Year 11 Mathematics Advanced

Topic: Trigonometry and Measure of Angles (1 of 5)

Review Questions (Worked Solutions): Right-angled trigonometry, bearings and elevation

Term 3, 2026

Right-angled trigonometry

1. Finding a side or angle in a right-angled triangle

Q1 Barker 2020 Trial Q19 · **2 marks**

A wire frame is made of 6 parts: AB, BC, CD, ED, AE and BD, with $AB = ED = 2$ cm and $BC = a$ cm. $ABDE$ is a rectangle and BCD is a triangle right-angled at D with $\angle CBD = \theta$. **(a)** Find BD and CD in terms of a and θ. **(b)** Find the total length L of wire in the frame.

Criteria	Marks
(a) Correct expressions $BD = a\cos\theta$ and $CD = a\sin\theta$	1
(b) Correct total length L (sum of all six parts)	1

Solution.

(a) In the triangle BCD, the right angle is at D and $\angle CBD = \theta$, with hypotenuse $BC = a$. Then BD is adjacent to θ and CD is opposite θ:

$$BD = a\cos\theta, \qquad CD = a\sin\theta.$$

(b) Since $ABDE$ is a rectangle, $AE = BD = a\cos\theta$. Adding the six parts,

$$L = \underbrace{AB}_{2} + \underbrace{BC}_{a} + \underbrace{CD}_{a\sin\theta} + \underbrace{ED}_{2} + \underbrace{AE}_{a\cos\theta} + \underbrace{BD}_{a\cos\theta},$$

$$L = 4 + a + a\sin\theta + 2a\cos\theta.$$

(The printed trial solution gives $L = 4 + 2a\cos\theta + a\sin\theta$, which omits the $BC = a$ term; the complete length includes it.)

Q2 Fort St 2022 Prelim Q4 · **1 mark**

A right-angled triangle has its right angle at the top vertex, with the two sides at the right angle of length $\sqrt{7}$ and 3. The angle α is at the base vertex adjacent to the side $\sqrt{7}$, so that the side 3 is opposite α. Calculate α, correct to the nearest second. (multiple choice)

Criteria	**Marks**
Correct option (C)	1

Solution.

The side of length 3 is opposite α and the side of length $\sqrt{7}$ is adjacent, so

$$\tan\alpha = \frac{3}{\sqrt{7}}, \qquad \alpha = \tan^{-1}\left(\frac{3}{\sqrt{7}}\right) = 48.5903\ldots^\circ.$$

Converting to degrees, minutes and seconds,

$$\alpha = 48^\circ 35' 25.36'' = 48^\circ 35' 25'' \text{ (nearest second)}.$$

The answer is **(C)**.

Q3 Riverview 2023 Trial Q11 · **4 marks**

A tower has top T and base A. The angle of elevation of T from A is $23°$; from B, which is 100 m closer to the tower on the same horizontal level, the elevation is $50°$. **(a)** Show $\angle BTA = 27°$. **(b)** Show $BT = \dfrac{100\sin 23°}{\sin 27°}$. **(c)** Find the height h of the tower (one decimal place).

Criteria	Marks
(a) Shows $\angle BTA = 27°$ using the exterior angle of $\triangle ABT$	1
(b) Correct sine rule statement giving $BT = \dfrac{100\sin 23°}{\sin 27°}$	1
(c) Uses the vertical right triangle to write $h = BT\sin 50°$	1
(c) Correct value $h \approx 65.9$ m	1

Solution.

(a) In $\triangle ABT$, $\angle TAB = 23°$. The angle $\angle TBX$ (elevation at B) is the exterior angle of the triangle at B, so

$$\angle TBX = \angle TAB + \angle BTA \quad \Rightarrow \quad 50° = 23° + \angle BTA,$$

$$\therefore \ \angle BTA = 27°.$$

(b) In $\triangle ABT$, side BT is opposite $\angle TAB = 23°$ and side $AB = 100$ is opposite $\angle BTA = 27°$. By the sine rule,

$$\frac{BT}{\sin 23°} = \frac{100}{\sin 27°} \quad \Rightarrow \quad BT = \frac{100\sin 23°}{\sin 27°}.$$

(c) Numerically $BT = 86.0659\ldots$ m. In the vertical right triangle at the foot of the tower, the elevation from B is $50°$, so

$$h = BT\sin 50° = 86.0659\ldots \times \sin 50° = 65.930\ldots \text{ m} \approx 65.9 \text{ m}.$$

Q4 Sydney Grammar 2025 Trial Q34 · **1 mark**

A and B are endpoints of a diameter of a circle, centre O, radius R. A point P is on the circumference with $\angle PAB = \theta$. M is the midpoint of chord AP, so $\angle OMA = \dfrac{\pi}{2}$. Find AM in terms of R and θ.

Criteria	Marks
Correct expression $AM = R\cos\theta$	1

Solution.

