Year 11 Mathematics Advanced

Topic: Trigonometry and Measure of Angles (4 of 5)

Review Questions (Worked Solutions): Radians: arc length, sectors and segments

Term 3, 2026

Radian measure and circle mensuration

1. Radian measure and converting degrees and radians

Q1 North Sydney Boys 2022 Trial Q11 · **1 mark**

Write 132° in radian measure.

Criteria	Marks
Correct radian measure $\dfrac{11\pi}{15}$	1

Solution.

Multiply by $\dfrac{\pi}{180}$ to convert from degrees to radians:

$$132^\circ = 132 \times \frac{\pi}{180} = \frac{132\pi}{180} = \frac{11\pi}{15}.$$

Q2 Kambala 2024 Prelim Q2 · **1 mark**

Which of the following angles is between 30° and 60° in size? (multiple choice)

Criteria	Marks
Correct option (C)	1

Solution.

Convert each option to degrees by multiplying by $\dfrac{180}{\pi}$:

$$0.3 \text{ rad} \approx 17.2^\circ, \quad 0.5 \text{ rad} \approx 28.6^\circ, \quad 0.8 \text{ rad} \approx 45.8^\circ, \quad 1.1 \text{ rad} \approx 63.0^\circ.$$

Only 0.8 radians lies between 30° and 60°. The answer is **(C)**.

Q3 St George Girls 2023 Prelim Q3 · 1 mark

What is 0.5 radians in degrees, when rounded to the nearest degree? (multiple choice)

Criteria	Marks
Correct option (A)	1

Solution.

Multiply by $\frac{180}{\pi}$ to convert from radians to degrees:

$$0.5 \times \frac{180}{\pi} = \frac{90}{\pi} = 28.6478\ldots^\circ \approx 29^\circ \text{ (nearest degree)}.$$

The answer is **(A)**.

Q4 Killara 2024 Prelim Q1 · 1 mark

135° converted to radians is: (multiple choice)

Criteria	Marks
Correct option (B)	1

Solution.

Multiply by $\frac{\pi}{180}$:

$$135^\circ = 135 \times \frac{\pi}{180} = \frac{135\pi}{180} = \frac{3\pi}{4}.$$

The answer is **(B)**.

Q5 Hurlstone 2021 Prelim Q1 · 1 mark

In radians, which of the following is equal to 150°? (multiple choice)

Criteria	Marks
Correct option (C)	1

Solution.

Multiply by $\frac{\pi}{180}$:

$$150^\circ = 150 \times \frac{\pi}{180} = \frac{150\pi}{180} = \frac{5\pi}{6}.$$

The answer is **(C)**.

Q6 Penrith 2023 Prelim Q15 · **1 mark**

Express $520°$ in radians. Simplify fully in terms of π.

Criteria	**Marks**
Correct simplified radian measure $\dfrac{26\pi}{9}$	1

Solution.

Multiply by $\dfrac{\pi}{180}$ and simplify (dividing numerator and denominator by 20):

$$520° = 520 \times \frac{\pi}{180} = \frac{520\pi}{180} = \frac{26\pi}{9}.$$

Q7 Danebank 2021 Trial Q24 · **4 marks**

A circle has centre O and radius 5 cm. Tangents from A and B meet at point C, with $CA = CB = 12$ cm. **(a)** Show that $\angle AOB = 2.4$ radians correct to two significant figures. **(b)** Hence, find the area of the shaded region.

Criteria	Marks
(a) Uses the tangent–radius right angle at A and finds $\angle AOC = \tan^{-1}\frac{12}{5}$	1
(a) Doubles to obtain $\angle AOB = 2.35\ldots \approx 2.4$ radians	1
(b) Computes the quadrilateral area $OACB = 60$ cm^2 and the sector area $\frac{1}{2}r^2\theta$	1
(b) Correct shaded area ≈ 30 cm^2	1

Solution.

(a) A tangent is perpendicular to the radius at the point of contact, so $\angle OAC = 90°$. In the right-angled triangle OAC, $OA = 5$ (radius) and $AC = 12$ (tangent), so

$$\tan(\angle AOC) = \frac{AC}{OA} = \frac{12}{5} \qquad \Rightarrow \qquad \angle AOC = \tan^{-1}\frac{12}{5} = 1.1760\ldots \text{ rad.}$$

By symmetry ($\triangle OAC \equiv \triangle OBC$), $\angle AOB = 2 \times \angle AOC$:

$$\angle AOB = 2\tan^{-1}\frac{12}{5} = 2.3520\ldots \text{ rad} \approx 2.4 \text{ radians (2 s.f.).}$$

(b) The shaded region is the quadrilateral $OACB$ with the sector OAB removed. The quadrilateral is made of two congruent right triangles:

$$\text{area } OACB = 2 \times \frac{1}{2} \times OA \times AC = 2 \times \frac{1}{2} \times 5 \times 12 = 60 \text{ cm}^2.$$

Using $\angle AOB = 2.4$ radians, the sector area is

$$\text{area sector} = \frac{1}{2}r^2\theta = \frac{1}{2} \times 5^2 \times 2.4 = 30 \text{ cm}^2.$$

Therefore the shaded area is

$$60 - 30 = 30 \text{ cm}^2.$$

2. Arc length

Q8 St George Girls 2022 Trial Q9 · **1 mark**

A sector, centred at O, has a radius of 6 cm and an arc length of 5 cm. What is the area of the sector? (multiple choice)

Criteria	Marks
Correct option (B)	1

Solution.

The area of a sector equals half the product of the radius and the arc length:

$$A = \tfrac{1}{2}rl = \tfrac{1}{2} \times 6 \times 5 = 15 \text{ cm}^2.$$

The answer is **(B)**.

Q9 Kambala 2023 Prelim Q5 · **1 mark**

In a circle with centre O, the arc AB subtends an angle of 150° at the centre and the radius is 12 cm. What is the length of arc AB? (multiple choice)

Criteria	Marks
Correct option (A)	1

Solution.

Convert the angle to radians:

$$\theta = 150^\circ \times \frac{\pi}{180^\circ} = \frac{5\pi}{6}.$$

Then the arc length is

$$l = r\theta = 12 \times \frac{5\pi}{6} = 10\pi \text{ cm}.$$

The answer is **(A)**.

