Year 11 Mathematics Advanced

Topic: Working with Functions (7 of 12)

Review Questions (Worked Solutions): Direct and Inverse Variation

Term 3, 2026

Direct and inverse variation

1. Recognise and set up the variation model (write the equation, find the constant k)

Q1 Killara 2023 Trial Q14(a) · **1 mark**

The owners of Wilson Tennis Racquets use the equation $pq = T$ to keep the price of their racquets appropriate to the market, where p is the price of a racquet, q is the quantity sold per year and T is a constant. Last year the price was $p = \$100$ and the quantity sold was $q = 4000$. Find T.

Criteria	Marks
Substitutes $p = 100$, $q = 4000$ and evaluates $T = 400000$	1

Solution.

Substitute the given values into $pq = T$:

$$T = pq = 100 \times 4000 = 400\,000.$$

Q2 Barker 2023 Trial Q13 · **2 marks**

The frequency F (in beats per minute) that a bird beats its wings is inversely proportional to the length L (in cm) of its wings. A bird with wings of length 14 cm beats them at a frequency of 9 beats per minute. Find an equation relating F and L.

Criteria	Marks
Writes the inverse-variation model $F = \dfrac{k}{L}$	1
Finds $k = 126$, giving $F = \dfrac{126}{L}$	1

Solution.

Inverse proportion means $F = \dfrac{k}{L}$ for some constant k. Substitute $L = 14$, $F = 9$ to find k:

$$9 = \frac{k}{14} \;\Rightarrow\; k = 126.$$

Hence the equation relating F and L is

$$F = \frac{126}{L}.$$

Q3 Blacktown Boys 2023 Trial Q13 · **2 marks**

The number of days D to complete a research project is inversely proportional to the number of researchers R who are working. The project takes 123 days to complete when there are 7 people working on it. Find the equation relating D and R.

Criteria	**Marks**
Writes the inverse-variation model $D = \dfrac{k}{R}$	1
Finds $k = 861$, giving $D = \dfrac{861}{R}$	1

Solution.

Inverse proportion means $D = \dfrac{k}{R}$ for some constant k. Substitute $D = 123$, $R = 7$:

$$123 = \frac{k}{7} \;\Rightarrow\; k = 861.$$

Hence the equation relating D and R is

$$D = \frac{861}{R}.$$

Q4 North Sydney Girls 2022 Prelim Q7 · **1 mark**

The number of hours h taken to clean an apartment block is inversely proportional to the square of the number of cleaners C. If it takes 6 cleaners 12.5 hours on average to clean an apartment block, which of the following equations represents this situation?

(A) $h = \dfrac{C^2}{75}$

(B) $C = \dfrac{75}{h^2}$

(C) $h = \dfrac{450}{C^2}$

(D) $C = \dfrac{450}{h^2}$

Criteria	Marks
Correct option (C)	1

Solution.

Inverse proportion to the square of C means $h = \dfrac{k}{C^2}$. Substitute $C = 6$, $h = 12.5$:

$$12.5 = \frac{k}{6^2} = \frac{k}{36} \Rightarrow k = 450.$$

Hence $h = \dfrac{450}{C^2}$, which is option (C).

Q5 North Sydney Girls 2024 Trial Q5 · **1 mark**

The cost \$$c$ of making a box of cookies is inversely proportional to the square of the number of boxes n made. Which of the following is a possible equation of the model?

(A) $cn^2 = 10$

(B) $cn^2 = -10$

(C) $c^2n = 10$

(D) $c^2n = -10$

Criteria	Marks
Correct option (A)	1

Solution.

Inverse proportion to the square of n means $c = \dfrac{k}{n^2}$, i.e. $cn^2 = k$. Since a cost is positive, the constant k must be positive, so $cn^2 = 10$, which is option (A).

2. Inverse variation: solve for a new value

Q6 Sydney Boys 2022 Trial Q1 · **1 mark**

The time taken for an orchard to be harvested varies inversely to the number of people picking fruit. If the orchard can be harvested by 35 people in 6 days, how many people would be needed to harvest the orchard in 5 days?

(A) 42

(B) 45

(C) 48

(D) 51

Criteria	Marks
Correct option (A)	1

Solution.

