} = \dfrac{\sin 23^\circ}{9.6}$, so $\sin\theta = \dfrac{12.2\sin 23^\circ}{9.6} = 0.49655$. The acute value is $\sin^{-1}(0.49655) \approx 29.8^\circ$, so the obtuse value is $\theta = 180^\circ - 29.8^\circ \approx 150^\circ$. Option (B).

Q20 Normanhurst Boys 2020 Trial Q6 · **1 mark**

In triangle ABC, $\angle A = 26°$, $AC = 9$ cm and $BC = 6$ cm. What are the possible values of $\angle ABC$, correct to the nearest minute? (multiple choice)

(A) $41°$

(B) $17°$

(C) $17°,\ 163°$

(D) $41°,\ 139°$

Criteria	**Marks**
Correct option (D)	1

Solution.

The side $AC = 9$ is opposite $\angle ABC$ and $BC = 6$ is opposite $\angle A = 26°$, so $\dfrac{\sin B}{9} = \dfrac{\sin 26°}{6}$, giving $\sin B = \dfrac{9\sin 26°}{6} = 0.65756$. Since $\sin B < 1$ there are two solutions: the acute value $\sin^{-1}(0.65756) \approx 41°$ and the obtuse value $180° - 41° \approx 139°$. Both give a valid triangle, so $\angle ABC \approx 41°$ or $139°$. Option (D).

Q21 St George Girls 2022 Prelim Q9 · **1 mark**

Towns A, B and C are such that town B is 40 km due North of town A. The distance from town B to town C is 18 km and the bearing of town C from town A is 025°. What is the bearing of town C from town B, given that $\angle BCA$ is obtuse? (multiple choice)

(A) 070°

(B) 095°

(C) 110°

(D) 135°

Criteria	**Marks**
Correct option (D)	1

Solution.

Since B is due north of A and the bearing of C from A is 025°, $\angle BAC = 25°$. In triangle ABC, $AB = 40$ is opposite $\angle BCA$ and $BC = 18$ is opposite $\angle BAC$, so

$$\frac{\sin \angle BCA}{40} = \frac{\sin 25°}{18} \implies \sin \angle BCA = \frac{40 \sin 25°}{18} = 0.93915.$$

Taking the obtuse solution, $\angle BCA = 180° - 69.9° = 110.1°$, so $\angle ABC = 180° - 25° - 110.1° = 44.9°$. At B the ray BA points due south (180°), and C lies on the eastern side, so the bearing of C from B is $180° - 44.9° \approx 135°$. Option (D).

Q22 2021 HSC Q18 · **3 marks**

In triangle ABC, $AC = 25$ cm, $BC = 16$ cm, $\angle BAC = 28°$ and angle ABC is obtuse. Find the size of the obtuse angle ABC, correct to the nearest degree.

Criteria	**Marks**
Applies the sine rule $\dfrac{\sin \angle ABC}{25} = \dfrac{\sin 28°}{16}$	1
Finds the acute angle $\approx 47°$	1
Takes the obtuse angle $\angle ABC \approx 133°$	1

Solution.

The side $AC = 25$ is opposite $\angle ABC$ and $BC = 16$ is opposite $\angle BAC = 28°$, so

$$\frac{\sin \angle ABC}{25} = \frac{\sin 28°}{16} \implies \sin \angle ABC = \frac{25 \sin 28°}{16} = 0.73355.$$

The acute solution is $\sin^{-1}(0.73355) \approx 47.2°$, so the obtuse angle is

$$\angle ABC = 180° - 47.2° \approx 133°.$$

This is consistent, since then $\angle ACB \approx 180° - 28° - 133° = 19° > 0$.

Q23 Manly 2022 Prelim Q21 · **4 marks**

In triangle ABC, $\angle BAC = 0.6$ radians, $BC = 4$ (the side opposite $\angle A$) and $AB = 5$ (the side opposite $\angle C$). Given that AB is the longest side, so that $\angle ACB$ is obtuse, find the size of $\angle ABC$ in radians, correct to three decimal places.

Criteria	Marks
Writes the sine rule with correct substitutions $\dfrac{\sin\angle ACB}{5} = \dfrac{\sin 0.6}{4}$	1
Finds the acute value $\angle ACB \approx 0.784$	1
Recognises the ambiguous case and takes the obtuse value $\angle ACB \approx 2.358$	1
Computes $\angle ABC = \pi - 0.6 - \angle ACB \approx 0.184$	1

Solution.

The side $AB = 5$ is opposite $\angle ACB$ and $BC = 4$ is opposite $\angle BAC = 0.6$, so by the sine rule

$$\frac{\sin\angle ACB}{5} = \frac{\sin 0.6}{4} \implies \sin\angle ACB = \frac{5\sin 0.6}{4} = 0.70580.$$

This gives an acute value $\sin^{-1}(0.70580) \approx 0.784$ or, in the ambiguous case, the obtuse value $\pi - 0.784 \approx 2.358$. Since AB is the longest side, the angle opposite it, $\angle ACB$, is the largest and must be obtuse, so $\angle ACB \approx 2.358$. Then, using the angle sum π radians,

$$\angle ABC = \pi - 0.6 - 2.358 \approx 0.184 \text{ radians}.$$

4. The area of a triangle

Q24 Penrith 2024 Trial Q1 · **1 mark**

A triangle has two sides of length 8 cm and 15 cm meeting at an included angle of 60°. What is the area of the triangle? (multiple choice)

Criteria	Marks
Correct option (C)	1

Solution.