Q10 Girraween 2025 Trial Q6 · **1 mark**

AOB is a sector of a circle with centre O and radius 5 cm, and area 10π cm^2. What is the arc length of the sector in exact form? (multiple choice)

Criteria	Marks
Correct option (B)	1

Solution.

From the sector area $A = \frac{1}{2}r^2\theta$,

$$10\pi = \tfrac{1}{2}(5)^2\theta \quad \Rightarrow \quad \theta = \frac{20\pi}{25} = \frac{4\pi}{5}.$$

Then the arc length is

$$l = r\theta = 5 \times \frac{4\pi}{5} = 4\pi \text{ cm}.$$

The answer is **(B)**.

Q11 Cherrybrook Tech 2021 Prelim Q13 · **2 marks**

A sector of a circle has centre A, with $\angle BAC = \dfrac{\pi}{5}$ radians. The arc BC has a length of 8π units. Find the area of the sector, giving your answer in terms of π.

Criteria	Marks
Uses $l = r\theta$ to find the radius $r = 40$	1
Correct area 160π units^2	1

Solution.

Using $l = r\theta$ to find the radius,

$$8\pi = r \times \frac{\pi}{5} \quad \Rightarrow \quad r = 8\pi \times \frac{5}{\pi} = 40.$$

The area of the sector is

$$A = \tfrac{1}{2}r^2\theta = \tfrac{1}{2}(40)^2 \times \frac{\pi}{5} = \tfrac{1}{2} \times 1600 \times \frac{\pi}{5} = 160\pi \text{ units}^2.$$

Q12 Girraween 2020 Trial Q18 · **4 marks**

A sector has radius r and angle θ, where $0 < \theta \leq 2\pi$, and arc length $\dfrac{10\pi}{3}$. **(a)** Find θ in terms of r, and hence show that $r \geq \dfrac{5}{3}$. **(b)** Calculate the area of the sector when $r = 4$.

Criteria	**Marks**
(a) Writes $\theta = \dfrac{10\pi}{3r}$ from $l = r\theta$	1
(a) Applies $\theta \leq 2\pi$ to establish $r \geq \dfrac{5}{3}$	1
(b) Finds $\theta = \dfrac{5\pi}{6}$ when $r = 4$	1
(b) Correct area $\dfrac{20\pi}{3}$	1

Solution.

(a) Since $l = r\theta$ and $l = \dfrac{10\pi}{3}$,

$$\theta = \frac{l}{r} = \frac{10\pi}{3r}.$$

The angle satisfies $0 < \theta \leq 2\pi$, so

$$\frac{10\pi}{3r} \leq 2\pi \quad \Rightarrow \quad 10\pi \leq 6\pi r \quad \Rightarrow \quad r \geq \frac{10\pi}{6\pi} = \frac{5}{3}.$$

(b) When $r = 4$,

$$\theta = \frac{10\pi}{3 \times 4} = \frac{10\pi}{12} = \frac{5\pi}{6}.$$

The area is

$$A = \tfrac{1}{2}r^2\theta = \tfrac{1}{2}(4)^2 \times \frac{5\pi}{6} = 8 \times \frac{5\pi}{6} = \frac{20\pi}{3} \text{ units}^2.$$

Q13 Knox 2021 Prelim Q11 · **2 marks**

The area of a sector of a circle is 4π units^2 and the arc length of this sector is $\dfrac{8\pi}{3}$ units. Find the radius of the circle and the angle that is subtended at the centre.

Criteria	**Marks**
Uses $A = \frac{1}{2}rl$ to find the radius $r = 3$	1
Correct angle $\theta = \dfrac{8\pi}{9}$	1

Solution.

The area of a sector can be written as $A = \frac{1}{2}rl$. With $A = 4\pi$ and $l = \dfrac{8\pi}{3}$,

$$4\pi = \tfrac{1}{2}\, r \times \frac{8\pi}{3} = \frac{4\pi r}{3} \quad \Rightarrow \quad r = 3.$$

The subtended angle then comes from $l = r\theta$:

$$\frac{8\pi}{3} = 3\theta \quad \Rightarrow \quad \theta = \frac{8\pi}{9}.$$

So the radius is 3 units and the angle is $\dfrac{8\pi}{9}$.

Q14 James Ruse 2025 Trial Q25 · **3 marks**

A car travels at 40 km/h on a circular curve whose radius is 500 m. **(a)** Find the distance, l km, that the car travels in one minute. **(b)** Calculate the size of the angle θ through which the car turns in 2 minutes, to the nearest degree.

Criteria	**Marks**
(a) Correct distance $l = \frac{2}{3}$ km	1
(b) Uses $l = r\theta$ with a consistent radius to find θ in radians	1
(b) Correct angle $\theta \approx 153°$	1

Solution.

(a) In one minute the car travels

$$l = 40 \times \frac{1}{60} = \frac{2}{3} \text{ km}.$$

(b) In 2 minutes the distance travelled (the arc length) is $\frac{4}{3}$ km. Working in kilometres, the radius is 500 m = 0.5 km. Using $l = r\theta$,

$$\theta = \frac{l}{r} = \frac{4/3}{0.5} = \frac{8}{3} \text{ radians}.$$

Converting to degrees,

$$\theta = \frac{8}{3} \times \frac{180°}{\pi} = 152.79° \ldots \approx 153° \text{ (nearest degree)}.$$

Q15 2023 HSC Q16 · **4 marks**

The shape $APQBCD$ consists of a rectangle $ABCD$ with $BC = 3.6$ m and $CD = 8.0$ m, and an arc PQ on side AB. The arc PQ is part of a circle with centre O and radius 2.1 m, with $\angle POQ = 110°$. Find the perimeter of the shape $APQBCD$, correct to one decimal place.

Criteria	**Marks**
Calculates the arc length PQ	1
Calculates the chord length PQ (cosine rule)	1
Adds the correct portions to form the perimeter	1
Correct value 23.8 m	1

Solution.

The perimeter runs $A \to P$ along AB, then along the arc PQ, then $Q \to B$ along AB, then $B \to C \to D \to A$ around the rectangle.

