# Year 11 Mathematics Advanced

**Topic: Working with Functions (10 of 12)**

**Review Questions (Worked Solutions): Piecewise-Defined Functions**

**Term 3, 2026**

# Piecewise-defined functions

## 1. Evaluate a piecewise function (numeric, algebraic, and nested)

### Q1 Baulkham Hills 2020 Trial Q12(d)(i) · **1 mark**

Consider the piecewise-defined function

$$f(x) = \begin{cases} x^2 - 1 & x \leq 1 \\ 4 - x & x > 1. \end{cases}$$

Find $f(1)$.

| Criteria | Marks |
|---|---|
| Correct value $f(1) = 0$ | 1 |

**Solution.**

Since $1 \leq 1$, use the first branch $f(x) = x^2 - 1$:

$$f(1) = 1^2 - 1 = 0.$$

### Q2 Hurlstone 2024 Prelim Q14(a) · **1 mark**

A function is defined as follows:

$$f(x) = \begin{cases} x^2 - x + 2 & \text{when } x < 0 \\ x^3 & \text{when } x \geq 0. \end{cases}$$

Evaluate $f(-2) + f(5)$.

| Criteria | Marks |
|---|---|
| Correct value $f(-2) + f(5) = 133$ | 1 |

**Solution.**

$-2 < 0$, so use $x^2 - x + 2$, and $5 \geq 0$, so use $x^3$:

$$f(-2) = (-2)^2 - (-2) + 2 = 4 + 2 + 2 = 8, \qquad f(5) = 5^3 = 125.$$

$$f(-2) + f(5) = 8 + 125 = 133.$$

## Q3 Cherrybrook Tech 2021 Prelim Q19(b) · **1 mark**

The function $y = f(x)$ is defined as follows:

$$f(x) = \begin{cases} -1 & x \leq -2 \\ 2x & -2 < x < 0 \\ 3 - x & x \geq 0. \end{cases}$$

Evaluate $f(-3) + f\left(\frac{1}{2}\right) + 2f(4)$.

| **Criteria** | **Marks** |
|---|---|
| Correct value $f(-3) + f\left(\frac{1}{2}\right) + 2f(4) = -\frac{1}{2}$ | 1 |

**Solution.**

Pick the branch for each input: $-3 \leq -2$, $0 \leq \frac{1}{2}$ and $0 \leq 4$:

$$f(-3) = -1, \qquad f\left(\tfrac{1}{2}\right) = 3 - \tfrac{1}{2} = \tfrac{5}{2}, \qquad f(4) = 3 - 4 = -1.$$

$$f(-3) + f\left(\tfrac{1}{2}\right) + 2f(4) = -1 + \tfrac{5}{2} + 2(-1) = -\tfrac{1}{2}.$$

## Q4 St George Girls 2022 Prelim Q17 · **2 marks**

A function is defined by the rule

$$f(x) = \begin{cases} 1 - x & x \leq -2 \\ -1 & -1 < x < 1 \\ x + 1 & x \geq 1. \end{cases}$$

Evaluate $f(2) + f(-2)$.

| **Criteria** | **Marks** |
|---|---|
| Chooses the correct branch for each: $f(2) = 3$ and $f(-2) = 3$ | 1 |
| Correct value $f(2) + f(-2) = 6$ | 1 |

**Solution.**

$2 \geq 1$, so use $x + 1$; $-2 \leq -2$, so use $1 - x$:

$$f(2) = 2 + 1 = 3, \qquad f(-2) = 1 - (-2) = 3.$$

$$f(2) + f(-2) = 3 + 3 = 6.$$

## Q5 Girraween 2022 Prelim Q7(d)(ii) · **2 marks**

A function $y = f(x)$ is defined as

$$f(x) = \begin{cases} x - 3 & x \leq -3 \\ 2x + 2 & -3 < x < 0 \\ x^2 & x \geq 0. \end{cases}$$

Evaluate $f(2) + f(-2) + f(-5)$.

| Criteria | Marks |
|---|---|
| Chooses the correct branch for each: $f(2) = 4$, $f(-2) = -2$, $f(-5) = -8$ | 1 |
| Correct value $f(2) + f(-2) + f(-5) = -6$ | 1 |