In the right-angled triangle OMA, the right angle is at M, the hypotenuse is $OA = R$ (a radius), and $\angle MAO = \angle PAB = \theta$. Since AM is adjacent to θ,

$$\cos\theta = \frac{AM}{R} \quad \Rightarrow \quad AM = R\cos\theta.$$

Q5 Killara 2021 Trial Q17 · **3 marks**

Points A, B, D lie on a level plane with B directly below the peak C. $AD = 3.5$ km, $\angle DAB = 15°$, $\angle BDA = 135°$, and the angle of inclination from A to C is $36°$. **(a)** Show $AB = \dfrac{3.5\sin 135°}{\sin 30°}$. **(b)** Find the angle of inclination from D to the peak (nearest minute).

Criteria	Marks
(a) Finds $\angle ABD = 30°$ and states the sine rule for AB	1
(b) Finds the tower height CB and the horizontal distance BD	1
(b) Correct angle of inclination $\approx 63°16'$	1

Solution.

(a) In $\triangle ABD$, the angle sum gives

$$\angle ABD = 180° - 15° - 135° = 30°.$$

Side AB is opposite $\angle BDA = 135°$ and $AD = 3.5$ is opposite $\angle ABD = 30°$. By the sine rule,

$$\frac{AB}{\sin 135°} = \frac{3.5}{\sin 30°} \quad \Rightarrow \quad AB = \frac{3.5\sin 135°}{\sin 30°} = 4.9497\ldots \text{ km}.$$

(b) The peak C is directly above B. From A, the elevation of C is $36°$, so in the vertical right triangle at B,

$$CB = AB\tan 36° = 4.9497\ldots \times \tan 36° = 3.5962\ldots \text{ km}.$$

The horizontal distance BD is found from $\triangle ABD$: side BD is opposite $\angle DAB = 15°$, so

$$BD = \frac{3.5\sin 15°}{\sin 30°} = 1.8117\ldots \text{ km}.$$

The angle of inclination from D satisfies

$$\tan\angle CDB = \frac{CB}{BD} = \frac{3.5962\ldots}{1.8117\ldots}, \qquad \angle CDB = \tan^{-1}\left(\frac{3.5962}{1.8117}\right) = 63.261\ldots° = 63°16' \text{ (nearest minute)}.$$

Q6 Danebank 2022 Trial Q24 · **5 marks**

A hiker walks 100 m north-east from P to Q. A peak R is directly above X, with $RX = h$. The angle of elevation of R is 10° from P and 5° from Q, and the horizontal triangle PQX has $\angle PXQ = 135^\circ$. **(a)** Show $XP = h\cot 10^\circ$ and find a similar expression for XQ. **(b)** Find h (nearest metre).

Criteria	Marks
(a) Shows $XP = h\cot 10^\circ$ from the vertical right triangle at P	1
(a) Correct $XQ = h\cot 5^\circ$	1
(b) Applies the cosine rule in $\triangle PQX$ with $\angle PXQ = 135^\circ$	1
(b) Solves for h^2 correctly	1
(b) Correct value $h \approx 6$ m	1

Solution.

(a) $RX = h$ is vertical, so $\triangle RXP$ is right-angled at X with elevation 10° at P:

$$\tan 10^\circ = \frac{h}{XP} \quad \Rightarrow \quad XP = \frac{h}{\tan 10^\circ} = h\cot 10^\circ.$$

Likewise in $\triangle RXQ$ (right-angled at X, elevation 5° at Q),

$$XQ = h\cot 5^\circ.$$

(b) In the horizontal triangle PQX, $PQ = 100$ and $\angle PXQ = 135^\circ$. By the cosine rule,

$$PQ^2 = XP^2 + XQ^2 - 2\,XP\cdot XQ\cos 135^\circ,$$

$$100^2 = h^2\cot^2 10^\circ + h^2\cot^2 5^\circ - 2h^2\cot 10^\circ\cot 5^\circ\cos 135^\circ.$$

Factoring out h^2,

$$10\,000 = h^2\left(\cot^2 10^\circ + \cot^2 5^\circ - 2\cot 10^\circ\cot 5^\circ\cos 135^\circ\right) = h^2(254.483\ldots).$$

Hence

$$h^2 = \frac{10\,000}{254.483\ldots} = 39.295\ldots, \qquad h = 6.268\ldots \text{ m} \approx 6 \text{ m (nearest metre)}.$$

Q7 Sydney Tech 2024 Trial Q28 · **6 marks**

A yacht leaves Port A on a bearing of 120° and sails three hours at 15 km/h to Y. At the same time a speed boat leaves A due south to Island B, 30 km from the port. **(a)** Find the distance of the yacht from B (nearest km). **(b)** Find the bearing of B from the yacht (nearest degree). **(c)** How far south of the port is the boat when it is directly west of the yacht?

Criteria	**Marks**
(a) Finds $AY = 45$ km and $\angle YAB = 60^\circ$, states the cosine rule	1
(a) Correct distance $YB \approx 40$ km	1
(b) Uses the sine rule to find $\angle AYB$	1
(b) Correct bearing $\approx 259^\circ$	1
(c) Sets up $45\cos 60^\circ$ (or equivalent) for the north-south component	1
(c) Correct distance 22.5 km	1

Solution.