Arc PQ. With radius 2.1 m and central angle 110°,

$$\text{arc } PQ = \frac{110}{360} \times 2\pi \times 2.1 = 4.0317\ldots \text{ m}.$$

Chord PQ. By the cosine rule in $\triangle OPQ$ (with $OP = OQ = 2.1$),

$$PQ = \sqrt{2.1^2 + 2.1^2 - 2(2.1)(2.1)\cos 110°} = 3.4404\ldots \text{ m}.$$

Along side AB (length 8.0 m), the two straight pieces AP and QB together measure AB minus the chord PQ:

$$AP + QB = 8.0 - 3.4404\ldots = 4.5596\ldots \text{ m}.$$

Perimeter. Adding the arc, the two straight pieces along AB, and the other three sides $BC = DA = 3.6$, $CD = 8.0$,

$$P = 4.0317\ldots + 4.5596\ldots + 2(3.6) + 8.0 = 23.7913\ldots \approx 23.8 \text{ m}.$$

3. Area of a sector

Q16 Sydney Grammar 2025 Trial Q31 · 1 mark

A planet completes a circular orbit of radius r around the sun in time T at a constant speed v. Express v in terms of r and T using speed $= \dfrac{\text{distance}}{\text{time}}$.

Criteria	Marks
Correct expression $v = \dfrac{2\pi r}{T}$	1

Solution.

In one orbit the planet travels the full circumference of the circle, a distance of $2\pi r$, in time T. Hence

$$v = \frac{\text{distance travelled}}{\text{time}} = \frac{2\pi r}{T}.$$

Q17 St George Girls 2021 Trial Q4 · 1 mark

A windscreen wiper traces the region $ABCD$ between two arcs, centre O, with radii 50 cm and 30 cm and $\angle AOB = 135°$. What is the area of $ABCD$ to the nearest square centimetre? (multiple choice)

Criteria	Marks
Correct option (C)	1

Solution.

The region is the difference of two sectors of the same angle $135° = \dfrac{3\pi}{4}$. In radians,

$$A = \tfrac{1}{2}R^2\theta - \tfrac{1}{2}r^2\theta = \tfrac{1}{2}\theta\left(R^2 - r^2\right) = \tfrac{1}{2}\cdot\frac{3\pi}{4}\left(50^2 - 30^2\right).$$

So

$$A = \tfrac{1}{2}\cdot\frac{3\pi}{4}\cdot 1600 = 600\pi = 1884.9\ldots \approx 1885 \text{ cm}^2.$$

The answer is **(C)**.

Q18 Sydney Boys 2023 Trial Q15 · **2 marks**

A sector has radius 4 cm and area 3π cm^2. Calculate the exact perimeter of the sector in terms of π.

Criteria	**Marks**
Uses $A = \frac{1}{2}r^2\theta$ to find $\theta = \dfrac{3\pi}{8}$ (or the arc length)	1
Correct perimeter $8 + \dfrac{3\pi}{2}$ cm	1

Solution.

Let the sector angle be θ. From $A = \frac{1}{2}r^2\theta$,

$$3\pi = \tfrac{1}{2}(4)^2\theta = 8\theta \quad \Rightarrow \quad \theta = \frac{3\pi}{8}.$$

The arc length is

$$\ell = r\theta = 4 \cdot \frac{3\pi}{8} = \frac{3\pi}{2} \text{ cm}.$$

The perimeter consists of the arc plus the two radii:

$$P = 2r + \ell = 8 + \frac{3\pi}{2} \text{ cm}.$$

Q19 Gosford 2022 Trial Q11 · 3 marks

Triangle ABC is right angled at B, with $AB = 8$ cm and $\angle BAC = \dfrac{\pi}{3}$. The arc BD has centre A and radius AB. **(i)** Find the exact area of sector ABD. **(ii)** Hence find the exact area of the shaded region BCD.

Criteria	Marks
(i) Correct sector area $\dfrac{32\pi}{3}$ cm^2	1
(ii) Finds $BC = 8\sqrt{3}$ (or area of $\triangle ABC$)	1
(ii) Correct shaded area $32\sqrt{3} - \dfrac{32\pi}{3}$ cm^2	1

Solution.

(i) Sector ABD has radius $AB = 8$ and angle $\dfrac{\pi}{3}$, so

$$A_{\text{sector}} = \tfrac{1}{2}r^2\theta = \tfrac{1}{2}(8)^2 \cdot \frac{\pi}{3} = \frac{32\pi}{3} \text{ cm}^2.$$

(ii) In the right-angled triangle, $\tan\dfrac{\pi}{3} = \dfrac{BC}{AB}$, so

$$BC = 8\tan\frac{\pi}{3} = 8\sqrt{3} \text{ cm}.$$

The triangle has area

$$A_{\triangle ABC} = \tfrac{1}{2} \cdot AB \cdot BC = \tfrac{1}{2} \cdot 8 \cdot 8\sqrt{3} = 32\sqrt{3} \text{ cm}^2.$$

The shaded region is the triangle with the sector removed:

$$A_{BCD} = A_{\triangle ABC} - A_{\text{sector}} = 32\sqrt{3} - \frac{32\pi}{3} \text{ cm}^2.$$

Q20 Gosford 2024 Trial Q2 · **1 mark**

An athlete runs 500 m around a circular track of radius 200 m. How far, in a direct line, is she from her starting position? (multiple choice)

Criteria	Marks
Correct option (C)	1

Solution.

The arc length is $\ell = r\theta$, so the angle swept is

$$\theta = \frac{\ell}{r} = \frac{500}{200} = 2.5 \text{ radians.}$$

The direct distance x is the chord subtending this angle. By the cosine rule in the isosceles triangle with the two radii,

$$x^2 = 200^2 + 200^2 - 2(200)(200)\cos 2.5 = 144091.5\ldots,$$

$$x = 379.6\ldots \approx 380 \text{ m.}$$

The answer is **(C)**.

Q21 Hornsby Girls 2023 Trial Q28 · **2 marks**

The circle $x^2 + y^2 = 4$ is shown; OP to the point $(1, \sqrt{3})$ makes an angle θ with the positive x-axis. By considering θ, find the exact area of the shaded sector.

Criteria	Marks
Finds $\theta = \frac{\pi}{3}$ (and radius $r = 2$)	1
Correct area $\frac{2\pi}{3}$	1

Solution.

