Year 11 Mathematics Advanced

Topic: Working with Functions (12 of 12)

Review Questions (Worked Solutions): Mixed One-off Exam Questions

Term 3, 2026

Mixed one-off exam questions

Q1 Girraween 2023 Prelim Q6(a) · **1 mark**

Evaluate $\dfrac{2.13 - 29}{4.01^2 - 0.8^2}$ to 3 significant figures.

Criteria	Marks
Correct value to 3 significant figures	1

Solution.

$$\frac{2.13 - 29}{4.01^2 - 0.8^2} = \frac{-26.87}{16.0801 - 0.64} = \frac{-26.87}{15.4401} = -1.7402\ldots \approx -1.74.$$

Q2 Sydney Girls 2020 Trial Q25 · **3 marks**

(a) Sketch $y = |x|$ and $y = 1 - x^2$ on the same axes. (b) Shade the region where both $y \geq |x|$ and $y \leq 1 - x^2$.

Criteria	Marks
Correct V-shape $y = \|x\|$ with vertex at the origin	1
Correct parabola $y = 1 - x^2$ (y-intercept 1, x-intercepts ± 1)	1
Shades the lens-shaped region between the two curves	1

Solution.

(a) $y = |x|$ is a V-shape with vertex $(0, 0)$. $y = 1 - x^2$ is a concave-down parabola with vertex $(0, 1)$ and x-intercepts $x = \pm 1$.

(b) The two curves meet where $|x| = 1 - x^2$. For $x \geq 0$: $x = 1 - x^2 \Rightarrow x^2 + x - 1 = 0 \Rightarrow x = \dfrac{-1 + \sqrt{5}}{2}$; by symmetry the other meeting point is $x = -\dfrac{-1 + \sqrt{5}}{2}$. The required region ($y$ above the V and below the parabola) is the lens between the two curves for $-\dfrac{\sqrt{5} - 1}{2} \leq x \leq \dfrac{\sqrt{5} - 1}{2}$.

Q3 Barker 2021 Trial Q33(a) · **1 mark**

Deliah is in the water 1 km off the coast and wants to get to a town 3 km down the coast. She needs to swim to shore and then walk to the town. Deliah can swim at 2 km/hour and walk at 4 km/hour.

How long will it take Deliah to swim the 1 km to shore and then walk the 3 km to town?

Criteria	**Marks**
Correct total time $\frac{5}{4}$ hour = 1 h 15 min (75 minutes)	1

Solution.

Using time = $\dfrac{\text{distance}}{\text{speed}}$ for each leg:

$$\text{swim: } \frac{1}{2}\text{ h}, \qquad \text{walk: } \frac{3}{4}\text{ h}.$$

Total time = $\frac{1}{2} + \frac{3}{4} = \frac{5}{4}$ hour = 1 hour 15 minutes (75 minutes).

Q4 Sydney Grammar 2021 Prelim Q4 · **1 mark**

Which of the following describes the relation $y = |x - 3|$?

(A) one-to-one

(B) one-to-many

(C) many-to-one

(D) many-to-many

Criteria	**Marks**
Correct option (C)	1

Solution.

The graph of $y = |x - 3|$ is a "V" with vertex at $(3, 0)$. Each input x gives exactly one output y (it is a function), so it is not one-to-many or many-to-many. Two different inputs can give the same output, e.g. $x = 2$ and $x = 4$ both give $y = 1$, so it is many-to-one. The answer is (C).

Q5 Sydney Grammar 2022 Trial Q29 · **2 marks**

Consider the exact values $\sin\dfrac{\pi}{8}=\dfrac{\sqrt{2-\sqrt{2}}}{2}$ and $\cos\dfrac{\pi}{8}=\dfrac{\sqrt{2+\sqrt{2}}}{2}$.

Show that $-\sin\dfrac{\pi}{8}-\cos\dfrac{\pi}{8}=-\dfrac{\sqrt{4+2\sqrt{2}}}{2}$.

Criteria	**Marks**
Computes $\left(\sin\frac{\pi}{8}+\cos\frac{\pi}{8}\right)^2=\dfrac{4+2\sqrt{2}}{4}$ (squares the sum and simplifies the cross term)	1
Takes the positive square root and concludes $-\sin\frac{\pi}{8}-\cos\frac{\pi}{8}=-\dfrac{\sqrt{4+2\sqrt{2}}}{2}$	1

Solution.

Work with the sum $S=\sin\dfrac{\pi}{8}+\cos\dfrac{\pi}{8}$ and square it:

$$S^2=\sin^2\tfrac{\pi}{8}+2\sin\tfrac{\pi}{8}\cos\tfrac{\pi}{8}+\cos^2\tfrac{\pi}{8}.$$

Now $\sin^2\frac{\pi}{8}=\dfrac{2-\sqrt{2}}{4}$ and $\cos^2\frac{\pi}{8}=\dfrac{2+\sqrt{2}}{4}$, so $\sin^2\frac{\pi}{8}+\cos^2\frac{\pi}{8}=1$. For the cross term,

$$2\sin\tfrac{\pi}{8}\cos\tfrac{\pi}{8}=2\cdot\frac{\sqrt{2-\sqrt{2}}}{2}\cdot\frac{\sqrt{2+\sqrt{2}}}{2}=\frac{\sqrt{(2-\sqrt{2})(2+\sqrt{2})}}{2}=\frac{\sqrt{4-2}}{2}=\frac{\sqrt{2}}{2}.$$

Hence $S^2=1+\dfrac{\sqrt{2}}{2}=\dfrac{4+2\sqrt{2}}{4}$. Both $\sin\frac{\pi}{8}$ and $\cos\frac{\pi}{8}$ are positive, so $S>0$ and

$$S=\sqrt{\frac{4+2\sqrt{2}}{4}}=\frac{\sqrt{4+2\sqrt{2}}}{2}.$$

Therefore $-\sin\dfrac{\pi}{8}-\cos\dfrac{\pi}{8}=-S=-\dfrac{\sqrt{4+2\sqrt{2}}}{2}$, as required.