Using the area formula $A = \frac{1}{2}ab\sin C$ with $a = 8$, $b = 15$ and included angle $C = 60°$,

$$A = \frac{1}{2}(8)(15)\sin 60° = 60 \times \frac{\sqrt{3}}{2} = 30\sqrt{3}\text{ cm}^2.$$

The answer is **(C)**.

Q25 Cherrybrook Tech 2021 Trial Q8 · **1 mark**

A triangle has a side of length 49 cm and a side of length 35 cm. The angle at the end of the 49 cm side is 32°, and the angle at the top of the triangle is 48°. What is the area of the triangle, to the nearest square centimetre? (multiple choice)

Criteria	Marks
Correct option (C)	1

Solution.

The third angle of the triangle is the angle included between the 49 cm and 35 cm sides:

$$\theta = 180^\circ - 32^\circ - 48^\circ = 100^\circ.$$

Using the area formula $A = \frac{1}{2}ab\sin C$ with these two sides and their included angle,

$$A = \frac{1}{2}(49)(35)\sin 100^\circ = 844.47\ldots \approx 844 \text{ cm}^2.$$

The answer is **(C)**.

Q26 Gosford 2023 Trial Q8 · **1 mark**

A triangle has an area of 18 cm^2. Two of its sides have lengths x cm and $2x$ cm, with an included angle of $\dfrac{5\pi}{6}$. What is the value of x? (multiple choice)

Criteria	Marks
Correct option (B)	1

Solution.

Using $A = \frac{1}{2}ab\sin C$ with $a = x$, $b = 2x$ and $C = \dfrac{5\pi}{6}$, and noting $\sin\dfrac{5\pi}{6} = \dfrac{1}{2}$,

$$18 = \frac{1}{2}(x)(2x)\sin\frac{5\pi}{6} = x^2 \cdot \frac{1}{2} = \frac{x^2}{2}.$$

Hence $x^2 = 36$, so $x = 6$ cm (taking the positive root). The answer is **(B)**.

Q27 Knox 2023 Prelim Q14 · **2 marks**

The side length of an equilateral triangle is 4 cm. Find the exact area of the triangle.

Criteria	Marks
Uses $A = \frac{1}{2}ab\sin C$ with $a = b = 4$ and $C = 60°$	1
Correct exact area $4\sqrt{3}$ cm^2	1

Solution.

Every angle of an equilateral triangle is 60°, so taking two sides of length 4 with their included angle,

$$A = \frac{1}{2}(4)(4)\sin 60° = 8 \times \frac{\sqrt{3}}{2} = 4\sqrt{3}\text{ cm}^2.$$

Q28 Manly 2022 Trial Q25 · **3 marks**

In $\triangle ABC$, $AB = 12$ cm, $BC = 20$ cm and $AC = 28$ cm. Find the exact area of the triangle.

Criteria	Marks
Uses the cosine rule to find an angle (e.g. $\cos B = -\frac{1}{2}$, so $B = 120°$)	1
Applies $A = \frac{1}{2}ab\sin B$ with the two sides about that angle	1
Correct exact area $60\sqrt{3}$ cm^2	1

Solution.

Find the angle B at vertex B, which is included between $AB = 12$ and $BC = 20$ (opposite $AC = 28$). By the cosine rule,

$$\cos B = \frac{AB^2 + BC^2 - AC^2}{2\,(AB)(BC)} = \frac{12^2 + 20^2 - 28^2}{2(12)(20)} = \frac{144 + 400 - 784}{480} = \frac{-240}{480} = -\frac{1}{2}.$$

So $B = 120°$ and $\sin B = \dfrac{\sqrt{3}}{2}$. The area is

$$A = \frac{1}{2}(AB)(BC)\sin B = \frac{1}{2}(12)(20) \times \frac{\sqrt{3}}{2} = 60\sqrt{3}\text{ cm}^2.$$

Q29 St George Girls 2022 Trial Q29 · **2 marks**

The point $P(x, y)$ lies on the part of the circle $x^2 + y^2 = 25$ in the first quadrant, and O is the origin. M has coordinates $(2, 0)$ and N has coordinates $(0, 4)$, and $\angle MOP = \theta$ is measured in radians. Show that the area, A, of the quadrilateral $OMPN$ is given by $A = 5\sin\theta + 10\cos\theta$.

Criteria	**Marks**
Splits $OMPN$ into $\triangle OMP$ and $\triangle OPN$ and applies $A = \frac{1}{2}ab\sin C$	1
Reaches $A = 5\sin\theta + 10\cos\theta$	1

Solution.

The circle has radius 5, so $OP = 5$. Also $OM = 2$ (as $M = (2, 0)$) and $ON = 4$ (as $N = (0, 4)$). Split the quadrilateral $OMPN$ into $\triangle OMP$ and $\triangle OPN$ along the diagonal OP.

For $\triangle OMP$, the sides $OM = 2$ and $OP = 5$ include the angle $\angle MOP = \theta$:

$$\text{Area}(OMP) = \frac{1}{2}(2)(5)\sin\theta = 5\sin\theta.$$

OM lies along the positive x-axis and ON along the positive y-axis, so $\angle MON = \dfrac{\pi}{2}$. Hence the angle between OP and ON is $\angle PON = \dfrac{\pi}{2} - \theta$. For $\triangle OPN$, the sides $OP = 5$ and $ON = 4$ include this angle:

$$\text{Area}(OPN) = \frac{1}{2}(5)(4)\sin\left(\frac{\pi}{2} - \theta\right) = 10\cos\theta.$$

Adding the two areas,

$$A = 5\sin\theta + 10\cos\theta,$$

as required.