**Solution.**

$2 \geq 0$ uses $x^2$; $-3 < -2 < 0$ uses $2x + 2$; $-5 \leq -3$ uses $x - 3$:

$$f(2) = 2^2 = 4, \qquad f(-2) = 2(-2) + 2 = -2, \qquad f(-5) = -5 - 3 = -8.$$

$$f(2) + f(-2) + f(-5) = 4 - 2 - 8 = -6.$$

## Q6 Killara 2021 Prelim Q9(b) · **2 marks**

Given the piecewise function

$$f(x) = \begin{cases} 2x - 1 & x < -5 \\ 3 & -5 \leq x < 0 \\ (x-1)^2 + 2 & x \geq 0, \end{cases}$$

evaluate $f\left(a^2\right)$ in simplest form.

| Criteria | Marks |
|---|---|
| Recognises $a^2 \geq 0$, so uses the branch $(x-1)^2 + 2$ | 1 |
| Correct expansion $f\left(a^2\right) = a^4 - 2a^2 + 3$ | 1 |

**Solution.**

For every real $a$, $a^2 \geq 0$, so $f\left(a^2\right)$ uses the branch $(x-1)^2 + 2$ with $x = a^2$:

$$f\left(a^2\right) = \left(a^2 - 1\right)^2 + 2 = a^4 - 2a^2 + 1 + 2 = a^4 - 2a^2 + 3.$$

## Q7 Caringbah 2023 Trial Q7 · **1 mark**

A function is given by the rule

$$f(x) = \begin{cases} (x-2)^2 & x < 2 \\ 4x - 7 & x \geq 2. \end{cases}$$

What is the value of $f(f(0))$?

(A) 5

(B) $-7$

(C) 9

(D) 4

| **Criteria** | **Marks** |
|---|---|
| Correct option (C) | 1 |

**Solution.**

Work from the inside out. Since $0 < 2$, use $(x-2)^2$:

$$f(0) = (0-2)^2 = 4.$$

Now evaluate $f(4)$; since $4 \geq 2$, use $4x - 7$:

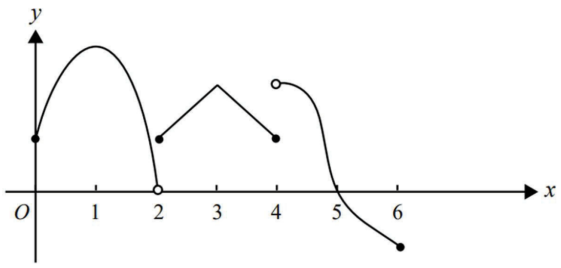
$$f(f(0)) = f(4) = 4(4) - 7 = 9.$$

The answer is (C).

## 2. Continuity and discontinuity: identify, justify, and reason

### Q8 North Sydney Boys 2022 Trial Q6 · **1 mark**

The graph of the function $f(x)$ with domain $x \in [0, 6]$ is shown below.




Which of the following is **not** true?

(A) The function is not continuous at $x = 2$ and $x = 4$.

(B) The function exists for all values of $x$ between 0 and 6.

(C) $f(x) = 0$ for $x = 2$ and $x = 5$.

(D) The function is positive for $x \in [0, 5)$.

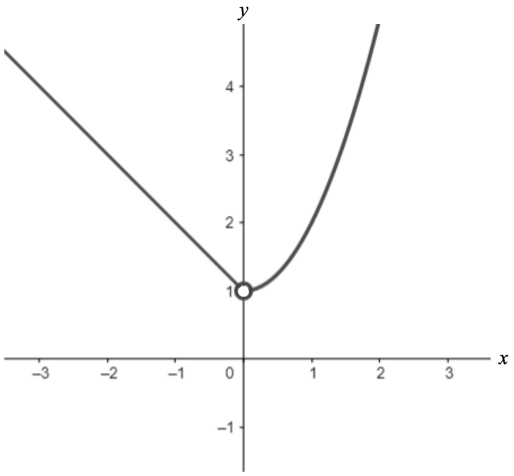
| **Criteria** | **Marks** |
|---|---|
| Correct option (C) | 1 |

**Solution.**

Read each statement against the graph. There are jump discontinuities at $x = 2$ and $x = 4$ (so A is true), the graph is defined at every $x$ in $[0, 6]$ (so B is true), and the curve stays above the $x$-axis on $[0, 5)$, touching the axis at $x = 5$ (so D is true). Statement C is false: at $x = 2$ the graph is *above* the axis ($f(2) > 0$), not zero: the only visible zero is at $x = 5$. The false statement is (C).

## Q9 Fort St 2021 Prelim Q6(a)(i) · **1 mark**

The graph of $y = f(x)$, where $x \neq 0$, is shown below.