(a) The yacht sails $3 \times 15 = 45$ km, so $AY = 45$ km. The bearing of Y from A is 120° and B is due south of A (bearing 180°), so

$$\angle YAB = 180^\circ - 120^\circ = 60^\circ.$$

By the cosine rule in $\triangle AYB$,

$$YB^2 = AY^2 + AB^2 - 2\,AY \cdot AB\cos 60^\circ = 45^2 + 30^2 - 2(45)(30)\left(\tfrac{1}{2}\right) = 1575,$$

$$YB = \sqrt{1575} = 39.686\ldots \text{ km} \approx 40 \text{ km}.$$

(b) By the sine rule in $\triangle AYB$,

$$\frac{\sin\angle AYB}{AB} = \frac{\sin\angle YAB}{YB} \quad\Rightarrow\quad \sin\angle AYB = \frac{30\sin 60^\circ}{39.686\ldots} = 0.6547\ldots,$$

$$\angle AYB = 40.9^\circ\ldots$$

The bearing of A from Y is $120^\circ + 180^\circ = 300^\circ$. Point B lies to the west of the line YA, so the bearing of B from Y is

$$300^\circ - 40.9^\circ\ldots = 259.1^\circ\ldots \approx 259^\circ.$$

(c) Being directly west of the yacht means the boat has the same north-south position as Y. The southward (north-south) component of AY is

$$45\cos 60^\circ = 45 \times \tfrac{1}{2} = 22.5 \text{ km}.$$

So the boat is 22.5 km south of the port.

Applications

2. Angles of elevation and depression

Q8 Kambala 2021 Prelim Q2 · **2 marks**

A lighthouse stands on top of a 50 metre high cliff. A yacht is 80 metres out to sea below the lighthouse. What is the angle of depression of the yacht from the base of the lighthouse, to the nearest degree?

Criteria	**Marks**
Correct trigonometric equation, e.g. $\tan\theta = \dfrac{50}{80}$	1
Correct angle $\theta \approx 32^\circ$	1

Solution.

The base of the lighthouse is at the top of the cliff, 50 m above sea level, and the yacht is 80 m out horizontally. The angle of depression θ equals the angle at the observer in the right-angled triangle with opposite side 50 (the vertical drop) and adjacent side 80 (the horizontal distance):

$$\tan\theta = \frac{50}{80} = \frac{5}{8}.$$

$$\theta = \tan^{-1}\left(\tfrac{5}{8}\right) = 32.0^\circ \approx 32^\circ.$$

Q9 Girraween 2020 Trial Q28 · **4 marks**

Declan walks 1600 metres due north along a straight road from point A to point B. The foot Y of a hill XY (where X is the top) lies on the same road, due south of A; the height of the hill above Y is h metres. The angle of elevation to X is 16° from A and 12° from B.

(a) Show that $BY = h\cot 12^\circ$.

(b) Hence, find the value of h, to the nearest metre.

Criteria	**Marks**
(a) Uses $\tan 12^\circ = \dfrac{h}{BY}$ to obtain $BY = h\cot 12^\circ$	1
(b) States $AY = h\cot 16^\circ$ and $BY - AY = 1600$	1
(b) Correct equation $h(\cot 12^\circ - \cot 16^\circ) = 1600$	1
(b) Correct value $h \approx 1314$ m	1

Solution.

(a) In the right-angled triangle BXY (right angle at Y), the angle of elevation at B is 12° and the vertical side is $XY = h$. Hence

$$\tan 12^\circ = \frac{h}{BY} \quad \Rightarrow \quad BY = \frac{h}{\tan 12^\circ} = h\cot 12^\circ.$$

(b) Similarly, from A the angle of elevation is 16°, so

$$AY = \frac{h}{\tan 16^\circ} = h\cot 16^\circ.$$

Since A lies between Y and B on the road, $BY = AY + AB$, so

$$h\cot 12^\circ = h\cot 16^\circ + 1600 \quad \Rightarrow \quad h\,(\cot 12^\circ - \cot 16^\circ) = 1600.$$

$$h = \frac{1600}{\cot 12^\circ - \cot 16^\circ} = \frac{1600}{4.7046\ldots - 3.4874\ldots} = 1314.47\ldots \approx 1314 \text{ m}.$$

Q10 Loreto Kirribilli 2022 Prelim Q34 · **3 marks**

A radio tower with top T is 85 metres tall on level ground. From Angela's point A the angle of elevation to T is $15.2°$; from Beth's point B it is $21.4°$. The angle $\angle ATB = 104.8°$. Find the area of triangle ATB, to the nearest square metre.

Criteria	**Marks**
Finds $TA = \dfrac{85}{\sin 15.2°} \approx 324.19$ m	1
Finds $TB = \dfrac{85}{\sin 21.4°} \approx 232.96$ m	1
Applies area rule to get ≈ 36509 m^2	1

Solution.