Q30 Knox 2022 Prelim Q15 · **3 marks**

A sector AOB has $OA = 6$ cm and $\angle AOB = \dfrac{5\pi}{12}$. Points C and D lie on OA and OB with $OC = OD$. **(i)** Find the exact area of sector AOB. **(ii)** If the area of triangle COD is half the area of sector AOB, find the length of OC (two significant figures).

Criteria	**Marks**
(i) Correct exact sector area $\dfrac{15\pi}{2}$ cm^2	1
(ii) Sets $\frac{1}{2}(OC)^2 \sin\frac{5\pi}{12} = \frac{1}{2}\times$ sector area	1
(ii) Correct value $OC \approx 4.9$ cm	1

Solution.

(i) The area of a sector is $\frac{1}{2}r^2\theta$ with $r = 6$ and $\theta = \dfrac{5\pi}{12}$:

$$\text{Area}(AOB) = \frac{1}{2}(6)^2 \times \frac{5\pi}{12} = \frac{1}{2} \times 36 \times \frac{5\pi}{12} = \frac{15\pi}{2} \text{ cm}^2.$$

(ii) Triangle COD has $OC = OD$ with the same included angle $\dfrac{5\pi}{12}$ as the sector. Let $OC = x$. Then

$$\text{Area}(COD) = \frac{1}{2}x^2 \sin\frac{5\pi}{12}.$$

Setting this equal to half the sector area,

$$\frac{1}{2}x^2 \sin\frac{5\pi}{12} = \frac{1}{2} \times \frac{15\pi}{2} \quad \Rightarrow \quad x^2 = \frac{15\pi/2}{\sin\frac{5\pi}{12}} = 24.393\ldots$$

Hence

$$x = \sqrt{24.393\ldots} = 4.938\ldots \approx 4.9 \text{ cm (2 s.f.)}.$$

Q31 Sydney Girls 2025 Trial Q39 · **2 marks**

A 21 m strip of tin is split into three 7 m segments: the middle segment is the horizontal base and the two outer segments are bent up by angle θ to form an isosceles trapezium of base 7 m and slant sides 7 m. Show that the cross-sectional area is $A = 49(\sin\theta + \sin\theta\cos\theta)$.

Criteria	Marks
Finds the height $7\sin\theta$ and the top parallel side $7 + 14\cos\theta$ (or splits into a rectangle and two right triangles)	1
Combines to reach $A = 49(\sin\theta + \sin\theta\cos\theta)$	1

Solution.

Each bent-up 7 m side rises a vertical height

$$h = 7\sin\theta$$

and projects a horizontal distance $7\cos\theta$ outwards. The cross-section is an isosceles trapezium: the lower parallel side is the base 7, and the upper parallel side is

$$7 + 2(7\cos\theta) = 7 + 14\cos\theta.$$

Using the trapezium area formula (half the sum of the parallel sides times the perpendicular height),

$$A = \frac{1}{2}\big(7 + (7 + 14\cos\theta)\big)(7\sin\theta) = \frac{1}{2}(14 + 14\cos\theta)(7\sin\theta).$$

Factoring 14 from the first bracket,

$$A = \frac{1}{2}\cdot 14(1 + \cos\theta)\cdot 7\sin\theta = 49\sin\theta(1 + \cos\theta) = 49(\sin\theta + \sin\theta\cos\theta),$$

as required.

Q32 Normanhurst Boys 2022 Trial Q18 · **6 marks**

A footpath has two parallel edges. CD is a vertical pole of height h m with base C on one edge. A and B are on the other edge with $AB = 7$ m and $\angle ACB = 60°$. The angles of elevation to the top D of the pole are $30°$ from A and $60°$ from B. **(a)** Show that $h = \sqrt{21}$ m. **(b)** By considering the area of $\triangle ABC$, find the exact width w of the footpath.

Criteria	Marks
(a) Expresses AC and BC in terms of h from the elevation angles	1
(a) Applies the cosine rule in $\triangle ABC$ to form an equation in h	1
(a) Solves to obtain $h = \sqrt{21}$ m	1
(b) Finds $AC = 3\sqrt{7}$, $BC = \sqrt{7}$ and the area of $\triangle ABC$	1
(b) Uses area $= \frac{1}{2}(AB)w$ to find w	1
(b) Correct exact width $w = \dfrac{3\sqrt{3}}{2}$ m	1

Solution.

(a) CD is vertical, so $\triangle DCA$ and $\triangle DCB$ are right-angled at C. From the elevation angles,

$$\tan 30° = \frac{h}{AC} \Rightarrow AC = \frac{h}{\tan 30°} = h\sqrt{3}, \qquad \tan 60° = \frac{h}{BC} \Rightarrow BC = \frac{h}{\tan 60°} = \frac{h}{\sqrt{3}}.$$

In $\triangle ABC$, apply the cosine rule with $AB = 7$ and the included angle $\angle ACB = 60°$:

$$AB^2 = AC^2 + BC^2 - 2(AC)(BC)\cos 60°.$$

Substituting,

$$49 = 3h^2 + \frac{h^2}{3} - 2(h\sqrt{3})\left(\frac{h}{\sqrt{3}}\right)\times\frac{1}{2} = 3h^2 + \frac{h^2}{3} - h^2 = \frac{7h^2}{3}.$$

Hence $h^2 = \dfrac{3\times 49}{7} = 21$, so $h = \sqrt{21}$ m.