The circle has radius $r = 2$. The point $(1, \sqrt{3})$ lies on it, and

$$\tan\theta = \frac{\sqrt{3}}{1} = \sqrt{3} \quad \Rightarrow \quad \theta = \frac{\pi}{3}.$$

The shaded region is the sector between the positive x-axis and OP, of radius 2 and angle $\frac{\pi}{3}$:

$$A = \tfrac{1}{2}r^2\theta = \tfrac{1}{2}(2)^2 \cdot \frac{\pi}{3} = \frac{2\pi}{3}.$$

Q22 SCEGGS 2022 Prelim Q21 · **3 marks**

Triangle ADB is inscribed in sector ABC, with $AC = CB = 6\sqrt{3}$ cm, $AD = DB = 6$ cm, $\angle ADB = 120°$ and $\angle ACB = 60°$. **(a)** Find the exact area of sector ACB. **(b)** Find the exact area of the shaded region.

Criteria	**Marks**
(a) Correct sector area 18π cm^2	1
(b) Correct area of $\triangle ADB = 9\sqrt{3}$ cm^2	1
(b) Correct shaded area $18\pi - 9\sqrt{3}$ cm^2	1

Solution.

(a) Sector ACB has radius $CB = 6\sqrt{3}$ and angle $60° = \dfrac{\pi}{3}$:

$$A_{\text{sector}} = \tfrac{1}{2}r^2\theta = \tfrac{1}{2}\left(6\sqrt{3}\right)^2 \cdot \frac{\pi}{3} = \tfrac{1}{2} \cdot 108 \cdot \frac{\pi}{3} = 18\pi \text{ cm}^2.$$

(b) Triangle ADB has $AD = DB = 6$ and included angle $\angle ADB = 120°$, so

$$A_{\triangle ADB} = \tfrac{1}{2} \cdot 6 \cdot 6 \cdot \sin 120° = 18 \cdot \frac{\sqrt{3}}{2} = 9\sqrt{3} \text{ cm}^2.$$

The shaded region is the sector with the inscribed triangle removed:

$$A_{\text{shaded}} = 18\pi - 9\sqrt{3} \text{ cm}^2.$$

Q23 Hornsby Girls 2025 Trial Q16 · **4 marks**

The shape $APQBCD$ is the rectangle $ABCD$ ($BC = 4.6$ m, $DC = 7.0$ m) with the sector POQ (centre O, radius 2.1 m, $\angle POQ = 120°$) removed. Find the area of $APQBCD$, correct to one decimal place.

Criteria	Marks
Correct rectangle area 32.2 m^2	1
Converts 120° to radians and states the sector-area formula	1
Correct sector area ≈ 4.62 m^2	1
Correct final area ≈ 27.6 m^2	1

Solution.

The rectangle has area

$$A_{\text{rect}} = BC \times DC = 4.6 \times 7.0 = 32.2 \text{ m}^2.$$

The sector removed has radius 2.1 m and angle $120° = \dfrac{2\pi}{3}$, so

$$A_{\text{sector}} = \tfrac{1}{2} r^2 \theta = \tfrac{1}{2}(2.1)^2 \cdot \frac{2\pi}{3} = 4.618\ldots \text{ m}^2.$$

Therefore

$$A_{APQBCD} = 32.2 - 4.618\ldots = 27.581\ldots \approx 27.6 \text{ m}^2.$$

Q24 St George Girls 2022 Prelim Q28 · **5 marks**

Isosceles triangle ABC has $AB = AC = 6$ cm and $BC = 6\sqrt{3}$ cm. Sector ABD (centre B, radius AB) is inscribed in the triangle. **(a)** Find $\angle ABC$ in radians. **(b)** Find the exact area of the shaded part of the triangle.

Criteria	Marks
(a) Correct cosine rule (or method) for $\angle ABC$	1
(a) Correct value $\angle ABC = \dfrac{\pi}{6}$	1
(b) Correct area of $\triangle ABC = 9\sqrt{3}$ cm^2	1
(b) Correct sector area $\dfrac{\pi}{6} \cdot 18 = 3\pi$ cm^2	1
(b) Correct shaded area $9\sqrt{3} - 3\pi$ cm^2	1

Solution.

(a) By the cosine rule at B,

$$\cos\angle ABC = \frac{AB^2 + BC^2 - AC^2}{2 \cdot AB \cdot BC} = \frac{6^2 + (6\sqrt{3})^2 - 6^2}{2 \cdot 6 \cdot 6\sqrt{3}} = \frac{108}{72\sqrt{3}} = \frac{\sqrt{3}}{2}.$$

Hence

$$\angle ABC = \frac{\pi}{6}.$$

(b) The triangle has $\angle ABC = \dfrac{\pi}{6}$, with sides $AB = 6$ and $BC = 6\sqrt{3}$ meeting at B:

$$A_{\triangle ABC} = \tfrac{1}{2} \cdot AB \cdot BC \cdot \sin\angle ABC = \tfrac{1}{2} \cdot 6 \cdot 6\sqrt{3} \cdot \sin\frac{\pi}{6} = \tfrac{1}{2} \cdot 6 \cdot 6\sqrt{3} \cdot \tfrac{1}{2} = 9\sqrt{3} \text{ cm}^2.$$

The sector ABD has centre B, radius $AB = 6$ and angle $\dfrac{\pi}{6}$:

$$A_{\text{sector}} = \tfrac{1}{2} r^2\theta = \tfrac{1}{2}(6)^2 \cdot \frac{\pi}{6} = 3\pi \text{ cm}^2.$$

The shaded part is the triangle with the sector removed:

$$A_{\text{shaded}} = 9\sqrt{3} - 3\pi \text{ cm}^2.$$

Q25 Manly 2024 Trial Q27 · **3 marks**

A square of side 12 m has two identical arcs on opposite sides, each an arc of a circle centred at the square's centre. The arc endpoints satisfy $AE = FB = CG = HD = 1$ m. Find the area of the shape, to the nearest square metre.

Criteria	**Marks**
Correct radius $r = \sqrt{61}$ and centre angle $\theta = \cos^{-1}\dfrac{11}{61}$ (or equivalent)	1
Sets up the shape as the square plus two segments (sector − triangle)	1
Correct final area 169 m^2	1

Solution.

