# Year 11 Mathematics Advanced

**Topic: Working with Functions (9 of 12)**

**Review Questions (Worked Solutions): Properties of Functions, Relations and Graphs**

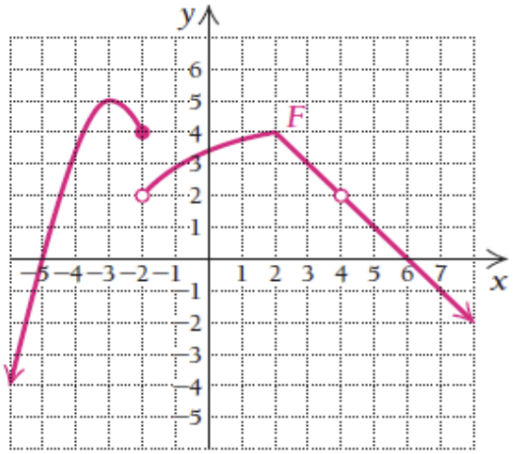
**Term 3, 2026**

# Domain, range and symmetry

## 1. Domain and range in interval notation (functions, relations, from graphs)

### Q1 Sydney Girls 2020 Trial Q14 · **2 marks**

Consider the function shown below.




Write down the domain and range using interval notation.

| Criteria | Marks |
|---|---|
| Correct domain in interval notation (horizontal extent of the graph) | 1 |
| Correct range in interval notation (vertical extent of the graph) | 1 |

**Solution.**

Read the two extents straight from the graph. The *domain* is the set of $x$-values the curve covers, read across the horizontal axis from the left-most to the right-most point (using a round bracket at an open endpoint / arrow and a square bracket at a filled endpoint). The *range* is the set of $y$-values the curve covers, read up the vertical axis from the lowest to the highest point with the same bracket convention. Write each as a single interval, e.g. $[a, b]$, in interval notation.

## Q2 Normanhurst Boys 2020 Trial Q13(b) · **2 marks**

A function is defined as

$$f(x) = \begin{cases} \log_2(x+4) & -4 < x \leq 4 \\ 2\,|x-5| + 1 & 4 < x < 7 \\ 5 & x \geq 7. \end{cases}$$

Using interval notation, state the domain and range of $f(x)$.

| **Criteria** | **Marks** |
|---|---|
| Correct domain $(-4, \infty)$ | 1 |
| Correct range $(-\infty, 5]$ | 1 |

**Solution.**

**Domain.** The pieces cover $-4 < x \leq 4$, then $4 < x < 7$, then $x \geq 7$, which join with no gaps to give all $x > -4$:

$$\text{Domain} = (-4, \infty).$$

**Range.** Take each piece. $\log_2(x+4)$ on $(-4, 4]$ increases from $-\infty$ (as $x \to -4^+$) up to $\log_2 8 = 3$, giving $(-\infty, 3]$. $2|x-5|+1$ on $(4, 7)$ has minimum 1 at $x = 5$ and rises towards $2|4-5|+1 = 3$ and $2|7-5|+1 = 5$ (both open ends), giving $[1, 5)$. The last piece contributes the single value 5. Uniting: $(-\infty, 3] \cup [1, 5) \cup \{5\} = (-\infty, 5]$, so

$$\text{Range} = (-\infty, 5].$$

## Q3 Gosford 2020 Trial Q23(a) · **3 marks**

The function $f$ is defined by $f(x) = 2 + \sqrt{x-3}$ for $x \geq 3$. Write the domain and range of the function $f$ using interval notation.

| **Criteria** | **Marks** |
|---|---|
| Recognises the domain restriction $x \geq 3$ (radicand $\geq 0$) | 1 |
| Correct domain $[3, \infty)$ | 1 |
| Correct range $[2, \infty)$ | 1 |

**Solution.**

The square root requires $x - 3 \geq 0$, i.e. $x \geq 3$ (and this matches the stated restriction), so

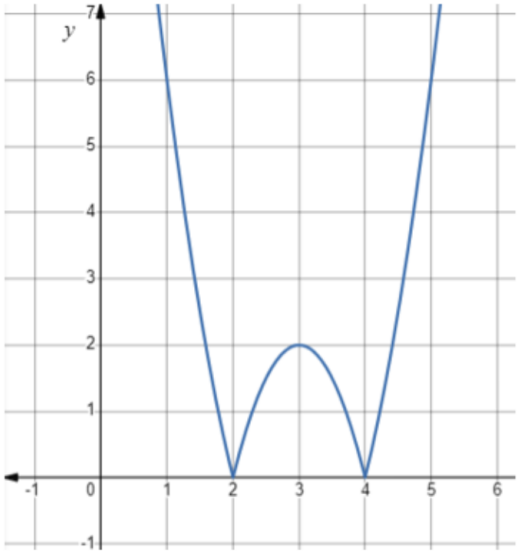
$$\text{Domain} = [3, \infty).$$

Since $\sqrt{x-3} \geq 0$ with equality at $x = 3$, the smallest value of $f$ is $f(3) = 2 + 0 = 2$, and $f$ increases without bound as $x \to \infty$. Hence

$$\text{Range} = [2, \infty).$$

## Q4 Hurlstone 2024 Prelim Q11 · **1 mark**

The graph of a function $y = f(x)$ is shown below.




State the range for the function $y = f(-x)$.

| **Criteria** | **Marks** |
|---|---|
| Correct range $[0, \infty)$ (equivalently $y \geq 0$) | 1 |

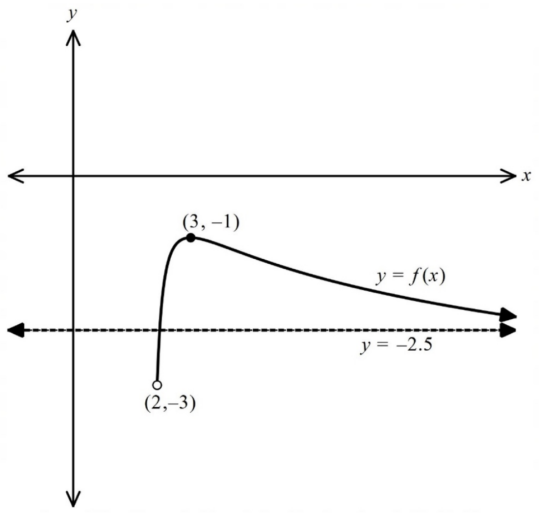
**Solution.**

Replacing $x$ by $-x$ reflects the graph in the $y$-axis. A reflection in the $y$-axis moves points horizontally only, so it changes the domain but leaves every $y$-value unchanged: $y = f(-x)$ has the *same range* as $y = f(x)$. Reading the vertical extent of the given graph gives

$$\text{Range} = [0, \infty).$$

## Q5 Barker 2024 Prelim Q8 · **1 mark**

The graph of a function $y = f(x)$ is shown below. It has an asymptote at $y = -2.5$. Which statement correctly describes its domain and range?




(A) Domain: $(2, \infty)$; Range: $[-2.5, -1]$

(B) Domain: $(-3, -1]$; Range: $(2, \infty)$

(C) Domain: $[2, \infty)$; Range: $[-3, -1)$

(D) Domain: $(2, \infty)$; Range: $(-3, -1]$

| **Criteria** | **Marks** |
|---|---|
| Correct option identified from the graph | 1 |

**Solution.**

Read the two extents off the graph, using the bracket rules: an open circle or an asymptote gives a round bracket (value not attained); a filled endpoint gives a square bracket. The domain is the horizontal extent and the range is the vertical extent. Because $y = -2.5$ is an asymptote the curve never reaches it, so any range that *includes* an endpoint the curve only approaches must be rejected. Match the horizontal and vertical extents (with the correct open/closed brackets) to the four listed pairs and select the option that agrees with both.

