Year 11 Mathematics Advanced

Topic: Working with Functions (11 of 12)

Review Questions (Worked Solutions): Absolute Value Functions

Term 3, 2026

Absolute value expressions and graphs

1. Evaluate and simplify absolute value expressions

Q1 International Grammar 2021 Prelim Q8 · **1 mark**

What is the value of $|3 - 5|$?

(A) 8

(B) 2

(C) -8

(D) -2

Criteria	Marks
Correct option (B)	1

Solution.

The absolute value of a number is its distance from 0, so it is never negative:

$$|3 - 5| = |-2| = 2.$$

The answer is (B).

Q2 Barker 2021 Prelim Q1 · **1 mark**

Which of the following is the value of $|5-7|+|-6|-|-5|+2|-3|$?

(A) -9

(B) -7

(C) 9

(D) 19

Criteria	Marks
Correct option (C)	1

Solution.

Evaluate each absolute value first, then combine left to right:

$$|5-7|+|-6|-|-5|+2|-3| = 2+6-5+2(3) = 2+6-5+6 = 9.$$

The answer is (C).

Q3 Gosford 2024 Prelim Q12 · **1 mark**

Evaluate $\big|6-|7\times(-4)|\times 3\big|$.

Criteria	Marks
Correct value 78	1

Solution.

Work from the inner absolute value outwards:

$$|7\times(-4)| = |-28| = 28, \qquad 28\times 3 = 84.$$

Then

$$\big|6-84\big| = |-78| = 78.$$

Q4 James Ruse 2022 Trial Q5 · **1 mark**

Which of the following is *not* equivalent to $\sqrt{(x-1)^2}$?

(A) $|1-x|$

(B) The distance from x to 1 on the number line

(C) $\begin{cases} x-1 & \text{when } x \geq 1 \\ 1-x & \text{when } x < 1 \end{cases}$

(D) $x-1$

Criteria	Marks
Correct option (D)	1

Solution.

Since $\sqrt{a^2} = |a|$ for all real a, we have $\sqrt{(x-1)^2} = |x-1|$. Option (A) $|1-x| = |-(x-1)| = |x-1|$, so it is equivalent. Option (B) is the definition of $|x-1|$ as a distance, so it is equivalent. Option (C) is exactly the piecewise definition of $|x-1|$, so it is equivalent. Option (D) $x-1$ is negative when $x < 1$ (for example at $x = 0$ it gives -1, whereas $|x-1| = 1$), so it is *not* equivalent. The answer is (D).

Q5 Normanhurst Boys 2023 Trial Q1 · **1 mark**

Simplify $\sqrt{x^2+2x+1} - \sqrt{x^2-2x+1}$ (for $x \geq 1$).

(A) -2

(B) 2

(C) $2x$

(D) $2x+2$

Criteria	Marks
Correct option (B)	1

Solution.

Each surd is a perfect square, so use $\sqrt{a^2} = |a|$:

$$\sqrt{x^2+2x+1} = \sqrt{(x+1)^2} = |x+1|, \qquad \sqrt{x^2-2x+1} = \sqrt{(x-1)^2} = |x-1|.$$

For $x \geq 1$ both $x+1 > 0$ and $x-1 \geq 0$, so $|x+1| = x+1$ and $|x-1| = x-1$. Hence

$$|x+1| - |x-1| = (x+1) - (x-1) = 2.$$

The answer is (B).

2. Sketch y = —ax + b— and transformations

Q6 Presbyterian 2023 Prelim Q34(a) · **1 mark**

On a number plane, graph $y = |x - 3|$.

Criteria	Marks
Correct V-shape: vertex $(3, 0)$ on the x-axis, opening upward, with correct gradients (± 1)	1

Solution.

This is $y = |x|$ shifted 3 units right. Write it as a piecewise function:

$$y = |x - 3| = \begin{cases} x - 3 & x \geq 3 \\ 3 - x & x < 3. \end{cases}$$

The graph is a "V" with its vertex at $(3, 0)$, gradient -1 to the left and $+1$ to the right. It cuts the y-axis at $(0, 3)$ and, by symmetry about $x = 3$, also passes through $(6, 3)$.

Q7 North Sydney Boys 2024 Prelim Q14(a) · **1 mark**

Sketch the graph of $y = |2x - 6|$.

Criteria	Marks
Correct V-shape: vertex $(3, 0)$, opening upward, gradients ± 2, y-intercept $(0, 6)$	1

Solution.

Note $|2x - 6| = 2|x - 3|$, so

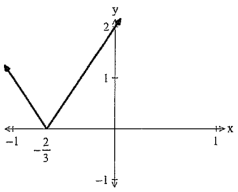
$$y = |2x - 6| = \begin{cases} 2x - 6 & x \geq 3 \\ 6 - 2x & x < 3. \end{cases}$$

The vertex is where $2x - 6 = 0$, i.e. $(3, 0)$. The graph is a "V" opening upward with gradients -2 (left) and $+2$ (right); the y-intercept is $(0, 6)$.

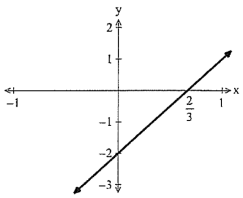
Q8 Killara 2024 Trial Q1 · **1 mark**

Which diagram is the correct sketch of $y = |3x - 2|$?

