Year 11 Mathematics Advanced

Topic: Working with Functions (4 of 12)

Review Questions (Worked Solutions): Quadratic and Cubic Functions

Term 3, 2026

Solving quadratics and the discriminant

1. Solving equations, and completing the square

Q1 Knox 2021 Trial Q3 · **1 mark**

The solutions to the quadratic equation $3x^2 - x - 4 = 0$ are:

(A) -1 and $-\dfrac{4}{3}$

(B) -1 and $\dfrac{4}{3}$

(C) 1 and $\dfrac{4}{3}$

(D) 1 and $-\dfrac{4}{3}$

Criteria	Marks
Correct option (B)	1

Solution.

Factorise the quadratic:

$$3x^2 - x - 4 = (3x - 4)(x + 1) = 0,$$

so $x = -1$ or $x = \dfrac{4}{3}$. The answer is (B).

Q2 Pymble 2022 Prelim Q22(b) · **2 marks**

Solve $x^2 + 24 = 11x$.

Criteria	Marks
Rearranges to $x^2 - 11x + 24 = 0$ and factorises to $(x - 3)(x - 8) = 0$	1
Correct solutions $x = 3$ and $x = 8$	1

Solution.

Bring every term to one side, then factorise:

$$x^2 + 24 = 11x \implies x^2 - 11x + 24 = 0 \implies (x - 3)(x - 8) = 0.$$

Hence $x = 3$ or $x = 8$.

Q3 Sydney Grammar 2021 Prelim Q12(c)(i) · **2 marks**

Express $x^2 - 4x + 8$ in the form $(x - p)^2 + q$.

Criteria	Marks
Completes the square: $x^2 - 4x = (x - 2)^2 - 4$	1
Correct final form $(x - 2)^2 + 4$	1

Solution.

Halve the coefficient of x and add/subtract its square:

$$x^2 - 4x + 8 = (x^2 - 4x + 4) - 4 + 8 = (x - 2)^2 + 4.$$

So $p = 2$ and $q = 4$.

Q4 Abbotsleigh 2024 Prelim Q25 · **2 marks**

Solve $x^2 - 6x - 6 = 0$ by completing the square. Leave your answer in exact form.

Criteria	Marks
Completes the square to $(x - 3)^2 = 15$	1
Correct exact solutions $x = 3 \pm \sqrt{15}$	1

Solution.

Move the constant across and complete the square:

$$x^2 - 6x = 6 \implies (x - 3)^2 - 9 = 6 \implies (x - 3)^2 = 15.$$

Taking square roots, $x - 3 = \pm\sqrt{15}$, so

$$x = 3 \pm \sqrt{15}.$$

Q5 Hornsby Girls 2022 Prelim Q12(a) · **2 marks**

Consider the quadratic function $f(x) = -2x^2 + 12x - 18$. Express $f(x)$ in the form $a(x-h)^2 + k$ and state the vertex of $y = f(x)$.

Criteria	**Marks**
Factors out -2 and completes the square inside the bracket	1
Correct form $-2(x-3)^2$ (i.e. $k = 0$) and vertex $(3, 0)$	1

Solution.

Take out the leading coefficient, then complete the square:

$$f(x) = -2\big(x^2 - 6x + 9\big) = -2(x-3)^2.$$

So $a = -2$, $h = 3$, $k = 0$, and the vertex of $y = f(x)$ is $(3, 0)$.

Q6 Girraween 2023 Prelim Q6(f) · **3 marks**

Use the quadratic formula to find the x-intercepts of $y = -7x^2 - 10x + 6$, in simplest surd form.

Criteria	**Marks**
Correct a, b, c and quadratic formula setup	1
Correct discriminant 268	1
Simplifies surd to $\dfrac{-5 \pm \sqrt{67}}{7}$	1

Solution.

With $a = -7,\ b = -10,\ c = 6$:

$$x = \frac{10 \pm \sqrt{(-10)^2 - 4(-7)(6)}}{2(-7)} = \frac{10 \pm \sqrt{268}}{-14} = \frac{10 \pm 2\sqrt{67}}{-14} = \frac{-5 \pm \sqrt{67}}{7}.$$

So the x-intercepts are $x = \dfrac{-5 + \sqrt{67}}{7}$ and $x = \dfrac{-5 - \sqrt{67}}{7}$.

Q7 Manly 2024 Trial Q9 · **1 mark**

If $f(x) = x(x-1)$, how many different solutions are there to the equation $f(x^2) = 0$?

(A) 1

(B) 2

(C) 3

(D) 4

Criteria	Marks
Correct option (C)	1

Solution.

Substitute x^2 into f:

$$f(x^2) = x^2\left(x^2 - 1\right) = 0.$$

This gives $x^2 = 0$ (so $x = 0$) or $x^2 = 1$ (so $x = \pm 1$). The distinct solutions are $x = -1,\ 0,\ 1$: three in total. The answer is (C).

Q8 Kambala 2023 Prelim Q22(a) · **2 marks**

Consider the equation $x^4 - 12x^2 + 32 = 0$. Solve for x by first letting $u = x^2$.

Criteria	Marks
Substitutes $u = x^2$ to get $u^2 - 12u + 32 = 0$ and solves $u = 4,\ u = 8$	1
Correct solutions $x = \pm 2,\ x = \pm 2\sqrt{2}$	1

Solution.

Let $u = x^2$, so the equation becomes

$$u^2 - 12u + 32 = 0 \implies (u-4)(u-8) = 0,$$

giving $u = 4$ or $u = 8$. Returning to x:

$$x^2 = 4 \implies x = \pm 2, \qquad x^2 = 8 \implies x = \pm 2\sqrt{2}.$$

Q9 Killara 2024 Trial Q11 · **2 marks**

Solve for x:

$$6(2x - 3) = 8 - 2(3x + 1).$$

Criteria	Marks
Expands both sides: $12x - 18 = 6 - 6x$	1
Correct solution $x = \frac{4}{3}$	1

Solution.

Expand each side and collect the x terms:

$$12x - 18 = 8 - 6x - 2 = 6 - 6x.$$

$$12x + 6x = 6 + 18 \implies 18x = 24 \implies x = \frac{24}{18} = \frac{4}{3}.$$

Q10 Ascham 2023 Trial Q11 · **1 mark**

Solve $\dfrac{3x}{x+2} = 4$.

Criteria	Marks
Correct solution $x = -8$	1

Solution.

Multiply both sides by $(x + 2)$ and solve:

$$3x = 4(x + 2) = 4x + 8 \implies -x = 8 \implies x = -8.$$

Q11 Sydney Grammar 2022 Prelim Q10(a) · **2 marks**

Solve $\dfrac{32}{x} - x = 4$.

