Year 11 Mathematics Advanced

Topic: Working with Functions (3 of 12)

Review Questions (Worked Solutions): Linear Functions

Term 3, 2026

Gradient, distance and angle

1. Gradient, distance and midpoint

Q1 Moriah 2021 Prelim Q16(a) · **1 mark**

A line passes through the points $A(2, -5)$ and $B(-1, 1)$ on the Cartesian plane. Find the gradient of the line AB.

Criteria	Marks
Correct gradient $m_{AB} = -2$	1

Solution.

Apply the gradient formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$:

$$m_{AB} = \frac{1 - (-5)}{-1 - 2} = \frac{6}{-3} = -2.$$

Q2 Sydney Tech 2023 Trial Q1 · **1 mark**

What is the gradient of the line with equation $3x - 7y + 1 = 0$?

(A) $-\dfrac{7}{3}$

(B) $-\dfrac{3}{7}$

(C) $\dfrac{3}{7}$

(D) $\dfrac{7}{3}$

Criteria	Marks
Correct option (C)	1

Solution.

Rearrange into gradient–intercept form $y = mx + b$:

$$3x - 7y + 1 = 0 \implies 7y = 3x + 1 \implies y = \frac{3}{7}x + \frac{1}{7}.$$

The gradient is $\dfrac{3}{7}$, so the answer is (C).

Q3 Sydney Grammar 2021 Prelim Q9(c) · **2 marks**

Find the distance between the points $(-5, 2)$ and $(4, -1)$, leaving your answer in simplified surd form.

Criteria	**Marks**
Correct substitution into the distance formula	1
Correct simplified surd $3\sqrt{10}$	1

Solution.

Apply the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$:

$$d = \sqrt{(4 - (-5))^2 + (-1 - 2)^2} = \sqrt{9^2 + (-3)^2} = \sqrt{81 + 9} = \sqrt{90}.$$

Simplify the surd: $\sqrt{90} = \sqrt{9 \times 10} = 3\sqrt{10}$.

Q4 Kambala 2021 Prelim Q1(a) · **1 mark**

$\triangle ABC$ has vertices $A(-3, 1)$, $B(5, -3)$ and $C(4, 5)$. Point D is the midpoint of AB. Determine the coordinates of the midpoint D.

Criteria	**Marks**
Correct midpoint $D = (1, -1)$	1

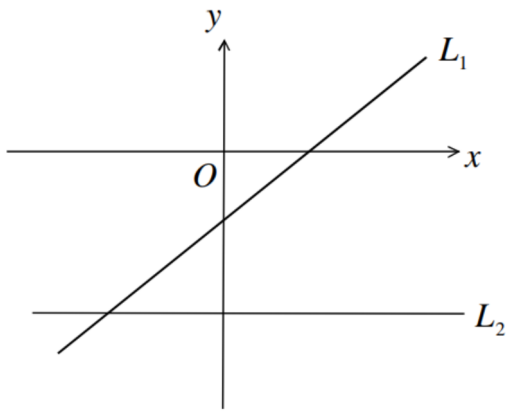
Solution.

Apply the midpoint formula $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ to $A(-3, 1)$ and $B(5, -3)$:

$$D = \left(\frac{-3 + 5}{2}, \frac{1 + (-3)}{2}\right) = \left(\frac{2}{2}, \frac{-2}{2}\right) = (1, -1).$$

Q5 Killara 2023 Prelim Q9 · **1 mark**

In the figure shown below, the equations of the straight lines L_1 and L_2 are $bx + ay = ac$ and $y = \frac{1}{c}$ respectively, where a, b and c are constants.



Which of the following statements are true?

$$\text{I. } c < 0 \qquad \text{II. The gradient of } L_1 = -\frac{b}{a} \qquad \text{III. } 0 < bc < 1$$

(A) I and II only

(B) I and III only

(C) II and III only

(D) I, II and III

Criteria	**Marks**
Correct option (A)	1

Solution.