Explain why $y = f(x)$ is continuous or discontinuous.

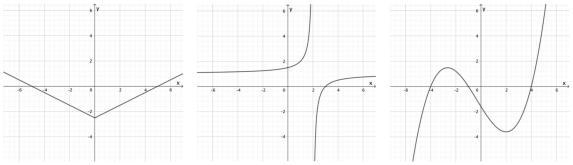
| **Criteria** | **Marks** |
|---|---|
| States discontinuous with a valid reason (e.g. $f(0)$ does not exist / there is a break in the curve at $x = 0$) | 1 |

**Solution.**

$y = f(x)$ is *discontinuous*. The domain excludes $x = 0$, so $f(0)$ does not exist; the graph has a break (gap) at $x = 0$ and cannot be drawn without lifting the pen there. Hence $f$ is not continuous.

## Q10 Hurlstone 2023 Prelim Q6 · **1 mark**

Three different functions are shown on the graphs below (Graph I, Graph II and Graph III).



Graph I Graph II Graph III

Which of the graphs is continuous for the domain $-4 \leq x \leq 4$?

(A) Graph I only

(B) Graph II only

(C) Graph III only

(D) Graphs I and III

| **Criteria** | **Marks** |
|---|---|
| Correct option (D) | 1 |

**Solution.**

A function is continuous on $-4 \leq x \leq 4$ if its graph can be drawn over that whole interval with no gaps or jumps. Graphs I and III are unbroken across $[-4, 4]$, while Graph II has a break (gap/jump). Hence the continuous graphs are I and III. The answer is (D).

## Q11 Sydney Tech 2022 Trial Q21(a) · **1 mark**

Consider the function

$$f(x) = \begin{cases} x^2 - 2x + 2, & -3 < x < 1, \\ x + 1, & 1 \leq x < 6. \end{cases}$$

Justify why the function is **not** continuous at $x = 1$.

| **Criteria** | **Marks** |
|---|---|
| Compares the two one-sided values at $x = 1$ (1 and 2) and concludes they differ, so $f$ is not continuous | 1 |

**Solution.**

For continuity at $x = 1$ the two branches must agree there. From the left ($x \to 1^-$) the first branch gives

$$x^2 - 2x + 2\Big|_{x=1} = 1 - 2 + 2 = 1,$$

while the value on the right branch is

$$f(1) = x + 1\Big|_{x=1} = 1 + 1 = 2.$$

Since $\lim_{x \to 1^-} f(x) = 1 \neq 2 = f(1)$, there is a jump at $x = 1$, so $f$ is not continuous there.

## Q12 Presbyterian 2022 Prelim Q6 · **1 mark**

A function is defined by the rule

$$f(x) = \begin{cases} 1, & x < 1, \\ x + 2, & x \geq 1. \end{cases}$$

Which statement is **incorrect**?

(A) The value of $f(-2)$ is 1.

(B) The graph is not continuous at $x = 1$.

(C) The domain is all real values of $x$.

(D) The range is $y \geq 1$.

| **Criteria** | **Marks** |
|---|---|
| Correct option (D) | 1 |

**Solution.**

Check each statement. Since $-2 < 1$, $f(-2) = 1$, so A is true. At $x = 1$ the left value is 1 but $f(1) = 1+2 = 3$, a jump, so B is true. Every real $x$ falls in one branch, so the domain is all reals and C is true. The values taken are $y = 1$ (from the first branch) together with $y = x + 2 \geq 3$ (from the second), i.e. the range is $\{1\} \cup [3, \infty)$. Values such as $y = 2$ are never attained, so the range is *not* $y \geq 1$. The incorrect statement is (D).

## Q13 Penrith 2023 Prelim Q24 · **2 marks**

Find the values of $x$ for which the graph of the function $f(x) = \dfrac{1}{x^2 - 9}$ is discontinuous.

| **Criteria** | **Marks** |
|---|---|
| Sets the denominator to zero: $x^2 - 9 = 0$ | 1 |
| Correct values $x = 3$ and $x = -3$ | 1 |

**Solution.**

A rational function is discontinuous exactly where its denominator is zero (the function is undefined there). Solve

$$x^2 - 9 = 0 \implies x^2 = 9 \implies x = \pm 3.$$

So $f$ is discontinuous at $x = 3$ and $x = -3$.

## Q14 North Sydney Girls 2022 Prelim Q9 · **1 mark**

Which statement is true of the function

$$f(x) = \begin{cases} x + 2, & x < 2, \\ x^2, & x \geq 2, \end{cases}$$

at the point $x = 2$?

