Year 11 Mathematics Advanced

Topic: Working with Functions (8 of 12)

Review Questions (Worked Solutions): Circles and Semicircles

Term 3, 2026

Centre, radius and equations

1. Centre and radius from standard form and completing the square

Q1 Sydney Grammar 2023 Prelim Q9(b) · **1 mark**

Write down the centre and radius of the circle with equation $(x-1)^2+y^2=4$.

Criteria	Marks
Correct centre $(1,0)$ and radius 2	1

Solution.

The equation is already in the standard form $(x-h)^2+(y-k)^2=r^2$ with $h=1$, $k=0$ and $r^2=4$.

$$\text{centre } (1,0), \qquad \text{radius } r=\sqrt{4}=2.$$

Q2 North Sydney Boys 2024 Prelim Q8 · **2 marks**

Find the centre and radius of the circle represented by the equation $x^2-6x+y^2+10y+18=0$.

Criteria	Marks
Completes the square in both variables to reach $(x-3)^2+(y+5)^2=16$	1
Correct centre $(3,-5)$ and radius 4	1

Solution.

Group and complete the square in x and in y:

$$x^2-6x+y^2+10y+18=0$$

$$(x^2-6x+9)+(y^2+10y+25)=-18+9+25$$

$$(x-3)^2+(y+5)^2=16.$$

Hence the centre is $(3,-5)$ and the radius is $r=\sqrt{16}=4$.

Q3 Presbyterian 2023 Prelim Q14 · **2 marks**

Find the centre and radius of the circle represented by the equation $x^2 - 4x + y^2 + 6y + 4 = 0$.

Criteria	Marks
Completes the square in both variables to reach $(x-2)^2 + (y+3)^2 = 9$	1
Correct centre $(2, -3)$ and radius 3	1

Solution.

Complete the square in x and in y:

$$x^2 - 4x + y^2 + 6y + 4 = 0$$

$$(x^2 - 4x + 4) + (y^2 + 6y + 9) = -4 + 4 + 9$$

$$(x-2)^2 + (y+3)^2 = 9.$$

Hence the centre is $(2, -3)$ and the radius is $r = \sqrt{9} = 3$.

Q4 Baulkham Hills 2024 Prelim Q16 · **2 marks**

State the centre and radius of the circle $2x^2 + 2y^2 - 12y - 4 = 0$.

Criteria	Marks
Divides by 2 and completes the square to reach $x^2 + (y-3)^2 = 11$	1
Correct centre $(0, 3)$ and radius $\sqrt{11}$	1

Solution.

Divide through by the leading coefficient 2, then complete the square in y:

$$2x^2 + 2y^2 - 12y - 4 = 0 \implies x^2 + y^2 - 6y - 2 = 0$$

$$x^2 + (y^2 - 6y + 9) = 2 + 9$$

$$x^2 + (y-3)^2 = 11.$$

Hence the centre is $(0, 3)$ and the radius is $r = \sqrt{11}$.

Q5 Girraween 2023 Prelim Q8(c) · **3 marks**

A circle has the equation $x^2 + y^2 - 5x + 3y - 1 = 0$. Find the radius and centre of the circle.

Criteria	**Marks**
Completes the square in x: $\left(x - \frac{5}{2}\right)^2$	1
Completes the square in y and reaches $\left(x - \frac{5}{2}\right)^2 + \left(y + \frac{3}{2}\right)^2 = \frac{19}{2}$	1
Correct centre $\left(\frac{5}{2}, -\frac{3}{2}\right)$ and radius $\frac{\sqrt{38}}{2}$	1

Solution.

Group by variable and complete the square, remembering to add the same amounts to the right-hand side:

$$x^2 - 5x + y^2 + 3y = 1$$

$$\left(x^2 - 5x + \tfrac{25}{4}\right) + \left(y^2 + 3y + \tfrac{9}{4}\right) = 1 + \tfrac{25}{4} + \tfrac{9}{4}$$

$$\left(x - \tfrac{5}{2}\right)^2 + \left(y + \tfrac{3}{2}\right)^2 = \tfrac{19}{2}.$$

Hence the centre is $\left(\dfrac{5}{2}, -\dfrac{3}{2}\right)$ and the radius is

$$r = \sqrt{\tfrac{19}{2}} = \frac{\sqrt{19}}{\sqrt{2}} = \frac{\sqrt{38}}{2}.$$

Q6 Penrith 2023 Prelim Q28 · **2 marks**

Give reasons why $x^2 + 4x + y^2 - 6y + 15 = 0$ is not an equation of a circle.