(b) With $h = \sqrt{21}$,

$$AC = \sqrt{3}\cdot\sqrt{21} = \sqrt{63} = 3\sqrt{7}, \qquad BC = \frac{\sqrt{21}}{\sqrt{3}} = \sqrt{7}.$$

The area of $\triangle ABC$ is

$$\text{Area} = \frac{1}{2}(AC)(BC)\sin 60° = \frac{1}{2}(3\sqrt{7})(\sqrt{7})\times\frac{\sqrt{3}}{2} = \frac{21\sqrt{3}}{4}\text{ m}^2.$$

The width w of the footpath is the perpendicular distance from C to the edge through A and B, i.e. the height of $\triangle ABC$ on base AB. Since area $= \frac{1}{2}(AB)w$,

$$w = \frac{2\times\text{Area}}{AB} = \frac{2\times\frac{21\sqrt{3}}{4}}{7} = \frac{21\sqrt{3}/2}{7} = \frac{3\sqrt{3}}{2}\text{ m}.$$

5. Finding sides and angles: choosing the rule

Q33 James Ruse 2024 Trial Q4 · 1 mark

A triangle has sides of length 34 cm, 27.4 cm and 24.3 cm. Which of the following gives the magnitude of the smallest angle in the triangle, to the nearest degree? (multiple choice)

Criteria	Marks
Correct option (B)	1

Solution.

The smallest angle lies opposite the shortest side, 24.3 cm. By the cosine rule, with θ opposite the 24.3 side,

$$\cos\theta = \frac{27.4^2 + 34^2 - 24.3^2}{2(27.4)(34)} = \frac{750.76 + 1156 - 590.49}{1863.2} = \frac{1316.27}{1863.2}.$$

So $\theta = \cos^{-1}(0.70645\ldots) = 45.05\ldots^\circ \approx 45^\circ$. The answer is **(B)**.

Q34 Barker 2025 Trial Q14 · 3 marks

In a triangle, the side of length 24 cm is opposite an angle of 67°, a second angle is 48°, and x cm is the side opposite the 48° angle. **(i)** Find x, to 2 decimal places. **(ii)** Hence find the area of the triangle, to 2 decimal places.

Criteria	Marks
(i) Correct sine rule statement for x	1
(i) Correct value $x = 19.38$ cm	1
(ii) Correct area ≈ 210.72 cm^2	1

Solution.

(i) The 24 cm side is opposite 67° and x is opposite 48°. By the sine rule,

$$\frac{x}{\sin 48^\circ} = \frac{24}{\sin 67^\circ}, \qquad x = \frac{24\sin 48^\circ}{\sin 67^\circ} = 19.3757\ldots \approx 19.38 \text{ cm}.$$

(ii) The third angle is $180^\circ - 67^\circ - 48^\circ = 65^\circ$, which is the angle included between the 24 cm side and the x cm side. Hence

$$\text{Area} = \tfrac{1}{2}(24)(19.3757\ldots)\sin 65^\circ = 210.7247\ldots \approx 210.72 \text{ cm}^2.$$

Q35 Sydney Boys 2023 Trial Q32 · **2 marks**

In triangle ABC, $BC \perp AC$, with $BC = a$, $AC = b$, $AB = c$. The point D on AB satisfies $CD \perp AB$, and $CD = x$. **(i)** Show that triangles ABC and ACD are similar. **(ii)** Hence show that $x = \dfrac{ab}{c}$.

Criteria	Marks
(i) Identifies a common angle and a right angle in each triangle, concluding similarity (AA)	1
(ii) Uses the ratio of matching sides of the similar triangles to reach $x = \dfrac{ab}{c}$	1

Solution.

(i) In triangle ABC, the angle at C is a right angle (since $BC \perp AC$). In triangle ACD, the angle at D is a right angle (since $CD \perp AB$). The two triangles also share the angle at A ($\angle BAC = \angle CAD$). Two pairs of equal angles give

$$\triangle ABC \;|||\; \triangle ACD \quad \text{(AA)}.$$

(ii) Since the triangles are similar, the ratios of matching sides are equal. Matching the hypotenuses and the sides opposite the shared angle at A,

$$\frac{CD}{BC} = \frac{AC}{AB} \qquad \Longrightarrow \qquad \frac{x}{a} = \frac{b}{c},$$

so $x = \dfrac{ab}{c}$, as required. (Equivalently, x is the altitude to the hypotenuse of the right triangle ABC, and equating areas gives $\frac{1}{2}c\,x = \frac{1}{2}a\,b$.)

Q36 Baulkham Hills 2022 Prelim Q19 · 3 marks

Asmita walks 270 m from her house H on a bearing of 330° to B, then 290 m in a straight line to C, which is due west of her house. **(a)** Draw a diagram. **(b)** Find the distance HC, to 1 decimal place.

Criteria	Marks
(a) Correct diagram: $HB = 270$ on bearing 330°, $BC = 290$, C due west of H	1
(b) Sets up the cosine rule with the included angle 60° at H	1
(b) Solves the resulting equation and reaches $HC \approx 306.5$ m	1

Solution.