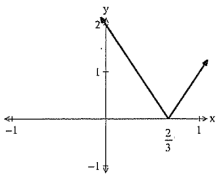
A.



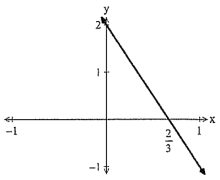
B.



C.



D.



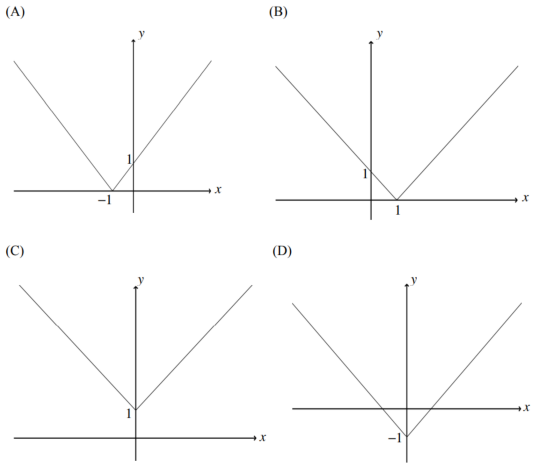
Criteria	Marks
Correct option (the "V" with vertex $\left(\frac{2}{3}, 0\right)$ on the x-axis and y-intercept $(0, 2)$)	1

Solution.

Since $|3x - 2| \geq 0$, the graph lies on or above the x-axis and is a "V" opening upward. The vertex is where $3x - 2 = 0$, i.e. at $\left(\frac{2}{3}, 0\right)$, touching the x-axis. The y-intercept is $y = |3(0) - 2| = 2$, so the graph passes through $(0, 2)$. The correct diagram is the one showing an upward "V" whose vertex sits on the x-axis at $x = \frac{2}{3}$ with y-intercept 2.

Q9 Knox 2022 Prelim Q6 · **1 mark**

Which graph shows $y = |x| - 1$?



Criteria	Marks
Correct option (the "V" shifted down 1: vertex $(0,-1)$, x-intercepts $(\pm 1, 0)$)	1

Solution.

Start from $y = |x|$ (a "V" with vertex at the origin) and shift it down 1 unit. The vertex moves to $(0,-1)$. The x-intercepts satisfy $|x| - 1 = 0$, i.e. $|x| = 1$, giving $x = \pm 1$, so the graph cuts the x-axis at $(-1, 0)$ and $(1, 0)$. The correct diagram is the upward "V" with vertex $(0,-1)$ passing through $(\pm 1, 0)$.

Q10 Sydney Girls 2023 Trial Q16 · **2 marks**

Sketch the curve $y = |x - 5| + 2$, showing all important features.

Criteria	Marks
Correct vertex $(5, 2)$ and correct shape (upward "V", gradients ± 1)	1
Correct y-intercept $(0, 7)$ / other key point marked	1

Solution.

This is $y = |x|$ shifted 5 right and 2 up:

$$y = |x - 5| + 2 = \begin{cases} x - 3 & x \geq 5 \\ 7 - x & x < 5. \end{cases}$$

The vertex (minimum point) is at $(5, 2)$. The graph is an upward "V" with gradients -1 (left) and $+1$ (right); the curve does not cross the x-axis (its minimum value is $2 > 0$). The y-intercept is $y = |0 - 5| + 2 = 7$, i.e. $(0, 7)$.

Q11 Baulkham Hills 2023 Trial Q28(b) · **2 marks**

On a number plane, graph the equation $y = 5 - 2|x + 2|$, showing all essential features.

Criteria	Marks
Correct vertex $(-2, 5)$ and correct shape (inverted "V", gradients ± 2)	1
Correct intercepts: x-intercepts $\left(-\frac{9}{2}, 0\right), \left(\frac{1}{2}, 0\right)$ and y-intercept $(0, 1)$	1

Solution.

The $-2|x + 2|$ term reflects and stretches, and the $+5$ shifts up:

$$y = 5 - 2|x + 2| = \begin{cases} 5 - 2(x + 2) = 1 - 2x & x \geq -2 \\ 5 + 2(x + 2) = 9 + 2x & x < -2. \end{cases}$$

Because the coefficient of the absolute value is negative, the graph is an inverted "V" (opening downward) with maximum point (vertex) at $(-2, 5)$ and gradients $+2$ (left) and -2 (right). The x-intercepts solve $5 - 2|x + 2| = 0$, i.e. $|x + 2| = \frac{5}{2}$, giving $x = -\frac{9}{2}$ and $x = \frac{1}{2}$. The y-intercept is $y = 5 - 2|0 + 2| = 1$, i.e. $(0, 1)$.

Q12 Hurlstone 2023 Prelim Q11(f) · **3 marks**

Sketch $y = f(x)$ for the function $f(x) = |x - 3| + 1$ on the domain $[-4, 1]$. Show all important features.