Criteria	**Marks**
Completes the square to reach $(x+2)^2 + (y-3)^2 = -2$	1
States that a circle needs $r^2 > 0$, but here $r^2 = -2 < 0$, so no such (real) circle exists	1

Solution.

Complete the square in both variables:

$$x^2 + 4x + y^2 - 6y + 15 = 0$$

$$(x^2 + 4x + 4) + (y^2 - 6y + 9) = -15 + 4 + 9$$

$$(x+2)^2 + (y-3)^2 = -2.$$

For a genuine circle $(x-h)^2 + (y-k)^2 = r^2$ the right-hand side must equal r^2, which cannot be negative (a sum of two real squares on the left is ≥ 0). Here the right-hand side is $-2 < 0$, so there are no real points (x, y) satisfying the equation. Hence it is not the equation of a circle.

Q7 Killara 2023 Prelim Q27 · **6 marks**

(a) Show the centre is $\left(\frac{1}{2}, -1\right)$. (b) Find the radius. (c) Hence show $\angle ACB$ is obtuse.

Criteria	Marks
(a) divides by 4 and completes the square	1
(a) correct centre	1
(b) correct radius	1
(c) finds A and B	1
(c) valid argument	1
(c) correct conclusion (obtuse)	1

Solution.

(a) Divide through by 4: $x^2 + y^2 - x + 2y - \dfrac{5}{4} = 0$. Complete the square:

$$\left(x - \frac{1}{2}\right)^2 + (y+1)^2 = \frac{5}{4} + \frac{1}{4} + 1 = \frac{10}{4}.$$

So the centre is $C\left(\dfrac{1}{2}, -1\right)$.

(b) The radius is $r = \sqrt{\dfrac{10}{4}} = \dfrac{\sqrt{10}}{2}$.

(c) The circle cuts the y-axis where $x = 0$: $4y^2 + 8y - 5 = 0$, i.e. $(2y+5)(2y-1) = 0$, giving $y = -\dfrac{5}{2}$ or $y = \dfrac{1}{2}$. So $A\left(0, -\dfrac{5}{2}\right)$ and $B\left(0, \dfrac{1}{2}\right)$.

From $C\left(\dfrac{1}{2}, -1\right)$: $\vec{CA} = \left(-\dfrac{1}{2}, -\dfrac{3}{2}\right)$ and $\vec{CB} = \left(-\dfrac{1}{2}, \dfrac{3}{2}\right)$.

Their dot product is $\vec{CA} \cdot \vec{CB} = \dfrac{1}{4} - \dfrac{9}{4} = -2$. Since this is negative, $\cos(\angle ACB) < 0$, so $\angle ACB$ is obtuse.

2. Domain and range of circles and semicircles

Q8 Baulkham Hills 2022 Prelim Q1 · **1 mark**

What is the domain and the range of the function $f(x) = \sqrt{4 - x^2}$?

(A) Domain: $[-2, 2]$, Range: $[-2, 2]$

(B) Domain: $[-2, 2]$, Range: $[0, 2]$

(C) Domain: $[0, 2]$, Range: $[-4, 4]$

(D) Domain: $[0, 2]$, Range: $[0, 4]$

Criteria	Marks
Correct option (B)	1

Solution.

This is the upper semicircle of $x^2 + y^2 = 4$ (radius 2). The domain needs $4 - x^2 \geq 0$, i.e. $-2 \leq x \leq 2$, so the domain is $[-2, 2]$. As $\sqrt{\cdot} \geq 0$, the largest value is $\sqrt{4} = 2$ (at $x = 0$) and the smallest is 0 (at $x = \pm 2$), so the range is $[0, 2]$. The answer is (B).

Q9 Fort St 2021 Prelim Q2(b)(i) · **2 marks**

Consider $f(x) = \sqrt{18 - x^2}$. State the domain and range of $f(x)$.

Criteria	Marks
Correct domain $[-3\sqrt{2},\ 3\sqrt{2}\,]$	1
Correct range $[0,\ 3\sqrt{2}\,]$	1

Solution.

This is the upper semicircle of $x^2 + y^2 = 18$ (radius $\sqrt{18} = 3\sqrt{2}$). The domain requires $18 - x^2 \geq 0$, i.e. $-\sqrt{18} \leq x \leq \sqrt{18}$, giving domain $[-3\sqrt{2},\ 3\sqrt{2}\,]$. Since $f(x) \geq 0$ with maximum $\sqrt{18} = 3\sqrt{2}$ at $x = 0$, the range is $[0,\ 3\sqrt{2}\,]$.