Let T be the time (days) and N the number of people. Inverse variation means $T = \dfrac{k}{N}$, i.e. $TN = k$ is constant. From the given data $k = 35 \times 6 = 210$. For $T = 5$ days,

$$N = \frac{210}{5} = 42.$$

The answer is **(A)**.

Q7 Cranbrook 2023 Trial Q13 · **2 marks**

The time it takes to complete a task varies inversely with the number of people assigned. It takes 5 people to complete a task in 4 hours. Find the amount of time it would take 8 people to complete the same task.

Criteria	Marks
Finds the constant $k = 20$ (or equivalent TN product)	1
Correct time $T = 2.5$ hours	1

Solution.

Let T be the time (hours) and N the number of people. Then $T = \dfrac{k}{N}$, so $k = TN$. From 5 people taking 4 hours,

$$k = 5 \times 4 = 20.$$

For $N = 8$,

$$T = \frac{20}{8} = 2.5 \text{ hours}.$$

Q8 St George Girls 2023 Prelim Q16 · **3 marks**

The intensity of a current (C) varies inversely with the resistance (R); $C = 19$ when $R = 12.5$. (a) Find the constant of variation k. (b) Find the intensity when $R = 30$ (to 2 d.p.).

Criteria	Marks
(a) Correct substitution into $C = \dfrac{k}{R}$	1
(a) Correct constant $k = 237.5$	1
(b) Correct intensity $C \approx 7.92$ (2 d.p.)	1

Solution.

(a) Inverse variation gives $C = \dfrac{k}{R}$, so $k = CR = 19 \times 12.5 = 237.5$.

(b) With $C = \dfrac{237.5}{R}$, at $R = 30$: $C = \dfrac{237.5}{30} = 7.91\overline{6} \approx 7.92$.

Q9 North Sydney Girls 2025 Trial Q20 · **2 marks**

It is given that a quantity y varies inversely with the square of x. When $x = 4$, $y = 10$. Find the value of y when $x = 5$.

Criteria	Marks
Sets up $y = \dfrac{k}{x^2}$ and finds $k = 160$	1
Correct value $y = 6.4$	1

Solution.

Inverse variation with the square of x gives $y = \dfrac{k}{x^2}$. Substituting $x = 4$, $y = 10$:

$$10 = \frac{k}{16} \quad \Rightarrow \quad k = 160.$$

When $x = 5$,

$$y = \frac{160}{25} = 6.4.$$

3. Direct variation and variation with a power (square / cube)

Q10 Sydney Girls 2020 Trial Q13 · **3 marks**

(a) Equation converting Earth weight E to moon weight M. (b) Moon weight of a 120 kg (on Earth) astronaut.

Criteria	Marks
(a) Writes $M = kE$ and substitutes $E = 90$, $M = 16.4$	1
(a) Finds $k = \frac{41}{225}$ and states $M = \frac{41}{225}E$	1
(b) Substitutes $E = 120$ to get $M \approx 21.87$ kg	1

Solution.

(a) "Directly proportional" means $M = kE$ for some constant k. Substitute $E = 90$, $M = 16.4$:

$$16.4 = k \times 90 \implies k = \frac{16.4}{90} = \frac{41}{225}.$$

Hence

$$M = \frac{41}{225}E.$$

(b) Substitute $E = 120$:

$$M = \frac{41}{225} \times 120 = \frac{4920}{225} \approx 21.87 \text{ kg}.$$

Q11 Killara 2024 Prelim Q16 · **4 marks**

(a) With $P \propto S^2$ and $P = 2$ when $S = 4$, find the constant of variation. (b) Find S needed for $P = 8$.

Criteria	Marks
(a) Sets up $P = kS^2$ and substitutes $P = 2$, $S = 4$	1
(a) Correct constant $k = \frac{1}{8}$	1
(b) Substitutes $P = 8$ into the model to obtain $S^2 = 64$	1
(b) Correct answer $S = 8$ hours	1

Solution.

(a) Direct variation with the square: $P = kS^2$. Substitute $P = 2$, $S = 4$:

$$2 = k(4)^2 = 16k \implies k = \frac{2}{16} = \frac{1}{8}.$$

So $P = \dfrac{1}{8}S^2$.