Criteria	Marks
Recognises that on $[-4, 1]$ the corner $x = 3$ is excluded, so $f(x) = (3 - x) + 1 = 4 - x$ (a straight line segment)	1
Correct endpoints: $(-4, 8)$ and $(1, 3)$, both included (closed dots)	1
Correct straight-line segment joining them with no part drawn outside $[-4, 1]$	1

Solution.

For the full function $f(x) = |x - 3| + 1$ the vertex is at $(3, 1)$, but $x = 3$ lies *outside* the domain $[-4, 1]$. On the whole interval $[-4, 1]$ we have $x < 3$, so $|x - 3| = 3 - x$ and

$$f(x) = (3 - x) + 1 = 4 - x, \qquad -4 \leq x \leq 1.$$

Hence on this domain the graph is a single straight line segment of gradient -1. Evaluating the endpoints: $f(-4) = 4 - (-4) = 8$ and $f(1) = 4 - 1 = 3$. Sketch the segment from the closed point $(-4, 8)$ down to the closed point $(1, 3)$, drawing nothing beyond $x = -4$ or $x = 1$.

Solving equations and inequalities

3. Solve absolute value equations —ax + b— = k

Q13 Hurlstone 2024 Prelim Q1 · **1 mark**

Which of the following is the solution to the equation $|x - 7| = 6$?

(A) $x = 13$

(B) $x = 1$ or $x = 13$

(C) $x = -13$ or $x = -1$

(D) $x = -13$ or $x = 13$

Criteria	Marks
Correct option (B)	1

Solution.

The two cases are $x - 7 = 6$ or $x - 7 = -6$, giving $x = 13$ or $x = 1$. The answer is **(B)**.

Q14 North Sydney Girls 2023 Trial Q11 · **2 marks**

Solve $|5 - x| = 3$.

Criteria	Marks
Sets up both cases $5 - x = \pm 3$	1
Both solutions $x = 2$ and $x = 8$	1

Solution.

Removing the absolute value gives $5 - x = 3$ or $5 - x = -3$.

$$5 - x = 3 \Rightarrow x = 2, \qquad 5 - x = -3 \Rightarrow x = 8.$$

So $x = 2$ or $x = 8$.

Q15 North Sydney Girls 2023 Prelim Q12 · **2 marks**

Solve the equation $|3x - 5| = 7$.

Criteria	Marks
Sets up both cases $3x - 5 = \pm 7$	1
Both solutions $x = 4$ and $x = -\frac{2}{3}$	1

Solution.

Removing the absolute value gives $3x - 5 = 7$ or $3x - 5 = -7$.

$$3x - 5 = 7 \implies 3x = 12 \implies x = 4,$$

$$3x - 5 = -7 \implies 3x = -2 \implies x = -\tfrac{2}{3}.$$

So $x = 4$ or $x = -\dfrac{2}{3}$.

Q16 Kambala 2024 Prelim Q11 · **2 marks**

Solve the equation $\left|\dfrac{x}{2} - 3\right| = 1$.

Criteria	Marks
Sets up both cases $\dfrac{x}{2} - 3 = \pm 1$	1
Both solutions $x = 8$ and $x = 4$	1

Solution.

Removing the absolute value gives $\dfrac{x}{2} - 3 = 1$ or $\dfrac{x}{2} - 3 = -1$.

$$\frac{x}{2} - 3 = 1 \implies \frac{x}{2} = 4 \implies x = 8,$$

$$\frac{x}{2} - 3 = -1 \implies \frac{x}{2} = 2 \implies x = 4.$$

So $x = 8$ or $x = 4$.

Q17 Sydney Grammar 2022 Prelim Q9(e) · **4 marks**

(i) Sketch $y = |x + 2|$ showing all intercepts. (ii) Hence solve $|x + 2| = 3$.

Criteria	Marks
(i) Correct V-shape with vertex at $(-2, 0)$	1
(i) Correct intercepts: x-intercept $(-2, 0)$, y-intercept $(0, 2)$	1
(ii) Sets up $x + 2 = \pm 3$ (or reads intersections with $y = 3$)	1
(ii) Both solutions $x = 1$ and $x = -5$	1

Solution.

(i) $y = |x + 2|$ is a V-shaped graph with vertex at $(-2, 0)$. It meets the x-axis at $(-2, 0)$ and the y-axis at $(0, 2)$; the arms have gradients -1 (left) and $+1$ (right).

(ii) Solving $|x + 2| = 3$ means the horizontal line $y = 3$ cuts the graph. Then

$$x + 2 = 3 \quad \text{or} \quad x + 2 = -3$$

$$x = 1 \quad \text{or} \quad x = -5.$$

4. Solve absolute value inequalities —ax + b— ¡ k or ¿= k

Q18 Hornsby Girls 2023 Trial Q6 · **1 mark**

The solution of $|x - 1| > 2$ is:

(A) $x > 3$

(B) $x < -1,\ x > 3$

(C) $x < -1$

(D) $x \leq -1,\ x \geq 3$

Criteria	Marks
Correct option (B)	1

Solution.

$|x - 1| > 2$ means the distance from x to 1 exceeds 2, so split into two cases:

$$x - 1 > 2 \quad \text{or} \quad x - 1 < -2,$$

giving $x > 3$ or $x < -1$. Both inequalities are strict, so the answer is (B).