Q10 Girraween 2023 Prelim Q7(b) · **2 marks**

Find the domain and range of $y = -\sqrt{\frac{25}{16} - x^2}$.

Criteria	Marks
correct domain	1
correct range	1

Solution.

(b) The expression under the root must be non-negative: $\frac{25}{16} - x^2 \geq 0$, so $x^2 \leq \frac{25}{16}$, giving $-\frac{5}{4} \leq x \leq \frac{5}{4}$.
This is the lower half of a circle of radius $\frac{5}{4}$ centred at the origin (the negative root), so y runs from $-\frac{5}{4}$ up to 0.
Domain: $-\frac{5}{4} \leq x \leq \frac{5}{4}$. Range: $-\frac{5}{4} \leq y \leq 0$.

Q11 Hornsby Girls 2022 Trial Q1 · **1 mark**

What is the range of the function $y = -2\sqrt{16 - x^2}$?

(A) $(-\infty, -8]$

(B) $[-4, 0]$

(C) $[-8, 0]$

(D) $[-8, \infty)$

Criteria	Marks
Correct option (C)	1

Solution.

The inner surd $\sqrt{16 - x^2}$ is the upper semicircle of radius 4, so it takes values in $[0, 4]$. Multiplying by -2 reflects and stretches this to $[-8, 0]$: the maximum 4 maps to -8 and the minimum 0 maps to 0. Hence the range is $[-8, 0]$. The answer is (C).

Q12 Riverview 2023 Trial Q5 · **1 mark**

What is the domain and range of a circle with the equation $x^2 + 6x + y^2 = 7$?

(A) domain $= [-3 - \sqrt{7},\ -3 + \sqrt{7}\,]$; range $= [-\sqrt{7}, \sqrt{7}\,]$

(B) domain $= [3 - \sqrt{7},\ 3 + \sqrt{7}\,]$; range $= [-\sqrt{7}, \sqrt{7}\,]$

(C) domain $= [-1, 7]$; range $= [-4, 4]$

(D) domain $= [-7, 1]$; range $= [-4, 4]$

Criteria	Marks
Correct option (D)	1

Solution.

Complete the square in x: $x^2 + 6x + y^2 = 7$ gives $(x + 3)^2 - 9 + y^2 = 7$, so $(x + 3)^2 + y^2 = 16$. This is a circle with centre $(-3, 0)$ and radius 4. The domain spans one radius either side of the centre in x: $[-3 - 4,\ -3 + 4] = [-7, 1]$; the range spans one radius either side in y: $[0 - 4,\ 0 + 4] = [-4, 4]$. The answer is (D).

Q13 Knox 2023 Prelim Q17 · **5 marks**

The circle $(x - 2)^2 + (y + 1)^2 = 4$ is reflected in the y-axis: (a) sketch the new circle; (b) write down its equation; (c) write down its domain and range.

Criteria	Marks
(a) Correctly plots a circle of radius 2 centred at $(-2, -1)$	1
(a) Sketch is a reflection of the original in the y-axis (labelled axes/centre)	1
(b) Correct equation $(x + 2)^2 + (y + 1)^2 = 4$	1
(c) Correct domain $[-4, 0]$	1
(c) Correct range $[-3, 1]$	1

Solution.

(a) The original circle has centre $(2, -1)$ and radius 2. Reflecting in the y-axis negates the x-coordinate of the centre, giving a circle of radius 2 centred at $(-2, -1)$. The sketch is this circle, the mirror image of the original across the y-axis.

(b) A circle with centre $(-2, -1)$ and radius 2 has equation

$$(x + 2)^2 + (y + 1)^2 = 4.$$

(c) With centre $(-2, -1)$ and radius 2, the domain spans one radius either side of the centre in x: $[-2 - 2,\ -2 + 2] = [-4, 0]$; the range spans one radius either side in y: $[-1 - 2,\ -1 + 2] = [-3, 1]$.

Graphs, domain and range

3. Identify or sketch a circle/semicircle from its equation or graph

Q14 Sydney Tech 2021 Trial Q7 · **1 mark**

The circle shown in the diagram has the equation given by one of the options below.

Diagram to be inserted:
`adv-working-with-functions-d8-q14.png`

Which of the following is the equation for the circle shown in the diagram above?

(A) $x^2 + 6x + y^2 + 2y + 1 = 0$

(B) $x^2 - 6x + y^2 + 2y - 1 = 0$

(C) $x^2 - 6x + y^2 - 2y + 1 = 0$

(D) $x^2 - 6x + y^2 - 2y - 1 = 0$

Criteria	Marks
Correct option (C)	1

Solution.