(A) Continuous and differentiable

(B) Continuous but not differentiable

(C) Not continuous but differentiable

(D) Neither continuous nor differentiable

| **Criteria** | **Marks** |
|---|---|
| Correct option (B) | 1 |

**Solution.**

Check the value and the gradient of each branch at $x = 2$. The two branches meet:

$$\lim_{x \to 2^-} (x + 2) = 4 = 2^2 = f(2),$$

so $f$ is continuous at $x = 2$. Their gradients differ, however: $\dfrac{d}{dx}(x + 2) = 1$ from the left, while $\dfrac{d}{dx}(x^2) = 2x = 4$ from the right. Since $1 \neq 4$, there is a corner and $f$ is not differentiable at $x = 2$. The answer is (B).

### Q15 Killara 2022 Prelim Q8 · **1 mark**

Let $f$ be the function defined by

$$f(x) = \begin{cases} \tan x, & x < 0, \\ -x^2, & 0 \le x \le 2, \\ x + 2, & 2 \le x. \end{cases}$$

For which $x$ value(s) is $f$ not continuous?

(A) 0 only

(B) 2 only

(C) Both 0 and 2

(D) Neither 0 nor 2

| Criteria | Marks |
|---|---|
| Correct option (B) | 1 |

**Solution.**

Test the two join points. At $x = 0$: from the left $\lim_{x\to 0^-} \tan x = 0$ and $f(0) = -(0)^2 = 0$, so the branches agree and $f$ is continuous at $x = 0$. At $x = 2$: the middle branch gives $-x^2\big|_{x=2} = -4$, but the right branch gives $x + 2\big|_{x=2} = 4$. Since $-4 \neq 4$ there is a jump, so $f$ is not continuous at $x = 2$. Thus $f$ fails to be continuous at $x = 2$ only. The answer is (B).

## 3. Sketch the graph of a piecewise function

### Q16 Sydney Grammar 2023 Prelim Q11(b)(i) · **2 marks**

Sketch the curve $y = f(x)$ where

$$f(x) = \begin{cases} -1, & \text{for } x < 0, \\ x^2, & \text{for } x \ge 0. \end{cases}$$

| Criteria | Marks |
|---|---|
| Horizontal ray $y = -1$ for $x < 0$, with an open circle at $(0, -1)$ | 1 |
| Right half of the parabola $y = x^2$ for $x \ge 0$, closed at the origin $(0, 0)$ | 1 |

**Solution.**

For $x < 0$ the graph is the horizontal line $y = -1$ (an open circle at $(0, -1)$, since $x = 0$ is not included). For $x \ge 0$ the graph is the right-hand half of the parabola $y = x^2$, starting with a closed dot at the origin $(0, 0)$ and increasing. There is a jump at $x = 0$: the value approaches $-1$ from the left but equals 0 at $x = 0$.

## Q17 Baulkham Hills 2023 Prelim Q28(i) · **2 marks**

The function $y = f(x)$ is defined as follows:

$$f(x) = \begin{cases} 3x + 1, & \text{for } x < 0, \\ x^2, & \text{for } x \geq 0. \end{cases}$$

Sketch the function $y = f(x)$, showing all important features.

| **Criteria** | **Marks** |
|---|---|
| Line $y = 3x + 1$ for $x < 0$ with $x$-intercept $\left(-\frac{1}{3}, 0\right)$ and open circle at $(0, 1)$ | 1 |
| Right half of parabola $y = x^2$ for $x \geq 0$, closed at origin $(0, 0)$ | 1 |

**Solution.**

For $x < 0$ the graph is the line $y = 3x + 1$, which meets the $x$-axis at $3x + 1 = 0 \Rightarrow x = -\frac{1}{3}$, i.e. $\left(-\frac{1}{3}, 0\right)$, and approaches the point $(0, 1)$ with an open circle (as $x = 0$ is excluded from this branch). For $x \geq 0$ the graph is the right half of $y = x^2$, with a closed dot at the origin $(0, 0)$. There is a jump discontinuity at $x = 0$ (the line approaches 1, the parabola gives 0).