(a) Mark the house H; draw $HB = 270$ m on a bearing of 330° (i.e. 30° west of north), then $BC = 290$ m to C, placing C due west of H.

(b) At H, the direction to B is bearing 330° and the direction to C is due west (bearing 270°), so the included angle is $\angle BHC = 330° - 270° = 60°$. Let $HC = d$. By the cosine rule in $\triangle BHC$, with BC opposite the angle at H,

$$290^2 = d^2 + 270^2 - 2(270)(d)\cos 60°,$$

$$d^2 - 270d - 11200 = 0 \qquad \Longrightarrow \qquad d = \frac{270 \pm \sqrt{270^2 + 4(11200)}}{2}.$$

Taking the positive root, $d = 306.53\ldots \approx 306.5$ m.

Q37 Presbyterian 2022 Prelim Q33 · 4 marks

A circle with centre O and radius r mm is divided into five equal sectors. One sector OAB has $\angle AOB = \theta$ radians and area 20π mm^2. **(a)** Find r. **(b)** Find the chord AB, to 1 decimal place.

Criteria	Marks
(a) Uses $\theta = \dfrac{2\pi}{5}$ with the sector-area formula	1
(a) Correct value $r = 10$ mm	1
(b) Correct cosine-rule (or chord) setup for AB	1
(b) Correct value $AB \approx 11.8$ mm	1

Solution.

(a) Five equal sectors make up the full angle, so $\theta = \dfrac{2\pi}{5}$. Using the sector-area formula $A = \frac{1}{2}r^2\theta$,

$$\frac{1}{2}r^2 \cdot \frac{2\pi}{5} = 20\pi \qquad \Longrightarrow \qquad \frac{\pi r^2}{5} = 20\pi \qquad \Longrightarrow \qquad r^2 = 100,$$

so $r = 10$ mm.

(b) In $\triangle OAB$, $OA = OB = 10$ and $\angle AOB = \dfrac{2\pi}{5}$. By the cosine rule,

$$AB^2 = 10^2 + 10^2 - 2(10)(10)\cos\frac{2\pi}{5} = 200\left(1 - \cos\frac{2\pi}{5}\right),$$

so $AB = 11.7557\ldots \approx 11.8$ mm.

Q38 Hurlstone 2022 Prelim Q13 · 8 marks

R, S, Q lie on horizontal ground; PQ is a vertical pole at Q. The rope $PS = 20$ m makes $30°$ with the ground at S; the rope PR makes $45°$ with the ground at R; and $\angle RPS = 60°$. **(i)** Show $PQ = 10$ m. **(ii)** Find PR (exact). **(iii)** Find RS, to 1 decimal place. **(iv)** Find $\angle RQS$, to the nearest degree. **(v)** Given R is due south of Q, find the bearing of Q from S, to the nearest degree.

Criteria	Marks
(i) Uses right triangle PQS to obtain $PQ = 20\sin 30° = 10$ m	1
(ii) Uses right triangle PQR to obtain $PR = 10\sqrt{2}$ m	1
(iii) Applies the cosine rule in $\triangle PRS$ with $\angle RPS = 60°$	1
(iii) Correct value $RS \approx 17.8$ m	1
(iv) Finds the ground lengths $QR = 10$ and $QS = 10\sqrt{3}$	1
(iv) Applies the cosine rule in $\triangle QRS$	1
(iv) Correct value $\angle RQS \approx 76°$	1
(v) Correct bearing $\approx 284°$	1

Solution.

(i) PQ is vertical, so $\triangle PQS$ is right-angled at Q with hypotenuse $PS = 20$ and $\angle PSQ = 30°$. Thus

$$PQ = 20\sin 30° = 10 \text{ m}.$$

(ii) In $\triangle PQR$ (right-angled at Q), $\angle PRQ = 45°$ and $PQ = 10$, so

$$PR = \frac{PQ}{\sin 45°} = \frac{10}{\sin 45°} = 10\sqrt{2} \text{ m}.$$

(iii) In $\triangle PRS$, $PR = 10\sqrt{2}$, $PS = 20$ and $\angle RPS = 60°$. By the cosine rule,

$$RS^2 = (10\sqrt{2})^2 + 20^2 - 2(10\sqrt{2})(20)\cos 60° = 200 + 400 - 200\sqrt{2} = 600 - 200\sqrt{2},$$

so $RS = 17.808\ldots \approx 17.8$ m.

(iv) The ground lengths at Q: in $\triangle PQS$, $QS = 20\cos 30° = 10\sqrt{3}$; in $\triangle PQR$ (isosceles, $45°$), $QR = PQ = 10$. In the ground triangle QRS, by the cosine rule,

$$\cos\angle RQS = \frac{QR^2 + QS^2 - RS^2}{2\,QR\,QS} = \frac{10^2 + (10\sqrt{3})^2 - (600 - 200\sqrt{2})}{2(10)(10\sqrt{3})} = \frac{200\sqrt{2} - 200}{200\sqrt{3}}.$$

Hence $\angle RQS = \cos^{-1}(0.23914\ldots) = 76.16\ldots° \approx 76°$.

(v) R is due south of Q, so QR points due south (bearing $180°$) and S lies to the west of the line QR. Measuring from north, the bearing of S from Q is $180° - 76° = 104°$ would be to the east; here S is to the west, so the bearing of Q from S works out as $360° - 76° = 284°$.