## Q6 Baulkham Hills 2023 Trial Q8 · **1 mark**

Which of the following is the domain and range of $y = \dfrac{x^2+1}{x^2-1}$?

(A) Domain: $x \in (-\infty,-1) \cup (-1,1) \cup (1,\infty)$; Range: $y \in (-\infty,-1) \cup (1,\infty)$

(B) Domain: $x \in (-\infty,-1) \cup (-1,1) \cup (1,\infty)$; Range: $y \in (0,\infty)$

(C) Domain: $x \in (-\infty,-1) \cap (-1,1) \cap (1,\infty)$; Range: $y \in (-\infty,-1) \cap (1,\infty)$

(D) Domain: $x \in (-\infty,-1) \cap (-1,1) \cap (1,\infty)$; Range: $y \in (0,\infty)$

| **Criteria** | **Marks** |
|---|---|
| Correct option (A) | 1 |

**Solution.**

**Domain.** The denominator $x^2-1=0$ at $x=\pm1$, so exclude these: the domain is the *union* $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$. This already rules out (C) and (D), which use intersection $\cap$ (an empty set). **Range.** Solve for $x^2$: from $y(x^2-1)=x^2+1$ we get $x^2(y-1)=y+1$, so $x^2=\dfrac{y+1}{y-1}$. A real $x$ needs $\dfrac{y+1}{y-1}\geq 0$, i.e. $y\leq -1$ or $y>1$. As $x\to\pm\infty$, $y\to 1^+$ (never 1), and at $x=0$, $y=\dfrac{1}{-1}=-1$ is attained. Hence

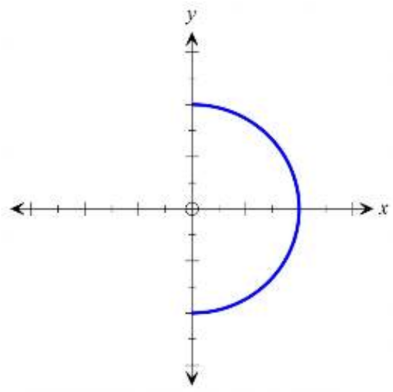
$$\text{Range} = (-\infty,-1]\cup(1,\infty).$$

Of the options, only (A) has the correct domain and the range approaching (but excluding) 1, so the answer is (A). (Strictly the value $y=-1$ is attained at $x=0$, so the endpoint at $-1$ should be closed; option (A) prints it open. The domain and the $y=1$ boundary still make (A) the correct choice.)

## 2. Classifying relations and functions (one-to-one, many-to-one, is-it-a-function)

### Q7 Barker 2021 Prelim Q5 · **1 mark**

What type of relation is the following graph?




(A) One-to-one

(B) One-to-many

(C) Many-to-one

(D) Many-to-many

| Criteria | Marks |
|---|---|
| Correct option (B) | 1 |

**Solution.**

The graph is a sideways parabola with its vertex on the $x$-axis, opening to the left. Any vertical line drawn through the opening cuts the curve *twice*, so a single $x$-value corresponds to two $y$-values: the vertical line test fails and this is not a function. A horizontal line, however, meets the curve only once, so each $y$-value comes from a single $x$-value. Hence each input $x$ maps to many outputs $y$: the relation is one-to-many. The answer is (B).

## Q8 Hunters Hill 2024 Trial Q2 · **1 mark**

Which of the following statements is true of the function $f(x) = -x(x+2)(x-2)$?

(A) $f(x)$ is a parabola that has been reflected and translated

(B) $f(x)$ is a one-to-one function

(C) $f(x)$ is an even function

(D) $f(x)$ is an odd function

| **Criteria** | **Marks** |
|---|---|
| Correct option (D) | 1 |

**Solution.**

Expanding, $f(x) = -x(x^2-4) = -x^3+4x$, a cubic (so it is not a parabola, ruling out (A)). Testing symmetry,

$$f(-x) = -(-x)^3 + 4(-x) = x^3 - 4x = -\left(-x^3+4x\right) = -f(x),$$

so $f$ is odd. Since $f'(x) = -3x^2 + 4$ has two zeros, the curve has two turning points and is not one-to-one, ruling out (B). The answer is (D).

## Q9 Hunters Hill 2022 Trial Q9 · **1 mark**

Let $f(x) = x^3 - (k^2-4)x + 1$. For which of the values of $k$ below is $f(x)$ many-to-one?

(A) $k = 2$

(B) $k = -2$

(C) $k = 1$

(D) $k = -4$

| **Criteria** | **Marks** |
|---|---|
| Correct option (D) | 1 |

**Solution.**

A cubic is many-to-one exactly when it has two turning points, i.e. when $f'(x) = 3x^2 - (k^2-4) = 0$ has two real solutions. This requires $k^2 - 4 > 0$, i.e. $|k| > 2$. Checking the options, only $k = -4$ gives $k^2 - 4 = 12 > 0$; the values $k = 2, -2, 1$ all give $k^2 - 4 \leq 0$, so $f$ is monotonic increasing (one-to-one) for those. The answer is (D).

## Q10 Baulkham Hills 2020 Trial Q9 · 1 mark

Given a function $f(x) = \dfrac{x}{x^2 - 5}$, which of the following statements is true?

(A) $f(x)$ is even and one-to-one.

(B) $f(x)$ is even and many-to-one.

(C) $f(x)$ is odd and one-to-one.

(D) $f(x)$ is odd and many-to-one.

| Criteria | Marks |
|---|---|
| Correct option (D) | 1 |

**Solution.**

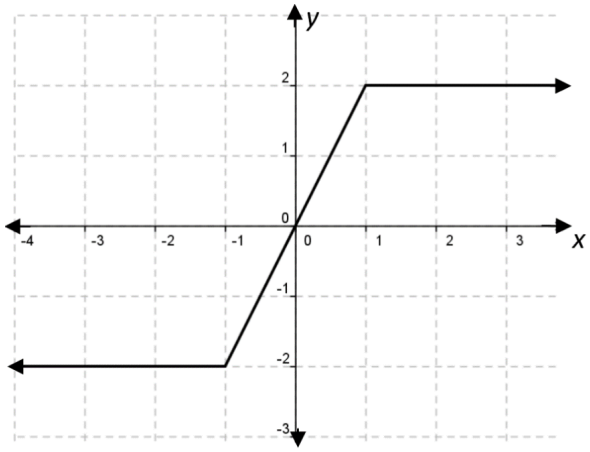
Replacing $x$ with $-x$,

$$f(-x) = \frac{-x}{(-x)^2 - 5} = \frac{-x}{x^2 - 5} = -f(x),$$

so $f$ is odd. It is not one-to-one: for example $f(x) = \frac{1}{10}$ gives $x^2 - 5 = 10x$, i.e. $x = 5 \pm \sqrt{30}$, two distinct inputs $x \approx -0.48$ and $x \approx 10.48$ producing the same output. So $f$ is odd and many-to-one. The answer is (D).

## Q11 Hurlstone 2023 Trial Q16(b) · **2 marks**

Consider the graph $y = f(x)$ shown below.




Determine if the graph drawn is even, odd or neither. Justify your answer.

| **Criteria** | **Marks** |
|---|---|
| Classifies the graph as odd | 1 |
| Justifies with the point (rotational) symmetry about the origin | 1 |

**Solution.**

The graph has point symmetry about the origin: rotating it 180° about $O$ maps the curve onto itself (equivalently, whenever $(x, y)$ lies on the graph so does $(-x, -y)$, e.g. $f(-3) = -2$ and $f(3) = 2$). This is exactly the condition $f(-x) = -f(x)$, so the graph is an *odd* function.