The tower is vertical, so for each observer the line to the top T, the vertical tower and the horizontal ground form a right-angled triangle in which 85 m is the side opposite the angle of elevation. The slant distances to T are therefore

$$TA = \frac{85}{\sin 15.2°} = 324.19\ldots \text{ m}, \qquad TB = \frac{85}{\sin 21.4°} = 232.96\ldots \text{ m}.$$

In triangle ATB the sides TA and TB include the angle $\angle ATB = 104.8°$, so by the area rule

$$\text{Area} = \tfrac{1}{2}(TA)(TB)\sin\angle ATB = \tfrac{1}{2}(324.19\ldots)(232.96\ldots)\sin 104.8° = 36508.5\ldots$$

$$\therefore\ \text{Area} \approx 36509 \text{ m}^2.$$

Q11 Killara 2022 Prelim Q28 · **4 marks**

A boat at B spots a 140 m vertical cliff (base C) on a true bearing of 312°, with angle of elevation to the top of 29°.

(a) Show that $d = \dfrac{140}{\tan 29^\circ}$, where d is the distance from the boat to the base.

(b) The boat travels due north to D, directly east of the base ($\angle BDC = 90^\circ$). Find the new angle of elevation, to the nearest degree.

Criteria	**Marks**
(a) Uses $\tan 29^\circ = \dfrac{140}{d}$ to obtain $d = \dfrac{140}{\tan 29^\circ}$	1
(b) Finds $d = BC \approx 252.57$ m	1
(b) Finds the new horizontal distance $DC = d \sin 48^\circ \approx 187.69$ m	1
(b) Correct new angle of elevation $\approx 37^\circ$	1

Solution.

(a) The cliff is vertical, so the boat B, the base C and the top form a right-angled triangle with the 140 m tower opposite the 29° elevation and horizontal distance $d = BC$:

$$\tan 29^\circ = \frac{140}{d} \quad \Rightarrow \quad d = \frac{140}{\tan 29^\circ} = 252.57\ldots \text{ m}.$$

(b) The base C is on a bearing of 312° from B, that is, 48° west of north (since $360^\circ - 312^\circ = 48^\circ$). Take C as the origin with east and north axes. Then C is 48° west of north of B, so relative to C the boat B lies 48° east of south, giving

$$B = (d \sin 48^\circ,\ -d \cos 48^\circ).$$

Travelling due north keeps the east-coordinate fixed, and D is directly east of C (north-coordinate 0), so

$$DC = d \sin 48^\circ = 252.57\ldots \times \sin 48^\circ = 187.69\ldots \text{ m}.$$

The new angle of elevation α satisfies $\tan \alpha = \dfrac{140}{DC}$:

$$\alpha = \tan^{-1} \frac{140}{187.69\ldots} = 36.72\ldots^\circ \approx 37^\circ.$$

Q12 Girraween 2023 Prelim Q12(b) · **4 marks**

The angle of elevation of the top of a tree of height h metres is 20° from a point X due south of its foot Z, and 28° from a point Y due east of Z. The distance XY is 500 metres.

(i) Write expressions for XZ and YZ in terms of h.

(ii) Calculate h, correct to one decimal place.

Criteria	**Marks**
(i) $XZ = h\cot 20^\circ$ and $YZ = h\cot 28^\circ$	1
(i) States $XY^2 = XZ^2 + YZ^2$ (right angle at Z)	1
(ii) Correct equation in h	1
(ii) Correct value $h \approx 150.2$ m	1

Solution.

(i) XZ and YZ are the horizontal distances from X and Y to the foot Z. Using the elevation angles in the two vertical right-angled triangles:

$$\tan 20^\circ = \frac{h}{XZ} \Rightarrow XZ = \frac{h}{\tan 20^\circ} = h\cot 20^\circ, \qquad \tan 28^\circ = \frac{h}{YZ} \Rightarrow YZ = h\cot 28^\circ.$$

(ii) X is due south of Z and Y is due east of Z, so $\angle XZY = 90^\circ$. By Pythagoras in triangle XZY,

$$XY^2 = XZ^2 + YZ^2 \quad \Rightarrow \quad 500^2 = h^2\cot^2 20^\circ + h^2\cot^2 28^\circ.$$

$$h^2 = \frac{250000}{\cot^2 20^\circ + \cot^2 28^\circ} = \frac{250000}{2.7475^2 + 1.8807^2}, \qquad h = 150.17\ldots$$

$$\therefore\ h \approx 150.2 \text{ m}.$$

Q13 Knox 2024 Trial Q23 · **5 marks**

Let B be the base and T the top of the Sydney Harbour Bridge, with T 50 m above B. From Darline's position D, B is on a bearing of 078° with elevation 35° to T. From Saher's position S, B is on a bearing of 140° with elevation 48° to T. Points D, S, B are on the same horizontal plane.

(a) Show that $\angle SBD = 62°$.

(b) Find the distance between Saher and Darline, to two decimal places.

Criteria	**Marks**
(a) Finds the bearings of D and S from B (reverse bearings 258° and 320°)	1
(a) Concludes $\angle SBD = 320° - 258° = 62°$	1
(b) Finds $DB = \dfrac{50}{\tan 35°} \approx 71.41$ m and $SB = \dfrac{50}{\tan 48°} \approx 45.02$ m	1
(b) Applies the cosine rule with $\angle SBD = 62°$	1
(b) Correct distance $SD \approx 64.09$ m	1

Solution.