Q39 Penrith 2023 Trial Q27 · 4 marks

Roberto R is directly in front of goalkeeper G, with $\angle AGR = 90°$. His kicking angle $\angle ARG$ is 5°; moving 5 m closer to G increases this angle to 8°. **(a)** Find the kicking distance to A from the 8° mark, to 2 decimal places. **(b)** Find how far Roberto was from the goalkeeper at the 5° mark, to 1 decimal place.

Criteria	Marks
(a) Sets up the sine rule using the 5 m gap and the 3° angle at A	1
(a) Correct value ≈ 8.33 m	1
(b) Correct method for the ground distance from G	1
(b) Correct value ≈ 13.2 m	1

Solution.

Let N be the near mark (8°) and let Roberto's far mark be R (5°), with $NR = 5$ m along the line towards G. In triangle NRA, the exterior geometry gives $\angle ANR = 180° - 8° = 172°$ and $\angle ARN = 5°$, so the angle at A is $180° - 172° - 5° = 3°$ (which is just $8° - 5°$).

(a) The 5 m side NR is opposite $\angle A = 3°$, and the required distance NA is opposite $\angle ARN = 5°$. By the sine rule,

$$NA = \frac{5 \sin 5°}{\sin 3°} = 8.3265\ldots \approx 8.33 \text{ m}.$$

(b) The distance RA is opposite $\angle ANR = 172°$, so $RA = \dfrac{5 \sin 172°}{\sin 3°} = 13.296\ldots$ m. Since $\angle AGR = 90°$, the ground distance from G to the far mark is

$$GR = RA \cos 5° = 13.296\ldots \times \cos 5° = 13.245\ldots \approx 13.2 \text{ m}.$$

Q40 Sydney Girls 2024 Trial Q16 · 5 marks

Kate rides 4 km from her house H on a bearing of 320° to the park P, then 5 km on a bearing of 250° to the beach B. **(i)** Show that $\angle HPB = 110°$. **(ii)** Find BH, to three decimal places. **(iii)** Find the bearing of H from B, to the nearest degree.

Criteria	Marks
(i) Uses the reverse bearing $P \to H$ of 140° and the bearing $P \to B$ of 250° to get $\angle HPB = 110°$	1
(ii) Applies the cosine rule in $\triangle HPB$ with the included angle 110°	1
(ii) Correct value $BH \approx 7.395$ km	1
(iii) Finds $\angle PBH \approx 30.55°$ (sine rule) and combines with the bearing of P from B	1
(iii) Correct bearing $\approx 101°$	1

Solution.

(i) Kate rides $H \to P$ on a bearing of 320°, so the reverse direction $P \to H$ has bearing $320° - 180° = 140°$. The direction $P \to B$ has bearing 250°. The angle between these two directions at P is

$$\angle HPB = 250° - 140° = 110°.$$

(ii) In $\triangle HPB$, $HP = 4$ and $PB = 5$ include the angle $\angle HPB = 110°$. By the cosine rule,

$$BH^2 = 4^2 + 5^2 - 2(4)(5)\cos 110° = 41 - 40\cos 110° = 54.6808\ldots,$$

so $BH = 7.39464\ldots \approx 7.395$ km.

(iii) By the sine rule in $\triangle HPB$, the angle $\angle PBH$ (at B, opposite $HP = 4$) satisfies

$$\frac{\sin\angle PBH}{4} = \frac{\sin 110°}{BH} \qquad \Longrightarrow \qquad \angle PBH = \sin^{-1}\left(\frac{4\sin 110°}{7.39464\ldots}\right) = 30.55\ldots°.$$

The bearing of B from P is 250°, so the reverse bearing of P from B is $250° - 180° = 070°$. Measuring clockwise from this direction to BH,

$$\text{bearing of } H \text{ from } B = 070° + 30.55\ldots° = 100.55\ldots° \approx 101°.$$

Q41 North Sydney Girls 2023 Prelim Q33 · 4 marks

In triangle ABC, $AC = 9$, $BC = 12$, $AB = 8$, with $\angle ABC = x$ and $\angle ACB = y$. **(a)** Express $\angle BAC$ in terms of x and y. **(b)** Hence show that $\sin x + \sin y = \dfrac{17\sin(x+y)}{12}$.

Criteria	Marks
(a) States $\angle BAC = 180^\circ - x - y$	1
(b) Applies the sine rule to relate the sides 9, 8, 12 to $\sin x$, $\sin y$, $\sin \angle BAC$	1
(b) Uses $\sin \angle BAC = \sin(x+y)$	1
(b) Combines to reach the required result	1

Solution.

(a) By the angle sum of a triangle,

$$\angle BAC = 180^\circ - x - y.$$

(b) By the sine rule, each side over the sine of its opposite angle is equal. The side $AC = 9$ is opposite $\angle ABC = x$, the side $AB = 8$ is opposite $\angle ACB = y$, and the side $BC = 12$ is opposite $\angle BAC$. Hence

$$\frac{9}{\sin x} = \frac{8}{\sin y} = \frac{12}{\sin \angle BAC}.$$

From part (a), $\sin \angle BAC = \sin\big(180^\circ - (x+y)\big) = \sin(x+y)$. Let the common ratio be $R = \dfrac{12}{\sin(x+y)}$. Then $\sin x = \dfrac{9}{R}$ and $\sin y = \dfrac{8}{R}$, so

$$\sin x + \sin y = \frac{9+8}{R} = \frac{17}{R} = \frac{17\sin(x+y)}{12},$$

as required.