Criteria	**Marks**
Multiplies by x to form the quadratic $x^2 + 4x - 32 = 0$	1
Correct solutions $x = 4$ and $x = -8$	1

Solution.

Multiply through by x (noting $x \neq 0$):

$$32 - x^2 = 4x \implies x^2 + 4x - 32 = 0 \implies (x-4)(x+8) = 0.$$

Hence $x = 4$ or $x = -8$.

Q12 Fort St 2022 Prelim Q7(b) · **2 marks**

Solve $x^2 - 12x + 9 = 0$ by the quadratic formula, in simplest surd form.

Criteria	**Marks**
Correct substitution into the quadratic formula	1
Correct simplified surd answer	1

Solution.

With $a = 1$, $b = -12$, $c = 9$,

$$x = \frac{12 \pm \sqrt{144 - 36}}{2} = \frac{12 \pm \sqrt{108}}{2} = \frac{12 \pm 6\sqrt{3}}{2} = 6 \pm 3\sqrt{3}.$$

Q13 Fort St 2023 Trial Q17 · **2 marks**

Complete the square to write the solution to $x^2 + 6x = 1$ in exact form.

Criteria	Marks
Completes the square to $(x+3)^2 = 10$	1
Correct exact solutions $x = -3 \pm \sqrt{10}$	1

Solution.

Add 9 to both sides to complete the square on the left:

$$x^2 + 6x + 9 = 1 + 9 \implies (x+3)^2 = 10.$$

Taking square roots, $x + 3 = \pm\sqrt{10}$, so

$$x = -3 \pm \sqrt{10}.$$

Q14 Fort St 2023 Trial Q2 · **1 mark**

Solve for x: $4^{x-2} \cdot 2^{2x+3} = 8^x$.

(A) $x = 2$

(B) $x = -2$

(C) $x = -1$

(D) $x = 1$

Criteria	Marks
Correct option (D)	1

Solution.

Write every term with base 2:

$$4^{x-2} \cdot 2^{2x+3} = 2^{2(x-2)} \cdot 2^{2x+3} = 2^{4x-1}, \qquad 8^x = 2^{3x}.$$

Equating indices: $4x - 1 = 3x$, so $x = 1$. The answer is (D).

Q15 Knox 2022 Prelim Q14(a)(i) · **2 marks**

Solve $\sqrt{x+1} = x - 2$ for x.

Criteria	Marks
Squares both sides and forms $x^2 - 5x + 3 = 0$	1
Solves and rejects the extraneous root, giving $x = \dfrac{5+\sqrt{13}}{2}$	1

Solution.

Squaring: $x + 1 = (x-2)^2 = x^2 - 4x + 4$, so $x^2 - 5x + 3 = 0$. Then

$$x = \frac{5 \pm \sqrt{25 - 12}}{2} = \frac{5 \pm \sqrt{13}}{2}.$$

The right-hand side $x - 2$ must be non-negative, so $x \geq 2$. Only $x = \dfrac{5+\sqrt{13}}{2} \approx 4.30$ satisfies this; $x = \dfrac{5-\sqrt{13}}{2} \approx 0.70$ is extraneous. Hence $x = \dfrac{5+\sqrt{13}}{2}$.

2. The discriminant: nature and number of roots (find the parameter)

Q16 Moriah 2021 Prelim Q2 · **1 mark**

Which statement best describes the solutions of the equation $x^2 - 4x - 3 = 0$?

(A) They are both real

(B) They are real and rational

(C) They are real and not rational

(D) They are equal

Criteria	Marks
Correct option (C)	1

Solution.

Compute the discriminant: $\Delta = (-4)^2 - 4(1)(-3) = 16 + 12 = 28$. Since $\Delta > 0$ the roots are real and distinct; since 28 is not a perfect square the roots are irrational. Hence the solutions are real and not rational, option (C).

Q17 Abbotsleigh 2024 Prelim Q2 · **1 mark**

Which of the following quadratic equations has two distinct, real and rational roots?

(A) $y = -x^2 - 3x + 5$

(B) $y = 4x^2 - x - 5$

(C) $y = -x^2 - 4$

(D) $y = 2x^2$

Criteria	Marks
Correct option (B)	1

Solution.

Two distinct rational roots require $\Delta > 0$ and Δ a perfect square. For (A) $\Delta = 9 + 20 = 29$ (not a perfect square); for (C) $\Delta = -16 < 0$; for (D) $\Delta = 0$ (equal roots). For (B) $\Delta = (-1)^2 - 4(4)(-5) = 1 + 80 = 81 = 9^2$, a positive perfect square, giving the distinct rational roots $x = -1,\ \frac{5}{4}$. Option (B).

Q18 Knox 2023 Prelim Q13(b) · **2 marks**

If $y = 3x^2 - 2x + k$ does not intersect the x-axis, find the range of possible values of k.

Criteria	Marks
Sets up the no-intersection condition $\Delta < 0$ using $\Delta = 4 - 12k$	1
Solves to $k > \frac{1}{3}$	1

Solution.

The curve does not intersect the x-axis when $3x^2 - 2x + k = 0$ has no real solutions, i.e. $\Delta < 0$. Here

$$\Delta = (-2)^2 - 4(3)(k) = 4 - 12k.$$

Requiring $\Delta < 0$ gives $4 - 12k < 0$, so $12k > 4$, i.e.

$$k > \tfrac{1}{3}.$$

Q19 North Sydney Girls 2023 Trial Q13 · **2 marks**

Find the values of m if the equation $4x^2 + (m+3)x + 9 = 0$ has two equal roots.

Criteria	Marks
Sets $\Delta = 0$: $(m+3)^2 - 144 = 0$	1
Solves to $m = 9$ or $m = -15$	1

Solution.

Two equal roots require $\Delta = 0$:

$$\Delta = (m+3)^2 - 4(4)(9) = (m+3)^2 - 144 = 0.$$

Then $(m+3)^2 = 144$, so $m + 3 = \pm 12$, giving

$$m = 9 \quad \text{or} \quad m = -15.$$

Q20 Presbyterian 2023 Prelim Q16 · **2 marks**

Show that the line $y = 5x - 2$ is a tangent to the parabola $y = x^2 + 3x - 1$.

Criteria	Marks
Equates the curves and forms $x^2 - 2x + 1 = 0$	1
Shows $\Delta = 0$, concluding one point of contact (tangent)	1

Solution.