Q6 Sydney Boys 2023 Trial Q11 · **2 marks**

Make b the subject of the formula $a(b+5) = 2a + b$.

Criteria	Marks
Expands and collects the b terms on one side: $b(a-1) = -3a$	1
Correct subject $b = \dfrac{-3a}{a-1} = \dfrac{3a}{1-a}$	1

Solution.

Expand the left side and gather the terms in b:

$$ab + 5a = 2a + b \implies ab - b = 2a - 5a \implies b(a-1) = -3a.$$

Divide by $(a-1)$ (with $a \neq 1$):

$$b = \frac{-3a}{a-1} = \frac{3a}{1-a}.$$

Q7 Penrith 2022 Prelim Q18 · **2 marks**

Describe the following solution set using interval notation.

Diagram to be inserted:
`adv-working-with-functions-d12-q7.png`

Criteria	Marks
Reads the endpoint(s) from the number line and correctly identifies open ($\circ$) versus closed ($\bullet$) ends	1
Writes the set correctly in interval notation, using round brackets for open/infinite ends and square brackets for included endpoints	1

Solution.

Read the diagram from left to right. A filled dot ($\bullet$) at an endpoint means that value is included (use a square bracket); an open dot ($\circ$) means it is excluded (use a round bracket); an arrow means the set continues to $\pm\infty$ (always a round bracket at the infinite end). Translate the shaded portion of the number line into interval form on that basis, e.g. a ray shaded from a (included) to the right is written $[a, \infty)$, and a segment shaded strictly between a and b is written (a, b).

Q8 Penrith 2022 Prelim Q28 · **2 marks**

Explain the difference between a one-to-one function and a many-to-one function, and provide an example of each.

Criteria	Marks
Correctly explains the difference (one-to-one: distinct inputs give distinct outputs / each output from exactly one input; many-to-one: at least two different inputs share an output)	1
Gives a correct example of each type of function	1

Solution.

In a *one-to-one* function, different inputs always give different outputs: no y-value is produced by more than one x-value, so the graph passes the horizontal line test. In a *many-to-one* function, at least two different inputs give the same output, so some horizontal line meets the graph more than once. (Both are functions, so every x still gives exactly one y.)

Examples. $f(x) = 2x + 1$ is one-to-one: if $f(p) = f(q)$ then $2p + 1 = 2q + 1$, so $p = q$. $g(x) = x^2$ is many-to-one: $g(-2) = g(2) = 4$, so two different inputs share the output 4.

Q9 Manly 2024 Trial Q12 · **2 marks**

The domains of two continuous functions, $f(x)$ and $g(x)$, are $[-3, 2)$ and $(0, 6]$ respectively. What is the domain of the sum function $f(x) + g(x)$?

Criteria	Marks
Recognises the domain of a sum is the intersection of the two domains	1
Correct domain $(0, 2)$ (open at both ends)	1

Solution.

The sum $f(x) + g(x)$ is defined only where *both* f and g are defined, i.e. on the intersection of the two domains:

$$[-3, 2) \cap (0, 6].$$

The overlap starts just after 0 (excluded, since g's domain is open at 0) and ends just before 2 (excluded, since f's domain is open at 2). Hence the domain of $f(x) + g(x)$ is

$$(0, 2).$$

Q10 Hurlstone 2021 Prelim Q15 · **1 mark**

For what values of x is the function $f(x) = \dfrac{1}{x^2 - 4x}$ not continuous?

(A) $x = 0,\ x = 4$

(B) $0 < x < 4$

(C) $x < 0,\ x > 4$

(D) Continuous for all values of x

Criteria	**Marks**
Correct option (A)	1

Solution.

A rational function is discontinuous exactly where its denominator is zero. Solve

$$x^2 - 4x = 0 \implies x(x-4) = 0 \implies x = 0 \text{ or } x = 4.$$

At these two values $f(x)$ is undefined, so it is not continuous there. The answer is **(A)**.

Q11 Baulkham Hills 2021 Prelim Q9 · **1 mark**

Below is an incomplete graph of the relation $y = f(x)$. What lies on the left of the y-axis has been obstructed from view.

Diagram to be inserted:
`adv-working-with-functions-d12-q11.png`

Given that $f(x)$ is an odd function, what type of relation is $y = f(x)$?

(A) One-to-one

(B) One-to-many

(C) Many-to-one

(D) Many-to-many

Criteria	**Marks**
Correct option (C)	1

Solution.

An odd function has 180° rotational symmetry about the origin, so the hidden left portion is the point-reflection of the visible right portion. The visible part already shows different x-values sharing the same y-value (a horizontal line would cut it more than once), so several inputs map to the same output: the relation is *many-to-one*. Because each x still gives exactly one y (it is a function), it is not one-to-many or many-to-many. The answer is **(C)**.

Q12 Hurlstone 2024 Prelim Q2 · **1 mark**

Given a set of coordinates $(-4,-2)$, $(-4,-5)$, $(-1,0)$, $(3,-2)$ and $(5,-1)$, which of the following best describes this relation?

(A) One-to-one

(B) One-to-many

(C) Many-to-one

(D) Many-to-many

Criteria	**Marks**
Correct option (D)	1

Solution.