Q19 Sydney Grammar 2020 Trial Q12 · **1 mark**

Solve $|2x - 1| > 5$.

Criteria	Marks
Correct solution $x < -2$ or $x > 3$	1

Solution.

$|2x - 1| > 5$ splits into $2x - 1 > 5$ or $2x - 1 < -5$:

$$2x - 1 > 5 \Rightarrow 2x > 6 \Rightarrow x > 3,$$

$$2x - 1 < -5 \Rightarrow 2x < -4 \Rightarrow x < -2.$$

Hence $x < -2$ or $x > 3$.

Q20 Caringbah 2021 Trial Q14 · **2 marks**

Solve $|7x - 12| \geq 16$.

Criteria	Marks
Splits into $7x - 12 \geq 16$ or $7x - 12 \leq -16$	1
Correct solution $x \leq -\frac{4}{7}$ or $x \geq 4$	1

Solution.

$|7x - 12| \geq 16$ means $7x - 12 \geq 16$ or $7x - 12 \leq -16$:

$$7x - 12 \geq 16 \Rightarrow 7x \geq 28 \Rightarrow x \geq 4,$$

$$7x - 12 \leq -16 \Rightarrow 7x \leq -4 \Rightarrow x \leq -\frac{4}{7}.$$

Hence $x \leq -\frac{4}{7}$ or $x \geq 4$.

Q21 Girraween 2021 Trial Q11(b) · **3 marks**

Solve $|3x + 2| < 5$ and draw the solution on the number line.

Criteria	Marks
Removes the modulus: $-5 < 3x + 2 < 5$	1
Correct solution $-\frac{7}{3} < x < 1$	1
Number line showing the open interval from $-\frac{7}{3}$ to 1	1

Solution.

$|3x + 2| < 5$ means the value inside the modulus lies between -5 and 5:

$$-5 < 3x + 2 < 5.$$

Subtract 2 throughout: $-7 < 3x < 3$. Divide by 3: $-\frac{7}{3} < x < 1$. On the number line this is the open interval from $-\frac{7}{3}$ to 1, drawn with open circles at both endpoints and the segment between them shaded.

Q22 Hurlstone 2024 Trial Q14(b) · **2 marks**

The graph of $y = 2|x - 1| - 3$ is a V-shape with vertex $(1, -3)$ and x-intercepts at $x = -\frac{1}{2}$ and $x = \frac{5}{2}$. Hence, or otherwise, solve the inequation $2|x - 1| - 3 < -1$.

Criteria	Marks
Reduces to $\lvert x - 1\rvert < 1$	1
Correct solution $0 < x < 2$	1

Solution.

Rearrange to isolate the modulus:

$$2|x - 1| - 3 < -1 \Rightarrow 2|x - 1| < 2 \Rightarrow |x - 1| < 1.$$

This means $-1 < x - 1 < 1$, so adding 1 throughout gives $0 < x < 2$. (Equivalently, on the graph from part (a) this is where the V lies below the line $y = -1$.)

5. Solve with the variable on both sides (—f— = line or —f— = —g—)

Q23 Girraween 2025 Trial Q12 · **3 marks**

Solve the equation $|2x - 3| = 9x$.

Criteria	Marks
Splits into $2x - 3 = 9x$ and $-(2x - 3) = 9x$	1
Solves both cases: $x = -\frac{3}{7}$ and $x = \frac{3}{11}$	1
Rejects $x = -\frac{3}{7}$ (RHS negative) and states $x = \frac{3}{11}$	1

Solution.

Since $|2x - 3| \geq 0$, a solution requires $9x \geq 0$, i.e. $x \geq 0$. Remove the modulus in two cases:

$$2x - 3 = 9x \Rightarrow -3 = 7x \Rightarrow x = -\tfrac{3}{7},$$

$$-(2x - 3) = 9x \Rightarrow 3 - 2x = 9x \Rightarrow 3 = 11x \Rightarrow x = \tfrac{3}{11}.$$

The first value $x = -\frac{3}{7}$ is negative, so $9x < 0$ and it does not satisfy the equation; it is rejected. Checking $x = \frac{3}{11}$: $|2 \cdot \frac{3}{11} - 3| = |-\frac{27}{11}| = \frac{27}{11}$ and $9 \cdot \frac{3}{11} = \frac{27}{11}$, so it is valid. Hence $x = \frac{3}{11}$.

Q24 Normanhurst Boys 2024 Trial Q11 · **3 marks**

(a) Sketch $y = |x - 1|$ on $[-4, 4]$. (b) Solve $|x - 1| = 2x + 4$.

Criteria	Marks
Correct V-shaped graph with vertex $(1, 0)$ over $[-4, 4]$	1
Considers both cases of the absolute value	1
Correct solution $x = -1$ (rejecting $x = -5$)	1

Solution.

(a) $y = |x - 1|$ is a V-shape with vertex at $(1, 0)$, passing through $(-4, 5)$ and $(4, 3)$ on the given domain; the left arm has gradient -1 and the right arm gradient $+1$.