Setting the line equal to the parabola,

$$x^2 + 3x - 1 = 5x - 2 \implies x^2 - 2x + 1 = 0.$$

The discriminant is $\Delta = (-2)^2 - 4(1)(1) = 4 - 4 = 0$. Since $\Delta = 0$ there is exactly one solution ($x = 1$), so the line meets the parabola at a single point and is therefore a tangent.

Q21 Sydney Grammar 2022 Prelim Q14(b) · **2 marks**

Show that $f(x) = (k+2)x^2 + 4x + k + 2$ has no x-intercepts for all $k > 0$.

Criteria	Marks
Computes $\Delta = 16 - 4(k+2)^2$ and shows $\Delta < 0$ for $k > 0$	1
Notes leading coefficient $k + 2 > 0$, so the parabola lies entirely above the x-axis	1

Solution.

The x-intercepts satisfy $(k+2)x^2 + 4x + (k+2) = 0$, with discriminant

$$\Delta = 4^2 - 4(k+2)(k+2) = 16 - 4(k+2)^2 = -4k(k+4).$$

For $k > 0$ both $k > 0$ and $k + 4 > 0$, so $-4k(k+4) < 0$, i.e. $\Delta < 0$: there are no real roots. Also $k + 2 > 0$, so the parabola opens upward. A upward parabola with no real roots lies entirely above the x-axis, hence $f(x)$ has no x-intercepts for all $k > 0$.

Q22 Penrith 2022 Prelim Q3 · **1 mark**

What is the value of the discriminant of the quadratic equation $x^2 - 3x - 10 = 0$?

(A) -5

(B) 5

(C) 7

(D) 49

Criteria	Marks
Correct option (D)	1

Solution.

$\Delta = b^2 - 4ac = (-3)^2 - 4(1)(-10) = 9 + 40 = 49$. Option (D).

Q23 Girraween 2023 Prelim Q3 · **1 mark**

For what values of k does the equation $x^2 - 6x - 3k = 0$ have real roots?

(A) $k \geq -3$

(B) $k \leq -3$

(C) $k \leq 3$

(D) $k \geq 3$

Criteria	Marks
Correct option (A)	1

Solution.

Real roots require $\Delta \geq 0$. Here $\Delta = (-6)^2 - 4(1)(-3k) = 36 + 12k$. Then $36 + 12k \geq 0$ gives $12k \geq -36$, i.e. $k \geq -3$. Option (A).

Q24 Manly 2022 Prelim Q17 · **2 marks**

For what value of k does the equation $x^2 - 3x + k = 0$ have equal roots?

Criteria	Marks
Sets $\Delta = 0$: $9 - 4k = 0$	1
Solves to $k = \frac{9}{4}$	1

Solution.

Equal roots require $\Delta = 0$:

$$\Delta = (-3)^2 - 4(1)(k) = 9 - 4k = 0.$$

Hence $4k = 9$, giving

$$k = \frac{9}{4}.$$

Q25 Presbyterian 2022 Prelim Q40 · **2 marks**

Use the discriminant to show that $x^2 - 2x + 4 > 0$ for all real x.

Criteria	Marks
Computes $\Delta = -12 < 0$ (no real roots)	1
States leading coefficient > 0, so the expression is positive for all x	1

Solution.

For $x^2 - 2x + 4$ the discriminant is

$$\Delta = (-2)^2 - 4(1)(4) = 4 - 16 = -12 < 0,$$

so the quadratic has no real zeros and never crosses the x-axis. Since the leading coefficient $1 > 0$ the parabola opens upward and therefore lies entirely above the x-axis. Hence $x^2 - 2x + 4 > 0$ for all real x. (Equivalently, $x^2 - 2x + 4 = (x - 1)^2 + 3 \geq 3 > 0$.)

Q26 Hurlstone 2025 Trial Q15 · **4 marks**

The quadratic equation $x^2 + (m + 6)x + 8m = 0$ has two distinct roots. By considering the discriminant or otherwise, determine all the possible values of m.

Criteria	Marks
Writes the condition for two distinct roots $\Delta > 0$	1
Forms $\Delta = (m + 6)^2 - 32m$ and simplifies to $m^2 - 20m + 36$	1
Factorises $(m - 2)(m - 18) > 0$	1
States $m < 2$ or $m > 18$	1

Solution.

Two distinct roots require $\Delta > 0$:

$$\Delta = (m + 6)^2 - 4(1)(8m) = (m + 6)^2 - 32m.$$

Expanding,

$$\Delta = m^2 + 12m + 36 - 32m = m^2 - 20m + 36 = (m - 2)(m - 18).$$

The condition $(m - 2)(m - 18) > 0$ holds when both factors are positive or both negative, i.e.

$$m < 2 \quad \text{or} \quad m > 18.$$

3. Solving quadratic and cubic inequalities

Q27 North Sydney Boys 2022 Trial Q4 · **1 mark**

The solution to the inequality $6 - x - x^2 \leq 0$ is

(A) $x \leq -3$ or $x \geq 2$

(B) $x \leq -2$ or $x \geq 3$

(C) $-3 \leq x \leq 2$

(D) $-2 \leq x \leq 3$

Criteria	**Marks**
Correct option (A)	1

Solution.

Rearrange into standard form: $6 - x - x^2 \leq 0$ becomes $x^2 + x - 6 \geq 0$, i.e. $(x+3)(x-2) \geq 0$. The parabola $y = x^2 + x - 6$ opens upward with zeros at $x = -3$ and $x = 2$, so it is ≥ 0 outside the roots:

$$x \leq -3 \quad \text{or} \quad x \geq 2.$$

This is option **(A)**.

Q28 Abbotsleigh 2020 Trial Q5 · **1 mark**

Which one of the following is the set of all solutions to $2x^2 - 5x + 2 \geq 0$?

(A) $\left[\frac{1}{2}, 2\right]$

(B) $\left(\frac{1}{2}, 2\right)$

(C) $\left(-\infty, \frac{1}{2}\right) \cup (2, \infty)$

(D) $\left(-\infty, \frac{1}{2}\right] \cup [2, \infty)$

Criteria	Marks
Correct option (D)	1

Solution.

Factorise: $2x^2 - 5x + 2 = (2x - 1)(x - 2)$, with zeros at $x = \frac{1}{2}$ and $x = 2$. The parabola opens upward, so $2x^2 - 5x + 2 \geq 0$ outside the roots, endpoints included since the inequality is $\geq$:

$$x \leq \tfrac{1}{2} \quad \text{or} \quad x \geq 2, \qquad \text{i.e.} \qquad \left(-\infty, \tfrac{1}{2}\right] \cup [2, \infty).$$

This is option **(D)**.

