Year 11 Mathematics Advanced

Topic: Trigonometry and Measure of Angles (3 of 5)

Review Questions (Worked Solutions): Trigonometry of any magnitude and exact values

Term 3, 2026

Angles of any magnitude

1. Exact values of special and any-magnitude angles

Q1 Sydney Grammar 2021 Prelim Q10 · **1 mark**

Write down the exact value of $\tan \dfrac{2\pi}{3}$.

Criteria	Marks
Correct exact value $-\sqrt{3}$	1

Solution.

The angle $\dfrac{2\pi}{3} = 120°$ lies in the second quadrant, where tangent is negative. Its related (acute) angle is $\pi - \dfrac{2\pi}{3} = \dfrac{\pi}{3} = 60°$, so

$$\tan \frac{2\pi}{3} = -\tan \frac{\pi}{3} = -\sqrt{3}.$$

Q2 Moriah 2021 Prelim Q3 · **1 mark**

What is the exact value of $2 \sin \dfrac{5\pi}{3}$? (multiple choice)

Criteria	Marks
Correct option (C)	1

Solution.

The angle $\dfrac{5\pi}{3} = 300°$ lies in the fourth quadrant, where sine is negative. Its related angle is $2\pi - \dfrac{5\pi}{3} = \dfrac{\pi}{3} = 60°$, so

$$\sin \frac{5\pi}{3} = -\sin \frac{\pi}{3} = -\frac{\sqrt{3}}{2}.$$

Therefore

$$2 \sin \frac{5\pi}{3} = 2 \times \left(-\frac{\sqrt{3}}{2}\right) = -\sqrt{3}.$$

The answer is **(C)**.

Q3 Sydney Tech 2022 Trial Q11 · **2 marks**

Write down the exact value of $\tan \dfrac{7\pi}{6}$.

Criteria	Marks
Identifies the quadrant and related angle (tangent positive, related angle $\frac{\pi}{6}$)	1
Correct exact value $\dfrac{\sqrt{3}}{3}$	1

Solution.

The angle $\dfrac{7\pi}{6} = 210°$ lies in the third quadrant, where tangent is positive. Its related angle is $\dfrac{7\pi}{6} - \pi = \dfrac{\pi}{6} = 30°$, so

$$\tan \frac{7\pi}{6} = +\tan \frac{\pi}{6} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}.$$

Q4 Presbyterian 2022 Prelim Q2 · **1 mark**

What is the exact value of $\cos \dfrac{11\pi}{6}$? (multiple choice)

Criteria	Marks
Correct option (D)	1

Solution.

The angle $\dfrac{11\pi}{6} = 330°$ lies in the fourth quadrant, where cosine is positive. Its related angle is $2\pi - \dfrac{11\pi}{6} = \dfrac{\pi}{6} = 30°$, so

$$\cos \frac{11\pi}{6} = +\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}.$$

The answer is **(D)**.

Q5 Gosford 2024 Prelim Q3 · **1 mark**

Given that $\cos\theta = \dfrac{4}{7}$ and $\sin\theta > 0$, which of the following statements is true? (multiple choice)

Criteria	Marks
Correct option (B)	1

Solution.

Since $\cos\theta > 0$ and $\sin\theta > 0$, the angle θ is in the first quadrant, so every ratio is positive. Using $\sin^2\theta + \cos^2\theta = 1$,

$$\sin\theta = \sqrt{1 - \left(\frac{4}{7}\right)^2} = \sqrt{1 - \frac{16}{49}} = \sqrt{\frac{33}{49}} = \frac{\sqrt{33}}{7}.$$

Checking the options, $\sin\theta = \dfrac{\sqrt{33}}{7}$, not $\dfrac{3}{7}$, so **(A) is false**. The reciprocal ratio is found directly:

$$\sec\theta = \frac{1}{\cos\theta} = \frac{7}{4}.$$

The answer is **(B)**.

Q6 Killara 2022 Prelim Q5 · **1 mark**

What is the exact value of $\cos 30° + \sin 135°$? (multiple choice)

Criteria	Marks
Correct option (D)	1

Solution.

From the exact-value triangles, $\cos 30° = \dfrac{\sqrt{3}}{2}$. The angle $135°$ is in the second quadrant, where sine is positive, with related angle $180° - 135° = 45°$, so $\sin 135° = \sin 45° = \dfrac{\sqrt{2}}{2}$. Therefore

$$\cos 30° + \sin 135° = \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} = \frac{\sqrt{3} + \sqrt{2}}{2}.$$

The answer is **(D)**.