6. Trigonometry in three dimensions

Q42 St George Girls 2022 Prelim Q22 · **3 marks**

The diagram shows a rectangular prism $ABCDEFGH$ (top face $ABCD$, bottom face $EFGH$, with E directly below B). The edge $BE = 9$ cm, the edge $EF = 12$ cm and the edge $GF = 17$ cm. Find the size of $\angle GBF$, to the nearest degree.

Criteria	**Marks**
Finds the diagonal $BF = \sqrt{BE^2 + EF^2} = 15$ cm	1
Recognises $\angle BFG = 90°$ and sets up $\tan \angle GBF = \dfrac{GF}{BF}$	1
Correct answer $\angle GBF \approx 49°$	1

Solution.

In the right-angled triangle BEF (right angle at E), BF is the diagonal of the face through B, E, F:

$$BF = \sqrt{BE^2 + EF^2} = \sqrt{9^2 + 12^2} = \sqrt{225} = 15 \text{ cm}.$$

The edge $GF = 17$ cm is perpendicular to the plane containing B, E, F, so triangle BFG has a right angle at F. Hence

$$\tan \angle GBF = \frac{GF}{BF} = \frac{17}{15} \implies \angle GBF = \tan^{-1}\left(\frac{17}{15}\right) \approx 48.58° \approx 49°.$$

Q43 Sydney Tech 2025 Trial Q10 · **1 mark**

A vertical tower TC is 40 metres high. The point A is due east of the base C of the tower, with an angle of elevation to the top T of $45°$. A second point B is on a different bearing from C (lying clockwise from A, on the southern side of the east line) and has an angle of elevation to the top of the tower of $30°$. The points A and B are 100 m apart. The bearing from C to B, correct to the nearest degree, is: (multiple choice)

(A) $131°$

(B) $162°$

(C) $221°$

(D) $311°$

Criteria	**Marks**
Correct option (C)	1

Solution.

From the elevations, the horizontal distances from the base C are

$$CA = \frac{40}{\tan 45°} = 40 \text{ m}, \qquad CB = \frac{40}{\tan 30°} = 40\sqrt{3} \approx 69.28 \text{ m}.$$

In the horizontal triangle ACB, apply the cosine rule to find $\angle ACB$:

$$\cos \angle ACB = \frac{CA^2 + CB^2 - AB^2}{2 \cdot CA \cdot CB} = \frac{40^2 + 4800 - 100^2}{2(40)(40\sqrt{3})} \approx -0.6495,$$

so $\angle ACB \approx 130.5°$. Since A is due east of C (bearing $090°$) and B is clockwise from A, the bearing of B from C is

$$090° + 130.5° \approx 221°.$$

Option (C).

Q44 2023 HSC Q22 · **3 marks**

In the rectangular prism shown, $AD = 7$ cm, $AE = 8$ cm and $EF = 6$ cm. The point M is the midpoint of CD. Find $\angle AEM$, to the nearest degree.

Criteria	**Marks**
Finds $AM = \sqrt{AD^2 + DM^2} = \sqrt{58}$ cm	1
Recognises $\angle EAM = 90°$ and sets up $\tan \angle AEM = \dfrac{AM}{AE}$	1
Correct answer $\angle AEM \approx 44°$	1

Solution.

Since M is the midpoint of CD and $CD = EF = 6$ cm, we have $DM = 3$ cm. The segment AM lies in the top face, in the right-angled triangle ADM (right angle at D):

$$AM = \sqrt{AD^2 + DM^2} = \sqrt{7^2 + 3^2} = \sqrt{58} \text{ cm}.$$

The edge $AE = 8$ cm is vertical, hence perpendicular to the top face and to AM, so triangle AEM has a right angle at A. Therefore

$$\tan \angle AEM = \frac{AM}{AE} = \frac{\sqrt{58}}{8} \implies \angle AEM = \tan^{-1}\left(\frac{\sqrt{58}}{8}\right) \approx 43.59° \approx 44°.$$

Q45 Girraween 2025 Trial Q29 · **4 marks**

A vertical tower of height h metres stands on horizontal ground. From a point P on the ground due east of the tower the angle of elevation of the top of the tower is $45°$. From a point Q on the ground due south of the tower the angle of elevation of the top of the tower is $30°$. If the distance PQ is 40 metres, find the exact height of the tower.

Criteria	**Marks**
Finds the horizontal distance to P: base $P = \dfrac{h}{\tan 45°} = h$	1
Finds the horizontal distance to Q: base $Q = \dfrac{h}{\tan 30°} = h\sqrt{3}$	1
Uses $PQ^2 = h^2 + 3h^2$ (right angle at the base, since P is east and Q is south)	1
Solves $4h^2 = 1600$ to get $h = 20$ metres exactly	1

Solution.

Let C be the base of the tower. From P (due east), the elevation is $45°$, so $CP = \dfrac{h}{\tan 45°} = h$. From Q (due south), the elevation is $30°$, so $CQ = \dfrac{h}{\tan 30°} = h\sqrt{3}$. Since P is due east and Q is due south of C, $\angle PCQ = 90°$, so by Pythagoras

$$PQ^2 = CP^2 + CQ^2 = h^2 + 3h^2 = 4h^2.$$

With $PQ = 40$, $4h^2 = 1600 \Rightarrow h^2 = 400 \Rightarrow h = 20$. The exact height of the tower is 20 metres.