Rearrange L_1 into gradient–intercept form: $bx+ay = ac \implies y = c-\frac{b}{a}x$, so the gradient of L_1 is $-\frac{b}{a}$, which makes **II** true. The line $L_2 : y = \frac{1}{c}$ is horizontal; the figure places it *below* the x-axis, so $\frac{1}{c} < 0$, giving $c < 0$, which makes **I** true. For **III**, the figure fixes only the signs of a, b, c (via the positive gradient and negative intercept of L_1), not their magnitudes: the product bc can take any positive value (e.g. $a = 2, b = -3, c = -1$ gives $bc = 3$), so $0 < bc < 1$ is not forced and **III** is false. Only I and II hold, so the answer is (A).

2. Angle of inclination and angle between lines

Q6 Sydney Grammar 2022 Trial Q1 · **1 mark**

Which of the following is the angle of inclination of the line $y = \frac{9}{8}x - 5$?

(A) $42°$

(B) $48°$

(C) $132°$

(D) $138°$

Criteria	Marks
Correct option (B)	1

Solution.

The angle of inclination θ satisfies $\tan\theta = m$, where m is the gradient. Here $m = \frac{9}{8} > 0$, so θ is acute:

$$\theta = \tan^{-1}\left(\tfrac{9}{8}\right) = 48° \text{ (nearest degree)}.$$

The answer is **(B)**.

Q7 Baulkham Hills 2022 Trial Q18 · **2 marks**

Determine the angle of inclination θ of the line $x + y + 1 = 0$.

Criteria	Marks
Finds the gradient $m = -1$	1
Correct angle of inclination $\theta = 135°$	1

Solution.

Rewrite in gradient form: $x + y + 1 = 0 \Rightarrow y = -x - 1$, so the gradient is $m = -1$. The angle of inclination satisfies $\tan\theta = m = -1$ with $0° \le \theta < 180°$. Since $m < 0$ the line slopes down to the right, so θ is obtuse:

$$\theta = 180° - 45° = 135°.$$

Q8 Gosford 2024 Prelim Q8 · **1 mark**

Find the acute angle between the lines $y = 3x - 4$ and $y = \dfrac{6}{5}x + 3$, correct to the nearest degree.

(A) $72°$

(B) $50°$

(C) $21°$

(D) $22°$

Criteria	**Marks**
Correct option (C)	1

Solution.

The acute angle θ between two lines with gradients m_1, m_2 satisfies

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|.$$

With $m_1 = 3$ and $m_2 = \dfrac{6}{5}$:

$$\tan\theta = \left|\frac{3 - \frac{6}{5}}{1 + 3\cdot\frac{6}{5}}\right| = \left|\frac{\frac{9}{5}}{\frac{23}{5}}\right| = \frac{9}{23},$$

so $\theta = \tan^{-1}\left(\frac{9}{23}\right) = 21°$ (nearest degree). The answer is **(C)**.

Equations and graphs of lines

3. Graphing a line and reading its equation from a graph

Q9 Sydney Grammar 2020 Trial Q1 · **1 mark**

Which of the following correctly expresses y as the subject of the formula $2x - 5y + 3 = 0$?

(A) $y = \dfrac{2x + 3}{2}$

(B) $y = \dfrac{x + 3}{5}$

(C) $y = \dfrac{2}{5}x + \dfrac{3}{5}$

(D) $y = \dfrac{2}{5}x - \dfrac{3}{5}$

Criteria	Marks
Correct option (C)	1

Solution.

Isolate the y-term, then divide by its coefficient:

$$2x - 5y + 3 = 0 \implies 5y = 2x + 3 \implies y = \frac{2x + 3}{5} = \frac{2}{5}x + \frac{3}{5}.$$

The answer is (C).

Q10 HSC 2025 Q2 · **1 mark**

Which graph could represent $y = 4x$?

Diagram to be inserted:
`adv-working-with-functions-d3-q10.png`

(A) Graph A

(B) Graph B

(C) Graph C

(D) Graph D

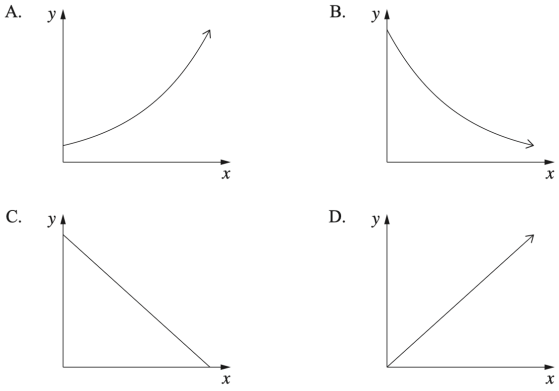
Criteria	**Marks**
Correct option (D)	1

Solution.