Place the centre O at the origin, so the square runs from -6 to 6 in each direction and each arc endpoint is 1 m in from a corner. One arc has endpoints 6 m along a side and 5 m up/down from the centre line, so the radius from O is

$$r = \sqrt{6^2 + 5^2} = \sqrt{61} \text{ m}.$$

The two endpoints of one arc lie at $(-6, 5)$ and $(-6, -5)$ relative to O; the angle between them at O is

$$\theta = \cos^{-1}\left(\frac{(-6)(-6) + (5)(-5)}{61}\right) = \cos^{-1}\frac{11}{61} = 1.3894\ldots \text{ rad}.$$

Each arc bulges outward, adding a circular segment equal to the sector minus the triangle joining O to the two endpoints:

$$A_{\text{segment}} = \tfrac{1}{2}r^2\theta - \tfrac{1}{2}r^2\sin\theta = \tfrac{1}{2}(61)(1.3894\ldots) - \tfrac{1}{2}(61)\sin(1.3894\ldots) = 42.379\ldots - 30 = 12.379\ldots \text{ m}^2.$$

The shape is the square plus two such segments:

$$A = 12^2 + 2(12.379\ldots) = 144 + 24.758\ldots = 168.758\ldots \approx 169 \text{ m}^2.$$

Q26 Hornsby Girls 2022 Trial Q23 · 6 marks

Triangle ABC is right angled at C, with $\angle BAC = \dfrac{\pi}{6}$ and $AB = 5$ cm. KP is an arc centre A, KQ an arc centre B, with K on AB and $AK = x$. Find x (one decimal place) for which sectors AKP and BKQ have equal areas.

Criteria	Marks
Sector AKP: centre A, radius x, angle $\dfrac{\pi}{6}$	1
Sector BKQ: centre B, radius $5-x$, angle $\dfrac{\pi}{3}$	2
Sets the areas equal and forms $x^2 = 2(5-x)^2$	2
Solves for the valid root $x = 10 - 5\sqrt{2} \approx 2.9$	1

Solution.

The angles of the triangle are $\angle A = \dfrac{\pi}{6}$ and $\angle B = \dfrac{\pi}{2} - \dfrac{\pi}{6} = \dfrac{\pi}{3}$. Sector AKP has centre A, radius $AK = x$ and angle $\dfrac{\pi}{6}$; sector BKQ has centre B, radius $BK = 5 - x$ and angle $\dfrac{\pi}{3}$. Setting the areas equal,

$$\tfrac{1}{2}x^2 \cdot \frac{\pi}{6} = \tfrac{1}{2}(5-x)^2 \cdot \frac{\pi}{3}.$$

Multiplying both sides by $\dfrac{12}{\pi}$,

$$x^2 = 2(5-x)^2.$$

Taking positive square roots (both lengths are positive), $x = \sqrt{2}\,(5-x)$, so

$$x\left(1+\sqrt{2}\right) = 5\sqrt{2} \quad \Rightarrow \quad x = \frac{5\sqrt{2}}{1+\sqrt{2}} = 5\sqrt{2}\left(\sqrt{2}-1\right) = 10 - 5\sqrt{2}.$$

Hence

$$x = 10 - 5\sqrt{2} = 2.928\ldots \approx 2.9 \text{ cm}.$$

(The other root $10 + 5\sqrt{2}$ exceeds 5 and is rejected.)

4. Area of a segment

Q27 Girraween 2021 Prelim Q2 · **2 marks**

A chord of a circle with radius 7 cm subtends an angle of 115° at the centre. Find the area of the minor segment cut off by the chord, correct to one decimal place.

Criteria	Marks
Correct segment formula with $r = 7$ and $\theta = 115°$ (or sector minus triangle)	1
Correct value ≈ 27.0 cm^2	1

Solution.

Work in radians: $115° = \dfrac{23\pi}{36}$. The minor segment equals the minor sector minus the triangle:

$$A = \tfrac{1}{2}r^2\theta - \tfrac{1}{2}r^2\sin\theta = \tfrac{1}{2}(7)^2\left(\tfrac{23\pi}{36}\right) - \tfrac{1}{2}(7)^2\sin 115°.$$

So

$$A = 49.199\ldots - 22.229\ldots = 26.970\ldots \approx 27.0 \text{ cm}^2.$$

Q28 Fort St 2025 Trial Q13 · **3 marks**

The shaded segment shown is formed by an arc BC of a circle with radius 1.6 m. The arc subtends an angle of $\dfrac{4\pi}{9}$ radians at its centre, A. Find the area of the shaded segment, correct to three significant figures.

Criteria	Marks
Correct expression for the sector area $\frac{1}{2}r^2\theta$	1
Subtracts the triangle area $\frac{1}{2}r^2\sin\theta$	1
Correct value ≈ 0.527 m^2	1

Solution.

The segment equals the sector minus the triangle. With $r = 1.6$ and $\theta = \dfrac{4\pi}{9}$,

$$A = \tfrac{1}{2}r^2\theta - \tfrac{1}{2}r^2\sin\theta = \tfrac{1}{2}(1.6)^2\left(\tfrac{4\pi}{9}\right) - \tfrac{1}{2}(1.6)^2\sin\tfrac{4\pi}{9}.$$

So

$$A = 1.7872\ldots - 1.2605\ldots = 0.5266\ldots \approx 0.527 \text{ m}^2.$$

Q29 Normanhurst Boys 2022 Trial Q7 · **1 mark**

The area of a minor segment of a circle is 100π square metres, and the chord subtends an angle of $\dfrac{\pi}{6}$ at the centre. Which of the following is the exact form of the radius? (multiple choice)

Criteria	Marks
Correct option (D)	1

Solution.

Using the segment area with $\theta = \dfrac{\pi}{6}$ (so $\sin\theta = \dfrac{1}{2}$),

$$\tfrac{1}{2}r^2(\theta - \sin\theta) = 100\pi \quad \Rightarrow \quad \tfrac{1}{2}r^2\left(\frac{\pi}{6} - \frac{1}{2}\right) = 100\pi.$$

Then

$$r^2 \cdot \frac{\pi - 3}{6} = 200\pi \quad \Rightarrow \quad r^2 = \frac{1200\pi}{\pi - 3},$$

$$r = \sqrt{\frac{1200\pi}{\pi - 3}} \text{ m}.$$

The answer is **(D)**.

Q30 Blacktown Boys 2025 Trial Q20 · **2 marks**

The area of a minor segment of a circle is 2.5 m^2, and the chord subtends an angle of $\dfrac{\pi}{3}$ at the centre. Find the radius of the circle, correct to two decimal places.

