Year 11 Mathematics Advanced

Topic: Working with Functions (6 of 12)

Review Questions (Worked Solutions): Constructing and Using Functions

Term 3, 2026

Building and using linear models

1. Write a linear cost/revenue function from a worded description

Q1 Sydney Tech 2024 Trial Q13(a) · **1 mark**

Jamie owns a company producing and selling pizzas. The income function is $y = 20x$, where x is the number of pizzas sold. The cost of producing these pizzas includes a set-up cost of \$50 and additional costs of \$10 per pizza. Write the cost function in the form $y = mx + c$.

Criteria	Marks
Cost function $y = 10x + 50$ (\$10 per pizza as gradient, \$50 set-up as intercept)	1

Solution.

The variable cost is \$10 per pizza, giving a gradient $m = 10$, and the fixed set-up cost of \$50 is the y-intercept, so $c = 50$. Hence

$$y = 10x + 50.$$

Q2 Hornsby Girls 2024 Trial Q11(a) · **1 mark**

The incoming revenue for a catering company, R, is based on a \$1 000 hiring fee plus \$20 per person, where x is the number of guests. Write the equation for R in terms of x.

Criteria	Marks
Correct equation $R = 20x + 1000$	1

Solution.

The revenue is a fixed hiring fee of \$1 000 plus \$20 for each of the x guests, so

$$R = 20x + 1000.$$

Q3 Normanhurst Boys 2023 Trial Q11(a) · **1 mark**

Sam and Jammy are planning a fund-raising dance. They can hire a hall for \$400 and a band for \$300. Refreshments will cost them \$12 per person. Write a formula for the cost (\$$C$) of running the dance for x people.

Criteria	**Marks**
Correct formula $C = 12x + 700$ (fixed cost \$700, \$12 per person)	1

Solution.

The fixed costs are the hall and the band: $400 + 300 = \$700$. Refreshments add \$12 for each of the x people, so

$$C = 12x + 700.$$

Q4 Killara 2024 Trial Q12(a) · **2 marks**

The population of a town in NSW has shown a linear decline in the years 2011 to 2019. In 2011 the population was 34 300 people. In 2019 it was 27 740 people. Write a linear equation expressing the population of the town, P, as a function of t, the number of years since 2011.

Criteria	**Marks**
Correct rate of change $m = -820$ (people per year) from the two data points	1
Correct linear equation $P = 34\,300 - 820t$	1

Solution.

Taking t as the number of years since 2011, the two data points are $(0,\ 34\,300)$ and $(8,\ 27\,740)$. The gradient is

$$m = \frac{27\,740 - 34\,300}{8 - 0} = \frac{-6\,560}{8} = -820.$$

Since $P = 34\,300$ when $t = 0$, the intercept is 34 300, so

$$P = 34\,300 - 820t.$$

Q5 North Sydney Girls 2021 Prelim Q1(c) · **3 marks**

(i) Linear formula for the charge \$$C$ for n sheets. (ii) Charge for 1025 sheets.

Criteria	Marks
(i) Finds the gradient $\dfrac{51-39}{800-600}=\dfrac{3}{50}=0.06$	1
(i) Finds the fixed cost and states $C=0.06n+3$	1
(ii) Substitutes $n=1025$ to get $C=\$64.50$	1

Solution.

(i) A linear charge has the form $C=an+b$. Using the points $(600,39)$ and $(800,51)$, the gradient is

$$a=\frac{51-39}{800-600}=\frac{12}{200}=\frac{3}{50}=0.06.$$

Substitute $(600,39)$: $39=0.06(600)+b=36+b$, so $b=3$. Hence

$$C=0.06n+3.$$

(ii) At $n=1025$:

$$C=0.06(1025)+3=61.5+3=64.5,$$

so the charge is \$64.50.

2. Use a given model: evaluate at a value and solve for the input

Q6 Riverview 2022 Trial Q11(a) · **1 mark**

A rainwater tank, initially full, is drained so that at time t minutes the volume of water V in litres is given by

$$V=500\left(1-\frac{t}{60}\right)^2,\qquad 0\le t\le 60.$$

How much water was initially in the tank?

Criteria	Marks
Substitutes $t=0$ to obtain $V=500$ litres	1

Solution.

"Initially" means $t=0$. Substituting,

$$V=500\left(1-\frac{0}{60}\right)^2=500(1)^2=500 \text{ litres}.$$

Q7 Sydney Girls 2020 Trial Q16(a) · **1 mark**

The population P, in thousands, of a small city is given by

$$P(t) = \frac{500t}{2t^2 + 9},$$

where t is the time in years. Find the population after 10 months.

Criteria	**Marks**
Uses $t = \frac{10}{12}$ years and evaluates $P \approx 40.11$ thousand	1

Solution.

Ten months is $t = \dfrac{10}{12} = \dfrac{5}{6}$ years. Substituting,

$$P = \frac{500 \cdot \frac{5}{6}}{2\left(\frac{5}{6}\right)^2 + 9} = \frac{\frac{2500}{6}}{\frac{50}{36} + 9} = \frac{7500}{187} \approx 40.11.$$

So the population is about 40.11 thousand, i.e. approximately 40 107 people.

Q8 Moriah 2021 Prelim Q18(a) · **1 mark**

Sophie is breeding mice for a laboratory. The number of mice M she has after t months is given by

$$M = 12 + 5t + 4t^2.$$

How many mice does she have initially?

Criteria	**Marks**
Substitutes $t = 0$ to obtain $M = 12$	1

Solution.

"Initially" means $t = 0$. Substituting,

$$M = 12 + 5(0) + 4(0)^2 = 12 \text{ mice.}$$

Q9 St George Girls 2022 Prelim Q34(a) · **1 mark**

The number of people in a stadium is approximated by

$$N = 416t^2 - 13t^3 + 88\,400,$$

where N is the number of people and t is the time in hours after 11 a.m. How many people are in the stadium at 2 p.m.?