## Q18 Roseville 2022 Trial Q22(a) · **2 marks**

Let

$$g(x) = \begin{cases} x - 1, & x < 0, \\ x^2 + 1, & 0 \leq x \leq 3. \end{cases}$$

Sketch $g(x)$, showing all essential features.

| **Criteria** | **Marks** |
|---|---|
| Line $y = x - 1$ for $x < 0$, with an open circle at $(0, -1)$ | 1 |
| Parabola arc $y = x^2 + 1$ from closed vertex $(0, 1)$ to closed endpoint $(3, 10)$ | 1 |

**Solution.**

For $x < 0$ the graph is the line $y = x - 1$, rising towards $(0, -1)$ which is drawn as an open circle (since $x = 0$ belongs to the other branch). For $0 \leq x \leq 3$ the graph is an arc of the parabola $y = x^2 + 1$, starting at the closed vertex $(0, 1)$ and rising to the closed endpoint $(3,\ 3^2 + 1) = (3, 10)$. There is a jump at $x = 0$ (from $-1$ up to 1).

## Q19 Cherrybrook Tech 2021 Prelim Q19(a) · **2 marks**

The function $y = f(x)$ is defined as follows:

$$f(x) = \begin{cases} -1, & \text{where } x \leq -2, \\ 2x, & \text{where } -2 < x < 0, \\ 3 - x, & \text{where } x \geq 0. \end{cases}$$

Draw a sketch of the graph of $y = f(x)$. Ensure your graph is large enough to clearly label the intercepts and any other relevant values.

| **Criteria** | **Marks** |
|---|---|
| Horizontal ray $y = -1$ for $x \leq -2$ (closed at $(-2, -1)$) and line $y = 2x$ for $-2 < x < 0$ (open circles at $(-2, -4)$ and $(0, 0)$) | 1 |
| Line $y = 3 - x$ for $x \geq 0$, closed at $(0, 3)$, with $x$-intercept $(3, 0)$ | 1 |

**Solution.**

For $x \leq -2$ the graph is the horizontal line $y = -1$, ending in a closed dot at $(-2, -1)$. For $-2 < x < 0$ the graph is the line $y = 2x$, running between the open circles $(-2, -4)$ and $(0, 0)$. For $x \geq 0$ the graph is the line $y = 3 - x$, starting at the closed dot $(0, 3)$ and crossing the $x$-axis at $3 - x = 0 \Rightarrow (3, 0)$. Note the jumps at $x = -2$ (from $-1$ down to $-4$) and at $x = 0$ (from 0 up to 3).

## Q20 Sydney Tech 2022 Trial Q21(c) · **3 marks**

Consider the function

$$f(x) = \begin{cases} x^2 - 2x + 2, & -3 \leq x < 1, \\ x + 1, & 1 \leq x \leq 6. \end{cases}$$

Sketch the function for the domain $-3 \leq x \leq 6$.

| **Criteria** | **Marks** |
|---|---|
| Parabola arc $y = x^2 - 2x + 2$ for $-3 \leq x < 1$: closed endpoint $(-3, 17)$, vertex $(1, 1)$ shown as an open circle | 1 |
| Line $y = x + 1$ for $1 \leq x \leq 6$: closed at $(1, 2)$, closed endpoint $(6, 7)$ | 1 |
| Correct behaviour at $x = 1$ (open circle at $(1, 1)$, closed dot at $(1, 2)$) showing the jump | 1 |

**Solution.**

Completing the square, $x^2 - 2x + 2 = (x - 1)^2 + 1$, a parabola with vertex $(1, 1)$. On $-3 \leq x < 1$ draw the left arc from the closed endpoint $(-3,\ 17)$ down to the vertex position $(1, 1)$, which is an open circle (as $x = 1$ belongs to the other branch); the $y$-intercept of this arc is $(0, 2)$. For $1 \leq x \leq 6$ draw the line $y = x + 1$ from the closed dot $(1, 2)$ up to the closed endpoint $(6, 7)$. Since the left branch approaches 1 at $x = 1$ but the right branch gives 2, there is a jump discontinuity at $x = 1$.