The equation $y = 4x$ is linear with y-intercept 0 and gradient 4, so its graph is a straight line passing through the origin with a positive (steep) slope, rising to the right. The only graph with these features is the straight line through the origin in the first and third quadrants, option (D).

Q11 Killara 2024 Prelim Q7 · **1 mark**

Determine the equation of the line shown below.



(A) $y = \dfrac{2\pi}{3}x + 3$

(B) $y = -\dfrac{2\pi}{3}x + 3$

(C) $y = -\sqrt{3}\,x + 3$

(D) $y = \sqrt{3}\,x + 3$

Criteria	**Marks**
Correct option (D)	1

Solution.

The line cuts the y-axis at $y = 3$, so all four options share the intercept +3; the equation has the form $y = mx + 3$. The line rises from left to right and cuts the x-axis at a negative value, so the gradient m is positive: this eliminates the negative-slope option (C) and the false "$2\pi/3$" options (a gradient is a number, not an angle in radians). The remaining exact value is $m = \sqrt{3}$ (the line is inclined at $60°$ to the positive x-axis, and $\tan 60° = \sqrt{3}$). Hence

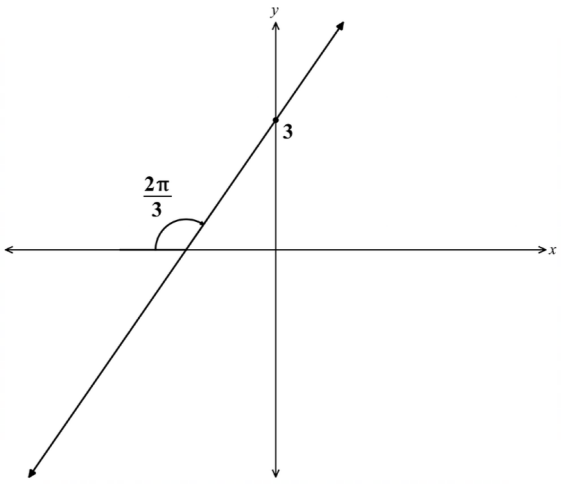
$$y = \sqrt{3}\,x + 3.$$

The answer is (D).

Q12 Killara 2024 Prelim Q10 · **1 mark**

The straight lines ℓ_1 and ℓ_2 shown below have equations $x + ay = b$ and $x + cy = d$ respectively. Which of the following are true?

I. $a < c$ II. $b > d$ III. $a + b < c + d$



(A) I and II

(B) I and III

(C) II and III

(D) I, II and III

Criteria	Marks
Correct option (A)	1

Solution.

Rewrite each line in gradient–intercept form. For $\ell_1:\ x + ay = b$ we get $y = -\frac{1}{a}x + \frac{b}{a}$, so its gradient is $-\frac{1}{a}$ and its x-intercept (put $y = 0$) is $x = b$. Likewise $\ell_2:\ x + cy = d$ has gradient $-\frac{1}{c}$ and x-intercept $x = d$.

From the graph both lines have *positive* gradient, so $-\frac{1}{a} > 0$ and $-\frac{1}{c} > 0$, giving $a < 0$ and $c < 0$. The line ℓ_1 is the shallower (less steep) of the two, so $\left|-\frac{1}{a}\right| < \left|-\frac{1}{c}\right|$, i.e. $|a| > |c|$; since both are negative this means $a < c$, so **I is true**.

Reading the x-intercepts, ℓ_1 crosses the x-axis to the right of the origin ($b > 0$) while ℓ_2 crosses to the left ($d < 0$), so $b > d$ and **II is true**.

For III, $a + b$ has $a < 0$ but $b > 0$, while $c + d$ has $c < 0$ and $d < 0$, so $c + d$ is clearly the smaller sum; hence $a + b > c + d$ and **III is false**. Only I and II hold, so the answer is (A).