### Q12 James Ruse 2024 Trial Q2 · **1 mark**

Consider the function $f(x) = \ln\left(x + \sqrt{x^2+1}\right)$. Which of the following describes $f(x)$?

(A) An even many-to-one function.

(B) An odd many-to-one function.

(C) An even one-to-one function.

(D) An odd one-to-one function.

| Criteria | Marks |
|---|---|
| Correct option (D) | 1 |

**Solution.**

For symmetry, add $f(x)$ and $f(-x)$ and use $\ln a + \ln b = \ln(ab)$:

$$f(x) + f(-x) = \ln\left(x + \sqrt{x^2+1}\right) + \ln\left(-x + \sqrt{x^2+1}\right) = \ln\left(\left(\sqrt{x^2+1}\right)^2 - x^2\right) = \ln 1 = 0,$$

so $f(-x) = -f(x)$ and $f$ is odd. Its derivative $f'(x) = \dfrac{1}{\sqrt{x^2+1}} > 0$ for all $x$, so $f$ is strictly increasing and therefore one-to-one. The answer is (D).

## 3. Even and odd functions: algebraic tests

### Q13 Girraween 2023 Prelim Q7(c) · **2 marks**

Test whether $f(x) = x^5 - 21x$ is even, odd or neither.

| Criteria | Marks |
|---|---|
| computes and simplifies $f(-x)$ | 1 |
| correct conclusion (odd) with justification | 1 |

**Solution.**

**(c)** $f(-x) = (-x)^5 - 21(-x) = -x^5 + 21x = -\left(x^5 - 21x\right) = -f(x)$.
Since $f(-x) = -f(x)$ for all $x$, the function is **odd**.

## Q14 Merewether 2025 Trial Q15(a) · **1 mark**

Consider the function $f(x) = \dfrac{x}{x^2+1}$. Show that $f(x)$ is odd.

| Criteria | Marks |
|---|---|
| Computes $f(-x) = -\dfrac{x}{x^2+1} = -f(x)$ and concludes $f$ is odd | 1 |

**Solution.**

Replace $x$ by $-x$:

$$f(-x) = \frac{-x}{(-x)^2+1} = \frac{-x}{x^2+1} = -\frac{x}{x^2+1} = -f(x).$$

Since $f(-x) = -f(x)$ for all $x$, the function $f(x) = \dfrac{x}{x^2+1}$ is odd.

## Q15 Pymble 2022 Prelim Q16 · **2 marks**

Show working to determine whether the function $f(x) = \dfrac{x^3}{x^2-1}$ is odd, even or neither.

| Criteria | Marks |
|---|---|
| Computes $f(-x) = -\dfrac{x^3}{x^2-1}$ | 1 |
| Concludes $f(-x) = -f(x)$, so $f$ is odd | 1 |

**Solution.**

Replace $x$ by $-x$:

$$f(-x) = \frac{(-x)^3}{(-x)^2-1} = \frac{-x^3}{x^2-1} = -\frac{x^3}{x^2-1} = -f(x).$$

Since $f(-x) = -f(x)$ for all $x$ in the domain, $f(x) = \dfrac{x^3}{x^2-1}$ is odd.

## Q16 Kambala 2021 Prelim Q8 · **3 marks**

(a) Find $g(f(x))$. (b) Determine, with justification, whether $g(f(x))$ is odd, even or neither.

| Criteria | Marks |
|---|---|
| (a) Correct composite $g(f(x)) = \sqrt{x^2-2}$ | 1 |
| (b) Evaluates $g(f(-x))$ correctly | 1 |
| (b) Concludes even with valid justification | 1 |

**Solution.**

**(a)** Substitute $f(x)$ into $g$:

$$g\big(f(x)\big) = \sqrt{f(x)} = \sqrt{x^2-2}.$$

**(b)** Test $g\big(f(-x)\big)$:

$$g\big(f(-x)\big) = \sqrt{(-x)^2-2} = \sqrt{x^2-2} = g\big(f(x)\big).$$

Since $g\big(f(-x)\big) = g\big(f(x)\big)$, the composite function is **even**.

## Q17 Abbotsleigh 2023 Trial Q13(c)(i) · **1 mark**

Consider the function $f(x) = 3x - \sin x$. Show that $f(x)$ is an odd function.

| Criteria | Marks |
|---|---|
| Uses $\sin(-x) = -\sin x$ to get $f(-x) = -3x + \sin x = -f(x)$, so $f$ is odd | 1 |

**Solution.**

Replace $x$ by $-x$ and use $\sin(-x) = -\sin x$:

$$f(-x) = 3(-x) - \sin(-x) = -3x + \sin x = -\left(3x - \sin x\right) = -f(x).$$

Since $f(-x) = -f(x)$ for all $x$, the function $f(x) = 3x - \sin x$ is odd.

### Q18 Penrith 2023 Trial Q17 · **3 marks**

Consider the curve given by $f(x) = x^2 \ln|x|$. Is $f(x) = x^2 \ln|x|$ an odd or an even function or neither? Justify your response.

| Criteria | Marks |
|---|---|
| Notes the domain is $x \neq 0$ and uses $(-x)^2 = x^2$ | 1 |
| Uses $\lvert -x\rvert = \lvert x\rvert$ to compute $f(-x) = x^2 \ln\lvert x\rvert$ | 1 |
| Concludes $f(-x) = f(x)$, so $f$ is even | 1 |

**Solution.**

The domain is $x \neq 0$, which is symmetric about 0. Replace $x$ by $-x$, using $(-x)^2 = x^2$ and $|-x| = |x|$:

$$f(-x) = (-x)^2 \ln|-x| = x^2 \ln|x| = f(x).$$

Since $f(-x) = f(x)$ for all $x \neq 0$, the function $f(x) = x^2 \ln|x|$ is even.

## 4. Even and odd functions from a graph, and even/odd problem-solving

### Q19 Sydney Grammar 2022 Trial Q3 · **1 mark**

Which of the following is an odd function?

(A) $y = x^2 + x$

(B) $y = x^2 + 1$

(C) $y = x^3 + x$

(D) $y = x^3 + 1$

| Criteria | Marks |
|---|---|
| Correct option (C) | 1 |

**Solution.**

A function is odd when $f(-x) = -f(x)$; every power in the rule must be odd. Testing (C) with $f(x) = x^3 + x$:

$$f(-x) = (-x)^3 + (-x) = -x^3 - x = -\left(x^3 + x\right) = -f(x).$$

Options (A), (B) and (D) each contain an even power ($x^2$) or a constant, so they are not odd. The answer is (C).

## Q20 Hurlstone 2022 Prelim Q5 · 1 mark

Which one of the following sets of ordered pairs represents an odd function?

(A) $\{(2,3),(3,2),(5,7),(7,5)\}$

(B) $\{(-2,3),(2,3),(-5,7),(7,5)\}$

(C) $\{(2,3),(-2,-3),(5,7),(-5,-7)\}$

(D) $\{(2,3),(3,5),(5,7),(7,2)\}$

| Criteria | Marks |
|---|---|
| Correct option (C) | 1 |

**Solution.**

For an odd function $f(-x) = -f(x)$, so whenever $(a, b)$ is a point, $(-a, -b)$ must also be a point. Only option (C) has this property: $(2, 3)$ pairs with $(-2, -3)$, and $(5, 7)$ pairs with $(-5, -7)$. The answer is (C).

## Q21 Gosford 2023 Prelim Q2 · 1 mark

Which of the following graphs represents an odd function?