Check both directions. The input $x = -4$ maps to two different outputs, $y = -2$ and $y = -5$, so one x-value gives many y-values (a "one-to-many" feature). The output $y = -2$ comes from two different inputs, $x = -4$ and $x = 3$, so many x-values share one y-value (a "many-to-one" feature). Since both features occur, the relation is *many-to-many*. The answer is **(D)**.

Q13 Baulkham Hills 2021 Prelim Q5 · **1 mark**

The points $P(3,1)$, $Q(2,5)$ and $R(a,b)$ are collinear. Which of the following statements is correct?

(A) $a+4b=7$

(B) $a-4b=-1$

(C) $4a+b=13$

(D) $4a-b=11$

Criteria	**Marks**
Correct option (C)	1

Solution.

Collinear means all three points share one gradient. The gradient of PQ is

$$m_{PQ}=\frac{5-1}{2-3}=\frac{4}{-1}=-4.$$

Setting the gradient of PR equal to this,

$$\frac{b-1}{a-3}=-4 \implies b-1=-4(a-3)=-4a+12 \implies 4a+b=13.$$

The answer is **(C)**.

Q14 Killara 2023 Trial Q14(b) · **2 marks**

The owners of Wilson Tennis Racquets use the equation $pq = T$, where p is the price of a racquet, q is the quantity sold per year and T is a constant. Given that a racquet priced at $p = \$100$ sold a quantity of $q = 4000$ (so that $T = 400\,000$), sketch the graph of the demand curve with q on the horizontal axis and p on the vertical axis.

Criteria	**Marks**
Correct shape: a rectangular hyperbola $p = \dfrac{400\,000}{q}$ in the first quadrant (decreasing, both axes as asymptotes)	1
Even scale with at least one point labelled, e.g. $(4000, 100)$	1

Solution.

With $T = 400\,000$ the demand relation is

$$p = \frac{400\,000}{q}, \qquad q > 0.$$

This is a rectangular hyperbola. Since price and quantity are positive, only the branch in the first quadrant is drawn. It is a decreasing curve: as q increases p falls, approaching the q-axis ($p \to 0$) but never touching it; as $q \to 0^+$ the price rises without bound, so the p-axis is a vertical asymptote. Plot and label a known point such as $(4000, 100)$ (and, e.g., $(1000, 400)$) and draw the smooth hyperbola through it. *(Common error: a straight line instead of a hyperbola.)*

Q15 Sydney Tech 2021 Trial Q38(a) · **2 marks**

Two straight roads meet at R at an angle of 90°. At time $t = 0$ car A leaves R on one road, and car B is 100 km from R on the other road. Car A travels away from R at a speed of 80 km/h, and car B travels towards R at a speed of 60 km/h. The distance between the cars at time t hours is r km. Show that $r^2 = 2000(5t^2 - 6t + 5)$.

Criteria	**Marks**
Correct distances along each road: car A is $80t$ from R, car B is $100 - 60t$ from R, and applies Pythagoras	1
Expands and factorises to obtain $r^2 = 2000(5t^2 - 6t + 5)$	1

Solution.

After t hours car A has travelled $80t$ km from R along one road, and car B is $100 - 60t$ km from R along the perpendicular road. Because the roads meet at 90°, the two cars and R form a right angle at R, so by Pythagoras

$$r^2 = (80t)^2 + (100 - 60t)^2.$$

Expanding,

$$r^2 = 6400t^2 + \left(10000 - 12000t + 3600t^2\right) = 10000t^2 - 12000t + 10000.$$

Taking out the common factor 2000,

$$r^2 = 2000\left(5t^2 - 6t + 5\right),$$

as required.

Q16 North Sydney Girls 2020 Trial Q7 · **1 mark**

The maximum speed of a train going up a hill is inversely proportional to the square root of its weight. A train weighing 3600 tonnes can go up a hill at 30 km/h. What is the maximum speed at which a train weighing 2500 tonnes could go up the same hill?

(A) 43.2 km/h

(B) 36 km/h

(C) 25 km/h

(D) 20.8 km/h

Criteria	Marks
Correct option (B)	1

Solution.

"Speed inversely proportional to the square root of weight" means

$$v = \frac{k}{\sqrt{w}}.$$

Use the given data $v = 30$ when $w = 3600$: since $\sqrt{3600} = 60$,

$$30 = \frac{k}{60} \Rightarrow k = 1800.$$

For $w = 2500$ (with $\sqrt{2500} = 50$),

$$v = \frac{1800}{50} = 36 \text{ km/h}.$$

The answer is **(B)**.

Q17 Abbotsleigh 2024 Prelim Q18(b) · **1 mark**

A function is defined by $f(x) = 2x$ for $x > 1$, $f(x) = 1 - x^2$ for $-1 \leq x \leq 1$, and $f(x) = -(2 + x)$ for $x < -1$. Explain why this is no longer a function if the third part is changed to $-(2 + x)$ for $x \leq 1$.

Criteria	Marks
Identifies that the domains of the second and third branches then overlap, so some x-values map to two outputs (fails the vertical line test)	1

Solution.

The second branch already covers $-1 \leq x \leq 1$, and the amended third branch $-(2 + x)$ now also covers $x \leq 1$. On the overlap $-1 \leq x \leq 1$ each input has two different outputs (for example at $x = 0$, $1 - x^2 = 1$ but $-(2 + x) = -2$). A function assigns exactly one output to each input, so this relation fails the vertical line test and is no longer a function.