4. Finding the equation of a line

Q13 Girraween 2020 Trial Q8 · **1 mark**

Let M be the midpoint of $(-1, 4)$ and $(5, 8)$. The equation of the line through M with gradient $-\dfrac{1}{2}$ is:

(A) $x + 2y - 10 = 0$

(B) $x + 2y - 14 = 0$

(C) $2x + y - 10 = 0$

(D) $2x - y - 14 = 0$

Criteria	**Marks**
Correct option (B)	1

Solution.

The midpoint is

$$M = \left(\frac{-1+5}{2}, \frac{4+8}{2}\right) = (2, 6).$$

Using point–gradient form with gradient $-\dfrac{1}{2}$ through $(2, 6)$:

$$y - 6 = -\tfrac{1}{2}(x - 2) \;\Rightarrow\; 2y - 12 = -(x - 2) \;\Rightarrow\; x + 2y - 14 = 0.$$

The answer is (B).

Q14 Kambala 2023 Prelim Q3 · **1 mark**

Which of the following is the equation of the line that passes through $(2, -1)$ and $(5, 3)$?

(A) $4x + 3y - 11 = 0$

(B) $3x - 4y + 11 = 0$

(C) $4x - 3y - 11 = 0$

(D) $3x - 4y - 11 = 0$

Criteria	Marks
Correct option (C)	1

Solution.

The gradient is

$$m = \frac{3 - (-1)}{5 - 2} = \frac{4}{3}.$$

Using point–gradient form through $(2, -1)$:

$$y + 1 = \tfrac{4}{3}(x - 2) \implies 3y + 3 = 4x - 8 \implies 4x - 3y - 11 = 0.$$

The answer is (C).

Q15 Girraween 2022 Prelim Q9(a)(i) · **2 marks**

The line ℓ_1 passes through the point $(9, -4)$ and has gradient $\dfrac{1}{3}$. Find the equation of ℓ_1 in general form.

Criteria	Marks
Writes point–gradient form $y + 4 = \frac{1}{3}(x - 9)$	1
Rearranges to general form $x - 3y - 21 = 0$	1

Solution.

Using point–gradient form with gradient $\dfrac{1}{3}$ through $(9, -4)$:

$$y - (-4) = \tfrac{1}{3}(x - 9) \implies 3(y + 4) = x - 9 \implies 3y + 12 = x - 9.$$

Rearranging into general form,

$$x - 3y - 21 = 0.$$

Q16 Sydney Girls 2023 Trial Q13 · **3 marks**

Two points have coordinates $P\left(4,\ b^2+2b+2\right)$ and $Q\left(3,\ 3b+3\right)$. It is given that the line PQ is parallel to the line $x+y=1$. Find the value(s) of b.

Criteria	**Marks**
States that parallel lines have equal gradient, so gradient of $PQ=-1$	1
Forms the gradient of PQ and sets up $b^2-b-1=-1$	1
Solves to give $b=0$ or $b=1$	1

Solution.

The line $x+y=1$ has gradient -1, and parallel lines have equal gradients, so the gradient of PQ must equal -1. Computing the gradient of PQ:

$$m_{PQ}=\frac{\left(b^2+2b+2\right)-(3b+3)}{4-3}=b^2-b-1.$$

Setting this equal to -1:

$$b^2-b-1=-1\ \Rightarrow\ b^2-b=0\ \Rightarrow\ b(b-1)=0.$$

Therefore $b=0$ or $b=1$.

5. Parallel and perpendicular lines

Q17 Barker 2024 Prelim Q1 · **1 mark**

Determine the gradient of the line perpendicular to $y = \frac{x}{2} - 5$.

(A) -2

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) 2

Criteria	Marks
Correct option (A)	1

Solution.

The line $y = \frac{x}{2} - 5$ has gradient $m = \frac{1}{2}$. Perpendicular gradients satisfy $m_1 m_2 = -1$, so

$$m_\perp = -\frac{1}{m} = -\frac{1}{\frac{1}{2}} = -2.$$

The answer is (A).

Q18 Sydney Grammar 2021 Prelim Q6 · **1 mark**

What is the equation of the line parallel to $y = 2x - 1$ with x-intercept $(-3, 0)$?

(A) $2x - y - 3 = 0$

(B) $x + 2y + 3 = 0$

(C) $x - 2y + 6 = 0$

(D) $2x - y + 6 = 0$

Criteria	Marks
Correct option (D)	1

Solution.