Diagram to be inserted:
`adv-working-with-functions-d9-q21.png`

(A) Graph A

(B) Graph B

(C) Graph C

(D) Graph D

| Criteria | Marks |
|---|---|
| Identifies the graph with point (origin) symmetry (odd) | 1 |

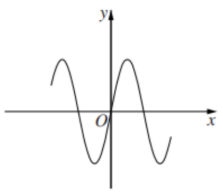
**Solution.**

A graph represents an odd function exactly when it has point symmetry about the origin: rotating the graph $180°$ about $(0, 0)$ maps it onto itself (equivalently, reflecting in the $y$-axis and then in the $x$-axis returns the original). Choose the option whose graph is unchanged by this $180°$ rotation about the origin; graphs that are symmetric about the $y$-axis are even, not odd.

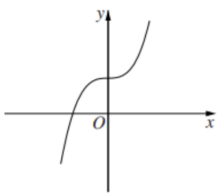
## Q22 Killara 2023 Prelim Q18 · **2 marks**

A part of an odd function $g(x)$ is shown on the diagram below. Complete the diagram for $g(x)$ over the domain $(0, \infty)$.

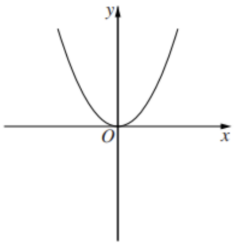
A.



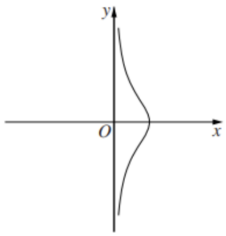

B.




C.




D.




| Criteria | Marks |
|---|---|
| Uses the odd property $g(-x) = -g(x)$ (rotates the given part 180° about the origin) | 1 |
| Draws the completed branch over $(0, \infty)$ with correct shape and endpoints | 1 |

**Solution.**

An odd function satisfies $g(-x) = -g(x)$, so its graph has point symmetry about the origin. The missing branch over $(0, \infty)$ is obtained by rotating the given part 180° about the origin (equivalently, reflect the shown part in the $y$-axis and then in the $x$-axis). Concretely, for each shown point $(-x_0,\ y_0)$ plot the image point $(x_0,\ -y_0)$, and join these to give the completed curve for $x > 0$.

## Q23 Baulkham Hills 2024 Trial Q9 · 1 mark

It is known that $f(x)$ is an odd function and $g(x)$ is an even function. Given that $f(2) = 2$ and $g(2) = -2$, what is the value of $f\big(g(-2)\big) + g\big(f(-2)\big)$?

(A) $-4$

(B) $-2$

(C) $0$

(D) $4$

| Criteria | Marks |
|---|---|
| Correct option (A) | 1 |

**Solution.**

Use $f(-x) = -f(x)$ and $g(-x) = g(x)$. Since $g$ is even, $g(-2) = g(2) = -2$; since $f$ is odd, $f(-2) = -f(2) = -2$. Then

$$f\big(g(-2)\big) = f(-2) = -2, \qquad g\big(f(-2)\big) = g(-2) = g(2) = -2.$$

Hence $f\big(g(-2)\big) + g\big(f(-2)\big) = -2 + (-2) = -4$. The answer is (A).

## Q24 North Sydney Girls 2022 Prelim Q21 · 2 marks

Let $f(x) = 1 - x^2$. Determine whether $f\big(f(x)\big)$ is odd, even, or neither. Show all working.

| Criteria | Marks |
|---|---|
| Forms $f\big(f(x)\big) = -x^4 + 2x^2$ | 1 |
| Shows $f\big(f(-x)\big) = f\big(f(x)\big)$ and concludes even | 1 |

**Solution.**

Compose the function with itself:

$$f\big(f(x)\big) = 1 - \big(f(x)\big)^2 = 1 - \left(1 - x^2\right)^2 = 1 - \left(1 - 2x^2 + x^4\right) = -x^4 + 2x^2.$$

Replacing $x$ by $-x$ leaves only even powers unchanged:

$$f\big(f(-x)\big) = -(-x)^4 + 2(-x)^2 = -x^4 + 2x^2 = f\big(f(x)\big).$$

Since $f\big(f(-x)\big) = f\big(f(x)\big)$ for all $x$, the composite is *even*.

## Q25 Sydney Boys 2025 Trial Q7 · **1 mark**

Let $f(x)$ be a continuous odd function. Given that $f(x-a)$ is an even function, which of the following must be true?

(A) $f(x-a) = f(a-x)$

(B) $f(x-a) + f(x+a) = 0$

(C) $f(0) > 0$

(D) None of the above

| **Criteria** | **Marks** |
|---|---|
| Correct option (B) | 1 |

**Solution.**

Let $h(x) = f(x-a)$. Because $h$ is even, $h(-x) = h(x)$, i.e. $f(-x-a) = f(x-a)$. Now $f$ is odd, so

$$f(-x-a) = f\big(-(x+a)\big) = -f(x+a).$$

Equating the two expressions gives $-f(x+a) = f(x-a)$, that is

$$f(x-a) + f(x+a) = 0,$$

so (B) must hold. (Option (A) fails because $f(a-x) = -f(x-a)$, and (C) fails because a continuous odd function has $f(0) = 0$, not $f(0) > 0$.) The answer is (B).

# Composite functions

## 5. Evaluate a composite function (numeric, from tables or graphs)

### Q26 Barker 2025 Trial Q3 · **1 mark**

The table shows the function values for $y = f(x)$ and $y = g(x)$.

| $x$ | $-1$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| $f(x)$ | 3 | 0 | 3 | 12 | 27 |
| $g(x)$ | 4 | 3 | 2 | 1 | 0 |

What is the value of $y = f\big(g(1)\big)$?

(A) 0

(B) 1

(C) 6

(D) 12

| **Criteria** | **Marks** |
|---|---|
| Correct option (D) | 1 |

**Solution.**

Work from the inside out, reading each value straight from the table. From the $g$ row, $g(1) = 2$. Then from the $f$ row, $f(2) = 12$. Hence

$$f\big(g(1)\big) = f(2) = 12.$$

The answer is (D).

## Q27 Barker 2023 Trial Q12(a) · **1 mark**

Given that $f(x) = 2x + 1$ and $h(x) = x^2 - x$, find $h\big(f(-2)\big)$.

| Criteria | Marks |
|---|---|
| Correct value $h\big(f(-2)\big) = 12$ (with $f(-2) = -3$) | 1 |

**Solution.**

Evaluate the inner function first, then substitute into $h$:

$$f(-2) = 2(-2) + 1 = -3,$$

$$h\big(f(-2)\big) = h(-3) = (-3)^2 - (-3) = 9 + 3 = 12.$$

## Q28 Ascham 2020 Trial Q19 · **1 mark**

Consider the functions $f(x)$ and $g(x)$ whose graphs are displayed below.

Diagram to be inserted:
`adv-working-with-functions-d9-q28.png`

Find the value of $g\big(f(-2)\big)$.

| Criteria | Marks |
|---|---|
| Correct value $g\big(f(-2)\big) = 4$ (reads $f(-2) = 3$ then $g(3) = 4$) | 1 |

**Solution.**

Read the inner value off the graph of $y = f(x)$ (the parabola $f(x) = x^2 - 1$): at $x = -2$, $f(-2) = 3$. Now read the graph of $y = g(x)$ (the line $g(x) = x + 1$) at that value:

$$g\big(f(-2)\big) = g(3) = 4.$$

## Q29 Knox 2023 Prelim Q18(a) · **2 marks**

A function is defined by $f(x) = 16 - \sqrt{x}$. Find $f\big(f(9)\big)$, leaving your answer in exact form.

| **Criteria** | **Marks** |
|---|---|
| Evaluates the inner value $f(9) = 16 - \sqrt{9} = 13$ | 1 |
| Correct exact answer $f\big(f(9)\big) = 16 - \sqrt{13}$ | 1 |

**Solution.**

Evaluate the inner function first:

$$f(9) = 16 - \sqrt{9} = 16 - 3 = 13.$$

Substitute this into $f$ (the 9 is a perfect square, but 13 is not, so the surd stays):

$$f\big(f(9)\big) = f(13) = 16 - \sqrt{13}.$$

## Q30 Newington 2024 Trial Q6 · **1 mark**

The table below shows the values of two functions, $f(x)$ and $g(x)$, for various values of $x$. It is known that $f(x)$ is an even function and $g(x)$ is an odd function.

| $x$ | 1 | 2 | 4 | 6 | 7 |
|---|---|---|---|---|---|
| $f(x)$ | 3 | 5 | 6 | 1 | 4 |
| $g(x)$ | 2 | 0 | 1 | 4 | 7 |

What is the value of $f\big(g(-4)\big)$?