## Q21 Abbotsleigh 2024 Prelim Q18(a) · **3 marks**

A function $f(x)$ is defined as follows:

$$f(x) = \begin{cases} 2x, & \text{for } x > 1, \\ 1 - x^2, & \text{for } -1 \leq x \leq 1, \\ -(2 + x), & \text{for } x < -1. \end{cases}$$

Draw the graph of $y = f(x)$.

| **Criteria** | **Marks** |
|---|---|
| Middle branch: downward parabola arc $y = 1-x^2$ with vertex $(0, 1)$, closed endpoints $(-1, 0)$ and $(1, 0)$ | 1 |
| Right branch: line $y = 2x$ for $x > 1$, open circle at $(1, 2)$ | 1 |
| Left branch: line $y = -(2 + x)$ for $x < -1$, open circle at $(-1, -1)$ | 1 |

**Solution.**

For $-1 \leq x \leq 1$ draw the downward parabola $y = 1 - x^2$: vertex $(0, 1)$ and closed endpoints where $1 - x^2 = 0$, i.e. $(-1, 0)$ and $(1, 0)$. For $x > 1$ draw the line $y = 2x$, beginning just past the open circle at $(1, 2)$ and rising. For $x < -1$ draw the line $y = -(2 + x) = -x - 2$, approaching the open circle at $(-1, -1)$ from the left. There is a jump at $x = 1$ (parabola gives 0, line jumps to 2) and at $x = -1$ (parabola gives 0, the left line approaches $-1$).

## Q22 Normanhurst Boys 2020 Trial Q13(a) · **4 marks**

A function is defined as

$$f(x) = \begin{cases} \log_2(x+4), & -4 < x \leq 4, \\ 2\,|x-5| + 1, & 4 < x < 7, \\ 5, & x \geq 7. \end{cases}$$

Sketch $f(x)$, showing all intercepts with the axes.

| **Criteria** | **Marks** |
|---|---|
| Correct axis intercepts: $x$-intercept $(-3, 0)$ and $y$-intercept $(0, 2)$ | 1 |
| Logarithm shape $y = \log_2(x+4)$ on $-4 < x \leq 4$ with vertical asymptote $x = -4$, closed at $(4, 3)$ | 1 |
| Absolute-value branch $y = 2\|x-5\| + 1$ on $4 < x < 7$ with vertex $(5, 1)$ (open circles at $(4, 3)$ and $(7, 5)$) | 1 |
| Horizontal line $y = 5$ for $x \geq 7$, closed at $(7, 5)$ | 1 |

**Solution.**

**Branch 1** $(-4 < x \leq 4)$: $y = \log_2(x+4)$ has a vertical asymptote at $x = -4$. Its $x$-intercept is where $x + 4 = 1$, i.e. $x = -3$, giving $(-3, 0)$; its $y$-intercept is $\log_2 4 = 2$, giving $(0, 2)$. It rises to the closed point $(4, \log_2 8) = (4, 3)$.

**Branch 2** $(4 < x < 7)$: $y = 2|x-5| + 1$ is a V-shape with vertex $(5, 1)$. As $x \to 4^+$ it approaches $2(1) + 1 = 3$ (open circle at $(4, 3)$) and as $x \to 7^-$ it approaches $2(2) + 1 = 5$ (open circle at $(7, 5)$).

**Branch 3** $(x \geq 7)$: the horizontal line $y = 5$, starting at the closed dot $(7, 5)$.

## 4. Analyse a piecewise function (domain, range, extrema, solve f(x)=k)

### Q23 Knox 2022 Trial Q12 · **2 marks**

The function $y = f(x)$ is defined as

$$f(x) = \begin{cases} 1 & \text{for } -2 \le x \le 0, \\ -x^2 + 1 & \text{for } x \ge 0. \end{cases}$$

By sketching $y = f(x)$ or otherwise, write the domain and range of $y = f(x)$ using interval notation.

| **Criteria** | **Marks** |
|---|---|
| Correct domain $[-2, \infty)$ | 1 |
| Correct range $(-\infty, 1]$ | 1 |

**Solution.**

The first piece is the horizontal segment $y = 1$ over $[-2, 0]$; the second piece is the downward parabola $y = -x^2 + 1$ for $x \ge 0$, starting at $(0, 1)$ and decreasing without bound.

**Domain:** the two pieces cover $[-2, 0]$ and $[0, \infty)$, so

$$\text{Domain} = [-2, \infty).$$

**Range:** the first piece contributes only the value 1; the second piece starts at 1 (when $x = 0$) and decreases to $-\infty$, giving $(-\infty, 1]$. The union is

$$\text{Range} = (-\infty, 1].$$

## Q24 Baulkham Hills 2020 Trial Q12(d)(ii) · **2 marks**

Consider the piecewise-defined function

$$f(x) = \begin{cases} x - 1 & \text{for } x \leq 1, \\ 4 - x^2 & \text{for } x > 1. \end{cases}$$

Find $x$ if $f(x) = 0$.

| **Criteria** | **Marks** |
|---|---|
| Solves the first branch: $x = 1$ (valid since $x \leq 1$) | 1 |
| Solves the second branch: $x = 2$ (valid since $x > 1$) | 1 |

**Solution.**

Solve each branch on its own domain.