Criteria	Marks
Correct segment equation $\frac{1}{2}r^2\left(\frac{\pi}{3} - \sin\frac{\pi}{3}\right) = 2.5$	1
Correct value $r \approx 5.25$ m	1

Solution.

Using the segment area with $\theta = \dfrac{\pi}{3}$,

$$\tfrac{1}{2}r^2\left(\frac{\pi}{3} - \sin\frac{\pi}{3}\right) = 2.5.$$

Since $\dfrac{\pi}{3} - \dfrac{\sqrt{3}}{2} = 0.18117\ldots$,

$$r^2 = \frac{2 \times 2.5}{0.18117\ldots} = 27.598\ldots \quad \Rightarrow \quad r = 5.2533\ldots \approx 5.25 \text{ m}.$$

Q31 Shore 2023 Trial Q21 · **5 marks**

A circle has centre O and radius 30 cm, with A and B on the circle such that $\angle AOB = 80°$. **(a)** Convert $80°$ to radians. **(b)** Show the area of sector AOB is 628 cm^2 (nearest square centimetre). **(c)** Find the perimeter of the shaded segment cut off by chord AB (one decimal place).

Criteria	**Marks**
(a) $80° = \dfrac{4\pi}{9}$	1
(b) Substitutes into $\frac{1}{2}r^2\theta$ to obtain 628 cm^2	1
(c) Correct arc length $AB \approx 41.9$ cm	1
(c) Correct chord length $AB \approx 38.6$ cm	1
(c) Correct perimeter ≈ 80.5 cm	1

Solution.

(a) $80° = 80 \times \dfrac{\pi}{180} = \dfrac{4\pi}{9}$.

(b) The sector area is

$$A = \tfrac{1}{2}r^2\theta = \tfrac{1}{2}(30)^2\left(\tfrac{4\pi}{9}\right) = 200\pi = 628.318\ldots \approx 628 \text{ cm}^2.$$

(c) The perimeter of the segment is the arc AB plus the chord AB. The arc is

$$\ell = r\theta = 30 \times \tfrac{4\pi}{9} = 41.887\ldots \text{ cm},$$

and the chord (by the isosceles triangle, or the cosine rule) is

$$AB = 2r\sin\tfrac{\theta}{2} = 2(30)\sin 40° = 38.567\ldots \text{ cm}.$$

Hence the perimeter is

$$41.887\ldots + 38.567\ldots = 80.455\ldots \approx 80.5 \text{ cm}.$$

Q32 Sydney Tech 2025 Trial Q23 · 3 marks

APB is an arc of a circle with centre O and radius 8 cm, such that the chord AB bisects the radius OP at right angles. The shaded region is the segment between chord AB and arc APB. **(i)** Show that $\angle AOB = \dfrac{2\pi}{3}$. **(ii)** Find the shaded area in simplest exact form.

Criteria	Marks
(i) Uses $OM = \frac{1}{2}r$ and the right angle to get $\cos\angle AOM = \frac{1}{2}$	1
(i) Concludes $\angle AOB = 2 \times \dfrac{\pi}{3} = \dfrac{2\pi}{3}$	1
(ii) Correct exact segment $\dfrac{64\pi}{3} - 16\sqrt{3}\ \text{cm}^2$	1

Solution.

(i) Let M be the point where AB meets OP. Since AB bisects OP, $OM = \frac{1}{2}(8) = 4$, and $AB \perp OP$, so $\triangle OMA$ is right-angled at M with hypotenuse $OA = 8$. Thus

$$\cos\angle AOM = \frac{OM}{OA} = \frac{4}{8} = \frac{1}{2} \quad \Rightarrow \quad \angle AOM = \frac{\pi}{3}.$$

By symmetry $\angle BOM = \dfrac{\pi}{3}$ as well, so

$$\angle AOB = 2 \times \frac{\pi}{3} = \frac{2\pi}{3}.$$

(ii) The shaded segment equals the sector minus the triangle:

$$A = \tfrac{1}{2}r^2(\theta - \sin\theta) = \tfrac{1}{2}(8)^2\left(\frac{2\pi}{3} - \sin\frac{2\pi}{3}\right) = 32\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right).$$

Hence

$$A = \frac{64\pi}{3} - 16\sqrt{3}\ \text{cm}^2.$$

Q33 Baulkham Hills 2022 Prelim Q26 · **4 marks**

A garden bed is a circle with centre O and radius 5 m, with a minor segment removed. The straight edge AB (the chord bounding the removed segment) is $5\sqrt{3}$ m. Find the exact area of the garden bed.

Criteria	**Marks**
Uses the cosine rule to find $\cos\angle AOB = -\dfrac{1}{2}$	1
Obtains $\angle AOB = \dfrac{2\pi}{3}$	1
Correct expression for the removed segment	1
Correct exact area $\dfrac{50\pi}{3} + \dfrac{25\sqrt{3}}{4}$ m^2	1

Solution.

The chord $AB = 5\sqrt{3}$ subtends $\angle AOB$ at the centre, with $OA = OB = 5$. By the cosine rule,

$$\cos\angle AOB = \frac{5^2 + 5^2 - (5\sqrt{3})^2}{2(5)(5)} = \frac{25 + 25 - 75}{50} = -\frac{1}{2} \quad \Rightarrow \quad \angle AOB = \frac{2\pi}{3}.$$

The removed minor segment has area

$$\tfrac{1}{2}r^2(\theta - \sin\theta) = \tfrac{1}{2}(5)^2\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right) = \frac{25\pi}{3} - \frac{25\sqrt{3}}{4}.$$

The garden bed is the whole circle minus this segment:

$$A = \pi(5)^2 - \left(\frac{25\pi}{3} - \frac{25\sqrt{3}}{4}\right) = 25\pi - \frac{25\pi}{3} + \frac{25\sqrt{3}}{4} = \frac{50\pi}{3} + \frac{25\sqrt{3}}{4} \text{ m}^2.$$

5. Perimeter and composite mensuration

Q34 Blacktown Boys 2022 Prelim Q22 · **4 marks**

ABC is a right-angled isosceles triangle with $\angle ABC = 90°$ and $AB = BC = 12$ cm. The arc BP, with centre C and radius CB, meets CA at P. **(a)** Find the exact area of the shaded region ABP. **(b)** Find the exact perimeter of the shaded region ABP.