(A) 2

(B) 3

(C) 4

(D) 5

| **Criteria** | **Marks** |
|---|---|
| Correct option (B) | 1 |

**Solution.**

The table only lists positive $x$, so use the even/odd properties to reach the negative arguments. Since $g$ is odd, $g(-4) = -g(4) = -1$. Then since $f$ is even, $f(-1) = f(1) = 3$. Hence

$$f\big(g(-4)\big) = f(-1) = f(1) = 3.$$

The answer is (B).

### Q31 Caringbah 2022 Trial Q7 · 1 mark

Consider the functions $f(x) = x^2$ and $g(x) = x + 2$. Which of the following expressions is equal to $f\big(g(x)\big) - g\big(f(x)\big)$?

(A) $4x + 2$

(B) $4x + 4$

(C) $x^2 - x - 2$

(D) $2$

| Criteria | Marks |
|---|---|
| Correct option (A) | 1 |

**Solution.**

Form each composite by substituting one function into the other, then subtract:

$$f\big(g(x)\big) = (x+2)^2 = x^2 + 4x + 4, \qquad g\big(f(x)\big) = x^2 + 2.$$

$$f\big(g(x)\big) - g\big(f(x)\big) = \big(x^2 + 4x + 4\big) - \big(x^2 + 2\big) = 4x + 2.$$

The answer is (A).

## 6. Form and simplify the composite function f(g(x))

### Q32 Loreto Kirribilli 2022 Prelim Q23(a) · 1 mark

If $f(x) = x^2 - 1$ and $g(x) = \sqrt{x}$, find the composite function $f(g(x))$.

| Criteria | Marks |
|---|---|
| Substitutes $g$ into $f$ and simplifies to $f(g(x)) = x - 1$ (for $x \geq 0$) | 1 |

**Solution.**

Substitute $g(x) = \sqrt{x}$ into $f$:

$$f(g(x)) = \big(\sqrt{x}\big)^2 - 1 = x - 1, \qquad x \geq 0.$$

## Q33 Normanhurst Boys 2022 Trial Q11 · **2 marks**

Given the functions $f(x) = 2x^2 - 7x + 5$ and $g(x) = 3x + 2$, find $f(g(x))$ in simplest form.

| Criteria | Marks |
|---|---|
| Substitutes $g$ into $f$: $2(3x+2)^2 - 7(3x+2) + 5$ | 1 |
| Expands and simplifies to $18x^2 + 3x - 1$ | 1 |

**Solution.**

Replace every $x$ in $f$ by $g(x) = 3x + 2$:

$$f(g(x)) = 2(3x+2)^2 - 7(3x+2) + 5.$$

Expand $(3x+2)^2 = 9x^2 + 12x + 4$:

$$= 2\left(9x^2 + 12x + 4\right) - 21x - 14 + 5 = 18x^2 + 24x + 8 - 21x - 9 = 18x^2 + 3x - 1.$$

## Q34 Sydney Girls 2021 Prelim Q4(b)(i) · **1 mark**

Consider the functions $f(x) = e^x$ and $g(x) = \ln(x-2)$. Find $f(g(x))$.

| Criteria | Marks |
|---|---|
| Uses $e^{\ln u} = u$ to obtain $f(g(x)) = x - 2$ (for $x > 2$) | 1 |

**Solution.**

Substitute $g(x) = \ln(x-2)$ into $f(x) = e^x$ and use that $e^x$ and ln are inverse operations:

$$f(g(x)) = e^{\ln(x-2)} = x - 2, \qquad x > 2.$$

## Q35 North Sydney Boys 2024 Prelim Q5 · **1 mark**

The functions $g(x)$ and $h(x)$ are defined as follows:

$$g(x) = \frac{x^2}{8}, \qquad h(x) = \sqrt{2x+3}.$$

Which is the correct expression for $h(g(x))$?

(A) $\frac{x}{2} + 3$

(B) $\frac{x}{2} - 3$

(C) $\sqrt{\frac{2x}{3}}$

(D) $\sqrt{\frac{x}{2} + 3}$

| **Criteria** | **Marks** |
|---|---|
| Correct option (D) | 1 |

**Solution.**

Substitute $g(x) = \frac{x^2}{8}$ into $h(x) = \sqrt{2x+3}$:

$$h(g(x)) = \sqrt{2 \cdot \frac{x^2}{8} + 3} = \sqrt{\frac{x^2}{4} + 3} = \frac{\sqrt{x^2+12}}{2}.$$

Only option (D) keeps the square root of a fraction plus 3; the answer is (D). (The other options drop the surd or use the wrong radicand.)

## Q36 Cherrybrook Tech 2021 Prelim Q11 · **2 marks**

Given that $f(x) = 2x^2 - x$ and $g(x) = 2x - 3$, write an expression for $f(g(x))$ in the form $ax^2 + bx + c$, where $a$, $b$ and $c$ are integers.

| Criteria | Marks |
|---|---|
| Substitutes $g$ into $f$: $2(2x-3)^2 - (2x-3)$ | 1 |
| Expands and simplifies to $8x^2 - 26x + 21$ | 1 |

**Solution.**

Replace every $x$ in $f$ by $g(x) = 2x - 3$:

$$f(g(x)) = 2(2x-3)^2 - (2x-3).$$

Expand $(2x-3)^2 = 4x^2 - 12x + 9$:

$$= 2\left(4x^2 - 12x + 9\right) - 2x + 3 = 8x^2 - 24x + 18 - 2x + 3 = 8x^2 - 26x + 21.$$

So $a = 8$, $b = -26$, $c = 21$.

## Q37 James Ruse 2022 Trial Q13(b) · **2 marks**

Let $f(x) = -x^2 + x + 4$ and $g(x) = x^2 - 2$. Express $f(g(x))$ in the form $ax^4 + bx^2 + c$, where $a$, $b$ and $c$ are non-zero integers.

| Criteria | Marks |
|---|---|
| Substitutes $g$ into $f$: $-(x^2-2)^2 + (x^2-2) + 4$ | 1 |
| Expands and simplifies to $-x^4 + 5x^2 - 2$ | 1 |

**Solution.**

Replace every $x$ in $f$ by $g(x) = x^2 - 2$:

$$f(g(x)) = -(x^2-2)^2 + (x^2-2) + 4.$$

Expand $(x^2-2)^2 = x^4 - 4x^2 + 4$:

$$= -\left(x^4 - 4x^2 + 4\right) + x^2 - 2 + 4 = -x^4 + 4x^2 - 4 + x^2 + 2 = -x^4 + 5x^2 - 2.$$

So $a = -1$, $b = 5$, $c = -2$.