Q7 Sydney Grammar 2023 Prelim Q9 · **1 mark**

Find the exact value of $\tan \dfrac{5\pi}{6}$.

Criteria	Marks
Correct exact value $-\dfrac{\sqrt{3}}{3}$	1

Solution.

The angle $\dfrac{5\pi}{6} = 150°$ lies in the second quadrant, where tangent is negative. Its related angle is $\pi - \dfrac{5\pi}{6} = \dfrac{\pi}{6} = 30°$, so

$$\tan \frac{5\pi}{6} = -\tan \frac{\pi}{6} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}.$$

2. Given one ratio and the quadrant, find another exact ratio

Q8 Abbotsleigh 2020 Trial Q12 · **2 marks**

If $\tan\theta = \dfrac{4}{5}$ and θ is acute, find the exact value of $\sin\theta$.

Criteria	Marks
Finds the hypotenuse $\sqrt{41}$ (or correct use of $1 + \tan^2\theta = \sec^2\theta$)	1
Correct exact value $\sin\theta = \dfrac{4}{\sqrt{41}}$	1

Solution.

Since θ is acute, all ratios are positive. Taking a right-angled triangle with the side opposite θ equal to 4 and the adjacent side equal to 5 (so that $\tan\theta = \frac{4}{5}$), the hypotenuse is

$$\sqrt{4^2 + 5^2} = \sqrt{41}.$$

Hence

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{4}{\sqrt{41}} = \frac{4\sqrt{41}}{41}.$$

Q9 Pymble 2022 Prelim Q5 · **1 mark**

If $\tan\theta = \dfrac{2}{3}$ and θ is acute, which of the following is the exact value of $\cos\theta$? (multiple choice)

Criteria	**Marks**
Correct option (C)	1

Solution.

As θ is acute, take a right-angled triangle with opposite side 2 and adjacent side 3, giving hypotenuse

$$\sqrt{2^2 + 3^2} = \sqrt{13}.$$

Then

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{3}{\sqrt{13}}.$$

The answer is **(C)**.

Q10 Killara 2023 Prelim Q6 · **1 mark**

If $\cos\theta = x$ and $0° < \theta < 90°$, then $\tan\theta$ is equal to: (multiple choice)

Criteria	**Marks**
Correct option (C)	1

Solution.

With θ acute and $\cos\theta = x = \dfrac{x}{1}$, take a right-angled triangle with adjacent side x and hypotenuse 1. The opposite side is

$$\sqrt{1^2 - x^2} = \sqrt{1 - x^2}.$$

Hence

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{\sqrt{1 - x^2}}{x}.$$

The answer is **(C)**.

Q11 Fort St 2023 Trial Q22 · **2 marks**

Given that $\sin\theta = \dfrac{5}{13}$ and $\cos\theta < 0$, find the exact value of $\tan\theta$.

Criteria	Marks
Finds $\cos\theta = -\dfrac{12}{13}$ (negative root, since $\cos\theta < 0$)	1
Correct exact value $\tan\theta = -\dfrac{5}{12}$	1

Solution.

Using $\sin^2\theta + \cos^2\theta = 1$,

$$\cos^2\theta = 1 - \left(\frac{5}{13}\right)^2 = 1 - \frac{25}{169} = \frac{144}{169}.$$

So $\cos\theta = \pm\dfrac{12}{13}$. Since $\cos\theta < 0$ (and $\sin\theta > 0$), θ lies in the second quadrant, so $\cos\theta = -\dfrac{12}{13}$. Therefore

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{5/13}{-12/13} = -\frac{5}{12}.$$

Q12 SCEGGS 2022 Prelim Q11 · **2 marks**

Given that $\tan\theta = -\dfrac{8}{5}$ and $90° \le \theta \le 180°$, find the exact value of $\sin\theta$.

Criteria	Marks
Finds the hypotenuse $\sqrt{89}$ from the reference triangle	1
Chooses the correct sign (second quadrant): $\sin\theta = \dfrac{8}{\sqrt{89}}$	1

Solution.

For the reference triangle, tan has magnitude $\dfrac{8}{5}$, so take opposite 8 and adjacent 5; the hypotenuse is

$$\sqrt{8^2 + 5^2} = \sqrt{89}.$$

Since $90° \le \theta \le 180°$, θ is in the second quadrant, where sine is positive (and cosine negative, consistent with $\tan\theta < 0$). Hence

$$\sin\theta = \frac{8}{\sqrt{89}} = \frac{8\sqrt{89}}{89}.$$

Q13 Sydney Tech 2024 Trial Q11 · **2 marks**

Find the value of $\tan\theta$ if $\sin\theta = \dfrac{2}{\sqrt{19}}$ and θ is obtuse.