From the diagram the circle has centre $(3, 1)$ and radius 3, so its equation is

$$(x - 3)^2 + (y - 1)^2 = 9.$$

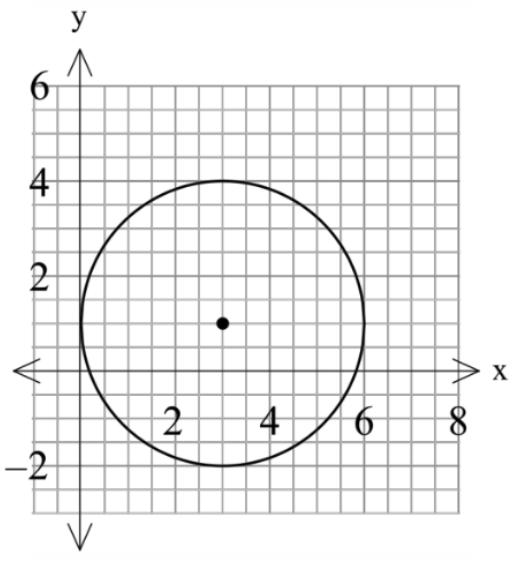
Expanding gives $x^2 - 6x + 9 + y^2 - 2y + 1 = 9$, i.e.

$$x^2 - 6x + y^2 - 2y + 1 = 0.$$

The answer is (C). (Checking the others: (A) is centre $(-3, -1)$, (B) is centre $(3, -1)$ with $r = \sqrt{11}$, (D) is centre $(3, 1)$ with $r = \sqrt{11}$.)

Q15 Normanhurst Boys 2021 Trial Q1 · **1 mark**

The equation of the semicircle illustrated below is given by



(A) $y = \sqrt{9 - x^2}$

(B) $y = -\sqrt{9 - x^2}$

(C) $y = \sqrt{3 - x^2}$

(D) $y = -\sqrt{3 - x^2}$

Criteria	Marks
Correct option (B)	1

Solution.

The semicircle has x-intercepts at $x = -3$ and $x = 3$ and dips to $(0, -3)$, so it is the lower half of the circle $x^2 + y^2 = 9$ (radius 3). Solving for the lower branch,

$$y = -\sqrt{9 - x^2}.$$

The answer is (B). (Options (C) and (D) have radius $\sqrt{3}$; option (A) is the upper semicircle.)

Q16 Sydney Girls 2021 Prelim Q3(c)(i) · **1 mark**

Sketch the following, showing all key features:

$$y = -\sqrt{25 - x^2}.$$

Criteria	Marks
Correct lower semicircle: centre O, radius 5, endpoints $(\pm 5, 0)$ and lowest point $(0, -5)$	1

Solution.

Squaring gives $y^2 = 25 - x^2$, i.e. $x^2 + y^2 = 25$, a circle of radius 5 centred at the origin. Since $y = -\sqrt{25 - x^2} \leq 0$, only the lower half is taken. The graph is the semicircle with endpoints $(-5, 0)$ and $(5, 0)$ and lowest point $(0, -5)$; domain $[-5, 5]$, range $[-5, 0]$.

Q17 Hornsby Girls 2022 Prelim Q17(a) · **1 mark**

Consider the semicircle defined by the function $f(x) = \sqrt{8 - x^2}$.
Sketch the graph of $y = f(x)$.

Criteria	Marks
Correct upper semicircle: centre O, radius $2\sqrt{2}$, endpoints $(\pm 2\sqrt{2}, 0)$ and highest point $(0, 2\sqrt{2})$	1

Solution.

Squaring gives $y^2 = 8 - x^2$, i.e. $x^2 + y^2 = 8$, a circle of radius $\sqrt{8} = 2\sqrt{2}$ centred at the origin. Since $y = \sqrt{8 - x^2} \geq 0$, only the upper half is taken. The graph is the semicircle with endpoints $(-2\sqrt{2}, 0)$ and $(2\sqrt{2}, 0)$ and highest point $(0, 2\sqrt{2})$; domain $[-2\sqrt{2}, 2\sqrt{2}]$, range $[0, 2\sqrt{2}]$.

Q18 Riverview 2021 Trial Q2(c)(i) · **2 marks**

Draw the graph of $y = -\sqrt{4 - x^2}$ and shade the area bounded by the function and the x-axis where $x \leq 0$.

Criteria	Marks
Correct lower semicircle: centre O, radius 2, endpoints $(\pm 2, 0)$ and lowest point $(0, -2)$	1
Correct region shaded: between the curve and the x-axis for $-2 \leq x \leq 0$ (the third-quadrant quarter)	1

Solution.