A parallel line has the same gradient $m = 2$. Using point–gradient form through $(-3, 0)$:

$$y - 0 = 2\big(x - (-3)\big) = 2x + 6.$$

Rearranging to general form gives $2x - y + 6 = 0$. The answer is (D).

Q19 Sydney Grammar 2023 Trial Q11 · **2 marks**

Find the equation of the line perpendicular to $y = 2x - 1$ passing through the point $A(0, -7)$.

Criteria	Marks
Perpendicular gradient $m = -\frac{1}{2}$	1
Correct equation $y = -\frac{1}{2}x - 7$ (or $x + 2y + 14 = 0$)	1

Solution.

The line $y = 2x - 1$ has gradient 2, so the perpendicular gradient is $m = -\frac{1}{2}$. The point $A(0, -7)$ is the y-intercept, so

$$y = -\frac{1}{2}x - 7,$$

which may also be written $x + 2y + 14 = 0$.

Q20 Barker 2024 Prelim Q12(c) · **3 marks**

Determine the equation of the line that is parallel to the interval joining the points $A(3,5)$ and $B(6,10)$, and passes through the point $C(-3,-1)$. Leave your answer in general form.

Criteria	**Marks**
Gradient of AB: $m = \dfrac{10-5}{6-3} = \dfrac{5}{3}$	1
Substitutes the gradient and $C(-3,-1)$ into point–gradient form	1
Correct general form $5x - 3y + 12 = 0$	1

Solution.

The required line is parallel to AB, so it has the same gradient:

$$m = \frac{10-5}{6-3} = \frac{5}{3}.$$

Using point–gradient form through $C(-3,-1)$:

$$y-(-1) = \frac{5}{3}\big(x-(-3)\big) \quad \Longrightarrow \quad y+1 = \frac{5}{3}(x+3).$$

Multiply through by 3 and collect terms:

$$3(y+1) = 5(x+3) \implies 3y+3 = 5x+15 \implies 5x-3y+12 = 0.$$

Q21 Knox 2022 Prelim Q15(e) · **4 marks**

The perpendicular bisector of the interval CD has equation $4x - 3y + 16 = 0$. If the point C has coordinates $(-9, 10)$, find the coordinates of D.

Criteria	**Marks**
Gradient of bisector $\frac{4}{3}$, so gradient of CD is $-\frac{3}{4}$; forms the equation of line CD through C	1
Finds the point of intersection (foot on the bisector), the midpoint $M(-1, 4)$	1
Uses M as the midpoint of CD to set up equations for D	1
Correct coordinates $D(7, -2)$	1

Solution.

The bisector $4x - 3y + 16 = 0$ can be written $y = \dfrac{4}{3}x + \dfrac{16}{3}$, so its gradient is $\dfrac{4}{3}$. Since CD is perpendicular to it,

$$m_{CD} = -\frac{3}{4}.$$

Line CD passes through $C(-9, 10)$:

$$y - 10 = -\frac{3}{4}(x + 9) \quad \Longrightarrow \quad 3x + 4y - 13 = 0.$$

The bisector meets CD at the midpoint M of CD. Solving simultaneously with $4x - 3y + 16 = 0$:

$$4x - 3y + 16 = 0, \qquad 3x + 4y - 13 = 0.$$

Solving gives $x = -1,\ y = 4$, so $M(-1, 4)$. As M is the midpoint of CD,

$$\frac{-9 + x_D}{2} = -1, \qquad \frac{10 + y_D}{2} = 4,$$

giving $x_D = 7$ and $y_D = -2$. Hence $D(7, -2)$.

Solving, modelling, regions and bisectors

6. Solving linear equations, inequalities and simultaneous systems

Q22 Knox 2023 Prelim Q11 · **2 marks**

Solve $t - \dfrac{t+4}{2} = 10$.

Criteria	Marks
Clears the fraction: $2t - (t+4) = 20$	1
Correct value $t = 24$	1

Solution.

Multiply both sides by 2:

$$2t - (t+4) = 20.$$

Expand and simplify: $2t - t - 4 = 20$, so $t - 4 = 20$ and

$$t = 24.$$

Q23 Trinity Grammar 2024 Trial Q1 · **1 mark**

What values of x satisfy $3 - 2x \geq 8$?