Criteria	Marks
Uses $t = 3$ and evaluates $N = 91\,793$ people	1

Solution.

2 p.m. is $t = 3$ hours after 11 a.m. Substituting,

$$N = 416(3)^2 - 13(3)^3 + 88\,400 = 3744 - 351 + 88\,400 = 91\,793 \text{ people.}$$

Q10 Sydney Tech 2020 Trial Q34(b) · **1 mark**

A large tank of liquid contains a toxic chemical which is being drained. The amount of chemical L litres in the tank after t minutes is given by

$$L = 110(20 - t)^2.$$

How long will it take for the tank to be completely empty?

Criteria	Marks
Sets $L = 0$ and solves for $t = 20$ minutes	1

Solution.

The tank is empty when $L = 0$:

$$110(20 - t)^2 = 0 \implies (20 - t)^2 = 0 \implies t = 20.$$

It takes 20 minutes.

Q11 Moriah 2021 Prelim Q18(d) · **2 marks**

The number of mice M after t months is given by

$$M = 12 + 5t + 4t^2.$$

Sophie's parents allow her to keep a maximum of 100 mice. After how many months will she reach this number? (Give your answer to 2 decimal places.)

Criteria	**Marks**
Sets $M = 100$ and forms $4t^2 + 5t - 88 = 0$	1
Solves for the positive root $t = \frac{-5+\sqrt{1433}}{8} \approx 4.11$ months	1

Solution.

Set $M = 100$:

$$12 + 5t + 4t^2 = 100 \implies 4t^2 + 5t - 88 = 0.$$

By the quadratic formula,

$$t = \frac{-5 \pm \sqrt{5^2 - 4(4)(-88)}}{2(4)} = \frac{-5 \pm \sqrt{1433}}{8}.$$

Taking the positive root,

$$t = \frac{-5 + \sqrt{1433}}{8} \approx 4.11 \text{ months}.$$

Q12 Moriah 2021 Trial Q12 · **2 marks**

A crystal growth furnace is controlled by adjusting the input power. The relationship is

$$T = 0.1w^2 + 2.155w + 20,$$

where T is the temperature in degrees Celsius and w is the (positive) input power in watts. How much power is needed to maintain the temperature at 200°C? (Give your answer to 2 decimal places.)

Criteria	**Marks**
Sets $T = 200$ and forms $0.1w^2 + 2.155w - 180 = 0$	1
Solves and takes the positive root $w \approx 33.00$ watts	1

Solution.

Set $T = 200$:

$$0.1w^2 + 2.155w + 20 = 200 \implies 0.1w^2 + 2.155w - 180 = 0.$$

By the quadratic formula,

$$w = \frac{-2.155 \pm \sqrt{2.155^2 - 4(0.1)(-180)}}{2(0.1)} = 32.998\ldots \text{ or } -54.548\ldots$$

Since the power is positive, reject the negative root:

$$w \approx 33.00 \text{ watts}.$$

Q13 Killara 2020 Trial Q19(a) · **3 marks**

A swimming pool is to be emptied for maintenance. The quantity of water Q litres remaining in the pool t minutes after it starts to drain is given by

$$Q(t) = 2000(25 - t)^2, \qquad t \geq 0.$$

How long will it take to remove half of the water from the pool? (Give your answer to the nearest minute.)

Criteria	**Marks**
Finds the initial quantity $Q(0) = 1\,250\,000$ and half of it, $625\,000$	1
Sets $Q = 625\,000$ and reduces to $(25 - t)^2 = 312.5$	1
Solves and rejects $t > 25$, giving $t = 25 - \sqrt{312.5} \approx 7$ minutes	1

Solution.

The initial quantity is $Q(0) = 2000(25)^2 = 1\,250\,000$ litres, so half of it is $625\,000$ litres. To remove half, the remaining amount is $625\,000$:

$$2000(25 - t)^2 = 625\,000 \implies (25 - t)^2 = 312.5.$$

$$25 - t = \pm\sqrt{312.5} \implies t = 25 \mp \sqrt{312.5}.$$

Since $t < 25$ (the pool empties at $t = 25$), take

$$t = 25 - \sqrt{312.5} \approx 7.32 \approx 7 \text{ minutes}.$$

3. Find the break-even point (algebraically and from a graph)

Q14 Baulkham Hills 2024 Trial Q1 · **1 mark**

A small business makes and sells bird houses. Technology was used to draw straight-line graphs to represent the cost of making bird houses (C) and the revenue from selling bird houses (R). The horizontal axis displays the number of bird houses and the vertical axis displays the cost/revenue in dollars.

Diagram to be inserted:
`adv-working-with-functions-d6-q14.png`

How many bird houses need to be sold to break even?

(A) 10

(B) 20

(C) 30

(D) 40

Criteria	Marks
Correct option (B)	1

Solution.

The break-even point is where the cost and revenue lines intersect. Reading off the graph, the two lines cross at $x = 20$ bird houses, so the answer is **(B)**.

Q15 Baulkham Hills 2023 Trial Q1 · **1 mark**

Stephanie bakes cupcakes to sell. The cost C, in dollars, for x cupcakes to be produced is given by the equation $C = 200 + 2x$. Each cupcake is then sold for \$4. How many cupcakes does Stephanie need to make to break even?

(A) 0

(B) 25

(C) 100

(D) 200

Criteria	**Marks**
Correct option (C)	1

Solution.

Break-even occurs when cost equals income. The income from selling x cupcakes at \$4 each is $4x$, so

$$200 + 2x = 4x \implies 200 = 2x \implies x = 100.$$

So Stephanie needs 100 cupcakes, which is option **(C)**.

Q16 Ascham 2020 Trial Q16 · **4 marks**

(a) Write equations for cost C and income I for the sale of x items. (b) Solve simultaneously to find the number of dolls needed to break even.

Criteria	Marks
(a) Correct cost equation $C = 60 + 16x$	1
(a) Correct income equation $I = 20x$	1
(b) Sets $C = I$ and solves for x	1
(b) Correct break-even value $x = 15$	1

Solution.