Squaring gives $y^2 = 4 - x^2$, i.e. $x^2 + y^2 = 4$, a circle of radius 2 centred at the origin; since $y \leq 0$ this is the lower semicircle with endpoints $(-2, 0)$ and $(2, 0)$ and lowest point $(0, -2)$. The required region is bounded above by the x-axis and below by the curve for $x \leq 0$, i.e. the quarter-disc in the third quadrant between $x = -2$ and $x = 0$; shade that region.

Q19 Killara 2022 Prelim Q6 · **1 mark**

The graph $y = -\sqrt{2 - x^2}$ can be described as

(A) One-to-one

(B) One-to-many

(C) Many-to-one

(D) Many-to-many

Criteria	Marks
Correct option (C)	1

Solution.

The graph $y = -\sqrt{2 - x^2}$ is the lower semicircle of radius $\sqrt{2}$. Each x in $[-\sqrt{2}, \sqrt{2}]$ gives exactly one y, so it is a function (rules out one-to-many and many-to-many). It is not one-to-one, since e.g. $x = 1$ and $x = -1$ both give $y = -1$. Hence the mapping is many-to-one. The answer is (C).

Q20 Presbyterian 2022 Prelim Q29 · **1 mark**

A semicircle has domain $[-1, 1]$ and range $[-1, 0]$. What is the equation of the semicircle?

Criteria	Marks
Correct equation $y = -\sqrt{1 - x^2}$ (lower unit semicircle)	1

Solution.

Domain $[-1, 1]$ with the range $[-1, 0]$ lying on or below the x-axis means the lower half of the unit circle $x^2 + y^2 = 1$. Solving for the lower branch,

$$y = -\sqrt{1 - x^2}.$$

Check: at $x = 0$, $y = -1$ (lowest point); at $x = \pm 1$, $y = 0$; domain $[-1, 1]$, range $[-1, 0]$, as required.

4. Equation or parameters of a circle/semicircle from points or conditions

Q21 Hurlstone 2023 Prelim Q11(e) · **2 marks**

Find the equation of the circle with centre $A(1, 0)$ that passes through the point $B(4, 1)$.

Criteria	Marks
Uses the distance AB to find $r^2 = 10$	1
Correct equation $(x-1)^2 + y^2 = 10$	1

Solution.

A circle with centre $A(1, 0)$ has the form $(x-1)^2 + (y-0)^2 = r^2$. Since it passes through $B(4, 1)$, the radius is the distance AB:

$$r^2 = (4-1)^2 + (1-0)^2 = 9 + 1 = 10.$$

Hence the equation of the circle is

$$(x-1)^2 + y^2 = 10.$$

Q22 Newington 2024 Trial Q30(a) · **1 mark**

The ellipse $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$ is shown. Show that for points in the top half of the ellipse, $y = \sqrt{9 - \dfrac{9}{4}x^2}$.

Criteria	Marks
Makes y^2 the subject and takes the positive square root for the top half	1

Solution.

Starting from the equation of the ellipse, make y^2 the subject:

$$\frac{x^2}{4} + \frac{y^2}{9} = 1 \implies \frac{y^2}{9} = 1 - \frac{x^2}{4}.$$

Multiplying both sides by 9,

$$y^2 = 9\left(1 - \frac{x^2}{4}\right) = 9 - \frac{9}{4}x^2.$$

The top half of the ellipse consists of the points with $y \geq 0$, so we take the positive square root:

$$y = \sqrt{9 - \frac{9}{4}x^2},$$

as required.

Q23 Gosford 2024 Trial Q23 · **3 marks**

Find two possible equations of a semicircle with range $[3, 5]$, given that it is an even function.

Criteria	**Marks**
Identifies radius 2 with centre on the y-axis (so the semicircle is even)	1
One correct equation, e.g. the upper semicircle $y = 3 + \sqrt{4 - x^2}$	1
A second correct equation, e.g. the lower semicircle $y = 5 - \sqrt{4 - x^2}$	1

Solution.

A function is even when its graph is symmetric about the y-axis, so the semicircle must have its centre on the y-axis: the centre is $(0, k)$ for some k. The range $[3, 5]$ has length 2, which equals the radius of the semicircle, so $r = 2$.