(A) $x \leq -\dfrac{11}{2}$

(B) $x \geq -\dfrac{11}{2}$

(C) $x \leq -\dfrac{5}{2}$

(D) $x \geq -\dfrac{5}{2}$

Criteria	Marks
Correct option (C)	1

Solution.

Subtract 3: $-2x \geq 5$. Dividing by -2 reverses the inequality:

$$x \leq -\frac{5}{2},$$

which is option **(C)**.

Q24 Baulkham Hills 2023 Prelim Q13 · **2 marks**

Solve the simultaneous equations.

$$2x + y = 3, \qquad x + 3y = -1.$$

Criteria	Marks
Eliminates a variable and solves for one unknown	1
Correct solution $x = 2$, $y = -1$	1

Solution.

From the first equation $y = 3 - 2x$. Substitute into the second:

$$x + 3(3 - 2x) = -1 \implies x + 9 - 6x = -1 \implies -5x = -10,$$

so $x = 2$ and $y = 3 - 2(2) = -1$. Thus

$$x = 2, \qquad y = -1.$$

Q25 Baulkham Hills 2022 Prelim Q5 · **1 mark**

Which of the following is the point of intersection of the two lines $3x - 4y + 6 = 0$ and $x - y - 1 = 0$?

(A) $(0, 0)$

(B) $(-2, -3)$

(C) $(10, 9)$

(D) $(11, 10)$

Criteria	Marks
Correct option (C)	1

Solution.

From the second line $x = y + 1$. Substitute into the first:

$$3(y + 1) - 4y + 6 = 0 \implies 3y + 3 - 4y + 6 = 0 \implies -y + 9 = 0,$$

so $y = 9$ and $x = 10$. The point of intersection is $(10, 9)$, which is option **(C)**.

Q26 Girraween 2023 Prelim Q8(d) · **4 marks**

Show that the three lines given by the equations $3x + 5y + 7 = 0$, $x + 2y + 2 = 0$, and $2x - y + 9 = 0$ are concurrent.

Criteria	**Marks**
Solves two of the lines simultaneously	2
Correct intersection point $(-4, 1)$	1
Verifies the point satisfies the third line, concluding concurrency	1

Solution.

Find where the first two lines meet. From $x + 2y + 2 = 0$ we have $x = -2y - 2$. Substitute into $3x + 5y + 7 = 0$:

$$3(-2y - 2) + 5y + 7 = 0 \implies -6y - 6 + 5y + 7 = 0 \implies -y + 1 = 0,$$

so $y = 1$ and $x = -2(1) - 2 = -4$. The lines $3x + 5y + 7 = 0$ and $x + 2y + 2 = 0$ meet at $(-4, 1)$.

Test this point in the third line $2x - y + 9$:

$$2(-4) - (1) + 9 = -8 - 1 + 9 = 0.$$

Since $(-4, 1)$ lies on all three lines, the three lines are concurrent.

7. Linear modelling: cost, income and break-even

Q27 Hills Grammar 2025 Trial Q1 · **1 mark**

Which equation describes the relationship between the two variables a and b?

Diagram to be inserted:
`adv-working-with-functions-d3-q27.png`

(A) $b = 2a + 3$

(B) $b = 3a + 2$

(C) $b = 6a + 3$

(D) $b = 6a + 15$

Criteria	Marks
Correct option (A)	1

Solution.

The line passes through $(0, 3)$ (the b-intercept) and $(6, 15)$. Its gradient is

$$m = \frac{15 - 3}{6 - 0} = \frac{12}{6} = 2,$$

and its b-intercept is 3, so $b = 2a + 3$. Check: at $a = 6$, $b = 2(6) + 3 = 15$. The answer is (A).

Q28 Knox 2024 Trial Q15(a) · **1 mark**

Company A charges a fixed cost of \$40 for the first 20 hours of hiring an electric bike, and an additional rate of \$2 per hour thereafter. How much does the company charge for renting an electric bike for 35 hours?

Criteria	Marks
Correct charge \$70	1

Solution.

The first 20 hours cost the fixed \$40. The remaining $35 - 20 = 15$ hours are charged at \$2 per hour, adding $15 \times \$2 = \$30$. Total charge

$$\$40 + \$30 = \$70.$$

Q29 Knox 2024 Trial Q15(c) · **1 mark**

Company A charges a fixed cost of \$40 for the first 20 hours of hiring an electric bike, and an additional rate of \$2 per hour thereafter. Company B charges a rate of \$4 per hour, with no fixed cost. Determine after how many hours Company B will cost more than Company A.