Q29 Sydney Grammar 2020 Trial Q20(c) · **2 marks**

Find the values of x for which $x^2 - 6 > x$.

Criteria	Marks
Rearranges to $x^2 - x - 6 > 0$ and factorises to $(x - 3)(x + 2) > 0$	1
Correct solution $x < -2$ or $x > 3$ with strict endpoints	1

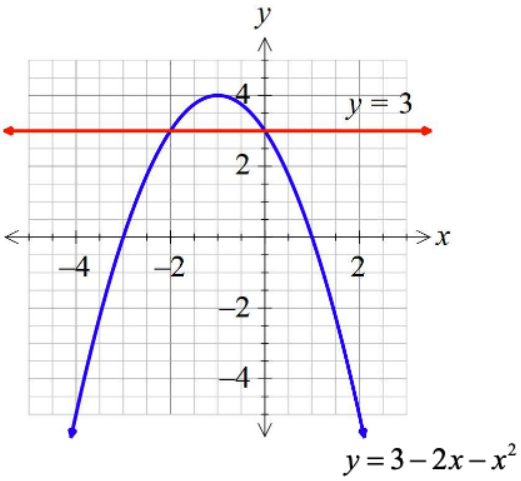
Solution.

Bring all terms to one side: $x^2 - x - 6 > 0$, i.e. $(x - 3)(x + 2) > 0$. The parabola $y = x^2 - x - 6$ opens upward with zeros at $x = -2$ and $x = 3$, so it is positive (strictly) outside the roots:

$$x < -2 \quad \text{or} \quad x > 3.$$

Q30 Ascham 2020 Trial Q7 · 1 mark

Use the graph below to solve the inequality $3 - 2x - x^2 < 3$.



(A) $x < 0$

(B) $-2 < x < 0$

(C) $x < -2$ or $x > 0$

(D) $x < -3$ or $x > 1$

Criteria	Marks
Correct option (C)	1

Solution.

The graph shows $y = 3 - 2x - x^2$; we need where it lies below the line $y = 3$. Algebraically, $3 - 2x - x^2 < 3$ gives $-2x - x^2 < 0$, i.e. $x^2 + 2x > 0$, so $x(x + 2) > 0$. This upward parabola is positive outside its roots $x = -2$ and $x = 0$:

$$x < -2 \quad \text{or} \quad x > 0.$$

This is option **(C)**.

Graphs of quadratics and cubics

4. Sketching parabolas, cubics and factored polynomials from the equation

Q31 Presbyterian 2021 Trial Q20(a) · **1 mark**

The diagram shows the graph of $y = x^2 + x - 6$ for $0 \leq x \leq 4$. The point A is where the curve meets the x-axis. Find the value of A.

Criteria	Marks
Correct value $A = 2$	1

Solution.

At the x-intercept $y = 0$, so $x^2 + x - 6 = 0$, i.e. $(x + 3)(x - 2) = 0$, giving $x = -3$ or $x = 2$. On the shown domain $0 \leq x \leq 4$ only $x = 2$ lies in range, so $A = 2$.

Q32 Gosford 2025 Trial Q11 · **2 marks**

Find the vertex of the parabola $y = x^2 - 6x + 10$.

Criteria	Marks
Completes the square (or finds axis of symmetry $x = 3$)	1
Correct vertex $(3, 1)$	1

Solution.

Complete the square:

$$y = x^2 - 6x + 10 = (x - 3)^2 - 9 + 10 = (x - 3)^2 + 1.$$

The vertex is at $(3, 1)$.

Q33 Loreto Kirribilli 2022 Prelim Q35(a) · **2 marks**

Draw a neat sketch of $y = x^2 - 5x + 14$, labelling the vertex and any axes intercepts.

Criteria	**Marks**
Correct vertex $\left(\frac{5}{2}, \frac{31}{4}\right)$ and y-intercept 14	1
Correct parabola (opens up, no x-intercepts as $\Delta < 0$)	1

Solution.

Complete the square:

$$y = x^2 - 5x + 14 = \left(x - \tfrac{5}{2}\right)^2 - \tfrac{25}{4} + 14 = \left(x - \tfrac{5}{2}\right)^2 + \tfrac{31}{4}.$$

So the vertex is $\left(\frac{5}{2}, \frac{31}{4}\right)$. The discriminant $\Delta = (-5)^2 - 4(1)(14) = -31 < 0$, so there are no x-intercepts and the curve lies entirely above the x-axis. The y-intercept is $y = 14$. Sketch an upward parabola with minimum $\left(\frac{5}{2}, \frac{31}{4}\right)$ passing through $(0, 14)$.

Q34 Baulkham Hills 2023 Prelim Q25 · **2 marks**

Sketch the graph of $y = (x - 1)^3 + 2$.

Criteria	**Marks**
Correct shape: basic cubic $y = x^3$ translated, point of inflexion at $(1, 2)$	1
Correct intercepts: y-intercept $(0, 1)$ and x-intercept where $x = 1 - \sqrt[3]{2} \approx -0.26$	1

Solution.

The graph is $y = x^3$ shifted right 1 and up 2, so it is an increasing cubic with a horizontal point of inflexion at $(1, 2)$. The y-intercept is $y = (0 - 1)^3 + 2 = 1$, i.e. $(0, 1)$. The x-intercept satisfies $(x - 1)^3 = -2$, so $x = 1 - \sqrt[3]{2} \approx -0.26$. Sketch the increasing cubic through $(1 - \sqrt[3]{2}, 0)$, $(0, 1)$ and $(1, 2)$.

Q35 Hornsby Girls 2020 Trial Q18 · **5 marks**

(a) Express $f(x)$ as $x(ax^2 + bx + c)$. **(b)** Hence factorise $f(x)$ completely. **(c)** Sketch $y = f(x)$, showing where it meets the axes.

Criteria	Marks
(a) Expands and factors out x to reach $f(x) = x(x^2 - 9x + 20)$	2
(b) Factorises the quadratic to give $f(x) = x(x-4)(x-5)$	1
(c) Correct positive cubic shape	1
(c) All three axis intercepts labelled: $(0,0)$, $(4,0)$, $(5,0)$	1

Solution.

(a) Take out the common factor x from the bracket first: $f(x) = (x^2-6x)(x-3)+2x = x\big[(x-6)(x-3)+2\big] = x\big(x^2 - 9x + 18 + 2\big) = x(x^2 - 9x + 20)$.

(b) Factorise the quadratic $x^2 - 9x + 20 = (x-4)(x-5)$, so $f(x) = x(x-4)(x-5)$.