(b) Split into cases. *Case* $x \geq 1$*:* $x - 1 = 2x + 4 \Rightarrow -x = 5 \Rightarrow x = -5$, which contradicts $x \geq 1$, so reject. *Case* $x < 1$*:* $-(x - 1) = 2x + 4 \Rightarrow 1 - x = 2x + 4 \Rightarrow -3 = 3x \Rightarrow x = -1$, which satisfies $x < 1$. Check: $|-1 - 1| = 2$ and $2(-1) + 4 = 2$. $\therefore x = -1$.

Q25 James Ruse 2023 Trial Q13 · **2 marks**

Solve $|4x - 8| = 3|x + 2|$.

Criteria	Marks
Squares (or uses cases) to obtain $7x^2 - 100x + 28 = 0$	1
Correct solutions $x = \frac{2}{7}$ and $x = 14$	1

Solution.

Both sides are non-negative, so square:

$$(4x - 8)^2 = 9(x + 2)^2.$$

Expanding, $16x^2 - 64x + 64 = 9x^2 + 36x + 36$, so

$$7x^2 - 100x + 28 = 0.$$

Factorising, $(7x - 2)(x - 14) = 0$, giving $x = \frac{2}{7}$ or $x = 14$. Both check: at $x = \frac{2}{7}$, $|4 \cdot \frac{2}{7} - 8| = \frac{48}{7}$ and $3|\frac{2}{7} + 2| = \frac{48}{7}$; at $x = 14$, $|48| = 48$ and $3|16| = 48$. Hence $x = \frac{2}{7}$ or $x = 14$.

Q26 Sydney Grammar 2023 Prelim Q12(c)(ii) · **1 mark**

By sketching $y = |x - 3|$ and $y = |2x + 6|$ on the same number plane, or otherwise, solve

$$|x - 3| = |2x + 6| \qquad \text{for } -3 \le x \le 3.$$

Criteria	Marks
Correct solution in the given domain: $x = -1$	1

Solution.

Squaring both sides (both are non-negative): $(x - 3)^2 = (2x + 6)^2$, so

$$x^2 - 6x + 9 = 4x^2 + 24x + 36 \Rightarrow 3x^2 + 30x + 27 = 0 \Rightarrow x^2 + 10x + 9 = 0.$$

Factorising, $(x + 1)(x + 9) = 0$, giving $x = -1$ or $x = -9$. Only $x = -1$ lies in $-3 \le x \le 3$. Checking: $|-1 - 3| = 4$ and $|2(-1) + 6| = 4$. Hence $x = -1$.

Q27 Hornsby Girls 2022 Prelim Q14(b) · **1 mark**

The graphs of $y = |x|$ and $y = 3 - \frac{x}{2}$ meet at $x = -6$ and $x = 2$. Hence, or otherwise, solve

$$|x| \leq 3 - \tfrac{x}{2}.$$

Criteria	Marks
Correct solution $-6 \leq x \leq 2$	1

Solution.

The inequality $|x| \leq 3 - \frac{x}{2}$ asks where the V-graph $y = |x|$ lies on or below the line $y = 3 - \frac{x}{2}$. The two graphs meet at $x = -6$ and $x = 2$, and between these values the V lies below the line. Hence the solution is $-6 \leq x \leq 2$. (Check: at $x = -6$, $|-6| = 6 = 3 - \frac{-6}{2}$; at $x = 2$, $|2| = 2 = 3 - \frac{2}{2}$.)

Q28 North Sydney Boys 2022 Trial Q18 · **5 marks**

(a) Sketch $y = |x - 3|$ and $y = \left|\frac{2}{5}x\right|$, giving their intersection points. **(b)** Hence solve $\left|\frac{2}{5}x\right| < |x - 3|$.

Criteria	Marks
(a) Correct V-shape of $y = \lvert x - 3\rvert$ with vertex $(3, 0)$	1
(a) Correct V-shape of $y = \left\lvert\frac{2}{5}x\right\rvert$ with vertex at the origin	1
(a) Both intersection points found: $\left(\frac{15}{7}, \frac{6}{7}\right)$ and $(5, 2)$	2
(b) Correct solution $x < \frac{15}{7}$ or $x > 5$	1

Solution.

(a) $y = |x - 3|$ is a V with vertex $(3, 0)$ and arms of gradient ± 1. $y = \left|\frac{2}{5}x\right|$ is a V with vertex at the origin and arms of gradient $\pm\frac{2}{5}$. To find where they meet, solve $|x-3| = \left|\frac{2}{5}x\right|$. On $x \geq 3$: $x - 3 = \frac{2}{5}x \Rightarrow \frac{3}{5}x = 3 \Rightarrow x = 5$, giving $(5, 2)$. On $x < 3$ (where $|x - 3| = 3 - x$, and $\frac{2}{5}x \geq 0$ needs $x \geq 0$): $3 - x = \frac{2}{5}x \Rightarrow 3 = \frac{7}{5}x \Rightarrow x = \frac{15}{7}$, giving $\left(\frac{15}{7}, \frac{6}{7}\right)$. The graphs meet at $\left(\frac{15}{7}, \frac{6}{7}\right)$ and $(5, 2)$.

(b) $\left|\frac{2}{5}x\right| < |x - 3|$ holds where the graph of $y = \left|\frac{2}{5}x\right|$ lies below $y = |x - 3|$, i.e. outside the two intersection points. Hence $x < \frac{15}{7}$ or $x > 5$.