### Q38 Barker 2021 Prelim Q11(d) · **2 marks**

If $f(x) = \dfrac{3}{x^2}$, find an expression for $f(f(x))$.

| Criteria | Marks |
|---|---|
| Substitutes $f$ into itself: $\dfrac{3}{(3/x^2)^2}$ | 1 |
| Simplifies to $\dfrac{x^4}{3}$ | 1 |

**Solution.**

Substitute $f(x) = \dfrac{3}{x^2}$ into $f$, i.e. square it and divide 3 by the result:

$$f(f(x)) = \frac{3}{\left(\dfrac{3}{x^2}\right)^2} = \frac{3}{\dfrac{9}{x^4}} = 3 \cdot \frac{x^4}{9} = \frac{x^4}{3}.$$

## 7. Domain and range of a composite function

### Q39 Fort St 2022 Prelim Q8(b) · **2 marks**

Given $f(x) = \dfrac{3}{x}$ and $g(x) = x + 2$, state the domain and range of $f(g(x))$.

| Criteria | Marks |
|---|---|
| Domain: all real $x$ except $x = -2$ | 1 |
| Range: all real $y$ except $y = 0$ | 1 |

**Solution.**

First form the composite:

$$f(g(x)) = \frac{3}{g(x)} = \frac{3}{x+2}.$$

The composite is undefined only where the denominator is zero, so the **domain** is all real $x$ with $x \neq -2$. As $\dfrac{3}{x+2}$ can take every value except 0 (it approaches 0 but never equals it), the **range** is all real $y$ with $y \neq 0$.

## Q40 SCEGGS 2022 Prelim Q17(b) · **1 mark**

Let $f(x) = \sqrt{x}$ and $g(x) = x - 4$, so that $f(g(x)) = \sqrt{x-4}$. State the domain of $f(g(x))$.

| Criteria | Marks |
|---|---|
| Domain $x \geq 4$ | 1 |

**Solution.**

For $\sqrt{x-4}$ to be defined we require the radicand to be non-negative:

$$x - 4 \geq 0 \implies x \geq 4.$$

So the domain is $x \geq 4$ (that is, $[4, \infty)$).

## Q41 Penrith 2023 Prelim Q13(c) · **2 marks**

Let $f(x) = x + 2$ and $g(x) = -x^2 + 6x + 7$, so that $g(f(x)) = -x^2 + 2x + 15$. Find the domain and range of $g(f(x))$ in interval notation.

| Criteria | Marks |
|---|---|
| Domain $(-\infty, \infty)$ | 1 |
| Range $(-\infty, 16]$ (from the maximum value 16) | 1 |

**Solution.**

The composite $g(f(x)) = -x^2 + 2x + 15$ is a polynomial, so it is defined for every real number: the **domain** is $(-\infty, \infty)$. It is a concave-down parabola, so it has a maximum at its vertex. Completing the square,

$$-x^2 + 2x + 15 = -(x-1)^2 + 16,$$

so the vertex is $(1, 16)$ and the maximum value is 16. Hence the **range** is $(-\infty, 16]$.

## Q42 Barker 2024 Prelim Q18(c) · **2 marks**

Given that $f(x) = x^2 + 1$ and $g(x) = \sqrt{x-3}$, determine the equation of $y = f(g(x))$ and state the domain of $f(g(x))$.

| **Criteria** | **Marks** |
|---|---|
| Equation $y = x - 2$ | 1 |
| Domain $x \geq 3$ | 1 |

**Solution.**

Substitute $g(x)$ into $f$:

$$f(g(x)) = \left(\sqrt{x-3}\right)^2 + 1 = (x-3) + 1 = x - 2.$$

Although the simplified rule $y = x - 2$ is a line, the **domain** is inherited from the inner function $g$: we need $x - 3 \geq 0$, i.e. $x \geq 3$. So $y = x - 2$ with domain $x \geq 3$.

## Q43 Killara 2024 Trial Q21 · **3 marks**

Given the two functions $f(x) = x^2 - 3$ and $g(x) = \sqrt{2-x}$, find the domain and range of $f(g(x))$.

| **Criteria** | **Marks** |
|---|---|
| Domain $x \leq 2$ (from the inner function $g$) | 1 |
| Simplify $f(g(x)) = -x - 1$ (equivalently $= (2-x) - 3$) | 1 |
| Range $[-3, \infty)$ | 1 |

**Solution.**

The inner function $g(x) = \sqrt{2-x}$ requires $2 - x \geq 0$, i.e. $x \leq 2$; this fixes the **domain** of the composite as $x \leq 2$. Substituting,

$$f(g(x)) = \left(\sqrt{2-x}\right)^2 - 3 = (2-x) - 3 = -x - 1.$$

For the **range**, note $g(x) = \sqrt{2-x}$ takes all values in $[0, \infty)$ as $x$ ranges over $x \leq 2$. Writing $f(g(x)) = (\text{output of } g)^2 - 3$, the smallest value occurs when $g = 0$ (at $x = 2$), giving $-3$, and the value grows without bound as $x \to -\infty$. Hence the range is $[-3, \infty)$.

## Q44 Killara 2024 Prelim Q22(a) · 3 marks

Let $f(x) = \sqrt{x}$ and $g(x) = x^4 + 1$. Let $h(x) = f(g(x))$. State the domain and range of $h(x)$.

| Criteria | Marks |
|---|---|
| Form $h(x) = \sqrt{x^4 + 1}$ | 1 |
| Domain: all real $x$ | 1 |
| Range $[1, \infty)$ (minimum at $x = 0$) | 1 |

**Solution.**

Form the composite:

$$h(x) = f(g(x)) = \sqrt{x^4 + 1}.$$

Since $x^4 + 1 \geq 1 > 0$ for every real $x$, the radicand is always non-negative, so the **domain** is all real $x$, i.e. $(-\infty, \infty)$. The expression $x^4 + 1$ has minimum value 1 (at $x = 0$) and increases without bound, so $\sqrt{x^4 + 1}$ has minimum $\sqrt{1} = 1$ and is unbounded above. Hence the **range** is $[1, \infty)$.

## Q45 North Sydney Boys 2024 Trial Q9 · 1 mark

Let $f(x) = \sqrt{x}$ and $g(x) = \dfrac{1}{x - 1}$. Which is the range of $g(f(x))$?

(A) $[1, \infty)$

(B) $[0, 1) \cup (1, \infty)$

(C) $(-\infty, -1] \cup (0, \infty)$

(D) $(0, \infty)$

| Criteria | Marks |
|---|---|
| Correct option (C) | 1 |

**Solution.**

The composite is $g(f(x)) = \dfrac{1}{\sqrt{x} - 1}$. Let $t = f(x) = \sqrt{x}$; as $x$ ranges over the domain $x \geq 0$, $t$ takes all values in $[0, \infty)$, and we must exclude $t = 1$ (i.e. $x = 1$) so the denominator is non-zero. Now consider $\dfrac{1}{t - 1}$:

$$t \in [0, 1) \Rightarrow t - 1 \in [-1, 0) \Rightarrow \tfrac{1}{t-1} \in (-\infty, -1], \qquad t \in (1, \infty) \Rightarrow t - 1 \in (0, \infty) \Rightarrow \tfrac{1}{t-1} \in (0, \infty).$$

At $t = 0$ (i.e. $x = 0$) the value is $\dfrac{1}{-1} = -1$, which is included. Combining the two branches, the range is $(-\infty, -1] \cup (0, \infty)$, which is option (C).

## Q46 North Sydney Girls 2023 Prelim Q10 · **1 mark**

The graphs of $y = f(x)$ and $y = g(x)$ are shown.

Diagram to be inserted:
`adv-working-with-functions-d9-q46.png`

What is the domain and range of $y = f(g(x))$?