Criteria	Marks
Finds $\cos\theta$, taking the negative root for an obtuse angle: $\cos\theta = -\dfrac{\sqrt{15}}{\sqrt{19}}$	1
Correct value $\tan\theta = -\dfrac{2}{\sqrt{15}}$	1

Solution.

Using $\sin^2\theta + \cos^2\theta = 1$,

$$\cos^2\theta = 1 - \left(\frac{2}{\sqrt{19}}\right)^2 = 1 - \frac{4}{19} = \frac{15}{19}.$$

Since θ is obtuse (second quadrant), cosine is negative:

$$\cos\theta = -\frac{\sqrt{15}}{\sqrt{19}}.$$

Therefore

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{2/\sqrt{19}}{-\sqrt{15}/\sqrt{19}} = -\frac{2}{\sqrt{15}} = -\frac{2\sqrt{15}}{15}.$$

Q14 Knox 2023 Prelim Q30 · **2 marks**

Given that $\sin\theta = \dfrac{4}{5}$, where θ is an obtuse angle, find the exact value of $\cos\theta$.

Criteria	Marks
Uses $\sin^2\theta + \cos^2\theta = 1$ to obtain $\cos^2\theta = \dfrac{9}{25}$	1
Chooses the negative root for an obtuse angle: $\cos\theta = -\dfrac{3}{5}$	1

Solution.

Using $\sin^2\theta + \cos^2\theta = 1$,

$$\cos^2\theta = 1 - \left(\frac{4}{5}\right)^2 = 1 - \frac{16}{25} = \frac{9}{25}.$$

So $\cos\theta = \pm\dfrac{3}{5}$. Since θ is obtuse (second quadrant), cosine is negative, so

$$\cos\theta = -\frac{3}{5}.$$

Q15 Killara 2024 Prelim Q8 · **1 mark**

If $\cot\theta = \dfrac{24}{7}$ and $\pi < \theta \leq \dfrac{3\pi}{2}$, find the exact value of $\sin\theta$. (multiple choice)

Criteria	**Marks**
Correct option (B)	1

Solution.

Since $\cot\theta = \dfrac{\cos\theta}{\sin\theta} = \dfrac{24}{7}$, the reference triangle has the side associated with cosine equal to 24 and the side associated with sine equal to 7, giving hypotenuse

$$\sqrt{24^2 + 7^2} = \sqrt{625} = 25.$$

The interval $\pi < \theta \leq \dfrac{3\pi}{2}$ is the third quadrant, where sine is negative. Hence

$$\sin\theta = -\frac{7}{25}.$$

The answer is **(B)**.

Q16 St George Girls 2023 Prelim Q13 · **3 marks**

If $\cos\theta = -\dfrac{9}{15}$ and $180° < \theta < 360°$, find the exact value of $\tan\theta$ and the possible value/s for θ to the nearest degree.

Criteria	**Marks**
Identifies the correct quadrant (third) from $\cos\theta < 0$ and the given interval	1
Correct exact value $\tan\theta = \dfrac{4}{3}$	1
Correct angle $\theta \approx 233°$	1

Solution.

First simplify $\cos\theta = -\dfrac{9}{15} = -\dfrac{3}{5}$. Since $\cos\theta < 0$, θ lies in the second or third quadrant; the interval $180° < \theta < 360°$ rules out the second quadrant, so θ is in the **third quadrant**, where both sine and cosine are negative and tangent is positive.

Using $\sin^2\theta + \cos^2\theta = 1$,

$$\sin^2\theta = 1 - \frac{9}{25} = \frac{16}{25} \quad \Rightarrow \quad \sin\theta = -\frac{4}{5} \text{ (negative in the third quadrant).}$$

Hence

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-4/5}{-3/5} = \frac{4}{3}.$$

The related (acute) angle is

$$\cos^{-1}\left(\frac{3}{5}\right) = 53°08' \ (\approx 53°).$$

In the third quadrant,

$$\theta = 180° + 53°08' \approx 233° \text{ (nearest degree).}$$

3. Simplifying with related and complementary angles

Q17 Sydney Boys 2025 Trial Q1 · **1 mark**

Which of the following is the same as $\operatorname{cosec}(\pi + \theta)$? (multiple choice)

Criteria	Marks
Correct option (A)	1

Solution.