(b) Set $P = 8$:

$$8 = \frac{1}{8}S^2 \implies S^2 = 64 \implies S = 8 \text{ (taking the positive root).}$$

Sam needs 8 hours of sleep.

Q12 Hornsby Girls 2023 Trial Q12 · **4 marks**

W varies directly with r^3; a radius of 2.5 cm gives a weight of 1.25 kg. (a) Find the radius for a weight of 5.12 kg. (b) Graph W against r for $0 \le r \le 10$.

Criteria	Marks
(a) Writes $W = kr^3$ and finds $k = \dfrac{2}{25} = 0.08$	1
(a) Solves $5.12 = 0.08\,r^3$ for $r = 4$ cm	1
(b) Correct cubic shape of $W = 0.08\,r^3$ from the origin, increasing on $0 \le r \le 10$	1
(b) Curve on $0 \le r \le 10$ with correct scale (endpoint $(10, 80)$)	1

Solution.

(a) Variation with the cube of r means $W = kr^3$. Using $r = 2.5$, $W = 1.25$: $k = \dfrac{1.25}{2.5^3} = \dfrac{1.25}{15.625} = \dfrac{2}{25} = 0.08$. Setting $W = 5.12$: $5.12 = 0.08\,r^3 \Rightarrow r^3 = 64 \Rightarrow r = 4$ cm.

(b) $W = 0.08\,r^3$: plot the cubic on $0 \le r \le 10$, through $(0, 0)$, $(2.5, 1.25)$, $(5, 10)$ and the endpoint $(10, 80)$: the increasing right branch of a cubic, drawn only from $r = 0$ to $r = 10$.

4. Model, tabulate and graph a variation relationship

Q13 Hunters Hill 2025 Trial Q1 · **1 mark**

Which of the following represents an inverse variation relationship?

(A) $y = \dfrac{4}{x}$

(B) $y = 4x$

(C) $y = 4$

(D) $y = -4x$

Criteria	Marks
Correct option (A)	1

Solution.

Inverse variation has the form $y = \dfrac{k}{x}$: as x increases, y decreases in proportion. Options (B) and (D) are direct variation ($y = kx$) and (C) is constant. Only (A), $y = \dfrac{4}{x}$, has the form $y = \dfrac{k}{x}$. The answer is (A).

Q14 Manly 2024 Trial Q11 · **3 marks**

(a) Find the equation relating x and y. (b) Graph the relationship showing two different points.

Criteria	Marks
(a) correct equation	1
(b) correct shape (hyperbola)	1
(b) two correct points shown	1

Solution.

(a) Inverse proportion means $xy = k$ for a constant k. Using $x = 8$, $y = 2$: $k = 8 \times 2 = 16$. Hence $y = \dfrac{16}{x}$.

(b) The graph is a rectangular hyperbola in the first and third quadrants (since $k > 0$), with the x- and y-axes as asymptotes. Plot two points, ideally one on each branch, e.g. $(8, 2)$ and $(-8, -2)$ (also $(4, 4)$, $(16, 1)$).

Q15 2022 HSC Q12 · **4 marks**

(a) Find the equation relating M and T. (b) Complete the table of values and graph the relationship for $5 \le T \le 30$.

Criteria	**Marks**
(a) Writes $M = \dfrac{k}{T}$ and substitutes $T = 15$, $M = 12$	1
(a) Finds $k = 180$ and states $M = \dfrac{180}{T}$	1
(b) Completes the table: $M = 36,\ 12,\ 6$ at $T = 5,\ 15,\ 30$	1
(b) Plots the three points and draws a smooth decreasing curve through them for $5 \le T \le 30$	1

Solution.

(a) "Varies inversely" means $M = \dfrac{k}{T}$ for some constant k. Substitute the observation $T = 15$, $M = 12$:

$$12 = \frac{k}{15} \implies k = 12 \times 15 = 180.$$

Hence the equation is

$$M = \frac{180}{T}.$$

(b) Substitute each T into $M = \dfrac{180}{T}$:

$$T = 5 \Rightarrow M = \frac{180}{5} = 36, \quad T = 15 \Rightarrow M = \frac{180}{15} = 12, \quad T = 30 \Rightarrow M = \frac{180}{30} = 6.$$

The completed table is

T	5	15	30
M	36	12	6

Plot the points $(5, 36)$, $(15, 12)$ and $(30, 6)$ and join them with a smooth curve that decreases steeply then levels off (one branch of the hyperbola $M = \dfrac{180}{T}$) over $5 \le T \le 30$.