6. Solve graphically: absolute value against a line or curve

Q29 Girraween 2020 Trial Q25(a) · **2 marks**

On the same set of axes, sketch the graphs of $y = |x - 1|$ and $y = 2x + 4$ for $-4 \leq x \leq 4$.

Criteria	Marks
Correct V-shaped graph of $y = \|x - 1\|$ (vertex $(1, 0)$, arms of gradient ± 1)	1
Correct line $y = 2x + 4$ drawn on the same axes, meeting the V at $(-1, 2)$	1

Solution.

The graph $y = |x - 1|$ is a "V" with vertex at $(1, 0)$; for $x \geq 1$ it is the line $y = x - 1$ and for $x < 1$ it is $y = -(x - 1) = 1 - x$. The line $y = 2x + 4$ has y-intercept 4 and x-intercept -2. Setting the two equal, for $x < 1$: $1 - x = 2x + 4 \Rightarrow -3 = 3x \Rightarrow x = -1$, giving the point $(-1, 2)$; for $x \geq 1$: $x - 1 = 2x + 4 \Rightarrow x = -5$, which is outside $x \geq 1$ and is rejected. So the two graphs meet only at $(-1, 2)$.

Q30 Presbyterian 2023 Prelim Q34(b) · **1 mark**

The graph of $y = |x - 3|$ has been drawn. By also drawing the line $y = x + 1$ on the same axes, solve $x + 1 = |x - 3|$ graphically.

Criteria	Marks
Reads off the single intersection to give $x = 1$	1

Solution.

The line $y = x + 1$ meets the "V" $y = |x - 3|$ where the two curves cross. On the left branch ($x < 3$), $|x - 3| = 3 - x$, so $x + 1 = 3 - x \Rightarrow 2x = 2 \Rightarrow x = 1$ (point $(1, 2)$). On the right branch ($x \geq 3$), $x + 1 = x - 3$ has no solution. Hence the graphs meet once and

$$x = 1.$$

Q31 Fort St 2022 Prelim Q7(d) · **5 marks**

(i) Solve $|x-2|=4$. (ii) Sketch $y=|x-2|$ and $y=4$, marking intersections and intercepts. (iii) Solve $|x-2|<4$.

Criteria	Marks
Solves $\lvert x-2\rvert=4$ to get $x=-2$ and $x=6$	1
Correct graph of $y=\lvert x-2\rvert$ with vertex and intercepts	1
Draws $y=4$ and marks intersections $(-2,4)$ and $(6,4)$	1
Reads/uses the graph to solve the inequality	1
States the solution $-2<x<4$ region correctly as $-2<x<6$	1

Solution.

(i) $|x-2|=4$ gives $x-2=4$ or $x-2=-4$, so $x=6$ or $x=-2$.

(ii) $y=|x-2|$ is a V-shape with vertex $(2,0)$, x-intercept $x=2$ and y-intercept 2. The horizontal line $y=4$ meets it at $(-2,4)$ and $(6,4)$.

(iii) $|x-2|<4$ holds where the V lies below the line $y=4$, i.e. between the two intersection points:

$$-2<x<6.$$

Q32 Hornsby Girls 2022 Prelim Q14(a) · **2 marks**

Given that $f(x)=|x|$ and $g(x)=3-\dfrac{x}{2}$, sketch the graphs of $y=f(x)$ and $y=g(x)$ on the same number plane, labelling the point(s) of intersection.

Criteria	Marks
Correct V-graph $y=\lvert x\rvert$ (vertex origin) and line $y=3-\frac{x}{2}$ (y-intercept 3, x-intercept 6)	1
Both intersection points labelled: $(-6,6)$ and $(2,2)$	1

Solution.

$y=|x|$ is a "V" with vertex at the origin. The line $y=3-\frac{x}{2}$ has y-intercept 3 and x-intercept 6, sloping down to the right. For the intersections solve $|x|=3-\frac{x}{2}$: on the right branch ($x\geq 0$) $x=3-\frac{x}{2}\Rightarrow\frac{3x}{2}=3\Rightarrow x=2$ (point $(2,2)$); on the left branch ($x<0$) $-x=3-\frac{x}{2}\Rightarrow-\frac{x}{2}=3\Rightarrow x=-6$ (point $(-6,6)$). The graphs meet at

$$(-6,6)\quad\text{and}\quad(2,2).$$

Q33 Sydney Grammar 2024 Trial Q27 · **5 marks**

(a) Sketch $f(x) = 3 - x$ and $g(x) = |2x - 4|$ on one plane with intercepts and intersections. (b) Hence solve $|2x - 4| < 3 - x$. (c) Find all a so $|2x - 4| < a - x$ has real solutions.

Criteria	**Marks**
(a) Correct line $y = 3 - x$ with intercepts	1
(a) Correct V-graph $y = \|2x - 4\|$ with vertex and intercept	1
(a) Marks both points of intersection $(1, 2)$ and $\left(\frac{7}{3}, \frac{2}{3}\right)$	1
(b) Correct solution $1 < x < \frac{7}{3}$	1
(c) Correct condition $a > 2$	1

Solution.