(a) Fixed cost \$60 plus \$16 per doll, so

$$C = 60 + 16x.$$

Income is \$20 per doll, so

$$I = 20x.$$

(b) Break even where $C = I$:

$$60 + 16x = 20x$$

$$60 = 4x$$

$$x = 15.$$

Ali needs to sell 15 dolls to break even.

Q17 Presbyterian Ladies 2023 Prelim Q21 · **3 marks**

(a) Graph her costs and revenue on the grid below. (b) Use your graphs to find the number of donuts Jenny needs to sell to break even.

Criteria	**Marks**
(a) Correct cost line $C = 1.5x + 140$ (y-intercept 140)	1
(a) Correct revenue line $R = 5x$ through the origin	1
(b) Correct break-even value $x = 40$ donuts	1

Solution.

(a) The cost of making x donuts is the \$140 start-up cost plus \$1.50 each, so

$$C = 1.5x + 140,$$

a line with y-intercept 140 and gradient 1.5. The revenue from selling x donuts at \$5.00 each is

$$R = 5x,$$

a line through the origin with gradient 5. **(b)** Break-even is where revenue equals cost, i.e. where the two lines cross:

$$5x = 1.5x + 140 \implies 3.5x = 140 \implies x = 40.$$

Jenny needs to sell 40 donuts to break even.

Q18 North Sydney Girls 2022 Prelim Q16 · **5 marks**

(a) Cost function C. (b) Break-even number of units. (c) Profit/loss on 24 units. (d) Draw the Cost graph and mark the break-even point. (e) New Revenue equation keeping the same break-even, with fixed costs \$900.

Criteria	**Marks**
(a) $C = 600 + 15u$	1
(b) Break-even $u = 40$	1
(c) Loss of \$240	1
(d) Cost line drawn with break-even point labelled	1
(e) $R = 37.5u$	1

Solution.

(a) Fixed cost \$600 plus \$15 per unit: $C = 600 + 15u$.

(b) Revenue is $R = 30u$. Break-even where $R = C$: $30u = 600 + 15u \Rightarrow 15u = 600 \Rightarrow u = 40$ units.

(c) At $u = 24$: $R = 30(24) = 720$, $C = 600 + 15(24) = 960$. Profit $= R - C = 720 - 960 = -240$, i.e. a loss of \$240.

(d) The Cost line $C = 600 + 15u$ starts at $(0, 600)$ and rises with gradient 15; it meets the Revenue line $R = 30u$ at the break-even point $(40, 1200)$.

(e) With fixed costs now \$900, the new cost is $C = 900 + 15u$. To keep the break-even at $u = 40$, the revenue at $u = 40$ must equal $C = 900 + 15(40) = 1500$. So the new selling price is $\dfrac{1500}{40} = 37.5$, giving $R = 37.5u$.

Q19 North Sydney Girls 2023 Trial Q5 · **1 mark**

A bus company runs tours around North Sydney. Each tour costs the company \$273 in fuel plus \$48 per person in other costs. Tourists are charged \$65 per person for the tour. What is the minimum number of tourists the bus company needs to recruit for the tour to break even?

(A) 3 tourists

(B) 5 tourists

(C) 16 tourists

(D) 17 tourists

Criteria	**Marks**
Correct option (D)	1

Solution.

With x tourists, the cost is $273 + 48x$ and the revenue is $65x$. Break-even needs revenue $\geq$ cost:

$$65x \geq 273 + 48x \implies 17x \geq 273 \implies x \geq \frac{273}{17} = 16.06\ldots$$

Since the number of tourists must be a whole number, the minimum is 17 tourists, which is option **(D)**.

Q20 Pymble 2022 Prelim Q20 · **3 marks**

Consider the *We are Jeans* store. It costs the business \$29 to make a pair of jeans (this includes materials and labour). The fixed costs come to \$1000. The store sells the jeans for \$49 per pair. How many pairs of jeans do they have to sell to start making a profit on them?

Criteria	**Marks**
Sets up cost $C = 1000 + 29x$ and revenue $R = 49x$	1
Finds the break-even value $x = 50$	1
Concludes a profit requires selling 51 pairs (strictly more than break-even)	1

Solution.

Let x be the number of pairs sold. The total cost is $C = 1000 + 29x$ and the revenue is $R = 49x$. Break-even occurs when $C = R$:

$$1000 + 29x = 49x \implies 1000 = 20x \implies x = 50.$$

A profit is made only when revenue exceeds cost, i.e. $x > 50$. Since x must be a whole number, the store must sell 51 pairs of jeans to start making a profit.

4. Profit/loss at a quantity, and quantity for a target profit

Q21 Baulkham Hills 2025 Trial Q13 · **3 marks**

(a) Write an equation for the cost C of hosting the event for n people. (b) Estimate the number of people required to break even. (c) What is the profit if 75 people attend the dinner?

Criteria	Marks
(a) Correct cost equation $C = 80n + 2000$	1
(b) Break-even at $n = 50$ people	1
(c) Correct profit \$1 000	1

Solution.

(a) Each person costs \$80 and the room is a fixed \$2 000, so for n people

$$C = 80n + 2000.$$

(b) At \$120 per ticket the income is $I = 120n$. Break-even is where income equals cost:

$$120n = 80n + 2000 \implies 40n = 2000 \implies n = 50.$$

From the graph the income and cost lines cross at $n = 50$ (where $C = \$6\,000$), so 50 people are required to break even. **(c)** At $n = 75$ the income is $120 \times 75 = \$9\,000$ and the cost is $80 \times 75 + 2000 = \$8\,000$ (the same values read off the graph). Hence

$$\text{Profit} = 9000 - 8000 = \$1\,000.$$

Q22 Normanhurst Boys 2023 Trial Q11(c) · **1 mark**

Sam and Jammy are planning a fund-raising dance. They can hire a hall for \$400 and a band for \$300, and refreshments cost \$12 per person, so the cost of running the dance for x people is $C = 12x + 700$. Tickets are sold at \$20 each, giving an income of $20x$ dollars. How much profit will be made if 150 people attend the dance?