(c) $y = f(x)$ is a positive (upward-tailed) cubic with single roots at $x = 0, 4, 5$. It crosses the x-axis at $(0,0)$, $(4,0)$ and $(5,0)$, and the y-intercept is also $(0,0)$. Sketch the standard cubic passing left-to-bottom, rising through $(0,0)$, turning to dip below the axis between $x = 4$ and $x = 5$, then rising through $(5,0)$.

Q36 St George Girls 2023 Prelim Q11 · **3 marks**

Without using calculus, sketch the polynomial below, clearly showing the x- and y-intercepts.

$$P(x) = (x+1)(3-x)^2$$

Criteria	Marks
Correct x-intercepts: crosses at $(-1,0)$, touches at $(3,0)$	1
Correct y-intercept $(0,9)$	1
Correct shape/concavity (cubic, positive leading coefficient with $(3-x)^2$)	1

Solution.

The factors give roots $x = -1$ (single, so the graph crosses) and $x = 3$ (repeated, so the graph touches the axis). Expanding, $P(x) = (x+1)(x-3)^2$ has a positive leading coefficient (leading term x^3), so the curve rises to the right and falls to the left. The y-intercept is $P(0) = (1)(3)^2 = 9$, i.e. $(0,9)$. Sketch a positive cubic crossing at $(-1,0)$, passing through $(0,9)$, and touching the x-axis at $(3,0)$.

Q37 Glenwood 2023 Trial Q27 · 3 marks

By factorising or otherwise, sketch the graph of $y = x^3 - 5x^2 - 4x + 20$, showing all intercepts with the axes. You do not need to find any stationary points.

Criteria	Marks
Factorises to $(x-2)(x+2)(x-5)$	1
Correct intercepts $(-2,0)$, $(2,0)$, $(5,0)$ and $(0,20)$	1
Correct positive cubic shape through the three simple roots	1

Solution.

Factorise by grouping:

$$x^3 - 5x^2 - 4x + 20 = x^2(x-5) - 4(x-5) = (x^2-4)(x-5) = (x-2)(x+2)(x-5).$$

The x-intercepts are $x = -2,\ 2,\ 5$ (all simple, so the curve crosses at each). The y-intercept is $y = 20$, i.e. $(0,20)$. Sketch a positive cubic crossing at $(-2,0)$, $(2,0)$ and $(5,0)$, and passing through $(0,20)$.

Q38 Barker 2024 Prelim Q15(b) · 2 marks

Sketch the graph of $f(x) = -(x-1)^2(x+2)(x+5)$, showing only the x- and y-intercepts.

Criteria	Marks
Correct x-intercepts: touches at $(1,0)$, crosses at $(-2,0)$ and $(-5,0)$	1
Correct y-intercept $(0,-10)$ and shape (negative leading coefficient, quartic)	1

Solution.

The factor $(x-1)^2$ is repeated, so the graph touches the x-axis at $(1,0)$; the simple factors give crossings at $x = -2$ and $x = -5$. The leading term is $-x^4$ (negative leading coefficient of a degree-4 polynomial), so both ends point downward. The y-intercept is $f(0) = -(0-1)^2(0+2)(0+5) = -(1)(2)(5) = -10$, i.e. $(0,-10)$. Sketch a negative quartic falling on both ends, crossing at $(-5,0)$ and $(-2,0)$, through $(0,-10)$, and just touching the axis at $(1,0)$.

5. Matching a graph to its equation

Q39 Trinity Grammar 2020 Trial Q1 · 1 mark

Which equation best represents the given graph?

Diagram to be inserted:
`adv-working-with-functions-d4-q39.png`

(A) $f(x) = x^2 + 1$

(B) $f(x) = x^2 - 1$

(C) $f(x) = (x+1)^2$

(D) $f(x) = (x-1)^2$

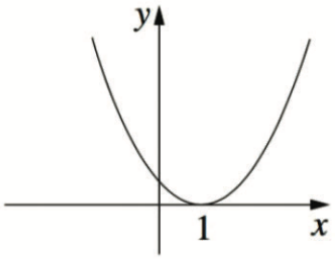
Criteria	Marks
Correct option (D)	1

Solution.

The parabola opens upward and just touches the x-axis once (a single, repeated root) at $x = 1$. A curve that touches the axis at $x = 1$ has a double factor $(x-1)^2$, so its vertex is $(1, 0)$ and the equation is $f(x) = (x-1)^2$. Options $x^2 + 1$ has no x-intercept, $x^2 - 1$ cuts at $x = \pm 1$, and $(x+1)^2$ touches at $x = -1$, so the answer is (D).

Q40 Girraween 2023 Prelim Q4 · **1 mark**

The graph of $y = 1 - x^3$ could be



NOT TO SCALE

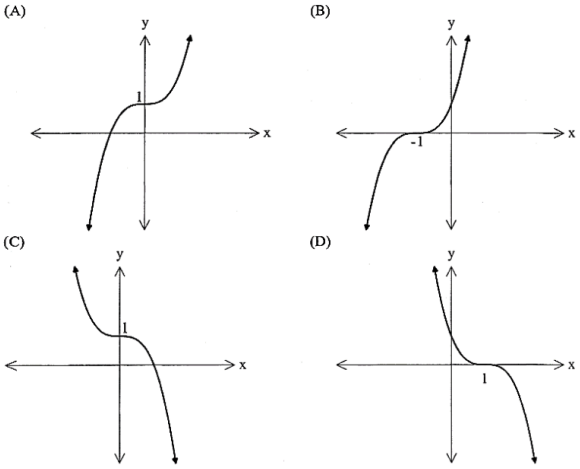
Criteria	Marks
Correct option (C)	1

Solution.

The coefficient of x^3 is negative, so the cubic is decreasing overall: as $x \to -\infty$, $y \to +\infty$, and as $x \to +\infty$, $y \to -\infty$. The y-intercept is $y = 1$ and the x-intercept is $x = 1$ (from $1 - x^3 = 0$). The point of inflection is where $y'' = -6x = 0$, i.e. at $x = 0$, giving the point $(0, 1)$ on the y-axis. The falling curve whose inflection sits at $(0, 1)$ and which cuts the x-axis at $x = 1$ is graph (C).

Q41 2020 HSC Q5 · 1 mark

Which of the following could represent the graph of $y = -x^2 + bx + 1$, where $b > 0$?



Criteria	Marks
Correct option (D)	1

Solution.