Upper semicircle. Take the top half of the circle of radius 2 centred at $(0, 3)$. Its lowest point is at $y = 3$ and its highest point is at $y = 3 + 2 = 5$, giving range $[3, 5]$:

$$y = 3 + \sqrt{4 - x^2}.$$

Lower semicircle. Take the bottom half of the circle of radius 2 centred at $(0, 5)$. Its highest point is at $y = 5$ and its lowest point is at $y = 5 - 2 = 3$, again giving range $[3, 5]$:

$$y = 5 - \sqrt{4 - x^2}.$$

Both are even (symmetric in the y-axis) with domain $[-2, 2]$ and range $[3, 5]$.

Finding equations and problems

5. Tangency: find the parameter so an axis is tangent

Q24 Gosford 2020 Trial Q2 · 1 mark

What is the value of c for which the circle $(x-3)^2+(y-2)^2=c$ touches the x-axis?

(A) 2

(B) 3

(C) 4

(D) 9

Criteria	Marks
Correct option (C)	1

Solution.

The circle $(x-3)^2+(y-2)^2=c$ has centre $(3,2)$ and radius $\sqrt{c}$. It touches (is tangent to) the x-axis when the radius equals the perpendicular distance from the centre to the x-axis, namely 2:

$$\sqrt{c}=2 \implies c=4.$$

The answer is (C).

Q25 Knox 2022 Trial Q3 · **1 mark**

A circle has the equation $(x+2)^2+(y+3)^2=d$. What is the value of d such that the x-axis is a tangent to the circle?

(A) 2

(B) 3

(C) 4

(D) 9

Criteria	Marks
Correct option (D)	1

Solution.

The circle $(x+2)^2+(y+3)^2=d$ has centre $(-2,-3)$ and radius $\sqrt{d}$. The x-axis is a tangent when the radius equals the distance from the centre to the x-axis, which is $|-3|=3$:

$$\sqrt{d}=3 \implies d=9.$$

The answer is (D).

Q26 St George Girls 2022 Trial Q2 · **1 mark**

A circle has the equation $(x+2)^2+(y+3)^2=r^2$. What is the value of r^2 such that the x-axis is a tangent to the circle?

(A) 2

(B) 3

(C) 4

(D) 9

Criteria	Marks
Correct option (D)	1

Solution.

The circle $(x+2)^2+(y+3)^2=r^2$ has centre $(-2,-3)$ and radius r. The x-axis is a tangent when the radius equals the distance from the centre to the x-axis, which is $|-3|=3$:

$$r=3 \implies r^2=9.$$

The answer is (D).

Q27 Normanhurst Boys 2020 Trial Q3 · **1 mark**

What is the value of a for which the y-axis is tangent to the circle $(x-2)^2+(y-5)^2=a$?

(A) 2

(B) 4

(C) 5

(D) 25

Criteria	**Marks**
Correct option (B)	1

Solution.

The circle $(x-2)^2+(y-5)^2=a$ has centre $(2,5)$ and radius $\sqrt{a}$. The y-axis is a tangent when the radius equals the distance from the centre to the y-axis, which is $|2|=2$:

$$\sqrt{a}=2 \implies a=4.$$

The answer is (B).

Q28 Sydney Grammar 2023 Prelim Q8 · **1 mark**

For what values of r does the graph of $x^2 + 4x + y^2 - 2y + 5 = r^2$ intersect the x-axis but not the y-axis?

(A) $r < 1$

(B) $1 < r < 2$

(C) $2 < r < \sqrt{5}$

(D) $r > \sqrt{5}$

Criteria	Marks
Completes the square to $(x+2)^2 + (y-1)^2 = r^2$: centre $(-2, 1)$, radius r	1

Solution.

Complete the square in both x and y:

$$x^2 + 4x + y^2 - 2y + 5 = (x+2)^2 - 4 + (y-1)^2 - 1 + 5 = (x+2)^2 + (y-1)^2.$$

So the circle has centre $(-2, 1)$ and radius r. The distance from the centre to the x-axis is $|1| = 1$ and to the y-axis is $|-2| = 2$.

The circle *intersects* the x-axis (cuts it in two points) when its radius exceeds 1: substituting $y = 0$ gives $(x+2)^2 = r^2 - 1$, which has real solutions iff $r > 1$.

The circle does *not* intersect the y-axis when its radius is less than 2: substituting $x = 0$ gives $(y-1)^2 = r^2 - 4$, which has no real solution iff $r < 2$.

Combining, $1 < r < 2$. The answer is (B).

6. Coordinate-geometry problems on circles

Q29 Penrith 2024 Trial Q28 · **5 marks**

(a) Find the bomb's position (centre) and blast radius from $x^2 - 20x + y^2 + 10y = -76$. (b) Determine whether the enemy at $(2, -4)$ is killed.