Q18 Girraween 2023 Prelim Q8(a) · **6 marks**

(i) Find $f(2) + f(0)$. (ii) Find the values of the x-intercepts of $f(x)$. (iii) Sketch the function showing all intercepts with the axes.

Criteria	Marks
(i) Correct value $f(2) + f(0) = -1$	1
(ii) Solves each branch on its own domain	1
(ii) Correct x-intercepts $x = -1,\ 1,\ 2$	1
(iii) Correct shape of each branch on its domain	2
(iii) All intercepts and the branch endpoints marked correctly	1

Solution.

(i) Since $2 > 1$, use $f(x) = 4 - x^2$: $f(2) = 4 - 4 = 0$. Since $0 \leq 1$, use $f(x) = x^2 - 1$: $f(0) = 0 - 1 = -1$. Hence

$$f(2) + f(0) = 0 + (-1) = -1.$$

(ii) Set each branch to zero on its own domain. For $x \leq 1$: $x^2 - 1 = 0 \Rightarrow x = \pm 1$, and both -1 and 1 satisfy $x \leq 1$. For $x > 1$: $4 - x^2 = 0 \Rightarrow x = \pm 2$, and only $x = 2$ satisfies $x > 1$. Hence the x-intercepts are $x = -1$, $x = 1$ and $x = 2$. **(iii)** For $x \leq 1$ draw the upward parabola $y = x^2 - 1$ (vertex $(0, -1)$), ending at the included point $(1, 0)$. For $x > 1$ draw the downward parabola $y = 4 - x^2$, starting just after $x = 1$ (where $y \to 3$) and cutting the x-axis at $x = 2$. Mark the x-intercepts $(-1, 0)$, $(1, 0)$, $(2, 0)$ and the y-intercept $(0, -1)$.

Q19 St George Girls 2021 Prelim Q8(a)(i) · **1 mark**

Let $f(x) = |x - 2|$. Explain why this function is continuous at $x = 2$.

Criteria	Marks
States the left-hand and right-hand values agree with $f(2)$ (no break/jump), so f is continuous at $x = 2$	1

Solution.

As $x \to 2^-$, $f(x) = |x - 2| \to 0$, and as $x \to 2^+$, $f(x) = |x - 2| \to 0$, while $f(2) = |2 - 2| = 0$. The limit from both sides equals the function value, so there is no jump or hole: the graph can be drawn through $(2, 0)$ without lifting the pen. Hence f is continuous at $x = 2$.

Q20 Knox 2022 Prelim Q14(c) · **2 marks**

Find the exact length of the hypotenuse x of a right-angled triangle with shorter sides $3-\sqrt{5}$ and $\sqrt{3}+\sqrt{15}$.

Criteria	Marks
Applies Pythagoras and expands both squares	1
Simplifies to $x = 4\sqrt{2}$	1

Solution.

By Pythagoras,

$$x^2 = (3-\sqrt{5})^2 + (\sqrt{3}+\sqrt{15})^2.$$

Now $(3-\sqrt{5})^2 = 9-6\sqrt{5}+5 = 14-6\sqrt{5}$, and $(\sqrt{3}+\sqrt{15})^2 = 3+2\sqrt{45}+15 = 18+6\sqrt{5}$ (since $\sqrt{45} = 3\sqrt{5}$). Adding,

$$x^2 = (14-6\sqrt{5}) + (18+6\sqrt{5}) = 32,$$

so $x = \sqrt{32} = 4\sqrt{2}$.

Q21 Penrith 2023 Prelim Q14 · **5 marks**

(a) Expand $\left(p-\frac{1}{p}\right)^2$. (b) Show $p-\frac{1}{p} = -6\sqrt{2}$ for $p = \sqrt{19}-3\sqrt{2}$. (c) Hence find $p^2+\frac{1}{p^2}$.

Criteria	Marks
(a) Correct expansion $p^2-2+\dfrac{1}{p^2}$	1
(b) Rationalises $\dfrac{1}{p}$ to $\sqrt{19}+3\sqrt{2}$	1
(b) Shows $p-\dfrac{1}{p} = -6\sqrt{2}$	1
(c) Uses part (a) with the result of (b)	1
(c) Correct value 74	1

Solution.

(a) $\left(p-\dfrac{1}{p}\right)^2 = p^2-2\cdot p\cdot\dfrac{1}{p}+\dfrac{1}{p^2} = p^2-2+\dfrac{1}{p^2}$. **(b)** $\dfrac{1}{p} = \dfrac{1}{\sqrt{19}-3\sqrt{2}} = \dfrac{\sqrt{19}+3\sqrt{2}}{(\sqrt{19})^2-(3\sqrt{2})^2} = \dfrac{\sqrt{19}+3\sqrt{2}}{19-18} = \sqrt{19}+3\sqrt{2}$. Then

$$p-\frac{1}{p} = (\sqrt{19}-3\sqrt{2}) - (\sqrt{19}+3\sqrt{2}) = -6\sqrt{2}.$$

(c) From (a), $p^2+\dfrac{1}{p^2} = \left(p-\dfrac{1}{p}\right)^2+2 = (-6\sqrt{2})^2+2 = 72+2 = 74$.

Q22 North Sydney Girls 2023 Trial Q28 · **5 marks**

(a) List the transformations taking $f(x)$ to $g(x)$. (b) With $u(x) = f(x)g(x)$, find $u'(1)$.

Criteria	Marks
(a) Identifies horizontal reflection and horizontal shift	1
(a) Identifies vertical reflection	1
(a) Identifies vertical shift, full correct sequence	1
(b) Applies product rule $u'(x) = f'(x)g(x) + f(x)g'(x)$	1
(b) Reads values from graph and evaluates $u'(1) = -10.5$	1

Solution.