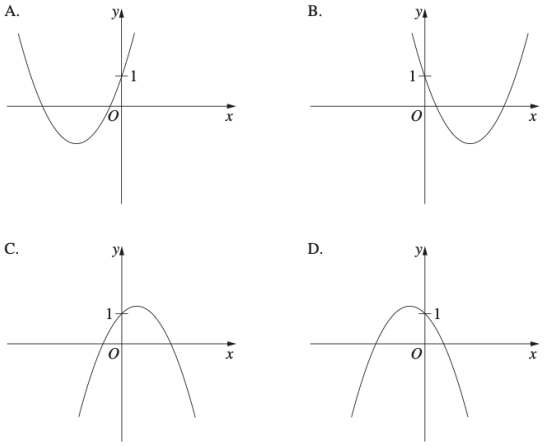
The coefficient of x^2 is $-1 < 0$, so the parabola is concave down. The y-intercept is $y = 1$. The axis of symmetry is

$$x = \frac{-b}{2(-1)} = \frac{b}{2} > 0 \quad (\text{since } b > 0),$$

so the vertex lies to the right of the y-axis. A concave-down parabola with y-intercept 1 and vertex to the right of the y-axis is graph (D).

Q42 2025 HSC Q4 · **1 mark**

Which of the following best represents the graph of $y = -5x(x-2)(3-x)$?



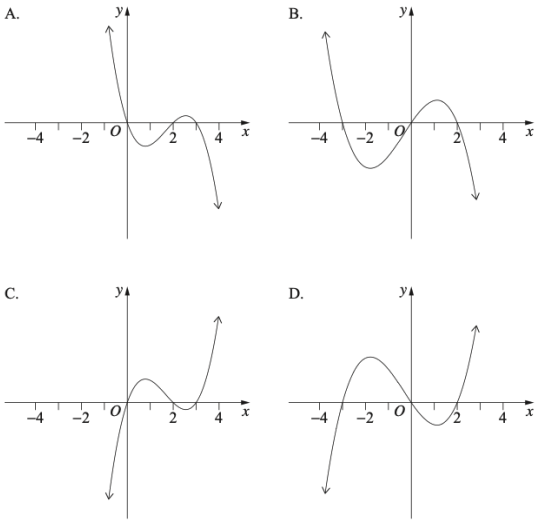
Criteria	Marks
Correct option (C)	1

Solution.

Rewrite $y = -5x(x-2)(3-x) = 5x(x-2)(x-3) = 5x^3 - 25x^2 + 30x$. The leading coefficient is positive, so as $x \to -\infty$, $y \to -\infty$ and as $x \to +\infty$, $y \to +\infty$. The zeros are $x = 0$, 2, 3 (all simple). Testing signs: $y < 0$ on $(-\infty, 0)$, $y > 0$ on $(0, 2)$ (e.g. $y(1) = 10$), $y < 0$ on $(2, 3)$ (e.g. $y(\frac{5}{2}) = -\frac{25}{8}$), and $y > 0$ on $(3, \infty)$. The rising cubic with simple roots at 0, 2 and 3 is graph (C).

Q43 2023 HSC Q4 · 1 mark

The graph of a polynomial is shown.



Which row of the table is correct for this polynomial?

	Equation	Value of b	Value of c
A.	$y = -(x-b)(x-c)^2$	$b > 0$	$c < 0$
B.	$y = -(x-b)(x-c)^2$	$b < 0$	$c > 0$
C.	$y = -x(x-b)(x-c)$	$b > 0$	$c < 0$
D.	$y = -x(x-b)(x-c)$	$b < 0$	$c > 0$

Criteria	**Marks**
Correct option (B)	1

Solution.

As $x \to -\infty$, $y \to +\infty$ and as $x \to +\infty$, $y \to -\infty$, so the leading coefficient is negative. The curve crosses the x-axis once (a simple root) at a negative value of x, and touches the x-axis (a double root) at a positive value of x; it does not pass through the origin, so neither cubic in options C and D (each has a factor x, forcing a root at the origin) can be correct. Hence the equation has the form $y = -(x-b)(x-c)^2$, where b is the simple root (negative, so $b < 0$) and c is the double root (positive, so $c > 0$). This is row (B).

6. Finding a function's equation from a graph, points or conditions

Q44 Killara 2023 Prelim Q23(a) · **1 mark**

Let $g(x) = x^2 + bx + 11$. The point $(-1, 8)$ lies on the graph of $g(x)$. Show that $b = 4$.

Criteria	Marks
Substitutes $(-1, 8)$ and solves to reach $b = 4$	1

Solution.

Since $(-1, 8)$ lies on $y = g(x)$, substitute $x = -1$, $g(-1) = 8$:

$$(-1)^2 + b(-1) + 11 = 8 \implies 1 - b + 11 = 8 \implies 12 - b = 8 \implies b = 4.$$

Q45 Girraween 2022 Trial Q5 · **1 mark**

The graph of the curve $y = (x + a)^2 + b$ is shown below.

Diagram to be inserted:
`adv-working-with-functions-d4-q45.png`

What are the values of a and b?

(A) $a = 2,\ b = 1$

(B) $a = -2,\ b = -1$

(C) $a = -2,\ b = 1$

(D) $a = 2,\ b = -1$

Criteria	Marks
Correct option (D)	1

Solution.

In vertex form $y = (x + a)^2 + b$ the vertex is at $(-a, b)$. The parabola shown has its minimum (vertex) at $(-2, -1)$, so $-a = -2$ and $b = -1$, giving $a = 2$, $b = -1$. This is option **(D)**.

Q46 North Sydney Girls 2022 Prelim Q30(a) · **2 marks**

A parabola with equation $y = f(x)$ has a vertex at $(-1, 3)$ and passes through the origin. Write an equation for $f(x)$.

Criteria	Marks
Uses vertex form $f(x) = a(x+1)^2 + 3$	1
Substitutes $(0, 0)$ to find $a = -3$ and states the equation	1

Solution.

With vertex $(-1, 3)$, write in vertex form

$$f(x) = a(x+1)^2 + 3.$$

The curve passes through the origin $(0, 0)$, so

$$0 = a(0+1)^2 + 3 \implies 0 = a + 3 \implies a = -3.$$

Hence $f(x) = -3(x+1)^2 + 3$ (equivalently $f(x) = -3x^2 - 6x$).

Q47 Fort St 2023 Trial Q20 · **3 marks**

The function $P(x) = x^3 + bx^2 + cx + d$ has x-intercepts at 3, 0 and -3. Find b, c and d.

Criteria	Marks
Writes $P(x)$ in factored form $x(x-3)(x+3)$ (leading coefficient 1)	1
Expands to $x^3 - 9x$	1
States $b = 0$, $c = -9$, $d = 0$	1

Solution.