Criteria	Marks
Correct profit \$500 (income \$3000 less cost \$2500)	1

Solution.

Profit = income − cost $= 20x - (12x + 700) = 8x - 700$. At $x = 150$,

$$\text{Profit} = 8(150) - 700 = 1200 - 700 = \$500.$$

Q23 Hornsby Girls 2024 Trial Q11(d) · 1 mark

A catering company has revenue $R = 1000 + 20x$ (a \$1 000 hiring fee plus \$20 per guest) and cost $C = 1200 + 10x$, where x is the number of guests. Find the value of x when a profit of \$500 is made.

Criteria	Marks
Correct value $x = 70$	1

Solution.

Profit $= R - C = (1000 + 20x) - (1200 + 10x) = 10x - 200$. Set this equal to 500:

$$10x - 200 = 500 \implies 10x = 700 \implies x = 70.$$

Q24 Moriah 2021 Trial Q15 · 5 marks

(a) If the production cost is \$800, how many skateboards have been manufactured? (b) Graph both equations on the axes provided. (c) What is the minimum number of skateboards that Nicky must sell to make a profit?

Criteria	Marks
(a) Correct number $x = 10$	1
(b) Correct straight line $C = 20x + 600$ (y-intercept 600)	1
(b) Correct straight line $I = 50x$ through the origin	1
(c) Sets up the profit inequality $50x > 20x + 600$ and solves to $x > 20$	1
(c) States the minimum whole number $x = 21$	1

Solution.

(a) Set the cost equal to \$800:

$$800 = 20x + 600 \implies 20x = 200 \implies x = 10.$$

So 10 skateboards have been manufactured. **(b)** Both are straight lines. The cost line $C = 20x + 600$ has y-intercept 600 and gradient 20; the income line $I = 50x$ passes through the origin with gradient 50. They intersect at $x = 20$ (where $C = I = 1000$). **(c)** To make a profit the income must exceed the cost:

$$50x > 20x + 600 \implies 30x > 600 \implies x > 20.$$

Since x must be a whole number, the smallest value is $x = 21$. Nicky must sell at least 21 skateboards to make a profit.

Q25 Normanhurst Boys 2023 Trial Q11(d) · **2 marks**

Sam and Jammy run a fund-raising dance whose cost for x people is $C = 12x + 700$ dollars (hall \$400, band \$300, refreshments \$12 per person). They plan to sell 200 tickets and want to make a profit of \$1500. What should be the price of a ticket, assuming all 200 tickets are sold?

Criteria	**Marks**
Finds the cost of 200 people $C = 12(200) + 700 = \$3100$ and the income needed \$4600	1
Correct ticket price \$23	1

Solution.

With $x = 200$ the cost is $C = 12(200) + 700 = \$3\,100$. To make a profit of \$1500 the income must be

$$\text{income} = 3100 + 1500 = \$4\,600.$$

Spread over 200 tickets, the price is

$$\frac{4600}{200} = \$23 \text{ per ticket.}$$

Q26 Abbotsleigh 2020 Trial Q12(c) · **4 marks**

(a) How many face masks to break even? (b) By first forming equations for C and R, find how many masks give a profit of \$1500.

Criteria	Marks
(a) Reads the break-even point where the lines cross: $x = 200$ masks	1
(b) Reads the cost line: y-intercept 200, gradient $\frac{1}{2}$, so $C = \frac{1}{2}x + 200$	1
(b) Reads the revenue line through the origin with gradient $\frac{3}{2}$, so $R = \frac{3}{2}x$	1
(b) Solves $R - C = 1500$ to get $x = 1700$	1

Solution.

(a) The break-even point is where the revenue and cost lines cross (revenue equals cost). From the graph this occurs at $x = 200$, so Terry must sell 200 face masks to break even.

(b) From the graph the cost line has y-intercept 200 and rises by 100 over 200 masks (gradient $\frac{1}{2}$), so

$$C = \tfrac{1}{2}x + 200.$$

The revenue line passes through the origin and reaches 300 at $x = 200$ (gradient $\frac{3}{2}$), so

$$R = \tfrac{3}{2}x.$$

For a profit of \$1500, revenue − cost = 1500:

$$\tfrac{3}{2}x - \left(\tfrac{1}{2}x + 200\right) = 1500 \implies x - 200 = 1500 \implies x = 1700.$$

Terry must sell 1700 face masks.

Graphs, systems and optimisation

5. Graph or sketch a model, then read a value off it

Q27 Hornsby Girls 2024 Trial Q11(b) · 1 mark

A catering company's cost, measured in dollars, is modelled by $C = 1200 + 10x$, where x is the number of guests. On a set of axes (guests x on the horizontal axis, dollars on the vertical axis), draw the graph of this cost model and label it C.

Criteria	Marks
Straight line with vertical intercept $(0, 1200)$ and gradient 10, correctly labelled C	1

Solution.

The cost model is a straight line. Its vertical intercept is $C(0) = 1200$ and its gradient is 10 (the cost rises \$10 per guest), so plot $(0, 1200)$ and join it to a second point such as

$$C(100) = 1200 + 10(100) = 2200,$$

giving the point $(100, 2200)$. Draw the line through these points and label it C.

Q28 North Sydney Girls 2020 Trial Q14(a) · 2 marks

A brick company has an annual production capacity of 100 000 standard bricks. The cost of production is fixed at \$60 000 plus \$0.60 per brick. Find the equation for the cost C (in dollars) of producing x bricks, and plot this graph on a set of axes for $0 \leq x \leq 100\,000$.

Criteria	Marks
Correct cost equation $C = 60\,000 + 0.6x$	1
Line plotted with intercept $(0, 60\,000)$ and endpoint $(100\,000, 120\,000)$	1

Solution.