(a) Reading the two graphs, $g(x)$ is obtained from $f(x)$ by, in turn:

- a reflection in the y-axis (horizontal reflection),
- a horizontal translation to the right,
- a reflection in the x-axis (vertical reflection),
- a vertical translation upwards.

(That is, $g(x) = -f\big(-(x-h)\big) + k$ for the shift amounts read from the diagram.)

(b) By the product rule,

$$u'(x) = f'(x)\,g(x) + f(x)\,g'(x).$$

Reading the gradients and values at $x = 1$ from the graph gives

$$u'(1) = f'(1)\,g(1) + f(1)\,g'(1) = (-3)(4.5) + 3 = -13.5 + 3 = -10.5.$$

Q23 Baulkham Hills 2021 Prelim Q10 · **1 mark**

The lines $ax + by + c = 0$ and $dx - ey - f = 0$ (all coefficients real, non-zero) are perpendicular. Which statement is correct?

Criteria	Marks
Correct option (A)	1

Solution.

The gradients are $-\dfrac{a}{b}$ (from $ax + by + c = 0$) and $\dfrac{d}{e}$ (from $dx - ey - f = 0$, i.e. $y = \frac{d}{e}x - \frac{f}{e}$). Perpendicular lines satisfy $m_1 m_2 = -1$:

$$\left(-\frac{a}{b}\right)\left(\frac{d}{e}\right) = -1 \;\Rightarrow\; \frac{ad}{be} = 1 \;\Rightarrow\; ad = be \;\Rightarrow\; ad - be = 0.$$

The answer is **(A)**.

Q24 2022 HSC Q31(a) · **2 marks**

A line through $P(1, 2)$ meets the axes at $X(x, 0)$ and $Y(0, y)$, with $x > 1$. Show that $y = \dfrac{2x}{x - 1}$.

Criteria	Marks
Uses collinearity of X, P, Y (equal gradients, or the intercept form $\frac{X}{x} + \frac{Y}{y} = 1$ through P)	1
Correctly rearranges to $y = \dfrac{2x}{x - 1}$	1

Solution.

The line has x-intercept x and y-intercept y, so in intercept form $\dfrac{X}{x} + \dfrac{Y}{y} = 1$. Since $P(1, 2)$ lies on the line,

$$\frac{1}{x} + \frac{2}{y} = 1 \;\Rightarrow\; \frac{2}{y} = 1 - \frac{1}{x} = \frac{x - 1}{x} \;\Rightarrow\; y = \frac{2x}{x - 1},$$

as required. (Equivalently, equate the gradients of XP and PY: $\dfrac{2 - 0}{1 - x} = \dfrac{y - 2}{0 - 1}$, which rearranges to the same result.)

Q25 Hornsby Girls 2023 Trial Q28(b) · **2 marks**

The parabola $x = ay^2$ passes through $(0, 0)$ and $(1, \sqrt{3})$. Find a and show the upper half has equation $y = \sqrt{3}\,\sqrt{x}$.

Criteria	Marks
Substitutes $(1, \sqrt{3})$ to find $a = \frac{1}{3}$	1
Rearranges to $y^2 = 3x$ and takes the positive root to give $y = \sqrt{3}\,\sqrt{x}$	1

Solution.

Substituting the point $(1, \sqrt{3})$ into $x = ay^2$: $1 = a(\sqrt{3})^2 = 3a$, so $a = \dfrac{1}{3}$. Then $x = \dfrac{y^2}{3}$, i.e. $y^2 = 3x$. Taking the positive square root for the upper half ($y \geq 0$),

$$y = \sqrt{3x} = \sqrt{3}\,\sqrt{x},$$

as required.

Q26 Fort St 2023 Trial Q11 · **3 marks**

Part of the graph of the function $y = \dfrac{a}{x-b} + c$ is shown below.

Diagram to be inserted:
`adv-working-with-functions-d12-q26.png`

Find the values of a, b and c.

Criteria	**Marks**
Reads the vertical asymptote $x = 3$, giving $b = 3$	1
Reads the horizontal asymptote $y = 2$, giving $c = 2$	1
Uses a point on the curve to find $a = -1$	1

Solution.

For a curve $y = \dfrac{a}{x-b} + c$ the vertical asymptote is $x = b$ and the horizontal asymptote is $y = c$. From the graph the branches blow up either side of $x = 3$, so $b = 3$, and the curve levels off towards $y = 2$, so $c = 2$. Thus

$$y = \frac{a}{x-3} + 2.$$

To find a, substitute a clearly-marked point. The curve passes through $(4, 1)$:

$$1 = \frac{a}{4-3} + 2 \implies 1 = a + 2 \implies a = -1.$$

(As a check, at $x = 2$ the rule gives $y = \dfrac{-1}{-1} + 2 = 3$, matching the graph.) Hence $a = -1$, $b = 3$, $c = 2$.

Q27 Hurlstone 2022 Prelim Q12(c) · **5 marks**

(i) Show $\dfrac{2x^2+x-1}{x^2-1} = 2 + \dfrac{1}{x-1}$. (ii) Sketch $y = \dfrac{2x^2+x-1}{x^2-1}$, labelling intercepts, discontinuities and key features.

Criteria	**Marks**
Factorises and simplifies the numerator over x^2-1	1
Reaches $2 + \dfrac{1}{x-1}$	1
Correct shape with asymptotes $x=1$ and $y=2$	1
Intercepts labelled: $(0,1)$ and $\left(\frac{1}{2},0\right)$	1
Removable discontinuity (open circle) at $\left(-1,\frac{3}{2}\right)$	1

Solution.