By definition $\operatorname{cosec}(\pi + \theta) = \dfrac{1}{\sin(\pi + \theta)}$. Since $\pi + \theta$ is a third-quadrant angle, $\sin(\pi + \theta) = -\sin\theta$, so

$$\operatorname{cosec}(\pi + \theta) = \frac{1}{-\sin\theta} = \frac{-1}{\sin\theta}.$$

The answer is **(A)**.

Q18 Hurlstone 2021 Prelim Q3 · **1 mark**

When simplified, $\sin(180^\circ + \theta) + \cos(360^\circ - \theta)$ is equal to which of the following? (multiple choice)

Criteria	Marks
Correct option (D)	1

Solution.

Using the related-angle results, $\sin(180^\circ + \theta) = -\sin\theta$ (third quadrant) and $\cos(360^\circ - \theta) = \cos\theta$ (fourth quadrant). Hence

$$\sin(180^\circ + \theta) + \cos(360^\circ - \theta) = -\sin\theta + \cos\theta.$$

The answer is **(D)**.

Q19 Loreto Kirribilli 2022 Prelim Q19 · **1 mark**

Simplify $\sin(-\theta)\cos(90^\circ - \theta)$.

Criteria	Marks
Correct simplification $-\sin^2\theta$	1

Solution.

Since sine is odd, $\sin(-\theta) = -\sin\theta$, and by the complementary-angle result $\cos(90^\circ - \theta) = \sin\theta$. Therefore

$$\sin(-\theta)\cos(90^\circ - \theta) = (-\sin\theta)(\sin\theta) = -\sin^2\theta.$$

Q20 Knox 2022 Prelim Q13 · **1 mark**

Simplify $\dfrac{\sin(2\pi-\theta)}{\sin\left(\frac{\pi}{2}-\theta\right)}$.

Criteria	Marks
Correct simplification $-\tan\theta$	1

Solution.

In the numerator, $\sin(2\pi-\theta) = -\sin\theta$ (fourth quadrant), and in the denominator $\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta$ (complementary angle). Therefore

$$\frac{\sin(2\pi-\theta)}{\sin\left(\frac{\pi}{2}-\theta\right)} = \frac{-\sin\theta}{\cos\theta} = -\tan\theta.$$

Q21 Hornsby Girls 2021 Prelim Q10 · **2 marks**

Simplify $\tan(2\pi-x)\operatorname{cosec}(\pi-x)$.

Criteria	Marks
Simplifies $\tan(2\pi-x) = -\tan x$ and $\operatorname{cosec}(\pi-x) = \operatorname{cosec} x$	1
Reaches $-\sec x$	1

Solution.

Using the related-angle results, $\tan(2\pi-x) = -\tan x$ (fourth quadrant) and $\sin(\pi-x) = \sin x$, so $\operatorname{cosec}(\pi-x) = \dfrac{1}{\sin(\pi-x)} = \dfrac{1}{\sin x} = \operatorname{cosec} x$. Hence

$$\tan(2\pi-x)\operatorname{cosec}(\pi-x) = (-\tan x)\left(\frac{1}{\sin x}\right) = -\frac{\sin x}{\cos x}\cdot\frac{1}{\sin x} = -\frac{1}{\cos x} = -\sec x.$$

Q22 Sydney Girls 2024 Trial Q9 · **1 mark**

Simplify $\cot(270^\circ + x^\circ)\cos(-x^\circ) + \cos(90^\circ + x^\circ)$. (multiple choice)

Criteria	**Marks**
Correct option (A)	1

Solution.

Take each related angle in turn. For $\cot(270^\circ + x^\circ)$, the angle $270^\circ + x^\circ$ is in the fourth quadrant where tangent (and cotangent) is negative, and 270° interchanges sine and cosine, so $\cot(270^\circ + x^\circ) = -\tan x^\circ$. Cosine is even, so $\cos(-x^\circ) = \cos x^\circ$. Finally $\cos(90^\circ + x^\circ) = -\sin x^\circ$ (second quadrant). Substituting,

$$\cot(270^\circ + x^\circ)\cos(-x^\circ) + \cos(90^\circ + x^\circ) = (-\tan x^\circ)(\cos x^\circ) + (-\sin x^\circ).$$

Since $\tan x^\circ \cos x^\circ = \dfrac{\sin x^\circ}{\cos x^\circ} \cdot \cos x^\circ = \sin x^\circ$, this becomes

$$-\sin x^\circ - \sin x^\circ = -2\sin x^\circ.$$

The answer is **(A)**.