The fixed cost is \$60 000 and each brick adds \$0.60, so

$$C = 60\,000 + 0.6x.$$

This is a straight line with vertical intercept $(0, 60\,000)$. At the production capacity,

$$C(100\,000) = 60\,000 + 0.6(100\,000) = 120\,000,$$

so plot the line from $(0, 60\,000)$ to $(100\,000, 120\,000)$.

Q29 Newington 2024 Trial Q12 · **4 marks**

(a) With $V \propto L^2$ and $V = 29.44$ at $L = 8$, find the equation. (b) Complete the table and graph V against L for $L = 10$ to $L = 50$.

Criteria	Marks
(a) Sets up $V = kL^2$ and substitutes $L = 8$, $V = 29.44$	1
(a) Correct equation $V = 0.46L^2$	1
(b) Completes the table of values correctly	1
(b) Plots the points and draws a smooth parabolic curve	1

Solution.

(a) Direct variation with the square: $V = kL^2$. Substitute $L = 8$, $V = 29.44$:

$$29.44 = k(8)^2 = 64k \implies k = \frac{29.44}{64} = 0.46.$$

So $V = 0.46L^2$.

(b) Evaluate $V = 0.46L^2$:

L	10	20	30	40	50
V	46	184	414	736	1150

Plot these five points $(10, 46), (20, 184), (30, 414), (40, 736), (50, 1150)$ and join them with a smooth increasing parabolic curve (concave up, through the origin).

Q30 Glenwood 2021 Trial Q14 · **5 marks**

(a) Graph the stone $h = 20t - 5t^2 + 60$. (b) Graph the glider and find its equation. (c) Find the glider's height when the stone reaches the ground.

Criteria	Marks
Correct parabola for the stone (vertex, intercepts) labelled Stone	2
Correct glider equation $h = 60 - 5t$ and line labelled Glider	2
Correct glider height when the stone lands	1

Solution.

(a) $h = 20t - 5t^2 + 60 = -5(t^2 - 4t - 12) = -5(t - 6)(t + 2)$. It is a concave-down parabola with h-intercept $(0, 60)$, vertex at $t = 2$ giving $h = 20(2) - 5(4) + 60 = 80$, and it meets $h = 0$ at $t = 6$ (taking $t \geq 0$). Plot and label Stone.

(b) The glider starts at height 60 and loses height at 5 m/s, so $h = 60 - 5t$. This is a straight line from $(0, 60)$ with gradient -5; label it Glider.

(c) The stone reaches the ground when $h = 0$: $-5(t - 6)(t + 2) = 0$, so $t = 6$ (rejecting $t = -2$). The glider's height then is $h = 60 - 5(6) = 30$ metres.

Q31 Roseville 2023 Trial Q11(b) · **2 marks**

Chloe builds a rectangular chicken enclosure against an existing wall using 16 metres of fencing, with dimensions x and y satisfying $2x + y = 16$. The area is $A = 16x - 2x^2$, whose graph is shown below.

Diagram to be inserted:
`adv-working-with-functions-d6-q31.png`

Use the graph to determine the value(s) of x that would give the enclosure an area of 24 m^2, and give the dimensions of the enclosure in each case.

Criteria	**Marks**
Both values $x = 2$ and $x = 6$ (from $A = 24$)	1
Correct dimensions in each case: 2 m $\times$ 12 m and 6 m $\times$ 4 m	1

Solution.

Reading across from $A = 24$ on the graph meets the curve at two points. Algebraically, set $A = 24$:

$$16x - 2x^2 = 24 \implies 2x^2 - 16x + 24 = 0 \implies x^2 - 8x + 12 = 0 \implies (x-2)(x-6) = 0,$$

so $x = 2$ or $x = 6$. Using $y = 16 - 2x$: when $x = 2$, $y = 12$; when $x = 6$, $y = 4$. The two possible enclosures are 2 m $\times$ 12 m and 6 m $\times$ 4 m.

6. Simultaneous linear models: intersection, meeting and comparison problems

Q32 Sydney Boys 2025 Trial Q11(b) · **2 marks**

Jayme has \$100 and saves \$25 each week, so his savings after t weeks are modelled by $S = 100 + 25t$, where S is the number of dollars saved and t is the time in weeks. Jorge, Jayme's brother, starts saving 8 weeks after Jayme. He begins with no money and saves \$40 each week. By drawing a line on the same grid, or otherwise, find the value of t when the two brothers have the same amount of money.

Criteria	Marks
Sets up Jorge's model $40(t-8)$ and equates it to Jayme's $100+25t$	1
Solves to obtain $t = 28$ weeks	1

Solution.

Jayme's savings after t weeks are $100 + 25t$. Jorge begins 8 weeks after Jayme with no money and saves \$40 per week, so after t weeks (of which he has been saving for $t-8$) his savings are $40(t-8)$. The brothers have the same amount when

$$100 + 25t = 40(t-8).$$

Expanding and collecting terms,

$$100 + 25t = 40t - 320 \implies 420 = 15t \implies t = 28.$$

So the brothers have the same amount of money after $t = 28$ weeks (each having saved \$800).

Q33 St George Girls 2022 Trial Q16(b) · **2 marks**

Chris and Ann take part in a charity bicycle ride between Canberra and Liverpool. Chris leaves Canberra and rides toward Liverpool, a distance of 250 km, at 20 km/h; his distance from Liverpool is modelled by $C = 250 - 20t$, where t is the time in hours he has been riding. Ann rides in the opposite direction, leaving from Berrima (90 km from Liverpool) at the same time as Chris and riding at 12 km/h towards Canberra. By drawing a line on the grid, or otherwise, find the value of t when Chris and Ann pass each other.

Criteria	**Marks**
Writes Ann's distance from Liverpool as $A = 90 + 12t$ and equates to $C = 250 - 20t$	1
Solves to obtain $t = 5$ hours	1

Solution.