The x-intercepts are $x = 3$, $x = 0$, $x = -3$, and the coefficient of x^3 is 1, so

$$P(x) = (x-3)(x-0)(x+3) = x\left(x^2 - 9\right) = x^3 - 9x.$$

Comparing with $x^3 + bx^2 + cx + d$ gives $b = 0$, $c = -9$, $d = 0$.

Q48 Knox 2022 Trial Q20 · **3 marks**

A basketball player shoots a ball from a height of 2 metres into a hoop 14 metres away. The hoop is 3 metres from the ground. After moving 5 metres horizontally, the ball reaches a height of 4 metres from the ground. The path of the ball can be modelled by the quadratic function

$$y = ax^2 + bx + c,$$

where y represents the height of the ball from the ground, x is the horizontal displacement from the basketball player, and a, b and c are constants. Find the values of a, b and c that best model the path of the ball.

Criteria	**Marks**
Reads off the three points $(0, 2)$, $(5, 4)$, $(14, 3)$ and finds $c = 2$	1
Sets up the two simultaneous equations $25a + 5b = 2$ and $196a + 14b = 1$	1
Solves to get $a = -\frac{23}{630}$, $b = \frac{367}{630}$	1

Solution.

The ball starts at $(0, 2)$, passes through $(5, 4)$, and reaches the hoop at $(14, 3)$.

Substituting $(0, 2)$: $c = 2$.

Substituting $(5, 4)$: $25a + 5b + 2 = 4$, so $25a + 5b = 2$.

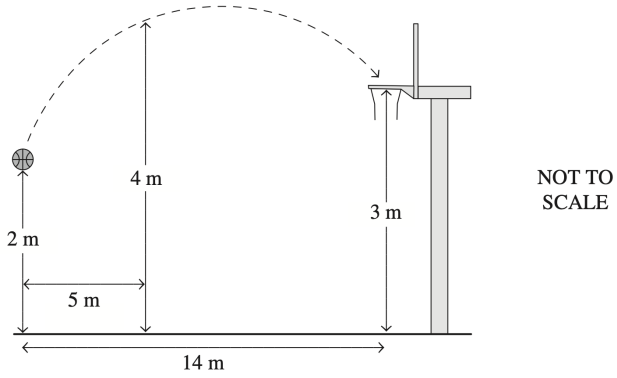
Substituting $(14, 3)$: $196a + 14b + 2 = 3$, so $196a + 14b = 1$.

Solving these simultaneously:

$$a = -\frac{23}{630}, \qquad b = \frac{367}{630}, \qquad c = 2.$$

Q49 Penrith 2024 Trial Q14 · **3 marks**

The graph of a polynomial of degree 6 is shown below. It has x-intercepts at $(-1, 0)$, $(1, 0)$ and $(4, 0)$, and a y-intercept at $(0, 8)$.



Find the equation of the polynomial.

Criteria	Marks
Identifies the roots and their multiplicities (single at -1, triple at 1, double at 4)	1
Writes $y = a(x+1)(x-1)^3(x-4)^2$ for the degree-6 form	1
Substitutes $(0, 8)$ to find $a = -\frac{1}{2}$ and states the equation	1

Solution.

From the graph the curve crosses the x-axis at $x = -1$ (a simple crossing), crosses with a point of inflection at $x = 1$ (a triple root), and touches the axis at $x = 4$ (a double root). These multiplicities $1 + 3 + 2 = 6$ give a degree-6 polynomial:

$$y = a(x+1)(x-1)^3(x-4)^2.$$

Using the y-intercept $(0, 8)$:

$$8 = a(0+1)(0-1)^3(0-4)^2 = a(1)(-1)(16) = -16a \;\Rightarrow\; a = -\frac{1}{2}.$$

Hence

$$y = -\frac{1}{2}(x+1)(x-1)^3(x-4)^2.$$

Intersections and features

7. Intersections and simultaneous equations (line-and-curve / two curves)

Q50 Hurlstone 2023 Trial Q19(a) · **2 marks**

Find the coordinates where the line $y = 2x - 3$ intersects the parabola $y = x^2 - 2x - 3$.

Criteria	Marks
Equates the two expressions and solves for x ($x = 0,\ 4$)	1
States both intersection points as coordinates	1

Solution.

At intersection the y-values are equal, so

$$2x - 3 = x^2 - 2x - 3.$$

Rearranging: $x^2 - 4x = 0$, i.e. $x(x - 4) = 0$, so $x = 0$ or $x = 4$. Substituting into $y = 2x - 3$: when $x = 0$, $y = -3$; when $x = 4$, $y = 5$. The points of intersection are $(0, -3)$ and $(4, 5)$.

Q51 2024 HSC Q14(a) · **1 mark**

The curves $y = (x - 1)^2$ and $y = 5 - x^2$ intersect at two points. Find the x-coordinates of the points of intersection of the two curves.

Criteria	Marks
Correct x-coordinates $x = -1$ and $x = 2$	1

Solution.

Setting the curves equal:

$$(x - 1)^2 = 5 - x^2.$$

Expanding: $x^2 - 2x + 1 = 5 - x^2$, so $2x^2 - 2x - 4 = 0$, i.e. $x^2 - x - 2 = 0$. Factoring: $(x - 2)(x + 1) = 0$, so $x = -1$ or $x = 2$.

Q52 Cherrybrook Tech 2021 Prelim Q16 · **3 marks**

Use algebra to solve the following simultaneous equations:

$$y = x - 2$$

$$y^2 + x^2 = 10$$

Criteria	Marks
Substitutes $y = x - 2$ into the second equation	1
Solves the resulting quadratic for x ($x = -1,\ 3$)	1
States both solutions as coordinate pairs	1

Solution.

Substitute $y = x - 2$ into $y^2 + x^2 = 10$:

$$(x - 2)^2 + x^2 = 10.$$

Expanding: $x^2 - 4x + 4 + x^2 = 10$, so $2x^2 - 4x - 6 = 0$, i.e. $x^2 - 2x - 3 = 0$. Factoring: $(x - 3)(x + 1) = 0$, so $x = 3$ or $x = -1$. Using $y = x - 2$: when $x = 3$, $y = 1$; when $x = -1$, $y = -3$. The solutions are $(3, 1)$ and $(-1, -3)$.

Q53 Loreto Kirribilli 2022 Prelim Q35(b) · **3 marks**

The line $y = x + 5$ is a tangent to the curve $y = x^2 - 5x + 14$. Find the coordinates of the point where the tangent touches the curve.

Criteria	Marks
Equates line and curve and forms the quadratic $x^2 - 6x + 9 = 0$	1
Obtains the double root $x = 3$	1
States the point of contact $(3, 8)$	1

Solution.