Criteria	Marks
Correct threshold: Company B costs more once $t > 10$ hours	1

Solution.

Let t be the number of hours. Company B costs $B = 4t$. For the first 20 hours Company A costs a flat $A = 40$. Setting $B = A$ over this interval:

$$4t = 40 \implies t = 10.$$

For $t > 10$ (and up to 20 hours) we have $B = 4t > 40 = A$, so Company B is already dearer. Beyond 20 hours Company A costs $40 + 2(t - 20) = 2t$, and since $B = 4t > 2t$ for $t > 0$, Company B remains dearer. Hence Company B costs more than Company A after 10 hours.

8. Area and regions bounded by lines and axes

Q30 Knox 2023 Prelim Q25 · **2 marks**

The line L with equation $3x + 4y - 24 = 0$ crosses the x axis at A and the y axis at B. Find the area of triangle OAB, where O is the origin.

Criteria	Marks
Both intercepts found: $A(8, 0)$ and $B(0, 6)$	1
Correct area 24 units^2	1

Solution.

For the x intercept A, set $y = 0$: $3x - 24 = 0$, so $x = 8$, giving $A(8, 0)$. For the y intercept B, set $x = 0$: $4y - 24 = 0$, so $y = 6$, giving $B(0, 6)$. Triangle OAB is right-angled at O with perpendicular sides $OA = 8$ and $OB = 6$, so

$$\text{Area} = \tfrac{1}{2} \times 8 \times 6 = 24 \text{ units}^2.$$

Q31 Gosford 2023 Trial Q14(a) · **3 marks**

Show that the points $A(17, 1)$, $B(2, -5)$ and $C(-4, 10)$ are the vertices of a right-angled triangle.

Criteria	Marks
Correct gradient $m_{AB} = \frac{2}{5}$	1
Correct gradient $m_{BC} = -\frac{5}{2}$	1
Shows $m_{AB} \times m_{BC} = -1$, hence $AB \perp BC$ and the triangle is right-angled at B	1

Solution.

Compute the gradients of two sides.

$$m_{AB} = \frac{-5-1}{2-17} = \frac{-6}{-15} = \frac{2}{5}, \qquad m_{BC} = \frac{10-(-5)}{-4-2} = \frac{15}{-6} = -\frac{5}{2}.$$

Then

$$m_{AB} \times m_{BC} = \frac{2}{5} \times \left(-\frac{5}{2}\right) = -1,$$

so $AB \perp BC$. Hence the angle at B is a right angle and $\triangle ABC$ is a right-angled triangle.

Q32 Girraween 2021 Trial Q20(a)(ii) · **2 marks**

The points A and C lie on the y axis and the point B lies on the x axis. The line AB has equation $y = \sqrt{3}x - 3$ and the line BC (which is perpendicular to AB) has equation $y = -\dfrac{1}{\sqrt{3}}x + 1$. Find the area of the triangle ABC.

Criteria	Marks
Coordinates found: $A(0, -3)$, $C(0, 1)$ (and $B(\sqrt{3}, 0)$)	1
Correct area $2\sqrt{3}$ units^2	1

Solution.

Point A is on the y axis, so from $y = \sqrt{3}x - 3$ with $x = 0$ we get $A(0, -3)$. Point C is on the y axis, so from $y = -\dfrac{1}{\sqrt{3}}x + 1$ with $x = 0$ we get $C(0, 1)$. Point B is on the x axis: setting $y = 0$ in $y = \sqrt{3}x - 3$ gives $x = \sqrt{3}$, so $B(\sqrt{3}, 0)$. Take AC as the base along the y axis: its length is $|1 - (-3)| = 4$. The perpendicular height is the horizontal distance from B to the y axis, namely $\sqrt{3}$. Hence

$$\text{Area} = \tfrac{1}{2} \times 4 \times \sqrt{3} = 2\sqrt{3} \text{ units}^2.$$

9. Perpendicular bisector of two points

Q33 Sydney Girls 2021 Prelim Q2(c) · **2 marks**

Given the points $A(10, 2)$ and $B(2, 8)$, find the equation of the perpendicular bisector of AB.