Measure each rider's distance from Liverpool. Chris is modelled by $C = 250 - 20t$. Ann starts 90 km from Liverpool and rides away from it (towards Canberra) at 12 km/h, so her distance from Liverpool is $A = 90 + 12t$. They pass each other when they are at the same place, i.e. $C = A$:

$$250 - 20t = 90 + 12t.$$

Collecting terms,

$$160 = 32t \implies t = 5.$$

Chris and Ann pass each other after $t = 5$ hours (both 150 km from Liverpool).

Q34 Penrith 2024 Trial Q13(b) · **2 marks**

Owais has two balloons, Balloon A and Balloon B. Balloon A starts full with 800 cm^3 of air and deflates at a constant rate, becoming empty after 40 seconds; its volume is modelled by $V_A = 800 - 20t$, where t is the time in seconds. Balloon B starts empty. After 10 seconds of Balloon A deflating, Owais' mum begins to inflate Balloon B at a rate of 40 cm^3/s. Find the value of t when the two balloons contain the same amount of air.

Criteria	**Marks**
Writes Balloon B's volume as $V_B = 40(t - 10)$ and equates to $V_A = 800 - 20t$	1
Solves to obtain $t = 20$ seconds	1

Solution.

Balloon A loses 800 cm^3 over 40 seconds, i.e. 20 cm^3/s, so $V_A = 800 - 20t$. Balloon B is empty until $t = 10$, after which it fills at 40 cm^3/s, so for $t \geq 10$ its volume is $V_B = 40(t - 10)$. The balloons contain the same amount of air when $V_A = V_B$:

$$800 - 20t = 40(t - 10).$$

Expanding and collecting terms,

$$800 - 20t = 40t - 400 \implies 1200 = 60t \implies t = 20.$$

The two balloons hold the same amount of air after $t = 20$ seconds (each containing 400 cm^3).

Q35 St George Girls 2022 Trial Q16(d) · **2 marks**

Chris and Ann take part in a charity bicycle ride. Chris rides from Canberra toward Liverpool at 20 km/h and Ann rides from Berrima toward Canberra at 12 km/h, both starting at the same time, where t is the time in hours since they started. Find the value of t when the two riders have ridden a combined total of 264 km.

Criteria	**Marks**
Forms the combined-distance equation $20t + 12t = 264$	1
Solves to obtain $t = 8.25$ hours	1

Solution.

In t hours Chris rides $20t$ km and Ann rides $12t$ km. Their combined distance ridden is $20t + 12t = 32t$. Setting this equal to 264 km,

$$32t = 264 \implies t = \frac{264}{32} = 8.25.$$

The riders have ridden a combined total of 264 km after $t = 8.25$ hours.

Q36 Danebank 2021 Trial Q13(a) · **3 marks**

A tennis arena has a maximum capacity of 15 000 spectators. The number of spectators, N, and the ticket price in dollars, P, are modelled by the linear relationship $N = m \times P + c$, where m and c are constants. The arena's records show that when the price of a ticket is \$50 the attendance averages 12 500 spectators, and when the price is \$35 the attendance averages 14 000 spectators. By developing and solving a pair of simultaneous equations, show that the number of spectators as a function of ticket price can be expressed as $N = -100P + 17\,500$.

Criteria	**Marks**
Forms the two equations $50m + c = 12\,500$ and $35m + c = 14\,000$	1
Eliminates c and solves for $m = -100$	1
Finds $c = 17\,500$ and states $N = -100P + 17\,500$	1

Solution.

Substituting each pair of values into $N = mP + c$ gives the simultaneous equations

$$50m + c = 12\,500 \qquad (1)$$

$$35m + c = 14\,000 \qquad (2).$$

Subtracting (2) from (1) eliminates c:

$$(50m + c) - (35m + c) = 12\,500 - 14\,000 \implies 15m = -1500 \implies m = -100.$$

Substituting $m = -100$ into (1):

$$50(-100) + c = 12\,500 \implies -5000 + c = 12\,500 \implies c = 17\,500.$$

Therefore the number of spectators as a function of ticket price is

$$N = -100P + 17\,500,$$

as required.

7. Construct a quadratic/rational function from a geometric setup (”show that”)

Q37 Roseville 2023 Trial Q11(a) · **1 mark**

Chloe plans to build a rectangular enclosure for her new chickens against an existing wall, using 16 metres of wire fencing. She calls the dimensions of the enclosure x and y (the side along the wall does not need fencing). She reasons that $2x + y = 16$. Write y in terms of x and hence show that the area of the enclosure can be written as

$$A = 16x - 2x^2.$$

Criteria	Marks
Rearranges to $y = 16 - 2x$ and substitutes into $A = xy$ to obtain $A = 16x - 2x^2$	1

Solution.

Rearrange the fencing constraint $2x + y = 16$ to make y the subject:

$$y = 16 - 2x.$$

The enclosure is a rectangle with sides x and y, so its area is

$$A = xy = x(16 - 2x) = 16x - 2x^2.$$

Q38 James Ruse 2024 Trial Q27 · **5 marks**

A 28 cm sheet is folded into a gutter with sides of height x cm: (a) show $A = 28x - 2x^2$; (b) explain why $x \in (0, 14)$; (c) find the maximum value of A.

Criteria	**Marks**
(a) Identifies base $= 28 - 2x$ and height $= x$, giving $A = 28x - 2x^2$	1
(b) Explains both bounds: $x > 0$ and $x < 14$ (the two turned-up sides cannot exceed the 28 cm width)	1
(c) Finds the vertex of the parabola at $x = 7$	1
(c) Substitutes to obtain the maximum area	1
(c) States maximum $A = 98$ cm^2	1

Solution.