(i) Factor the numerator: $2x^2+x-1 = (2x-1)(x+1)$, and $x^2-1 = (x-1)(x+1)$. So

$$\frac{2x^2+x-1}{x^2-1} = \frac{(2x-1)(x+1)}{(x-1)(x+1)} = \frac{2x-1}{x-1} = \frac{2(x-1)+1}{x-1} = 2 + \frac{1}{x-1}.$$

(ii) The graph is $y = 2 + \dfrac{1}{x-1}$: a hyperbola with vertical asymptote $x=1$ and horizontal asymptote $y=2$. y-intercept: $x=0 \Rightarrow y = 2-1 = 1$, i.e. $(0,1)$. x-intercept: $y=0 \Rightarrow \dfrac{1}{x-1} = -2 \Rightarrow x = \frac{1}{2}$, i.e. $\left(\frac{1}{2},0\right)$. Because the original function is undefined at $x=-1$ (the cancelled factor $x+1$), there is a removable discontinuity: an open circle at $\left(-1,\ 2+\dfrac{1}{-2}\right) = \left(-1,\frac{3}{2}\right)$.

Q28 Penrith 2022 Prelim Q30 · **3 marks**

A cyclist goes up a long slope at a constant speed of 15 km/h. He turns around and comes down the slope at a constant speed of 40 km/h. Find his average speed over a full circuit.

Criteria	**Marks**
Writes average speed as $\dfrac{\text{total distance}}{\text{total time}}$ with the two times $\dfrac{d}{15}$ and $\dfrac{d}{40}$	1
Forms $\dfrac{2d}{\frac{d}{15}+\frac{d}{40}}$ and cancels d	1
Correct average speed $\dfrac{240}{11} \approx 21.8$ km/h	1

Solution.

Let the slope have length d km. Time up $= \dfrac{d}{15}$ h and time down $= \dfrac{d}{40}$ h, over a total distance of $2d$ km. Average speed is total distance over total time:

$$\text{avg} = \frac{2d}{\frac{d}{15}+\frac{d}{40}} = \frac{2}{\frac{1}{15}+\frac{1}{40}} = \frac{2}{\frac{8+3}{120}} = \frac{2\times 120}{11} = \frac{240}{11} \approx 21.8 \text{ km/h}.$$

(The average speed is the harmonic mean of 15 and 40, not their arithmetic mean.)

Q29 Penrith 2022 Prelim Q20 · **3 marks**

A group of friends hire a bus for \$2100 to take them to a music festival. Seven friends can no longer make the festival and withdraw from the bus hire. The remaining friends must each pay an additional \$10 to cover the cost of the bus hire. How many friends originally agreed to go on the bus?

Criteria	**Marks**
Sets up the cost-per-person equation $\dfrac{2100}{n-7} - \dfrac{2100}{n} = 10$	1
Reduces to the quadratic $n^2 - 7n - 1470 = 0$	1
Solves and rejects the negative root: $n = 42$ friends	1

Solution.

Let n be the number who originally agreed. Originally each pays $\dfrac{2100}{n}$; after 7 withdraw each of the $n-7$ remaining pays $\dfrac{2100}{n-7}$, which is \$10 more:

$$\frac{2100}{n-7} - \frac{2100}{n} = 10.$$

Multiply through by $n(n-7)$:

$$2100n - 2100(n-7) = 10n(n-7) \implies 14700 = 10n^2 - 70n,$$

so $n^2 - 7n - 1470 = 0$, giving $(n-42)(n+35) = 0$. Rejecting $n = -35$, we get $n = 42$. (Check: 42 pay \$50 each; 35 remaining pay \$60 each, i.e. \$10 more.) So 42 friends originally agreed.

Q30 Sydney Boys 2022 Trial Q29(a) · 3 marks

Relative to a suitable set of coordinate axes, the line $2x-y+5=0$ intersects the circle $x^2+y^2-6x-2y-30=0$ at the points P and Q. Given that the coordinates of P are $(-3,-1)$, find the coordinates of Q.

Criteria	Marks
Substitutes $y = 2x + 5$ into the circle equation	1
Obtains $x^2 + 2x - 3 = 0$ (i.e. $5x^2 + 10x - 15 = 0$) and solves for x	1
Correct second point $Q = (1, 7)$	1

Solution.

From the line, $y = 2x + 5$. Substitute into the circle:

$$x^2 + (2x + 5)^2 - 6x - 2(2x + 5) - 30 = 0.$$

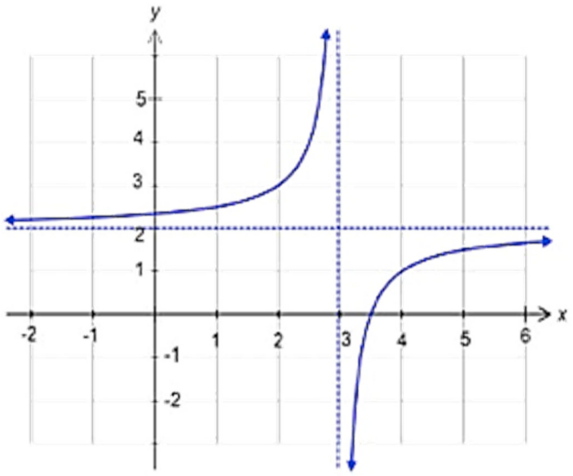
Expanding, $x^2 + 4x^2 + 20x + 25 - 6x - 4x - 10 - 30 = 0$, i.e.

$$5x^2 + 10x - 15 = 0 \implies x^2 + 2x - 3 = 0 \implies (x + 3)(x - 1) = 0.$$

So $x = -3$ (giving P) or $x = 1$. When $x = 1$, $y = 2(1) + 5 = 7$. Hence $Q = (1, 7)$.