Criteria	Marks
(a) completes the square in x and y	1
(a) correct centre	1
(a) correct radius	1
(b) computes distance from centre to $(2, -4)$	1
(b) correct comparison and conclusion	1

Solution.

(a) Complete the square: $x^2 - 20x + y^2 + 10y = -76$ becomes

$$(x - 10)^2 - 100 + (y + 5)^2 - 25 = -76,$$

so $(x - 10)^2 + (y + 5)^2 = 49$. The bomb is placed at $(10, -5)$ with blast radius $r = \sqrt{49} = 7$.

(b) The distance from the centre $(10, -5)$ to the enemy $(2, -4)$ is

$$d = \sqrt{(2 - 10)^2 + (-4 - (-5))^2} = \sqrt{(-8)^2 + (1)^2} = \sqrt{65} \approx 8.06.$$

Since $d = \sqrt{65} > 7 = r$, the enemy lies outside the blast zone and will **not** be killed.

Q30 Manly 2022 Trial Q30 · **4 marks**

(a) Find the centre and radius of the circle $x^2 - 2x + y^2 - 4y - 15 = 0$. (b) Find the point on the circle furthest from the origin.

Criteria	Marks
(a) Completes the square in x and y	1
(a) Correct centre $(1, 2)$ and radius $2\sqrt{5}$	1
(b) Identifies furthest point lies on line OC produced, distance $\lvert OC\rvert + r$ from O	1
(b) Correct coordinates $(3, 6)$	1

Solution.

(a) Complete the square:

$$(x^2 - 2x + 1) + (y^2 - 4y + 4) = 15 + 1 + 4$$

$$(x - 1)^2 + (y - 2)^2 = 20.$$

The centre is $(1, 2)$ and the radius is $\sqrt{20} = 2\sqrt{5}$.

(b) The point furthest from the origin $O(0, 0)$ lies on the line through O and the centre $C(1, 2)$, on the far side of the circle. Here

$$|OC| = \sqrt{1^2 + 2^2} = \sqrt{5},$$

and the unit vector from O towards C is $\dfrac{1}{\sqrt{5}}(1, 2)$. The furthest point is

$$C + 2\sqrt{5} \cdot \frac{1}{\sqrt{5}}(1, 2) = (1, 2) + 2(1, 2) = (3, 6).$$

So the required point is $(3, 6)$, at a distance $|OC| + r = \sqrt{5} + 2\sqrt{5} = 3\sqrt{5}$ from the origin.

Q31 Shore 2023 Prelim Q10 · **1 mark**

A point lies on the circle $x^2 + y^2 - 12x + 16y + 91 = 0$. What is the furthest distance, in units, the point could be from the origin?

(A) 9

(B) 11

(C) 13

(D) 14

Criteria	**Marks**
Correct option (C)	1

Solution.

Complete the square:

$$x^2 + y^2 - 12x + 16y + 91 = 0 \implies (x-6)^2 + (y+8)^2 = -91 + 36 + 64 = 9,$$

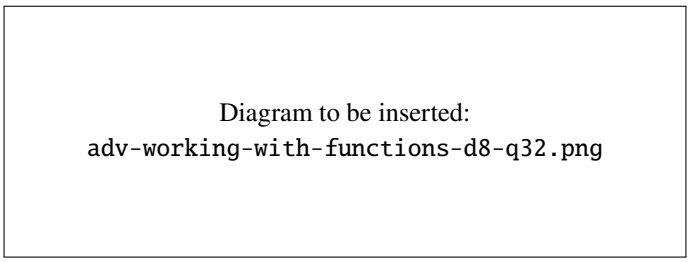
so the circle has centre $C(6, -8)$ and radius 3. The distance from the origin to the centre is $OC = \sqrt{6^2 + (-8)^2} = \sqrt{100} = 10$. The furthest a point on the circle can be from O is

$$OC + r = 10 + 3 = 13.$$

The answer is (C).

Q32 North Sydney Girls 2025 Trial Q29(a) · **1 mark**

The diagram below shows a rectangle $PQRS$ drawn in a sector OAB of a circle with radius 6. Here $OR = x$, $QR = 2x$ and $RS = y$, with R and S on the radius OA and the vertex P on the arc. All units are in metres.



Show that $(x + y)^2 = 36 - 4x^2$.

Criteria	Marks
Applies Pythagoras to $\triangle OSP$ (or equivalent), using $OP = 6$, horizontal distance $OS = x + y$ and height $SP = QR = 2x$, and rearranges to $(x + y)^2 = 36 - 4x^2$	1

Solution.