(a) The bearing of B from D is 078°, so the reverse bearing of D from B is $078° + 180° = 258°$. The bearing of B from S is 140°, so the bearing of S from B is $140° + 180° = 320°$. The angle at B between BS and BD is therefore

$$\angle SBD = 320° - 258° = 62°.$$

(b) The bridge is vertical of height 50 m, so the horizontal distances from each observer to the base B are

$$DB = \frac{50}{\tan 35°} = 71.407\ldots \text{ m}, \qquad SB = \frac{50}{\tan 48°} = 45.020\ldots \text{ m}.$$

Applying the cosine rule in triangle SBD with included angle $\angle SBD = 62°$:

$$SD^2 = DB^2 + SB^2 - 2\,(DB)(SB)\cos 62° = 71.407^2 + 45.020^2 - 2(71.407)(45.020)\cos 62° = 4107.3\ldots$$

$$SD = \sqrt{4107.3\ldots} = 64.088\ldots \approx 64.09 \text{ m}.$$

Q14 St George Girls 2023 Prelim Q21 · **6 marks**

Anthony is at A, 230 m due east of the base O of a tower OT of height h m; the angle of elevation of T from A is $15°$. Brian is at B, due south of the tower; the angle of elevation of T from B is $10°$.

(a) Show that $OA = h\tan 75°$.

(b) By finding a similar expression for OB, find the height of the tower.

(c) What is the bearing of Brian from Anthony, to the nearest degree?

Criteria	**Marks**
(a) Uses $\tan 15° = \dfrac{h}{OA}$ and $\cot 15° = \tan 75°$ to show $OA = h\tan 75°$	1
(a) Correct completed reasoning	1
(b) States $OB = h\tan 80°$	1
(b) Uses $OA = 230$ to find $h \approx 61.6$ m	1
(c) Sets up the angle at A using OA and OB (right angle at O)	1
(c) Correct bearing $\approx 213°$	1

Solution.

(a) OA is the horizontal distance from A to the base O. In the right-angled triangle OAT (right angle at O),

$$\tan 15° = \frac{OT}{OA} = \frac{h}{OA} \quad \Rightarrow \quad OA = \frac{h}{\tan 15°} = h\cot 15° = h\tan 75°,$$

since $\cot 15° = \tan(90° - 15°) = \tan 75°$.

(b) By the same reasoning at B (elevation $10°$),

$$OB = \frac{h}{\tan 10°} = h\cot 10° = h\tan 80°.$$

Anthony is 230 m due east of O, so $OA = 230$. From part (a),

$$230 = h\tan 75° \quad \Rightarrow \quad h = \frac{230}{\tan 75°} = 230\tan 15° = 61.6\ldots \text{ m}.$$

Exactly, $h = 460 - 230\sqrt{3} \approx 61.6$ m. (The stated XY-type separation of 300 m in the original is not needed and is not consistent with these figures.)

(c) A is due east of O and B is due south of O, so $\angle AOB = 90°$. With $OA = 230$ m and

$$OB = h\tan 80° = 61.6\ldots \times \tan 80° = 349.5\ldots \text{ m},$$

Brian lies to the south-west of Anthony. Measuring from due south at A towards the west, the offset angle is

$$\tan^{-1}\frac{OA}{OB} = \tan^{-1}\frac{230}{349.5\ldots} = 33.35\ldots{}^\circ.$$

Hence the bearing of Brian from Anthony is

$$180^\circ + 33.35\ldots^\circ = 213.3\ldots^\circ \approx 213^\circ.$$

3. Bearings

Q15 Shore 2023 Trial Q4 · **1 mark**

In the diagram, a compass cross is drawn at the point A, with north pointing up. The point B lies below and to the right of A, so that the line AB makes an angle of 37° with the southward direction at A. What is the true bearing of A from B? (multiple choice)

Criteria	Marks
Correct option (D)	1

Solution.

At A, the line AB makes 37° with the south direction, on the eastern side. Measuring the bearing of B from A clockwise from north:

$$\text{bearing of } B \text{ from } A = 180^\circ - 37^\circ = 143^\circ.$$

The bearing of A from B is the reverse bearing:

$$143^\circ + 180^\circ = 323^\circ.$$

The answer is **(D)**.

Q16 Girraween 2022 Trial Q3 · **1 mark**

A plane flies on a bearing of 150° from A to B. What is the bearing of A from B? (multiple choice)

Criteria	Marks
Correct option (D)	1

Solution.

The bearing of A from B is the reverse of the bearing of B from A. Since the bearing is less than 180°, add 180°:

$$150^\circ + 180^\circ = 330^\circ.$$

The answer is **(D)**.

Q17 Cherrybrook Tech 2020 Trial Q13 · **3 marks**

Three towns A, B and C form a triangle. Town A is 80 km from Town B, and Town C is 40 km from Town A. The bearing of Town B from Town A is 130°, and the bearing of Town C from Town A is 240°. Find the distance between Town B and Town C, to the nearest kilometre.

Criteria	**Marks**
Finds the angle $\angle BAC = 110°$ from the two bearings	1
Correct substitution into the cosine rule	1
Correct value $BC \approx 101$ km	1

Solution.