(a) $f(x) = 3 - x$ is a line with intercepts $(3, 0)$ and $(0, 3)$. $g(x) = |2x - 4|$ is a V with vertex $(2, 0)$ and y-intercept $(0, 4)$. Intersections: $2x - 4 = 3 - x \Rightarrow 3x = 7 \Rightarrow x = \frac{7}{3}$ (giving $\left(\frac{7}{3}, \frac{2}{3}\right)$), and $-(2x - 4) = 3 - x \Rightarrow -x = -1 \Rightarrow x = 1$ (giving $(1, 2)$). **(b)** $|2x - 4| < 3 - x$ holds where the V lies below the line, i.e. between the intersection points:

$$1 < x < \frac{7}{3}.$$

(c) $|2x - 4| < a - x$ has a solution iff $a > |2x - 4| + x$ for some x, i.e. iff a exceeds the minimum of $h(x) = |2x - 4| + x$. For $x < 2$, $h(x) = 4 - x$ (decreasing to 2); for $x \geq 2$, $h(x) = 3x - 4$ (increasing from 2). The minimum is $h(2) = 2$. Hence real solutions exist iff $\boxed{a > 2}$.

Q34 2023 HSC Q19 · **4 marks**

(a) Sketch $f(x) = |x-1|$ and $g(x) = (1-x)(3+x)$ showing x-intercepts. (b) Solve $|x-1| < (1-x)(3+x)$.

Criteria	**Marks**
Correct sketch of $f(x) = \|x-1\|$ with x-intercept	1
Correct sketch of $g(x) = (1-x)(3+x)$ with x-intercepts	1
Finds the points of intersection $x = -4$ and $x = 1$	1
States the solution $-4 < x < 1$	1

Solution.

(a) $f(x) = |x-1|$ is a V-shape with vertex $(1, 0)$ and x-intercept $x = 1$. $g(x) = (1-x)(3+x)$ is a concave-down parabola with x-intercepts $x = -3$ and $x = 1$ and y-intercept 3.

(b) The curves meet where $|x-1| = (1-x)(3+x)$. For $x \le 1$, $|x-1| = 1-x$, so $1-x = (1-x)(3+x)$, giving $(1-x)\big[(3+x)-1\big] = 0$, i.e. $(1-x)(x+2) = 0$, so $x = 1$ or $x = -2$. Checking the full picture (and the point $x = -4$ where g falls below f), the parabola lies above the V for $-4 < x < 1$.

Solving directly: on $x \le 1$, $1-x < (1-x)(3+x) \Rightarrow 0 < (1-x)(x+2) \Rightarrow -2 < x < 1$; on $x > 1$, $x-1 < (1-x)(3+x)$ has no solution since the RHS is negative there. Combined with the branch giving $x > -4$, the graph shows $g > f$ on

$$-4 < x < 1.$$

Finding equations and counting solutions

7. Find the equation/parameters and count solutions

Q35 Blacktown Boys 2020 Trial Q13(a) · **2 marks**

The graph of $f(x) = |ax + b|$ is shown. What are the values of a and b?

Criteria	Marks
Uses the x-intercept $(\frac{5}{2}, 0)$ (or y-intercept 5) to find one parameter	1
Correct $a = 2$ and $b = -5$	1

Solution.

The graph is a V with x-intercept at $x = \frac{5}{2}$ and y-intercept 5.

$$|a(\tfrac{5}{2}) + b| = 0 \implies \tfrac{5}{2}a + b = 0, \qquad |a(0) + b| = 5 \implies |b| = 5.$$

Since the vertex is to the right of the y-axis, take $b = -5$, giving $\frac{5}{2}a - 5 = 0$, so $a = 2$. Checking the marked point $(2, 1)$: $|2(2) - 5| = |-1| = 1$. Hence $a = 2$, $b = -5$, i.e. $f(x) = |2x - 5|$.

Q36 Barker 2020 Trial Q17(a) · **3 marks**

The function $f(x) = |x|$ is transformed to $y = k\,f(x + b) + c = k|x + b| + c$; the graph of the new function is shown. What are the values of k, b and c?

Criteria	Marks
Reads the vertex $(2, 1)$ to get $b = -2$ and $c = 1$	1
Uses a second point (e.g. the endpoint $(-4, -2)$ or intercept) to find $\lvert k\rvert$	1
Correct $k = -\frac{1}{2}$, $b = -2$, $c = 1$	1

Solution.

The graph is an inverted V with vertex $(2, 1)$. For $y = k|x+b|+c$ the vertex is at $x = -b$, so $-b = 2 \Rightarrow b = -2$, and the vertex height gives $c = 1$. The branches fall away from the vertex, so $k < 0$. Using the point $(0, 0)$ on the graph:

$$0 = k|0 - 2| + 1 \implies 2k + 1 = 0 \implies k = -\tfrac{1}{2}.$$

(Check: at $x = -4$, $y = -\frac{1}{2}|-6| + 1 = -2$, matching the graph.) Hence $k = -\frac{1}{2}$, $b = -2$, $c = 1$.

Q37 Barker 2024 Prelim Q6 · **1 mark**

Determine the value(s) of k for which $|2k-1| = k$.

Criteria	Marks
Correct option (D)	1

Solution.