The vertex P lies on the arc, so OP equals the radius 6. Dropping to the radius OA, the horizontal distance from O to the foot below P is $OS = OR + RS = x + y$, and the height of P above OA is $SP = QR = 2x$. By Pythagoras in the right-angled triangle with hypotenuse OP:

$$OS^2 + SP^2 = OP^2 \implies (x + y)^2 + (2x)^2 = 6^2.$$

Hence

$$(x + y)^2 = 36 - (2x)^2 = 36 - 4x^2,$$

as required.

Q33 Hornsby Girls 2024 Trial Q26 · **4 marks**

The circle $x^2 + ax + y^2 + by + c = 0$, which has a radius of 3 units, is shifted to the right by 1 unit. After the shifting, the circle has its centre at $(3, -4)$. Find the values of a, b and c.

Criteria	**Marks**
Undoes the shift: the original centre is $(3-1, -4) = (2, -4)$	1
Relates the general form to the centre: $a = -2(2) = -4$ and $b = -2(-4) = 8$	1
Uses the radius to find $c = 2^2 + (-4)^2 - 3^2$	1
Correct values $a = -4$, $b = 8$, $c = 11$	1

Solution.

The shift moves the circle 1 unit to the right, so the original centre is 1 unit to the left of $(3, -4)$, namely $(2, -4)$. Completing the square on the general form,

$$x^2 + ax + y^2 + by + c = 0 \;\Rightarrow\; \left(x + \tfrac{a}{2}\right)^2 + \left(y + \tfrac{b}{2}\right)^2 = \tfrac{a^2}{4} + \tfrac{b^2}{4} - c,$$

the centre is $\left(-\frac{a}{2}, -\frac{b}{2}\right)$. Matching to $(2, -4)$:

$$-\tfrac{a}{2} = 2 \;\Rightarrow\; a = -4, \qquad -\tfrac{b}{2} = -4 \;\Rightarrow\; b = 8.$$

The radius is unchanged by a translation, so it equals 3. Using $r^2 = \left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2 - c$ with centre $(2, -4)$:

$$c = (\text{centre } x)^2 + (\text{centre } y)^2 - r^2 = 2^2 + (-4)^2 - 3^2 = 4 + 16 - 9 = 11.$$

Therefore $a = -4$, $b = 8$ and $c = 11$.

Q34 Sydney Boys 2022 Trial Q29(b) · 3 marks

The line $2x - y + 5 = 0$ intersects the circle $x^2 + y^2 - 6x - 2y - 30 = 0$ at the points $P(-3, -1)$ and $Q(1, 7)$. The diagram below shows this circle together with a second congruent circle that also passes through P and Q.

Diagram to be inserted:
`adv-working-with-functions-d8-q34.png`

By considering the properties of rhombi, or otherwise, find the equation of the second circle.

Criteria	Marks
Identifies the first circle's centre $C_1(3, 1)$ and radius $\sqrt{40}$	1
Uses the rhombus C_1PC_2Q (its diagonals C_1C_2 and PQ bisect each other) to locate the second centre $C_2(-5, 5)$	1
States the second circle: $(x + 5)^2 + (y - 5)^2 = 40$ (i.e. $x^2 + y^2 + 10x - 10y + 10 = 0$)	1

Solution.

The first circle $x^2 + y^2 - 6x - 2y - 30 = 0$ has centre $C_1(3, 1)$ and radius $r = \sqrt{3^2 + 1^2 + 30} = \sqrt{40} = 2\sqrt{10}$. The second circle is congruent (same radius) and also passes through P and Q, so its centre C_2 satisfies $C_2P = C_2Q = \sqrt{40}$. The four points C_1, P, C_2, Q form a rhombus (all four sides equal the radius), whose diagonals C_1C_2 and PQ bisect each other. Hence the midpoint of C_1C_2 equals the midpoint of PQ:

$$M = \left(\frac{-3 + 1}{2}, \frac{-1 + 7}{2}\right) = (-1, 3).$$

Since M is also the midpoint of C_1C_2,

$$C_2 = 2M - C_1 = (2(-1) - 3,\ 2(3) - 1) = (-5, 5).$$

The second circle therefore has centre $(-5, 5)$ and radius $\sqrt{40}$:

$$(x + 5)^2 + (y - 5)^2 = 40, \qquad \text{i.e.} \qquad x^2 + y^2 + 10x - 10y + 10 = 0.$$

(Check: both $P(-3, -1)$ and $Q(1, 7)$ satisfy this equation.)