(A) Domain $(-1, 2]$, Range $[-3, 3]$

(B) Domain $(-1, 2]$, Range $[-1, 2]$

(C) Domain $(-\infty, \infty)$, Range $[-3, 3)$

(D) Domain $(-\infty, \infty)$, Range $(-1, 2]$

| **Criteria** | **Marks** |
|---|---|
| Correct option (C) | 1 |

**Solution.**

Read the two graphs. The inner function $g$ is defined for all real $x$ and its outputs lie inside the domain of $f$, so $f(g(x))$ is defined for every real $x$: the **domain** is $(-\infty, \infty)$, eliminating options (A) and (B). As $g(x)$ sweeps across the domain of $f$, the composite $f(g(x))$ takes the output values of $f$ over that domain. Because $f$ is read from an interval that is open at its left endpoint, the corresponding extreme output value is excluded, giving the **range** $[-3, 3)$. Hence the answer is option (C).

## 8. Determine the equation of a composite / solve composite equations

### Q47 Penrith 2023 Prelim Q13(a) · **2 marks**

Let $f(x) = x + 2$ and $g(x) = -x^2 + 6x + 7$.
Show that $g(f(x)) = -x^2 + 2x + 15$.

| Criteria | Marks |
|---|---|
| Substitutes $f(x) = x + 2$ into $g$: forms $g(x + 2) = -(x + 2)^2 + 6(x + 2) + 7$ | 1 |
| Expands and simplifies to the required $-x^2 + 2x + 15$ | 1 |

**Solution.**

Replace the input of $g$ with $f(x) = x + 2$:

$$g(f(x)) = -(x + 2)^2 + 6(x + 2) + 7.$$

Expanding, $-(x^2 + 4x + 4) + 6x + 12 + 7 = -x^2 - 4x - 4 + 6x + 19 = -x^2 + 2x + 15$, as required.

### Q48 Barker 2024 Prelim Q10 · **1 mark**

Given that $f$ and $g$ are two functions such that $f(x) = 2x$ and $g(x + 2) = 3x + 1$, which function is $f(g(x))$?

(A) $6x - 5$

(B) $6x + 1$

(C) $6x - 10$

(D) $6x + 2$

| Criteria | Marks |
|---|---|
| Correct option (C) | 1 |

**Solution.**

First find $g(x)$. Let $u = x + 2$, so $x = u - 2$ and $g(u) = 3(u - 2) + 1 = 3u - 5$; hence $g(x) = 3x - 5$. Then

$$f(g(x)) = 2(3x - 5) = 6x - 10.$$

The answer is **(C)**.

## Q49 St George Girls 2022 Prelim Q26 · 3 marks

Consider the functions $f(x) = x^2 - 4$ and $g(x) = 4 - 2x$.
Solve $f(g(x)) = 0$.

| Criteria | Marks |
|---|---|
| Forms $f(g(x)) = (4-2x)^2 - 4$ and expands to $4x^2 - 16x + 12$ | 1 |
| Reduces to $x^2 - 4x + 3 = 0$ and factorises | 1 |
| Both solutions $x = 1$ and $x = 3$ | 1 |

**Solution.**

Substitute $g(x) = 4 - 2x$ into $f$:

$$f(g(x)) = (4-2x)^2 - 4 = 16 - 16x + 4x^2 - 4 = 4x^2 - 16x + 12.$$

Setting this equal to 0 and dividing by 4: $x^2 - 4x + 3 = 0$, so $(x-1)(x-3) = 0$. Hence $x = 1$ or $x = 3$.

## Q50 Girraween 2023 Prelim Q7(e) · 4 marks

(a)(i) Find $g(f(x))$. (ii) Find $f(g(x))$. (b) Hence find $b$ so that $g(f(x)) = f(g(x))$ for all $x$.

| Criteria | Marks |
|---|---|
| (a)(i) correct $g(f(x))$ | 1 |
| (a)(ii) correct $f(g(x))$ | 1 |
| (b) sets expressions equal | 1 |
| (b) correct value $b = 6$ | 1 |

**Solution.**

**(a)(i)** $g(f(x)) = 7(3x + 2) + b = 21x + 14 + b$.

**(a)(ii)** $f(g(x)) = 3(7x + b) + 2 = 21x + 3b + 2$.

**(b)** Setting $g(f(x)) = f(g(x))$: $21x + 14 + b = 21x + 3b + 2$. The $21x$ terms cancel, giving $14 + b = 3b + 2$, so $12 = 2b$ and $b = 6$.

## Q51 Knox 2023 Prelim Q18(b) · **3 marks**

A function is defined by $f(x) = 16 - \sqrt{x}$.

Solve $f(f(x)) = 14$.

| **Criteria** | **Marks** |
|---|---|
| Forms $f(f(x)) = 16 - \sqrt{16 - \sqrt{x}}$ and sets equal to 14 | 1 |
| Simplifies to $\sqrt{16 - \sqrt{x}} = 2$, then $\sqrt{x} = 12$ | 1 |
| Correct answer $x = 144$ | 1 |

**Solution.**

Apply $f$ twice:

$$f(f(x)) = 16 - \sqrt{f(x)} = 16 - \sqrt{16 - \sqrt{x}} = 14.$$

So $\sqrt{16 - \sqrt{x}} = 2$, hence $16 - \sqrt{x} = 4$, giving $\sqrt{x} = 12$ and therefore $x = 144$.

## Q52 Baulkham Hills 2024 Trial Q31 · **3 marks**

Given $f(x) = \dfrac{x+1}{2x-1}$ and $f(g(x)) = \dfrac{3x+2}{3x-5}$, find $g(x)$.

| **Criteria** | **Marks** |
|---|---|
| Writes $f(g(x)) = \dfrac{g(x)+1}{2g(x)-1} = \dfrac{3x+2}{3x-5}$ | 1 |
| Cross-multiplies and collects terms in $g(x)$ | 1 |
| Solves $g(x) = \dfrac{2x-1}{x+3}$ | 1 |

**Solution.**

Since $f(t) = \dfrac{t+1}{2t-1}$, setting $t = g(x)$ gives

$$\frac{g(x)+1}{2g(x)-1} = \frac{3x+2}{3x-5}.$$

Write $g = g(x)$ and cross-multiply: $(g+1)(3x-5) = (3x+2)(2g-1)$, i.e.

$$3gx - 5g + 3x - 5 = 6gx + 4g - 3x - 2.$$

Collecting the $g$ terms on the left and everything else on the right:

$$3gx - 5g - 6gx - 4g = -3x - 2 - 3x + 5, \qquad -3gx - 9g = -6x + 3.$$

Factor: $g(-3x-9) = -6x+3$, so

$$g(x) = \frac{-6x+3}{-3x-9} = \frac{6x-3}{3x+9} = \frac{2x-1}{x+3}.$$

### Q53 North Sydney Girls 2024 Trial Q27(b) · **2 marks**

Consider the function $f\left(\frac{x}{3}-1\right)=\frac{1}{x-2}-6$.
Find the equation of $y=f(x)$.

| Criteria | Marks |
|---|---|
| Substitutes $u=\frac{x}{3}-1$ so $x=3u+3$ into the given expression | 1 |
| Obtains $f(x)=\frac{1}{3x+1}-6$ | 1 |

**Solution.**

Let $u=\frac{x}{3}-1$, so $x=3(u+1)=3u+3$. Substituting into the right-hand side,

$$f(u)=\frac{1}{(3u+3)-2}-6=\frac{1}{3u+1}-6.$$

Renaming $u$ as $x$, the equation of the function is

$$y=f(x)=\frac{1}{3x+1}-6.$$

## 9. Sketch or identify the graph of a composite function

### Q54 Trinity Grammar 2022 Trial Q22(b) · **1 mark**

Let $f(x)=\sqrt{4-x}$ and $g(x)=x^2$. Sketch the graph of $y=f\big(g(x)\big)$.

| Criteria | Marks |
|---|---|
| Correct graph of $y=\sqrt{4-x^2}$ (upper semicircle, radius 2, on $[-2,2]$) | 1 |

**Solution.**

Substitute $g$ into $f$:

$$f\big(g(x)\big)=\sqrt{4-g(x)}=\sqrt{4-x^2}.$$

This is the upper half of the circle $x^2+y^2=4$: a semicircle of radius 2 centred at the origin, with domain $[-2,2]$ and range $[0,2]$. It meets the $x$-axis at $(\pm2,0)$ and has maximum point $(0,2)$.