The angle at A between AB and AC is the difference of the two bearings:

$$\angle BAC = 240° - 130° = 110°.$$

By the cosine rule, with $AB = 80$ and $AC = 40$,

$$BC^2 = 80^2 + 40^2 - 2(80)(40)\cos 110° = 10188.9\ldots,$$

$$BC = \sqrt{10188.9\ldots} = 100.94\ldots \approx 101 \text{ km}.$$

Q18 Pymble 2022 Prelim Q17 · **3 marks**

The plane flies from a town T to a property P, which is 83 km away on a bearing of 129°. It then flies to a property Q, which is 91 km from P and which is due east of T. **(a)** Mark the diagram with all the given information. **(b)** Find the bearing of Q from P, correct to the nearest minute.

Criteria	Marks
(a) Marks the diagram: $TP = 83$, bearing 129°, $PQ = 91$, Q due east of T	1
(b) Finds $\angle PTQ = 39°$ and applies the sine rule to find $\angle TQP$	1
(b) Correct bearing 054°58′	1

Solution.

(a) On the diagram, mark $TP = 83$ km at bearing 129° from T, mark $PQ = 91$ km, and mark Q due east of T.

(b) Since Q is due east of T, the bearing of Q from T is 090°. The angle at T inside $\triangle TPQ$ is

$$\angle PTQ = 129° - 90° = 39°.$$

Side $TP = 83$ is opposite $\angle TQP$, and side $PQ = 91$ is opposite $\angle PTQ = 39°$. By the sine rule,

$$\frac{\sin \angle TQP}{83} = \frac{\sin 39°}{91} \quad \Rightarrow \quad \sin \angle TQP = \frac{83 \sin 39°}{91} = 0.5741\ldots,$$

$$\angle TQP = 35.03°\ldots = 35°02'.$$

At Q, the direction $Q \to T$ is due west (270°), so the direction $Q \to P$ is 35°02′ south of west:

$$\text{bearing of } P \text{ from } Q = 270° - 35°02' = 234°58'.$$

Hence the bearing of Q from P is the reverse:

$$234°58' - 180° = 054°58'.$$

Q19 Moriah 2021 Prelim Q25 · **4 marks**

Four towns Q, R, S, T: Q is due east of R with $RQ = 75$ km, T is due south of S with $ST = 125$ km, and $QT = 65$ km. The bearing of S from R is 058°, and the bearing of Q from T is 328°. **(a)** Calculate SQ to the nearest km. **(b)** Find $\angle RSQ$ to the nearest degree, and hence the bearing of Q from S.

Criteria	Marks
(a) Finds $\angle STQ = 32°$ and applies the cosine rule in $\triangle SQT$	1
(a) Correct value $SQ \approx 78$ km	1
(b) Finds $\angle RSQ \approx 31°$ using the sine rule	1
(b) Correct bearing of Q from S ($\approx 206°$)	1

Solution.

(a) In $\triangle SQT$, the direction $T \to S$ is due north (000°) and the direction $T \to Q$ is on bearing 328°, so

$$\angle STQ = 360° - 328° = 32°.$$

With $ST = 125$ and $QT = 65$, the cosine rule gives

$$SQ^2 = 125^2 + 65^2 - 2(125)(65)\cos 32° = 6069.1\ldots,$$

$$SQ = 77.9\ldots \approx 78 \text{ km}.$$

(b) Since Q is due east of R, the direction $R \to Q$ is due east (090°), and S is on bearing 058° from R, so

$$\angle SRQ = 90° - 58° = 32°.$$

In $\triangle RSQ$, side $RQ = 75$ is opposite $\angle RSQ$ and side $SQ = 77.9$ is opposite $\angle SRQ = 32°$. By the sine rule,

$$\frac{\sin \angle RSQ}{75} = \frac{\sin 32°}{77.9}, \qquad \sin \angle RSQ = \frac{75 \sin 32°}{77.9} = 0.5102\ldots,$$

$$\angle RSQ = 30.7°\ldots \approx 31°.$$

For the bearing of Q from S: in $\triangle SQT$ the angle at S is $\angle QST = 180° - 32° - \angle SQT$. Since $\dfrac{\sin \angle QST}{65} = \dfrac{\sin 32°}{77.9}$, we get $\angle QST = 26.2°\ldots$. As $S \to T$ points due south, the direction $S \to Q$ is 26.2° west of south, so

$$\text{bearing of } Q \text{ from } S = 180° + 26.2°\ldots \approx 206°.$$

Q20 Barker 2020 Trial Q11 · 5 marks

A balloon drifts 100 km from A to B on a bearing of 028°, then changes direction and drifts 50 km to C on a bearing of 114°. **(i)** Draw a diagram. **(ii)** Show that $AC = 115$ km to the nearest km. **(iii)** Find the bearing of A from C to the nearest degree.

Criteria	Marks
(i) Correct diagram with both legs, bearings and distances marked	1
(ii) Finds $\angle ABC = 94°$ and substitutes into the cosine rule	1
(ii) Reaches $AC \approx 115$ km	1
(iii) Finds the angle at C using the sine rule	1
(iii) Correct bearing 234°	1

Solution.

(i) From A, draw $AB = 100$ km on bearing 028°; from B, draw $BC = 50$ km on bearing 114°.