Criteria	**Marks**
(a) Correct area of the sector BPC	1
(a) Correct shaded area $(72 - 18\pi)$ cm^2	1
(b) Finds AP or the arc length BP	1
(b) Correct perimeter $\left(12\sqrt{2} + 3\pi\right)$ cm	1

Solution.

(a) Since $\triangle ABC$ is right-angled and isosceles with the right angle at B, the base angles are equal, so $\angle BCA = \angle BAC = 45° = \dfrac{\pi}{4}$. The sector BPC has centre C, radius $CB = 12$ and angle $\dfrac{\pi}{4}$, so its area is

$$A_{\text{sector}} = \frac{1}{2}r^2\theta = \frac{1}{2}(12)^2 \cdot \frac{\pi}{4} = 18\pi \text{ cm}^2.$$

The area of the triangle is

$$A_{\triangle} = \frac{1}{2} \times 12 \times 12 = 72 \text{ cm}^2.$$

Hence the shaded region ABP has area

$$A_{ABP} = 72 - 18\pi \text{ cm}^2.$$

(b) The boundary of ABP consists of the interval AB, the interval AP and the arc BP. The hypotenuse is

$$CA = \sqrt{12^2 + 12^2} = 12\sqrt{2},$$

and $CP = 12$ (a radius), so

$$AP = CA - CP = 12\sqrt{2} - 12.$$

The arc has length

$$BP = r\theta = 12 \times \frac{\pi}{4} = 3\pi.$$

Therefore

$$P = AB + AP + BP = 12 + \left(12\sqrt{2} - 12\right) + 3\pi = 12\sqrt{2} + 3\pi \text{ cm}.$$

Q35 Sydney Girls 2024 Trial Q11 · **5 marks**

A shape consists of three sides of the square $ABCD$ (side length 2 cm) together with a sector ODC, where O is the midpoint of AB. **(a)** Find the exact length of DO. **(b)** If $\angle DOC = \theta$, show that $\cos\theta = \dfrac{3}{5}$. **(c)** Hence find the perimeter of the shape $ABCD$, correct to 2 decimal places.

Criteria	Marks
(a) Correct length $DO = \sqrt{5}$ cm	1
(b) Applies the cosine rule in $\triangle DOC$	1
(b) Establishes $\cos\theta = \dfrac{3}{5}$	1
(c) Correct arc length $DC = \sqrt{5}\,\theta$	1
(c) Correct perimeter ≈ 8.07 cm	1

Solution.

(a) O is the midpoint of AB, so $AO = 1$, and $DA = 2$ with $\angle DAO = 90°$. By Pythagoras,

$$DO = \sqrt{DA^2 + AO^2} = \sqrt{2^2 + 1^2} = \sqrt{5}\text{ cm}.$$

(b) By symmetry $OC = \sqrt{5}$ as well, and $DC = 2$. Applying the cosine rule in $\triangle DOC$ to the angle $\theta = \angle DOC$,

$$\cos\theta = \frac{DO^2 + OC^2 - DC^2}{2 \cdot DO \cdot OC} = \frac{5 + 5 - 4}{2\sqrt{5} \cdot \sqrt{5}} = \frac{6}{10} = \frac{3}{5}.$$

(c) The perimeter consists of the three sides DA, AB, BC (each 2 cm) together with the arc DC of the sector, which has radius $\sqrt{5}$ and angle $\theta = \cos^{-1}\dfrac{3}{5}$. So

$$\text{arc } DC = r\theta = \sqrt{5}\,\cos^{-1}\frac{3}{5} = 2.0735\ldots$$

Hence

$$P = 2 + 2 + 2 + \sqrt{5}\,\cos^{-1}\frac{3}{5} = 6 + 2.0735\ldots = 8.0735\ldots \approx 8.07\text{ cm}.$$

Q36 North Sydney Girls 2020 Trial Q22 · **5 marks**

A circle has centre O and radius r cm. Points A and B lie on the circle, AT is a tangent to the circle, OBT is a straight line and $\angle AOB = \theta$ radians. The shaded region is bounded by the tangent AT, the straight interval TB and the arc AB. **(a)** Express the area of the shaded region in terms of r and θ. **(b)** Given that $r = 3$ and $\theta = 1.2$, find the perimeter of the shaded region (2 decimal places).

Criteria	**Marks**
(a) Uses right angle at A to find $AT = r\tan\theta$ and the area of $\triangle OAT$	1
(a) Correct shaded area $\frac{1}{2}r^2(\tan\theta - \theta)$	1
(b) Correct tangent length AT and external interval TB	1
(b) Correct arc length $AB = r\theta$	1
(b) Correct perimeter ≈ 16.60 cm	1

Solution.

(a) Since AT is a tangent at A, the radius $OA \perp AT$, so $\triangle OAT$ is right-angled at A with $\angle AOT = \theta$. Thus

$$AT = r\tan\theta, \qquad \text{area of } \triangle OAT = \frac{1}{2}\cdot OA \cdot AT = \frac{1}{2}r\cdot r\tan\theta = \frac{1}{2}r^2\tan\theta.$$

The sector OAB has area $\frac{1}{2}r^2\theta$. The shaded region is the triangle with the sector removed, so

$$A_{\text{shaded}} = \frac{1}{2}r^2\tan\theta - \frac{1}{2}r^2\theta = \frac{1}{2}r^2(\tan\theta - \theta).$$

(b) The boundary of the shaded region is AT, then the interval TB, then the arc AB. In the right triangle OAT,

$$OT = \frac{r}{\cos\theta} \qquad \Rightarrow \qquad TB = OT - OB = \frac{r}{\cos\theta} - r.$$

With $r = 3$, $\theta = 1.2$:

$$AT = 3\tan 1.2 = 7.7165\ldots, \quad TB = \frac{3}{\cos 1.2} - 3 = 5.2791\ldots, \quad AB = r\theta = 3\times 1.2 = 3.6.$$

Hence

$$P = AT + TB + AB = 7.7165\ldots + 5.2791\ldots + 3.6 = 16.5955\ldots \approx 16.60 \text{ cm}.$$

Q37 North Sydney Boys 2024 Prelim Q27 · **4 marks**

Nick's swimming pool is made up of a sector of radius r and angle θ together with a semicircle whose diameter equals the radius r of the sector. **(a)** Find the perimeter of the pool in terms of r and θ. **(b)** If the area of the pool equals 1 square unit, show that the perimeter can be given by $P = \dfrac{2}{r} + r\left(1 + \dfrac{\pi}{4}\right)$.