## Q55 Loreto Kirribilli 2022 Prelim Q23(c) · **2 marks**

Let $f(x) = x^2 - 1$ and $g(x) = \sqrt{x}$, so that $f\big(g(x)\big) = x - 1$ with domain $x \geq 0$. Sketch the graph of $y = f\big(g(x)\big)$, showing any intercepts on the axes.

| Criteria | Marks |
|---|---|
| Correct straight-line shape $y = x - 1$ restricted to $x \geq 0$ (a ray from $(0, -1)$) | 1 |
| Both intercepts shown: $y$-intercept $(0, -1)$ and $x$-intercept $(1, 0)$ | 1 |

**Solution.**

Since $g(x) = \sqrt{x}$, we have

$$f\big(g(x)\big) = \big(\sqrt{x}\big)^2 - 1 = x - 1, \qquad x \geq 0.$$

The graph is the portion of the line $y = x - 1$ for $x \geq 0$ only: a ray beginning at the endpoint $(0, -1)$ and rising with gradient 1. It crosses the $x$-axis at $(1, 0)$.

## Q56 Baulkham Hills 2021 Prelim Q16 · **5 marks**

(a) Simplify $g(f(x))$. (b) State its domain and range. (c) Sketch $y = g(f(x))$.

| Criteria | Marks |
|---|---|
| (a) correct simplified expression | 1 |
| (b) correct domain and range | 2 |
| (c) correct graph with correct shape and endpoint | 2 |

**Solution.**

**(a)** $g(f(x)) = \left(\sqrt{x+3}\right)^2 - 2 = x + 3 - 2 = x + 1$, so $g(f(x)) = x + 1$.

**(b)** The inner function $f(x) = \sqrt{x+3}$ requires $x + 3 \geq 0$, i.e. $x \geq -3$. On this domain $g(f(x)) = x + 1$, giving $y \geq -2$.
Domain: $x \geq -3$. Range: $y \geq -2$.

**(c)** The graph is the half-line $y = x + 1$ starting at the endpoint $(-3, -2)$ (included) and rising to the right. It is a straight ray, not the full line.

## Q57 Killara 2022 Prelim Q33 · **3 marks**

Let $f(x) = \sqrt{x}$ and $g(x) = \dfrac{1}{x^2 - 3}$. By finding an expression for $g\big(f(x)\big)$, sketch the function $y = g\big(f(x)\big)$ over its natural domain.

| Criteria | Marks |
|---|---|
| Finds $g\big(f(x)\big) = \dfrac{1}{x-3}$ with natural domain $x \geq 0$ | 1 |
| Correct hyperbola shape with vertical asymptote $x = 3$ and horizontal asymptote $y = 0$ | 1 |
| Restricts to $x \geq 0$ and shows the $y$-intercept $\left(0, -\frac{1}{3}\right)$ | 1 |

**Solution.**

Substitute $f(x) = \sqrt{x}$ into $g$:

$$g\big(f(x)\big) = \frac{1}{\left(\sqrt{x}\right)^2 - 3} = \frac{1}{x-3}.$$

The natural domain comes from $f$: $x \geq 0$ (and $x \neq 3$). So $y = \dfrac{1}{x-3}$ for $x \geq 0$, $x \neq 3$. On $0 \leq x < 3$ the curve runs from the $y$-intercept $\left(0, -\frac{1}{3}\right)$ down to $-\infty$ as $x \to 3^-$; on $x > 3$ it comes down from $+\infty$ towards the horizontal asymptote $y = 0$. The vertical asymptote is $x = 3$.

## Q58 Manly 2020 Trial Q22 · **3 marks**

Given the functions $f(x) = x^2 - 1$ and $g(x) = \sqrt{4 - x^2}$, form an expression for $y = f\big(g(x)\big)$ and sketch it over its natural domain.

| Criteria | Marks |
|---|---|
| Forms $f\big(g(x)\big) = \left(\sqrt{4-x^2}\right)^2 - 1 = 3 - x^2$ | 1 |
| States the natural domain $-2 \leq x \leq 2$ (from $4 - x^2 \geq 0$) | 1 |
| Correct parabola arc: maximum $(0, 3)$, endpoints $(-2, -1)$ and $(2, -1)$ | 1 |

**Solution.**

Substitute $g$ into $f$:

$$f\big(g(x)\big) = \left(\sqrt{4 - x^2}\right)^2 - 1 = (4 - x^2) - 1 = 3 - x^2.$$

The domain is set by $g$: $4 - x^2 \geq 0 \Rightarrow -2 \leq x \leq 2$. So $y = 3 - x^2$ is drawn only on $[-2, 2]$: a concave-down parabola arc with maximum $(0, 3)$ and endpoints $(-2, -1)$ and $(2, -1)$.

## Q59 HSC 2022 Q10 · **1 mark**

The graphs of $y = f(x)$ and $y = g(x)$ are shown.

Diagram to be inserted:
`adv-working-with-functions-d9-q59.png`

Which graph best represents $y = g\big(f(x)\big)$?

(A) Graph A

(B) Graph B

(C) Graph C

(D) Graph D

| **Criteria** | **Marks** |
|---|---|
| Correct option (C) | 1 |

**Solution.**

Read off the composite point by point: for each $x$, first find $f(x)$ from the graph of $f$, then evaluate $g$ at that height using the graph of $g$. Tracking the sign and turning behaviour of $g\big(f(x)\big)$ across the domain matches graph (C).

## Q60 Killara 2025 Trial Q19 · **4 marks**

Given the two functions $f(x) = \dfrac{1}{x^2+2} - 3$ and $g(x) = \sqrt{x+2}$, sketch $y = f\big(g(x)\big)$ over its natural domain.

| **Criteria** | **Marks** |
|---|---|
| Forms $f\big(g(x)\big) = \dfrac{1}{x+4} - 3$ | 1 |
| States the natural domain $x \geq -2$ (from $x + 2 \geq 0$) | 1 |
| Identifies the horizontal asymptote $y = -3$ and the endpoint $\left(-2, -\frac{5}{2}\right)$ | 1 |
| Correct decreasing curve on $x \geq -2$ with $y$-intercept $\left(0, -\frac{11}{4}\right)$ | 1 |

**Solution.**

Substitute $g(x) = \sqrt{x+2}$ into $f$:

$$f\big(g(x)\big) = \frac{1}{\left(\sqrt{x+2}\right)^2 + 2} - 3 = \frac{1}{(x+2)+2} - 3 = \frac{1}{x+4} - 3.$$

The natural domain is fixed by $g$: $x + 2 \geq 0 \Rightarrow x \geq -2$. On this restricted domain $x + 4 > 0$, so $\dfrac{1}{x+4} > 0$ and the curve lies just above $y = -3$, decreasing as $x$ increases. Key features: the left endpoint at $x = -2$ is $\dfrac{1}{2} - 3 = -\dfrac{5}{2}$, giving $\left(-2, -\frac{5}{2}\right)$; the $y$-intercept ($x = 0$) is $\dfrac{1}{4} - 3 = -\dfrac{11}{4}$; and as $x \to \infty$ the graph approaches the horizontal asymptote $y = -3$ from above.