(ii) At B, the direction $B \to A$ is the reverse of 028°, i.e. 208°, and $B \to C$ is 114°, so

$$\angle ABC = 208° - 114° = 94°.$$

By the cosine rule,

$$AC^2 = 100^2 + 50^2 - 2(100)(50)\cos 94° = 13197.5\ldots,$$

$$AC = 114.88\ldots \approx 115 \text{ km}.$$

(iii) Using the sine rule to find $\angle ACB$ (opposite $AB = 100$):

$$\frac{\sin \angle ACB}{100} = \frac{\sin 94°}{114.88}, \qquad \angle ACB = 60.3°\ldots.$$

The direction $C \to B$ is the reverse of 114°, i.e. 294°. Point A lies clockwise from CB by $\angle ACB$, so

$$\text{bearing of } A \text{ from } C = 294° - 60.3°\ldots = 233.7°\ldots \approx 234°.$$

Q21 Fort St 2021 Prelim Q3 · **6 marks**

A hiker walks 800 m from A on a bearing of 053°T to B, then 1.35 km on a bearing of 165°T to C. **(i)** Draw a diagram. **(ii)** Find AC to the nearest metre. **(iii)** Find the bearing to walk back to A from C.

Criteria	Marks
(i) Correct diagram with both legs, bearings and distances marked	1
(ii) Finds $\angle ABC = 68°$ and substitutes into the cosine rule	1
(ii) Reaches $AC \approx 1286$ m	1
(iii) Uses the sine rule to find the angle at C	1
(iii) Interprets the angle as a true bearing	1
(iii) Correct bearing 309°46′	1

Solution.

(i) From A, draw $AB = 800$ m on bearing 053°; from B, draw $BC = 1350$ m on bearing 165°.

(ii) At B, the direction $B \to A$ is the reverse of 053°, i.e. 233°, and $B \to C$ is 165°, so

$$\angle ABC = 233° - 165° = 68°.$$

By the cosine rule, with $AB = 800$ and $BC = 1350$,

$$AC^2 = 800^2 + 1350^2 - 2(800)(1350)\cos 68° = 1653349\ldots,$$

$$AC = 1285.83\ldots \approx 1286 \text{ m}.$$

(iii) Using the sine rule to find $\angle ACB$ (opposite $AB = 800$):

$$\frac{\sin \angle ACB}{800} = \frac{\sin 68°}{1285.83}, \qquad \sin \angle ACB = \frac{800 \sin 68°}{1285.83} = 0.5768\ldots, \qquad \angle ACB = 35.23°\ldots.$$

The direction $C \to B$ is the reverse of 165°, i.e. 345°. Point A lies anticlockwise from CB by $\angle ACB$, so

$$\text{bearing of } A \text{ from } C = 345° - 35.23°\ldots = 309.77°\ldots = 309°46'.$$

(The printed trial answer of 309°47′ comes from rounding the intermediate angle to 35.22°; the exact value rounds to 309°46′.)

Q22 Barker 2024 Trial Q24 · 3 marks

An electricity tower BT is 50 m high (base B, top T). From S, due south of B, the elevation of T is $48°$. From D, on a bearing of $085°$ from B, the elevation of T is $35°$. **(i)** Find SB. **(ii)** Find SD to the nearest metre.

Criteria	Marks
(i) Correct value $SB \approx 45$ m	1
(ii) Finds $\angle SBD = 95°$ and DB, then applies the cosine rule	1
(ii) Correct value $SD \approx 88$ m	1

Solution.

(i) SB is the horizontal distance from S to the base B. In right triangle SBT,

$$\tan 48° = \frac{50}{SB} \quad \Rightarrow \quad SB = \frac{50}{\tan 48°} = 45.02\ldots \approx 45 \text{ m}.$$

(ii) Similarly the horizontal distance $DB = \dfrac{50}{\tan 35°} = 71.41\ldots$ m. Since S is due south of B (direction $B \to S$ is $180°$) and D is on bearing $085°$ from B, the angle between BS and BD is

$$\angle SBD = 180° - 85° = 95°.$$

By the cosine rule in $\triangle SBD$,

$$SD^2 = SB^2 + DB^2 - 2\,(SB)(DB)\cos 95° = 7686.2\ldots,$$

$$SD = 87.67\ldots \approx 88 \text{ m}.$$

Q23 Penrith 2024 Trial Q18 · **4 marks**

Henry walks 3 km from his car C on a bearing of 060°T to a lookout L, then 2 km on a bearing of 110°T to a rockpool R. Find the shortest distance back to the car, and the bearing he would walk from the rockpool (to the nearest degree).

Criteria	**Marks**
Finds $\angle CLR = 130°$ from the two bearings	1
Applies the cosine rule for CR	1
Correct distance $CR \approx 4.55$ km	1
Correct return bearing $\approx 260°$	1

Solution.