(a) Turning up a strip of height x cm on each side uses $2x$ cm of the 28 cm width, so the flat base of the gutter has width $28 - 2x$ cm. The cross-section is a rectangle of width $28 - 2x$ and height x, so

$$A = x(28 - 2x) = 28x - 2x^2.$$

(b) The height turned up must be positive, so $x > 0$. The two turned-up sides together use $2x$ cm of the 28 cm width, leaving a base of $28 - 2x$ cm which must be positive, so $28 - 2x > 0$, i.e. $x < 14$. Hence the model is valid only for $x \in (0, 14)$.

(c) $A = 28x - 2x^2 = -2x^2 + 28x$ is a concave-down parabola, so its maximum occurs at the vertex

$$x = -\frac{b}{2a} = -\frac{28}{2(-2)} = 7.$$

Then

$$A = 28(7) - 2(7)^2 = 196 - 98 = 98 \text{ cm}^2.$$

The maximum cross-sectional area is 98 cm^2 (at $x = 7$, which lies in $(0, 14)$).

Q39 2023 HSC Q24(a) · **1 mark**

A gardener wants to build a rectangular garden of area 50 m^2 against an existing wall. A concrete path of width 1 metre is to be built around the other three sides of the garden. Let x and y be the dimensions, in metres, of the outer rectangle (path included). Show that

$$y = \frac{50}{x-2} + 1.$$

Criteria	Marks
Expresses garden sides as $x-2$ and $y-1$, sets their product to 50, and rearranges to $y = \frac{50}{x-2} + 1$	1

Solution.

The path (width 1 m) runs along both ends in the x-direction and along one side in the y-direction (the wall closes the fourth side). So the garden itself, sitting inside the outer rectangle, has dimensions

$$(x-2) \text{ by } (y-1).$$

Its area is 50 m^2, so

$$(x-2)(y-1) = 50 \implies y - 1 = \frac{50}{x-2} \implies y = \frac{50}{x-2} + 1.$$

Q40 North Sydney Girls 2024 Trial Q28(a) · **1 mark**

A new logo is to be made by inscribing a rectangle inside a quadrant of radius 3 cm. The length and width of the rectangle are x and y cm respectively, and $x^2 + y^2 = 9$. Show that the area of the rectangle is given by

$$A = x\sqrt{9 - x^2}.$$

Criteria	Marks
Solves $x^2 + y^2 = 9$ for $y = \sqrt{9-x^2}$ and substitutes into $A = xy$	1

Solution.

From the constraint $x^2 + y^2 = 9$, and since $y > 0$,

$$y = \sqrt{9 - x^2}.$$

The area of the rectangle is

$$A = xy = x\sqrt{9 - x^2}.$$

Q41 2020 HSC Q25(a) · **3 marks**

A landscape gardener wants to build a garden bed in the shape of a rectangle attached to a quarter-circle. Let x and y be the dimensions of the rectangle in metres, and let the quarter-circle have radius x. The garden bed is required to have an area of 36 m^2. Let P metres be the perimeter of the garden bed. Show that

$$P = 2x + \frac{72}{x}.$$

Criteria	**Marks**
Writes the area equation $xy + \frac{1}{4}\pi x^2 = 36$ and makes y the subject	1
Writes the perimeter as straight edges $2x + 2y$ plus the quarter-arc $\frac{\pi x}{2}$	1
Substitutes and simplifies so the π terms cancel, giving $P = 2x + \frac{72}{x}$	1

Solution.

The garden bed is a rectangle (x by y) with a quarter-circle of radius x attached. Its total area is

$$xy + \tfrac{1}{4}\pi x^2 = 36 \implies y = \frac{36}{x} - \frac{\pi x}{4}.$$

The outer boundary consists of two straight edges of length x, two straight edges of length y, and the quarter-circle arc of length $\frac{1}{4}(2\pi x) = \frac{\pi x}{2}$. Hence

$$P = 2x + 2y + \frac{\pi x}{2} = 2x + 2\left(\frac{36}{x} - \frac{\pi x}{4}\right) + \frac{\pi x}{2} = 2x + \frac{72}{x} - \frac{\pi x}{2} + \frac{\pi x}{2} = 2x + \frac{72}{x}.$$

Q42 Baulkham Hills 2024 Trial Q30(a) · **2 marks**

A coffee jar is a right circular cylinder of radius x cm and height h cm, fitted with a lid of height 2 cm and the same radius x. The jar and lid are made of thin sheet metal with total metal 190 cm^2; that is, $2x(h+2)+2x^2=190$. Show that the volume of the jar, V cm^3, is given by

$$V=\pi\left(95x-2x^2-x^3\right).$$

Criteria	**Marks**
Uses the metal constraint to obtain $xh=95-2x-x^2$	1
Substitutes into $V=\pi x^2h$ to obtain $V=\pi(95x-2x^2-x^3)$	1

Solution.

From the total-metal constraint, divide through by 2:

$$2x(h+2)+2x^2=190 \implies x(h+2)+x^2=95 \implies xh+2x+x^2=95,$$

so that

$$xh=95-2x-x^2.$$

The volume of the cylindrical jar is $V=\pi x^2h=\pi x\,(xh)$. Substituting,

$$V=\pi x\,(95-2x-x^2)=\pi\left(95x-2x^2-x^3\right).$$

Q43 Penrith 2022 Prelim Q30 · **3 marks**

A cyclist goes up a long slope at a constant speed of 15 km/h. He turns around and comes down the same slope at a constant speed of 40 km/h. Find his average speed over a full circuit.

Criteria	**Marks**
Lets the slope length be d and writes the up and down times as $\frac{d}{15}$ and $\frac{d}{40}$	1
Applies average speed $= \dfrac{\text{total distance}}{\text{total time}} = \dfrac{2d}{\frac{d}{15}+\frac{d}{40}}$	1
Simplifies to $\dfrac{240}{11} \approx 21.8$ km/h	1

Solution.