Q31 Merewether 2025 Trial Q33(a) · **2 marks**

A circle centred at A with radius 1 unit intersects the parabola $y = 2x^2$ at the origin O and at the point B, as shown in the diagram. The line ℓ passes through O and B.



Show that the coordinates of B are $\left(\dfrac{\sqrt{3}}{2}, \dfrac{3}{2}\right)$.

Criteria	**Marks**
Uses centre $A(0,1)$, radius 1: circle $x^2 + (y-1)^2 = 1$, and substitutes $y = 2x^2$	1
Solves for the first-quadrant point: $x = \dfrac{\sqrt{3}}{2}$, $y = \dfrac{3}{2}$	1

Solution.

Since the circle has radius 1 and passes through O with centre A on the y-axis, $A = (0, 1)$ and the circle is

$$x^2 + (y-1)^2 = 1.$$

B also lies on $y = 2x^2$. Substituting,

$$x^2 + (2x^2 - 1)^2 = 1 \implies x^2 + 4x^4 - 4x^2 + 1 = 1 \implies 4x^4 - 3x^2 = 0,$$

so $x^2(4x^2 - 3) = 0$, giving $x = 0$ (the point O) or $x^2 = \dfrac{3}{4}$, i.e. $x = \pm\dfrac{\sqrt{3}}{2}$. From the diagram B is in the first quadrant, so $x = \dfrac{\sqrt{3}}{2}$ and $y = 2\left(\dfrac{\sqrt{3}}{2}\right)^2 = 2 \cdot \dfrac{3}{4} = \dfrac{3}{2}$. Hence $B = \left(\dfrac{\sqrt{3}}{2}, \dfrac{3}{2}\right)$, as required.

Q32 Normanhurst Boys 2021 Trial Q8 · **1 mark**

The circle $x^2 + y^2 - 4x - 4y + c = 0$ touches both the x-axis and the y-axis. What is the value of c?

(A) $c = 1$

(B) $c = 2$

(C) $c = 4$

(D) $c = 8$

Criteria	**Marks**
Correct option (C)	1

Solution.

Completing the square, $x^2 - 4x + y^2 - 4y + c = 0$ becomes $(x-2)^2 + (y-2)^2 = 8 - c$, so the centre is $(2, 2)$ and the radius is $\sqrt{8-c}$. A circle touches both axes when its radius equals the distance from the centre to each axis, i.e. $r = 2$. Then $8 - c = 4$, so $c = 4$. The answer is (C).

Q33 Manly 2023 Trial Q30(c) · **2 marks**

The graph of $y = h(x)$ is shown below. The graph is symmetrical about the line $x = 2$, and passes through $A(0, 3)$ and $B(2, 0)$.

Diagram to be inserted:
`adv-working-with-functions-d12-q33.png`

Find the number of solutions to the equation

$$h(x) + h(x + 2) = 3.$$

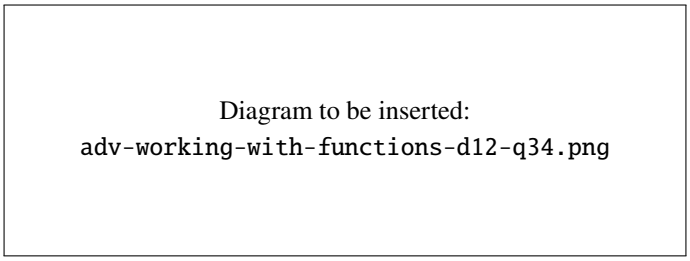
Criteria	**Marks**
Interprets $h(x + 2)$ as a horizontal shift of h left by 2 and adds the graphs (or the values)	1
Correct number of solutions: 2	1

Solution.

The equation $h(x) + h(x + 2) = 3$ is solved graphically. The graph of $y = h(x + 2)$ is the graph of h shifted 2 units to the left. Adding the two graphs point by point (ordinate addition) produces the graph of $y = h(x) + h(x + 2)$; the solutions of the equation are where this sum-graph meets the horizontal line $y = 3$. Reading the resulting graph, the line $y = 3$ cuts the sum-graph in exactly 2 places. Hence there are $\boxed{2}$ solutions.

Q34 Baulkham Hills 2023 Trial Q28(c) · **1 mark**

The graph below shows $y = \dfrac{9}{2x+1}$ together with $y = 5 - 2|x+2|$. The point of intersection of the two graphs is $(-5, -1)$.



Hence or otherwise solve

$$\frac{9}{2x+1} < 5 - 2|x+2|.$$

Criteria	**Marks**
Correct solution $-5 < x < -\dfrac{1}{2}$ (read from the graph, using the intersection $(-5,-1)$ and the asymptote $x = -\frac{1}{2}$)	1

Solution.

We want where the hyperbola $y = \dfrac{9}{2x+1}$ lies *below* the graph of $y = 5 - 2|x+2|$. The two curves meet only at $(-5,-1)$, and the hyperbola has a vertical asymptote at $2x+1 = 0$, i.e. $x = -\frac{1}{2}$. From the graph, the hyperbola is below the absolute-value graph exactly between the intersection point and the asymptote. Hence

$$-5 < x < -\tfrac{1}{2}.$$

(Check: at $x = -3$, LHS $= \dfrac{9}{-5} = -1.8$ and RHS $= 5 - 2 = 3$, so LHS $<$ RHS; at $x = 0$, LHS $= 9$ and RHS $= 1$, so the inequality fails.)