At L, the direction $L \to C$ is the reverse of 060°, i.e. 240°, and $L \to R$ is 110°, so

$$\angle CLR = 240° - 110° = 130°.$$

By the cosine rule, with $CL = 3$ and $LR = 2$,

$$CR^2 = 3^2 + 2^2 - 2(3)(2)\cos 130° = 20.71\ldots,$$

$$CR = 4.551\ldots \approx 4.55 \text{ km}.$$

For the return bearing (of C from R): using the sine rule to find $\angle LRC$,

$$\frac{\sin \angle LRC}{3} = \frac{\sin 130°}{4.551}, \qquad \angle LRC = 30.3°\ldots.$$

The direction $R \to L$ is the reverse of 110°, i.e. 290°; C lies 30.3° anticlockwise from RL, so

$$\text{bearing of } C \text{ from } R = 290° - 30.3°\ldots = 259.7°\ldots \approx 260°.$$

Q24 2020 HSC Q15 · **5 marks**

From site P: Mr Ali walks on bearing 035° for 7 km to A; Ms Brown walks on bearing 100° for 9 km to B. **(a)** Show $\angle APB = 65°$. **(b)** Find AB. **(c)** Find the bearing of Ms Brown's group (B) from Mr Ali's group (A), to the nearest degree.

Criteria	Marks
(a) Shows $\angle APB = 100° - 35° = 65°$	1
(b) Correct substitution into the cosine rule	1
(b) Correct value $AB \approx 8.76$ km	1
(c) Uses the sine rule to find the angle at A	1
(c) Correct bearing 146°	1

Solution.

(a) The two paths leave P on bearings 035° and 100°, so the angle between them is

$$\angle APB = 100° - 35° = 65°.$$

(b) By the cosine rule, with $PA = 7$ and $PB = 9$,

$$AB^2 = 7^2 + 9^2 - 2(7)(9)\cos 65° = 76.75\ldots,$$

$$AB = 8.76\ldots \approx 8.8 \text{ km}.$$

(c) Using the sine rule to find $\angle PAB$ (opposite $PB = 9$):

$$\frac{\sin \angle PAB}{9} = \frac{\sin 65°}{8.76}, \qquad \angle PAB = 68.6°\ldots.$$

At A, the direction $A \to P$ is the reverse of 035°, i.e. 215°. B lies clockwise from AP by $\angle PAB$, so

$$\text{bearing of } B \text{ from } A = 215° - 68.6°\ldots = 146.4°\ldots \approx 146°.$$

Q25 Normanhurst Boys 2025 Trial Q24 · **6 marks**

A vertical pole OT (base O, top T) is secured by ropes to pegs A and B with $AB = 10$ m. A is due west of O, and B is on a bearing of 240° from O. The rope $TA = 3\sqrt{3}$ m makes 60° with the ground. **(a)** Show $OB = \dfrac{9 + 11\sqrt{13}}{4}$ m. **(b)** Find the rope length $\ell = TB$ to the nearest metre.

Criteria	**Marks**
(a) Finds $AO = \frac{3\sqrt{3}}{2}$ and $\angle AOB = 30°$	1
(a) Applies the cosine rule in $\triangle AOB$	1
(a) Forms the quadratic $4(OB)^2 - 18(OB) - 373 = 0$	1
(a) Solves to $OB = \frac{9+11\sqrt{13}}{4}$	1
(b) Finds $OT = \frac{9}{2}$	1
(b) Correct value $\ell \approx 13$ m	1

Solution.

(a) O is the foot of the vertical pole, so $\triangle AOT$ is right-angled at O, with hypotenuse $TA = 3\sqrt{3}$ and $\angle TAO = 60°$. The horizontal leg is

$$AO = 3\sqrt{3}\cos 60° = \frac{3\sqrt{3}}{2}.$$

A is due west of O, so the bearing of A from O is 270°; B is on bearing 240° from O, so

$$\angle AOB = 270° - 240° = 30°.$$

Applying the cosine rule in $\triangle AOB$ with $AB = 10$:

$$10^2 = \left(\frac{3\sqrt{3}}{2}\right)^2 + OB^2 - 2\left(\frac{3\sqrt{3}}{2}\right)(OB)\cos 30°,$$

$$100 = \frac{27}{4} + OB^2 - \frac{9}{2}\,OB.$$

Multiplying by 4 and rearranging,

$$4(OB)^2 - 18\,(OB) - 373 = 0.$$

By the quadratic formula,

$$OB = \frac{18 \pm \sqrt{18^2 + 4(4)(373)}}{2(4)} = \frac{18 \pm \sqrt{6292}}{8} = \frac{18 \pm 22\sqrt{13}}{8} = \frac{9 \pm 11\sqrt{13}}{4}.$$

Since $OB > 0$,

$$OB = \frac{9 + 11\sqrt{13}}{4} \text{ metres.}$$

(b) From $\triangle AOT$, the vertical leg (the pole) is

$$OT = 3\sqrt{3}\sin 60^\circ = \frac{9}{2}.$$

Since OT is vertical and OB is horizontal, $\triangle TOB$ is right-angled at O, so

$$\ell^2 = TB^2 = OB^2 + OT^2 = \left(\frac{9 + 11\sqrt{13}}{4}\right)^2 + \left(\frac{9}{2}\right)^2,$$

$$\ell = \sqrt{147.99\ldots + 20.25} = 12.97\ldots \approx 13 \text{ m}.$$