Let the length of the slope be d km. Going up takes $\dfrac{d}{15}$ hours and coming down takes $\dfrac{d}{40}$ hours, over a total distance of $2d$ km. Average speed is total distance over total time:

$$\text{average speed} = \frac{2d}{\frac{d}{15}+\frac{d}{40}} = \frac{2}{\frac{1}{15}+\frac{1}{40}} = \frac{2}{\frac{8+3}{120}} = \frac{2\times 120}{11} = \frac{240}{11} \approx 21.8 \text{ km/h}.$$

(The average speed is the harmonic mean of the two speeds, which lies below their arithmetic mean because more time is spent at the slower speed.)

8. Maximum or optimum of a model

Q44 Fort St 2022 Prelim Q8(d) · **6 marks**

For $d = \dfrac{w^2}{800} - 200$: (a) show it is even; (b) find the maximum depth from the top of the span; (c) find the total width of the dish; (d) find the depth 100 m from an endpoint.

Criteria	Marks
(a) Shows $d(-w) = d(w)$, so the function is even	1
(b) Maximum depth = 200 m	1
(c) Sets $d = 0$ to find the rim at $w = \pm 400$	1
(c) Total width = 800 m	1
(d) Identifies the point 100 m in from the endpoint as $w = 300$	1
(d) Depth = 87.5 m below the top of the span	1

Solution.

(a) Let $d(w) = \dfrac{w^2}{800} - 200$. Then

$$d(-w) = \frac{(-w)^2}{800} - 200 = \frac{w^2}{800} - 200 = d(w),$$

so the function is even (its graph is symmetric about the d-axis).

(b) The parabola opens upward, so its lowest point (the bottom of the dish) is the vertex at $w = 0$, where $d = \dfrac{0}{800} - 200 = -200$. The top of the span is the rim, at $d = 0$. The maximum depth measured downward from the top of the span is

$$0 - (-200) = 200 \text{ m}.$$

(c) The rim of the dish is where $d = 0$:

$$\frac{w^2}{800} - 200 = 0 \implies w^2 = 160\,000 \implies w = \pm 400.$$

The dish spans from $w = -400$ to $w = 400$, so the total width is

$$400 - (-400) = 800 \text{ m}.$$

(d) An endpoint is at $w = 400$. A point 100 m in from that endpoint is at $w = 300$:

$$d = \frac{300^2}{800} - 200 = \frac{90\,000}{800} - 200 = 112.5 - 200 = -87.5.$$

The depth below the top of the span is 87.5 m.

Q45 Roseville 2023 Trial Q11(c) · **1 mark**

Chloe builds a rectangular enclosure against an existing wall using 16 metres of wire fencing, with dimensions x and y satisfying $2x+y=16$. The area of the enclosure is given by $A=16x-2x^2$. Determine the dimensions (both x and y) that will achieve the maximum possible area for the enclosure.

Criteria	Marks
Finds $x=4$ (vertex of $A=16x-2x^2$)	1

Solution.

$A=16x-2x^2=-2x^2+16x$ is a concave-down parabola, so its maximum occurs at the vertex

$$x=-\frac{b}{2a}=-\frac{16}{2(-2)}=4.$$

Then from $2x+y=16$, $y=16-2(4)=8$. The maximum area is achieved when $x=4$ m and $y=8$ m (giving $A=32$ m^2).

Q46 Knox 2021 Trial Q12(a) · **5 marks**

(i) Find the cost equation C for n bolts. **(ii)** State the practical meaning of its gradient. **(iii)** Find the break-even number of bolts. **(iv)** Find the maximum monthly profit.

Criteria	Marks
(i) Correct cost equation $C=50\,000+0.95n$	1
(ii) Gradient is the cost of production per bolt	1
(iii) Correct break-even $n=62\,500$	1
(iv) Correct profit expression $P=0.8n-50\,000$	1
(iv) Correct maximum profit \$46 000 at $n=120\,000$	1

Solution.

(i) Fixed cost \$50 000 plus 0.95 per bolt: $C=50\,000+0.95n$.

(ii) The gradient 0.95 is the cost of producing each additional bolt, i.e. the marginal (per-bolt) production cost.

(iii) Break even when revenue equals cost: $1.75n=50\,000+0.95n$, so $0.8n=50\,000$, giving $n=62\,500$ bolts.

(iv) Profit $P=R-C=1.75n-(50\,000+0.95n)=0.8n-50\,000$. This increases with n, so it is greatest at the maximum output $n=120\,000$: $P=0.8(120\,000)-50\,000=96\,000-50\,000=\$46\,000$.

Q47 Killara 2024 Prelim Q33 · **7 marks**

For $N = mP + c$ with $(P, N) = (50, 12500)$ and $(35, 14000)$: (a) show $N = -100P + 17\,500$; (b) find the maximum revenue $R = NP$; (c) show that filling the arena to 15 000 spectators reduces revenue by \$390 625.

Criteria	Marks
(a) Sets up both simultaneous equations	1
(a) Solves $m = -100$, $c = 17\,500$	1
(a) States $N = -100P + 17\,500$	1
(b) Forms $R = -100P^2 + 17\,500P$	1
(b) Maximum revenue \$765 625	1
(c) Finds $P = 25$, $R = 375\,000$ at $N = 15\,000$	1
(c) Shows the reduction of \$390 625	1

Solution.

(a) Substituting the points into $N = mP + c$: $50m + c = 12\,500$ and $35m + c = 14\,000$. Subtracting: $15m = -1\,500$, so $m = -100$; then $c = 12\,500 - 50(-100) = 17\,500$. Hence $N = -100P + 17\,500$.

(b) $R = NP = (-100P + 17\,500)P = -100P^2 + 17\,500P$, a concave-down parabola with maximum at $P = \dfrac{17\,500}{200} = 87.5$. Then $N = -100(87.5) + 17\,500 = 8\,750$ and $R = 8\,750 \times 87.5 = \$765\,625$.

(c) For $N = 15\,000$: $15\,000 = -100P + 17\,500 \Rightarrow P = 25$, so $R = 15\,000 \times 25 = \$375\,000$. The reduction is $765\,625 - 375\,000 = \$390\,625$, as required.