**Branch 1** ($x \leq 1$): $x - 1 = 0 \Rightarrow x = 1$, which satisfies $x \leq 1$, so $x = 1$ is a solution.

**Branch 2** ($x > 1$): $4 - x^2 = 0 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$. Only $x = 2$ satisfies $x > 1$; $x = -2$ is rejected.

$$\therefore \; x = 1 \text{ or } x = 2.$$

## Q25 Knox 2022 Prelim Q11(d) · **2 marks**

Find the range of the piecewise function $f(x)$.

| **Criteria** | **Marks** |
|---|---|
| Analyses one branch correctly | 1 |
| Combines both branches to state the range | 1 |

**Solution.**

For $x > 0$: as $x$ ranges over $(0, \infty)$, $x^2$ ranges over $(0, \infty)$, so $f(x) = x^2 + 1$ ranges over $(1, \infty)$.

For $x < 0$: as $x$ ranges over $(-\infty, 0)$, $x^2$ ranges over $(0, \infty)$, so $f(x) = -1 - x^2$ ranges over $(-\infty, -1)$.

The value $x = 0$ is not in the domain, so neither $f = 1$ nor $f = -1$ is attained. Combining the two branches, the range is

$$(-\infty, -1) \cup (1, \infty).$$

## Q26 Abbotsleigh 2024 Prelim Q27 · **2 marks**

Consider the following function:

$$f(x) = \begin{cases} x^2 & \text{for } x < 0, \\ 2x - 3 & \text{for } x \geq 0. \end{cases}$$

Find the value(s) of $x$ if $f(x) = 25$.

| **Criteria** | **Marks** |
|---|---|
| Solves the $x < 0$ branch: $x = -5$ | 1 |
| Solves the $x \geq 0$ branch: $x = 14$ | 1 |

**Solution.**

Solve each branch on its own domain.

**Branch $x < 0$:** $x^2 = 25 \Rightarrow x = \pm 5$. Only $x = -5$ satisfies $x < 0$.

**Branch $x \geq 0$:** $2x - 3 = 25 \Rightarrow 2x = 28 \Rightarrow x = 14$, which satisfies $x \geq 0$.

$$\therefore \; x = -5 \text{ or } x = 14.$$

## Q27 Hornsby Girls 2023 Trial Q22 · **3 marks**

Find the global maximum and minimum value of

$$y = \begin{cases} -2 & \text{for } x < -3, \\ x + 1 & \text{for } -3 \leq x < 1, \\ -x^3 - x^2 + 4 & \text{for } 1 \leq x \leq 1.5. \end{cases}$$

Show working or sketch a graph to justify your answer.

| **Criteria** | **Marks** |
|---|---|
| Analyses the linear piece over $[-3, 1)$: values in $[-2, 2)$ | 1 |
| Analyses the cubic piece on $[1, 1.5]$ (decreasing): values in $[-1.625,\ 2]$ | 1 |
| Concludes global maximum = 2 and global minimum = $-2$ | 1 |

**Solution.**

Consider each piece over its own interval.

**Piece 1** ($x < -3$): constant $y = -2$.

**Piece 2** ($-3 \leq x < 1$): $y = x + 1$ is increasing, running from $y = -2$ at $x = -3$ up towards (but not reaching) $y = 2$ as $x \to 1^-$; values $[-2, 2)$.

**Piece 3** ($1 \leq x \leq 1.5$): $y = -x^3 - x^2 + 4$, so $y' = -3x^2 - 2x = -x(3x + 2)$. On $[1, 1.5]$ this is negative, so the cubic is decreasing. Hence its maximum is at $x = 1$, $y = -1 - 1 + 4 = 2$, and its minimum is at $x = 1.5$, $y = -1.625$. (The stationary points $x = 0, -\frac{2}{3}$ lie outside $[1, 1.5]$.)

Collecting: the largest attained value is 2 (reached at $x = 1$ on Piece 3, since Piece 2 only approaches it), and the smallest attained value is $-2$ (reached for all $x < -3$ and at $x = -3$).

$$\text{global maximum} = 2, \qquad \text{global minimum} = -2.$$

## 5. Find parameter(s) that make a piecewise function continuous

### Q28 Gosford 2025 Trial Q5 · **1 mark**

Given that

$$f(x) = \begin{cases} 5-(x-3)^2, & x \leq 2 \\ mx+3, & x > 2 \end{cases}$$

what is the value of $m$ for which $f(x)$ is continuous at $x = 2$?