For $|2k-1| = k$ we need $k \geq 0$. Then $2k-1 = k \Rightarrow k = 1$, or $2k-1 = -k \Rightarrow 3k = 1 \Rightarrow k = \frac{1}{3}$. Both are non-negative and check out, so $k = 1$ or $k = \frac{1}{3}$. The answer is **(D)**.

Q38 Hornsby Girls 2022 Trial Q29 · **6 marks**

(a) Sketch $y = |2x+5|$ with intercepts. (b) Solve $|4x+10|+2 = 8$. (c) Values of m for which $|2x+5| = m$ has no solutions.

Criteria	Marks
(a) Correct V-shape with vertex at $\left(-\frac{5}{2}, 0\right)$	1
(a) Labels x-intercept $x = -\frac{5}{2}$ and y-intercept $y = 5$	1
(b) Reduces to $\lvert 4x+10\rvert = 6$	1
(b) Solves the two cases	1
(b) States $x = -1$ and $x = -4$	1
(c) States $m < 0$	1

Solution.

(a) $y = |2x+5|$ is a V-shaped graph. The vertex is where $2x+5 = 0$, i.e. $x = -\dfrac{5}{2}$, giving vertex $\left(-\dfrac{5}{2}, 0\right)$. The y-intercept is $y = |2(0)+5| = 5$, and the x-intercept is $x = -\dfrac{5}{2}$. Draw the two straight branches meeting at the vertex, rising to the right through $(0, 5)$.

(b) Rearrange:

$$|4x+10|+2 = 8 \;\Rightarrow\; |4x+10| = 6.$$

Either $4x+10 = 6 \Rightarrow 4x = -4 \Rightarrow x = -1$, or $4x+10 = -6 \Rightarrow 4x = -16 \Rightarrow x = -4$. Hence $x = -1$ or $x = -4$.

(c) The equation $|2x+5| = m$ has solutions exactly where the horizontal line $y = m$ meets the graph of $y = |2x+5|$. Since $|2x+5| \geq 0$, the line $y = m$ misses the graph only when $m < 0$. So there are no solutions for $m < 0$.

Q39 2023 HSC Q27(a) · **3 marks**

The graph of $y = f(x)$, where $f(x) = a|x - b| + c$, passes through $(3, -5)$, $(6, 7)$ and $(9, -5)$. Find a, b and c.

Criteria	**Marks**
Uses symmetry of $(3, -5)$ and $(9, -5)$ to get $b = 6$	1
Uses the vertex/turning point to get $c = 7$	1
Correct $a = -4$ (so $f(x) = -4\|x - 6\| + 7$)	1

Solution.

The points $(3, -5)$ and $(9, -5)$ have equal y-values, so the axis of symmetry is $x = \frac{3+9}{2} = 6$; hence $b = 6$. The vertex is then $(6, c)$, and since the graph passes through $(6, 7)$ we get $c = 7$. Using $(3, -5)$:

$$-5 = a|3 - 6| + 7 = 3a + 7 \implies 3a = -12 \implies a = -4.$$

Hence $a = -4$, $b = 6$, $c = 7$, so $f(x) = -4|x - 6| + 7$.

Q40 North Sydney Boys 2024 Prelim Q14(b) · **2 marks**

Determine the values of k for which $x + k = |2x - 6|$ has exactly two solutions.

Criteria	**Marks**
Identifies the line $y = x + k$ against $y = \|2x - 6\|$ (vertex $(3, 0)$, branch slopes ± 2) and the critical case	1
Correct answer $k > -3$	1

Solution.

Sketch $y = |2x - 6|$, a V with vertex $(3, 0)$ and branch gradients -2 (left) and $+2$ (right). The line $y = x + k$ has gradient 1, which lies strictly between -2 and 2, so once the line is high enough it meets each branch exactly once (two solutions). The changeover happens when the line passes through the vertex:

$$0 = 3 + k \implies k = -3,$$

which gives a single solution. For $k < -3$ the line lies below the vertex and meets neither branch (no solutions); for $k > -3$ it meets both branches. Hence there are exactly two solutions when $k > -3$.

Q41 2023 HSC Q27(b) · **2 marks**

With $f(x) = -4|x-6|+7$, the line $y = mx$ cuts $y = f(x)$ in two distinct places. Find all possible values of m.

Criteria	**Marks**
Finds a boundary gradient (line through the vertex $m = \frac{7}{6}$, or line parallel to a branch $m = -4$)	1
Correct answer $-4 < m < \frac{7}{6}$	1

Solution.

The graph is an inverted V with vertex $(6, 7)$ and branches $y = 4x - 17$ (for $x < 6$) and $y = -4x + 31$ (for $x \geq 6$). The line $y = mx$ passes through the origin, which sits below the graph.

As m increases from a large negative value, the line first meets only one branch until it becomes steep enough to catch the second branch. The line is parallel to the right branch when $m = -4$: for $m \leq -4$ there is only one intersection. The line passes through the vertex $(6, 7)$ when $6m = 7$, i.e. $m = \frac{7}{6}$, giving a single (touching) intersection; for $m \geq \frac{7}{6}$ (up to the left-branch slope) the line misses the graph.

$$\text{Two distinct intersections} \iff -4 < m < \tfrac{7}{6}.$$