5. Multi-step variation modelling (harder word problems)

Q16 Penrith 2022 Prelim Q16 · **2 marks**

Two electrical technicians can repair five electrical lines in three hours. After a storm, 50 electrical lines need repairing. How many electrical technicians are needed to repair the electrical lines in one eight-hour working day?

Criteria	Marks
Finds a work rate, e.g. $\frac{5}{6}$ lines per technician per hour, or 60 technician-hours needed for 50 lines	1
Divides by the 8-hour day and rounds up to a whole technician: 8 technicians	1

Solution.

Two technicians repair 5 lines in 3 hours, so the combined rate is $\frac{5}{3}$ lines per hour, i.e. each technician repairs

$$\frac{5}{2\times 3} = \frac{5}{6} \text{ lines per hour.}$$

Repairing 50 lines therefore requires

$$\frac{50}{5/6} = 60 \text{ technician-hours.}$$

Over a single 8-hour day this needs

$$\frac{60}{8} = 7.5 \text{ technicians.}$$

Since a technician is a whole person, round up: $\boxed{8 \text{ technicians}}$ are needed.

Q17 Barker 2024 Prelim Q13(c) · **3 marks**

(a) Find the inverse-variation model $M = \dfrac{k}{T}$. (b) Find how much longer melting takes at 10°C than at 25°C, to the nearest minute.

Criteria	Marks
Finds k and states $M = \dfrac{192}{T}$	1
Finds both melting times $M(10)$ and $M(25)$	1
Correct difference to the nearest minute	1

Solution.

(a) Inverse variation: $M = \dfrac{k}{T}$. At $T = 32$, $M = 6$, so $k = 6 \times 32 = 192$. Hence

$$M = \frac{192}{T}.$$

(b) At $T = 10$: $M = \dfrac{192}{10} = 19.2$ hours. At $T = 25$: $M = \dfrac{192}{25} = 7.68$ hours. Difference $= 19.2 - 7.68 = 11.52$ hours $= 11.52 \times 60 = 691.2$ minutes. So it takes about **691** minutes (11 hours 31 minutes) longer.

Q18 North Sydney Girls 2023 Prelim Q32 · **4 marks**

Stopping distance $d \propto v^2$, with $d = 43.2$ m at $v = 60$ km/h: (a) show $d = 58.8$ m at 70 km/h; (b) add a 1.5 s reaction time and find the new stopping distance at 70 km/h.

Criteria	Marks
(a) Sets up $d = kv^2$ and finds the constant $k = 0.012$ from $d(60) = 43.2$	1
(a) Substitutes $v = 70$ to obtain $d = 58.8$ m	1
(b) Finds the distance travelled during the 1.5 s reaction time: 70 km/h $= \frac{175}{9}$ m/s, so ≈ 29.17 m	1
(b) Adds the braking distance 58.8 m to get a total ≈ 87.97 m	1

Solution.

(a) Direct proportion to the square of the speed gives $d = kv^2$. Use $d = 43.2$ when $v = 60$:

$$43.2 = k \times 60^2 = 3600k \quad \Longrightarrow \quad k = \frac{43.2}{3600} = 0.012.$$

So $d = 0.012v^2$. At $v = 70$:

$$d = 0.012 \times 70^2 = 0.012 \times 4900 = 58.8 \text{ m},$$

as required.

(b) During the reaction time the car keeps moving at 70 km/h. Convert to metres per second:

$$70 \text{ km/h} = \frac{70 \times 1000}{3600} = \frac{175}{9} \approx 19.44 \text{ m/s}.$$

Distance covered in the 1.5 s reaction time:

$$\frac{175}{9} \times 1.5 = \frac{175}{6} \approx 29.17 \text{ m}.$$

Add the braking distance from part (a):

$$d_{\text{total}} = 58.8 + 29.17 \approx 87.97 \text{ m}.$$