(A) $\dfrac{7}{2}$

(B) $\dfrac{3}{2}$

(C) $\dfrac{1}{2}$

(D) $2$

| Criteria | Marks |
|---|---|
| Correct option (C) | 1 |

**Solution.**

For continuity at $x = 2$ the two pieces must agree there. From the top branch, $f(2) = 5-(2-3)^2 = 5-1 = 4$. Setting the bottom branch equal to this value:

$$m(2)+3 = 4 \quad \Rightarrow \quad 2m = 1 \quad \Rightarrow \quad m = \tfrac{1}{2}.$$

The answer is $\boxed{\text{C}}$.

## Q29 Manly 2022 Prelim Q19 · **2 marks**

Find the values of $a$ and $b$ for the function below, given that $g(x)$ is continuous for all $x$.

$$g(x) = \begin{cases} x^2 - 4, & x < -1 \\ ax + b, & -1 \leq x \leq 2 \\ 3x + 6, & x > 2 \end{cases}$$

| **Criteria** | **Marks** |
|---|---|
| Sets up continuity at $x = -1$ and $x = 2$ (i.e. $-a + b = -3$ and $2a + b = 12$) | 1 |
| Solves to $a = 5,\ b = 2$ | 1 |

**Solution.**

For $g$ to be continuous the pieces must agree at each join.

At $x = -1$: the left piece gives $(-1)^2 - 4 = -3$, so

$$a(-1) + b = -3 \quad \Rightarrow \quad -a + b = -3.$$

At $x = 2$: the right piece gives $3(2) + 6 = 12$, so

$$a(2) + b = 12 \quad \Rightarrow \quad 2a + b = 12.$$

Subtracting the first equation from the second: $3a = 15$, so $a = 5$, and then $b = a - 3 = 2$.

$$a = 5, \qquad b = 2.$$

## Q30 North Sydney Girls 2023 Prelim Q4 · **1 mark**

A function is given by

$$f(x) = \begin{cases} \dfrac{8}{\sqrt{x+1}}, & x \neq 3 \\ m, & x = 3 \end{cases}$$

For what value of $m$ is the function continuous at $x = 3$?

(A) 4

(B) $-1$

(C) 1

(D) $-4$

| **Criteria** | **Marks** |
|---|---|
| Correct option (A) | 1 |

**Solution.**

For continuity at $x = 3$ we need $m = \lim\limits_{x \to 3} \dfrac{8}{\sqrt{x+1}}$, which for this function is just its value there:

$$m = \frac{8}{\sqrt{3+1}} = \frac{8}{\sqrt{4}} = \frac{8}{2} = 4.$$

The answer is $\boxed{\text{A}}$.

## Q31 Gosford 2023 Prelim Q28 · **4 marks**

Consider the function defined by

$$f(x) = \begin{cases} ax^2 + x - b, & x > 2 \\ ax + b, & 2 \le x \le 5 \\ 2ax - 7, & x > 5 \end{cases}$$

Given the function is continuous at $f(2)$ and $f(5)$, find the values of $a$ and $b$.

| **Criteria** | **Marks** |
|---|---|
| Continuity at $x = 2$: $4a + 2 - b = 2a + b$, i.e. $b = a + 1$ | 1 |
| Continuity at $x = 5$: $5a + b = 10a - 7$, i.e. $b = 5a - 7$ | 1 |
| Solves the pair to $a = 2$ | 1 |
| States $b = 3$ | 1 |

**Solution.**

Continuity at each join means the adjacent pieces agree there.

At $x = 2$: equate the $ax^2 + x - b$ branch with the $ax + b$ branch,

$$a(2)^2 + 2 - b = a(2) + b \;\Rightarrow\; 4a + 2 - b = 2a + b \;\Rightarrow\; 2a + 2 = 2b \;\Rightarrow\; b = a + 1.$$

At $x = 5$: equate the $ax + b$ branch with the $2ax - 7$ branch,

$$a(5) + b = 2a(5) - 7 \;\Rightarrow\; 5a + b = 10a - 7 \;\Rightarrow\; b = 5a - 7.$$

Setting the two expressions for $b$ equal: $a + 1 = 5a - 7$, so $4a = 8$ and $a = 2$. Then $b = a + 1 = 3$.

$$a = 2, \qquad b = 3.$$