Criteria	Marks
(a) Correct perimeter $P = r + r\theta + \dfrac{\pi r}{2}$	1
(b) Correct area expression and sets it equal to 1	1
(b) Makes θ the subject	1
(b) Substitutes to obtain the required form	1

Solution.

(a) The semicircle has diameter r, hence radius $\dfrac{r}{2}$. The boundary of the pool is one straight radius of the sector (length r), the sector's arc (length $r\theta$) and the semicircular arc (length $\pi \cdot \dfrac{r}{2}$); the shared straight edge is internal. Therefore

$$P = r + r\theta + \frac{\pi r}{2}.$$

(b) The area of the pool is the sector area plus the semicircle area:

$$A = \frac{1}{2}r^2\theta + \frac{1}{2}\pi\left(\frac{r}{2}\right)^2 = \frac{1}{2}r^2\theta + \frac{\pi r^2}{8}.$$

Setting $A = 1$ and making θ the subject,

$$\frac{1}{2}r^2\theta = 1 - \frac{\pi r^2}{8} \qquad \Rightarrow \qquad \theta = \frac{2}{r^2} - \frac{\pi}{4}.$$

Substituting into the perimeter,

$$P = r + r\left(\frac{2}{r^2} - \frac{\pi}{4}\right) + \frac{\pi r}{2} = r + \frac{2}{r} - \frac{\pi r}{4} + \frac{\pi r}{2} = \frac{2}{r} + r + \frac{\pi r}{4} = \frac{2}{r} + r\left(1 + \frac{\pi}{4}\right),$$

as required.

Q38 North Sydney Girls 2023 Prelim Q27 · **4 marks**

Two circles with centres A and B intersect at P and Q. The radii are 6 cm and 4 cm respectively and $\angle PAQ = 45°$. The shaded region is the overlap of the two circles. **(a)** Show that $PQ = 6\sqrt{2-\sqrt{2}}$. **(b)** Given $\angle PBQ = 70.1°$, find the perimeter of the shaded region to the nearest centimetre.

Criteria	Marks
(a) Applies the cosine rule in $\triangle PAQ$	1
(a) Simplifies to $PQ = 6\sqrt{2-\sqrt{2}}$	1
(b) Adds the two arc lengths bounding the overlap	1
(b) Correct perimeter ≈ 10 cm	1

Solution.

(a) In $\triangle PAQ$, $AP = AQ = 6$ and $\angle PAQ = 45°$. By the cosine rule,

$$PQ^2 = 6^2 + 6^2 - 2(6)(6)\cos 45° = 72 - 72 \cdot \frac{\sqrt{2}}{2} = 72 - 36\sqrt{2} = 36\left(2-\sqrt{2}\right).$$

Taking the positive square root,

$$PQ = 6\sqrt{2-\sqrt{2}}.$$

(b) The shaded overlap is bounded by two arcs: the arc PQ of the circle centred at A (radius 6, angle 45°) and the arc PQ of the circle centred at B (radius 4, angle 70.1°). Converting the angles to radians, the arc lengths are

$$\ell_A = 6 \times \frac{45\pi}{180} = \frac{3\pi}{2} = 4.7124\ldots, \qquad \ell_B = 4 \times \frac{70.1\pi}{180} = 4.8939\ldots$$

Hence the perimeter is

$$P = \ell_A + \ell_B = 4.7124\ldots + 4.8939\ldots = 9.6063\ldots \approx 10 \text{ cm}.$$

Q39 Abbotsleigh 2024 Prelim Q29 · **5 marks**

The logo $OABCDEFO$ has: sector OAB (centre O, radius r); sector OFE congruent to OAB; sector ODC (centre O, radius $2r$); and AOF a straight line. The angle $COD = \theta$ radians. **(a)** Show that $\angle AOB = \dfrac{\pi - \theta}{2}$. **(b)** Show that the area of the logo is $\dfrac{1}{2}r^2(3\theta + \pi)$. **(c)** Find the perimeter of the logo in terms of r, θ and π.

Criteria	Marks
(a) Uses AOF straight, with congruent sectors either side of $\angle COD$	1
(b) Correct areas of the three sectors	1
(b) Sums to $\dfrac{1}{2}r^2(3\theta + \pi)$	1
(c) Correct arc lengths and straight radial segments	1
(c) Correct perimeter $r(\pi + \theta + 4)$	1

Solution.

(a) Since AOF is a straight line, the angles about O from OA to OF sum to π. These are $\angle AOB$, then $\angle COD = \theta$, then $\angle EOF$. As sectors OAB and OFE are congruent, $\angle AOB = \angle EOF$, so

$$\angle AOB + \theta + \angle EOF = \pi \quad \Rightarrow \quad 2\angle AOB + \theta = \pi \quad \Rightarrow \quad \angle AOB = \frac{\pi - \theta}{2}.$$

(b) The three sectors have areas

$$A_{OAB} = \frac{1}{2}r^2 \cdot \frac{\pi - \theta}{2}, \qquad A_{OFE} = \frac{1}{2}r^2 \cdot \frac{\pi - \theta}{2}, \qquad A_{ODC} = \frac{1}{2}(2r)^2\theta = 2r^2\theta.$$

Adding,

$$A = 2 \cdot \frac{1}{2}r^2 \cdot \frac{\pi - \theta}{2} + 2r^2\theta = \frac{1}{2}r^2(\pi - \theta) + 2r^2\theta = \frac{1}{2}r^2\pi + \frac{3}{2}r^2\theta = \frac{1}{2}r^2(3\theta + \pi).$$

(c) Following the boundary $O \to A \to B \to C \to D \to E \to F \to O$: the straight radii OA and FO each have length r; the radial steps BC and DE each bridge from radius r to radius $2r$, so each has length r. The arcs are AB and EF (radius r, angle $\dfrac{\pi - \theta}{2}$ each) and CD (radius $2r$, angle θ). Hence

$$P = \underbrace{r + r + r + r}_{OA,\, BC,\, DE,\, FO} + \underbrace{2 \cdot r \cdot \frac{\pi - \theta}{2}}_{\text{arcs } AB,\, EF} + \underbrace{2r \cdot \theta}_{\text{arc } CD} = 4r + r(\pi - \theta) + 2r\theta = r\,(\pi + \theta + 4)\,.$$