At the point of contact the y-values are equal:

$$x + 5 = x^2 - 5x + 14.$$

Rearranging: $x^2 - 6x + 9 = 0$, i.e. $(x - 3)^2 = 0$, giving the repeated root $x = 3$ (confirming tangency). Then $y = 3 + 5 = 8$, so the tangent touches the curve at $(3, 8)$.

Q54 2020 HSC Q30(a) · **2 marks**

Two parabolas $y = 4x - x^2$ and $y = ax^2$, where $a > 0$, intersect at the origin O and at a point A. Show that the x-coordinate of A is $\dfrac{4}{a+1}$.

Criteria	Marks
Equates the two parabolas and factorises to $x\big[(a+1)x - 4\big] = 0$	1
Rejects $x = 0$ (that is O) and concludes $x = \dfrac{4}{a+1}$ at A	1

Solution.

At an intersection the y-values are equal, so

$$ax^2 = 4x - x^2.$$

Bringing all terms to one side: $ax^2 + x^2 - 4x = 0$, i.e. $x\big[(a+1)x - 4\big] = 0$. Hence $x = 0$ (the origin O) or $(a+1)x - 4 = 0$. The point A is not the origin, so

$$x = \frac{4}{a+1},$$

as required.

Q55 Killara 2022 Prelim Q22 · **3 marks**

Determine the number of points of intersection between the following graphs:

$$y = 3x^2 + 7x + 4$$

$$y = -x^2 + 1$$

Criteria	Marks
Equates the two curves and forms $4x^2 + 7x + 3 = 0$	1
Evaluates the discriminant $\Delta = 1 > 0$ (or factors)	1
Concludes there are 2 points of intersection	1

Solution.

Setting the curves equal:

$$3x^2 + 7x + 4 = -x^2 + 1.$$

Rearranging: $4x^2 + 7x + 3 = 0$. The discriminant is

$$\Delta = 7^2 - 4(4)(3) = 49 - 48 = 1 > 0,$$

so the quadratic has two distinct real roots ($x = -1$ and $x = -\frac{3}{4}$). Therefore the two graphs meet at 2 points of intersection.

8. Domain, range, extreme values and polynomial language

Q56 Sydney Tech 2023 Trial Q14(a) · **1 mark**

A polynomial function $f(x) = x^3 - 5x^2 + 8x - 3$ is defined over the domain $0 \leq x \leq 4$. What is the degree of this polynomial?

Criteria	Marks
States the degree is 3	1

Solution.

The degree of a polynomial is the highest power of x that appears. The highest-power term is x^3, so the degree is 3.

Q57 St George Girls 2022 Prelim Q11 · 2 marks

State the domain and range of $y = 1 - 2x^2$.

Criteria	Marks
Correct domain: all real x	1
Correct range: $y \leq 1$	1

Solution.

The expression $1 - 2x^2$ is defined for every real number, so the domain is all real x. Since $x^2 \geq 0$ we have $-2x^2 \leq 0$, hence $y = 1 - 2x^2 \leq 1$, with the maximum $y = 1$ attained at $x = 0$. The range is

$$y \leq 1, \qquad \text{i.e. } (-\infty, 1].$$

Q58 North Sydney Girls 2022 Prelim Q18 · 2 marks

Find the minimum value of the quadratic function $k(x) = 2x^2 - 5x + 18$.

Criteria	Marks
Finds the axis of symmetry $x = \frac{5}{4}$ (or completes the square)	1
Correct minimum value $k\left(\frac{5}{4}\right) = \frac{119}{8}$	1

Solution.

Since the coefficient of x^2 is positive, k has a minimum at its vertex. The axis of symmetry is

$$x = -\frac{b}{2a} = -\frac{-5}{2(2)} = \frac{5}{4}.$$

The minimum value is

$$k\left(\tfrac{5}{4}\right) = 2\left(\tfrac{5}{4}\right)^2 - 5\left(\tfrac{5}{4}\right) + 18 = \frac{25}{8} - \frac{25}{4} + 18 = \frac{119}{8} = 14\tfrac{7}{8}.$$

Q59 Kambala 2024 Prelim Q8 · **1 mark**

For the polynomial $p(x) = 3x(x+4)(x+5)(x-1)(x-7)$, which statement is true?

(A) The degree of $p(x)$ is 5.

(B) The constant term of $p(x)$ is -140.

(C) The leading coefficient is 10.

(D) $p(x)$ has a zero at $x = 5$.

Criteria	Marks
Correct option (A)	1

Solution.

There are five linear factors, so the degree is 5: option A is true. Checking the others: the leading coefficient is 3 (not 10); the constant term is $p(0) = 0$ because of the factor x (not -140); and the zeros are $x = 0, -4, -5, 1, 7$, so $x = 5$ is not a zero. The correct answer is (A).

Q60 Killara 2023 Trial Q6 · **1 mark**

A function is defined as $f(x) = x^2 - 6x + 10$ for $x \in [-1, 3)$. What is its range?

(A) $y \in (1, \infty)$

(B) $y \in [1, \infty)$

(C) $y \in (1, 17]$

(D) $y \in [1, 17]$

Criteria	Marks
Correct option (C)	1

Solution.

Completing the square, $f(x) = (x-3)^2 + 1$, so the vertex is at $x = 3$ with minimum value 1. On the domain $[-1, 3)$ the function decreases from $f(-1) = 1 + 6 + 10 = 17$ down towards $f(3) = 1$. The maximum 17 is attained (at $x = -1$), but $x = 3$ is excluded, so the value 1 is approached but never reached. Hence the range is $(1, 17]$: option (C).

Q61 Abbotsleigh 2024 Prelim Q5 · **1 mark**

Let $f(x) = x^2 + ax$, where a is a constant and $f(10) = f(2)$. Which of the following statements is true?

(A) $f\left(\frac{1}{2}\right) < f(1) < f(3)$

(B) $f(1) < f\left(\frac{1}{2}\right) < f(3)$

(C) $f(1) < f(3) < f\left(\frac{1}{2}\right)$

(D) $f(3) < f(1) < f\left(\frac{1}{2}\right)$

Criteria	**Marks**
Correct option (D)	1

Solution.

From $f(10) = f(2)$: $100 + 10a = 4 + 2a$, so $8a = -96$ and $a = -12$. Thus $f(x) = x^2 - 12x$, with vertex at $x = 6$. Evaluating:

$$f\left(\tfrac{1}{2}\right) = -\tfrac{23}{4} = -5.75, \qquad f(1) = -11, \qquad f(3) = -27.$$

Ordering from smallest to largest gives $f(3) < f(1) < f\left(\frac{1}{2}\right)$: option (D).