Q35 North Sydney Boys 2024 Trial Q31(i) · **2 marks**

The function $y = \dfrac{x}{x^2+1}$ is shown below. It is an odd function.

For what values of m do the line $y = mx$ and the curve $y = \dfrac{x}{x^2+1}$ enclose a region?

Criteria	**Marks**
Sets $mx = \dfrac{x}{x^2+1}$ and finds the non-zero intersections require $x^2 = \dfrac{1}{m} - 1$	1
Concludes three intersection points (hence an enclosed region) iff $0 < m < 1$	1

Solution.

Intersections satisfy $mx = \dfrac{x}{x^2+1}$, i.e. $x\left(m - \dfrac{1}{x^2+1}\right) = 0$. So $x = 0$ (the origin, always a solution) or

$$m = \frac{1}{x^2+1} \implies x^2 = \frac{1}{m} - 1 = \frac{1-m}{m}.$$

The line and curve enclose a region only when there are intersection points either side of the origin, i.e. two further real solutions, so we need $x^2 = \dfrac{1-m}{m} > 0$. This holds precisely when

$$0 < m < 1.$$

(For $m \le 0$ or $m \ge 1$ the line meets the curve only at the origin, so no region is enclosed.)

Q36 James Ruse 2025 Trial Q12(b) · **3 marks**

A circle has equation $x^2 + 4x + y^2 + 2y + 4 = 0$. The line $y = mx + 2$ is a tangent to the circle. Determine the values of m.

Criteria	**Marks**
Centre $(-2,-1)$ and radius 1 by completing the square	1
Applies the tangent (perpendicular-distance = radius) condition to form $3m^2 - 12m + 8 = 0$	1
Correct values $m = 2 \pm \dfrac{2\sqrt{3}}{3}$	1

Solution.

Completing the square: $(x+2)^2 + (y+1)^2 = 4 + 1 - 4 = 1$, so the circle has centre $C(-2,-1)$ and radius $r = 1$. Write the line as $mx - y + 2 = 0$. It is a tangent iff the perpendicular distance from C to the line equals r:

$$\frac{|m(-2) - (-1) + 2|}{\sqrt{m^2+1}} = 1 \implies (-2m+3)^2 = m^2 + 1.$$

$$4m^2 - 12m + 9 = m^2 + 1 \implies 3m^2 - 12m + 8 = 0.$$

$$m = \frac{12 \pm \sqrt{144 - 96}}{6} = \frac{12 \pm 4\sqrt{3}}{6} = 2 \pm \frac{2\sqrt{3}}{3}.$$

Q37 Sydney Boys 2025 Trial Q38 · **3 marks**

(a) Find μ such that $n(n+1)(n+2)(n+3) = (n^2+\mu n)(n^2+\mu n+2)$. (b) Hence deduce that $2022\times 2023\times 2024\times 2025+1$ is a perfect square.

Criteria	**Marks**
(a) Correct value $\mu = 3$	1
(b) Writes the product $+1$ as $u(u+2)+1$ with $u = n^2+3n$	1
(b) Applies $n = 2022$: value $= \left(2022^2+3(2022)+1\right)^2 = 4094551^2$	1

Solution.

(a) Pair the outer and inner factors: $n(n+3) = n^2+3n$ and $(n+1)(n+2) = n^2+3n+2$. Hence $n(n+1)(n+2)(n+3) = (n^2+3n)(n^2+3n+2)$, so $\mu = 3$.

(b) Let $u = n^2+3n$. By part (a),

$$n(n+1)(n+2)(n+3)+1 = u(u+2)+1 = u^2+2u+1 = (u+1)^2 = (n^2+3n+1)^2.$$

So four consecutive integers multiplied, plus 1, is always a perfect square. For $2022, 2023, 2024, 2025$ (i.e. $n = 2022$),

$$2022\times 2023\times 2024\times 2025+1 = \left(2022^2+3(2022)+1\right)^2 = 4094551^2.$$

Q38 Sydney Girls 2025 Trial Q10 · **1 mark**

The straight line with equation $y = ax + 3$, where $a > 1$, is perpendicular to the line with equation $y = bx + 2$. The lines cut the x-axis at the points L and M respectively. The length of LM is 5 units. What is the value of $a + b$, given that $a > 1$?

(A) $-\dfrac{13}{6}$

(B) $\dfrac{13}{6}$

(C) $-\dfrac{5}{6}$

(D) $\dfrac{5}{6}$

Criteria	**Marks**
Correct option (D)	1

Solution.

Perpendicular lines give $ab = -1$, so $b = -\dfrac{1}{a}$. The x-intercepts are $L\left(-\dfrac{3}{a}, 0\right)$ (from $y = ax + 3$) and $M\left(-\dfrac{2}{b}, 0\right) = M(2a, 0)$ (from $y = bx + 2$, using $b = -\frac{1}{a}$). Then

$$LM = \left|2a - \left(-\frac{3}{a}\right)\right| = 2a + \frac{3}{a} = 5 \quad (a > 0).$$

$$2a^2 - 5a + 3 = 0 \implies (2a - 3)(a - 1) = 0 \implies a = \tfrac{3}{2} \text{ or } a = 1.$$

Since $a > 1$, take $a = \frac{3}{2}$, so $b = -\frac{2}{3}$ and

$$a + b = \frac{3}{2} - \frac{2}{3} = \frac{5}{6}.$$

The answer is **(D)**.