Q46 Knox 2023 Prelim Q20 · **5 marks**

A pillar BT of height h m is installed in a park, with B the base and T the top. Alex stands due south of the pillar and notices that the angle of elevation of the top is 45°. Chet stands due east of the pillar and notices that the angle of elevation of the top is 60°. If Alex and Chet are 20 m apart, find the height of the pillar to the nearest cm.

Criteria	**Marks**
From Alex (45°): base A distance $AB = \dfrac{h}{\tan 45^\circ} = h$	1
From Chet (60°): base C distance $BC = \dfrac{h}{\tan 60^\circ} = \dfrac{h}{\sqrt{3}}$	1
Recognises $\angle ABC = 90^\circ$ (Alex south, Chet east) and writes $AC^2 = AB^2 + BC^2$	1
Forms $\dfrac{4h^2}{3} = 400$, so $h^2 = 300$	1
Solves $h = 10\sqrt{3} \approx 17.32$ m (to nearest cm)	1

Solution.

Let B be the base of the pillar. Alex is due south, with elevation 45°, so the horizontal distance is $AB = \dfrac{h}{\tan 45^\circ} = h$. Chet is due east, with elevation 60°, so $BC = \dfrac{h}{\tan 60^\circ} = \dfrac{h}{\sqrt{3}}$. Since Alex is south and Chet is east of B, $\angle ABC = 90^\circ$, so by Pythagoras

$$AC^2 = AB^2 + BC^2 = h^2 + \frac{h^2}{3} = \frac{4h^2}{3}.$$

With $AC = 20$, $\dfrac{4h^2}{3} = 400 \Rightarrow h^2 = 300 \Rightarrow h = 10\sqrt{3} \approx 17.3205$ m. To the nearest cm, the pillar is 17.32 m high.

Q47 2025 HSC Q29 · 7 marks

The point T is the peak of a mountain and the point O is directly below the peak. The point Y is due east of O, and the angle of elevation of T from Y is $60°$. The point F is 4 km south-west of Y, with $YF = 4$ km. The points O, Y and F are on level ground, and the angle of elevation of T from F is $45°$. Let the height of the mountain be h. **(a)** Show that $OY = \dfrac{h}{\sqrt{3}}$. **(b)** Hence, or otherwise, find h, correct to 2 decimal places. **(c)** Find the bearing of O from F, correct to the nearest degree.

Criteria	Marks
(a) Uses $\tan 60° = \dfrac{h}{OY}$ to show $OY = \dfrac{h}{\sqrt{3}}$	1
(b) Finds $OF = \dfrac{h}{\tan 45°} = h$ and identifies $\angle OYF = 45°$	1
(b) Applies the cosine rule in $\triangle OYF$: $h^2 = \dfrac{h^2}{3} + 16 - \dfrac{8h}{\sqrt{3}} \cdot \dfrac{1}{\sqrt{2}}$	1
(b) Solves to $h = \sqrt{30} - \sqrt{6} \approx 3.03$ km	1
(c) Finds a relevant angle in $\triangle OYF$ (e.g. $\angle OFY$ via the sine rule)	1
(c) Combines with the direction of F to Y (bearing $045°$)	1
(c) Correct bearing $\approx 021°$	1

Solution.

(a) In the right-angled triangle TOY (vertical $TO = h$, horizontal OY), the elevation of T from Y is $60°$, so

$$\tan 60° = \frac{TO}{OY} = \frac{h}{OY} \implies OY = \frac{h}{\tan 60°} = \frac{h}{\sqrt{3}}.$$

(b) Similarly, the elevation of T from F is $45°$, so $OF = \dfrac{h}{\tan 45°} = h$. Now consider the horizontal triangle OYF. Since Y is due east of O, O is due west of Y; and F is south-west of Y, so $\angle OYF = 45°$ (between the westerly and south-westerly directions). By the cosine rule,

$$OF^2 = OY^2 + YF^2 - 2 \cdot OY \cdot YF \cos \angle OYF,$$

$$h^2 = \frac{h^2}{3} + 16 - 2 \cdot \frac{h}{\sqrt{3}} \cdot 4 \cdot \frac{1}{\sqrt{2}}.$$

This rearranges to $\dfrac{2h^2}{3} + \dfrac{8h}{\sqrt{6}} - 16 = 0$, whose positive solution is

$$h = \sqrt{30} - \sqrt{6} \approx 3.0277 \text{ km} \approx 3.03 \text{ km}.$$

(c) Place O at the origin with east $= +x$, north $= +y$. Then $Y = \left(\dfrac{h}{\sqrt{3}}, 0\right)$ and $F = Y + 4(\text{south-west}) = \left(\dfrac{h}{\sqrt{3}} - \dfrac{4}{\sqrt{2}}, -\dfrac{4}{\sqrt{2}}\right) \approx (-1.080, -2.828)$. The displacement from F to O is $(1.080, 2.828)$: O lies north-east

of F. The bearing is measured clockwise from north:

$$\text{bearing} = \tan^{-1}\left(\frac{1.080}{2.828}\right) \approx 20.9^\circ \approx 021^\circ.$$