Criteria	**Marks**
Midpoint $(6, 5)$ and gradient $m_{AB} = -\frac{3}{4}$, so $m_\perp = \frac{4}{3}$	1
Correct equation of the perpendicular bisector	1

Solution.

Midpoint of AB is $\left(\dfrac{10+2}{2}, \dfrac{2+8}{2}\right) = (6, 5)$. Gradient of AB is

$$m_{AB} = \frac{8-2}{2-10} = \frac{6}{-8} = -\frac{3}{4}, \qquad m_\perp = \frac{4}{3}.$$

The perpendicular bisector passes through $(6, 5)$ with gradient $\frac{4}{3}$:

$$y - 5 = \frac{4}{3}(x - 6) \implies y = \frac{4}{3}x - 3, \quad \text{i.e. } 4x - 3y - 9 = 0.$$

Q34 Sydney Grammar 2021 Prelim Q10(c) · 3 marks

Find the equation of the perpendicular bisector of the interval joining the points $(2, 6)$ and $(-1, 5)$, writing your final answer in general form.

Criteria	Marks
Midpoint $\left(\frac{1}{2}, \frac{11}{2}\right)$	1
Gradient of interval $= \frac{1}{3}$, so $m_\perp = -3$	1
Correct equation in general form	1

Solution.

Midpoint is $\left(\dfrac{2+(-1)}{2}, \dfrac{6+5}{2}\right) = \left(\dfrac{1}{2}, \dfrac{11}{2}\right)$. Gradient of the interval is

$$m = \frac{5-6}{-1-2} = \frac{-1}{-3} = \frac{1}{3}, \qquad m_\perp = -3.$$

Perpendicular bisector through $\left(\frac{1}{2}, \frac{11}{2}\right)$ with gradient -3:

$$y - \frac{11}{2} = -3\left(x - \frac{1}{2}\right) \implies y - \frac{11}{2} = -3x + \frac{3}{2}.$$

Multiplying by 2 and collecting terms gives the general form

$$6x + 2y - 14 = 0, \quad \text{i.e. } 3x + y - 7 = 0.$$

Q35 Sydney Tech 2025 Trial Q22 · **3 marks**

Find the equation of the line through the midpoint of the interval from $A(3,2)$ to $B(-7,8)$ perpendicular to the line $\dfrac{x}{6}+\dfrac{y}{2}=1$.

Criteria	Marks
Midpoint $(-2,5)$	1
Gradient of given line $=-\frac{1}{3}$, so required gradient $=3$	1
Correct equation of the line	1

Solution.

Midpoint of AB is $\left(\dfrac{3+(-7)}{2},\dfrac{2+8}{2}\right)=(-2,5)$. Rewrite the given line:

$$\frac{x}{6}+\frac{y}{2}=1 \implies y=2-\frac{x}{3},$$

which has gradient $-\frac{1}{3}$, so the perpendicular gradient is 3. The required line passes through $(-2,5)$ with gradient 3:

$$y-5=3(x+2) \implies y=3x+11, \quad \text{i.e. } 3x-y+11=0.$$

Q36 Barker 2021 Prelim Q13(d) · **3 marks**

Find the exact equation of the straight line which passes through the midpoint of $(0,-5)$ and $(4,-1)$ and is perpendicular to the line that makes an angle of 30° with the positive x-axis.

Criteria	Marks
Midpoint $(2,-3)$	1
Given line has gradient $\tan 30^\circ=\frac{1}{\sqrt{3}}$, so $m_\perp=-\sqrt{3}$	1
Correct exact equation of the line	1

Solution.

Midpoint is $\left(\dfrac{0+4}{2},\dfrac{-5+(-1)}{2}\right)=(2,-3)$. A line at 30° to the positive x-axis has gradient

$$\tan 30^\circ=\frac{1}{\sqrt{3}}, \qquad m_\perp=-\frac{1}{\tan 30^\circ}=-\sqrt{3}.$$

The required line passes through $(2,-3)$ with gradient $-\sqrt{3}$:

$$y-(-3)=-\sqrt{3}\,(x-2) \implies y=-\sqrt{3}\,x+2\sqrt{3}-3.$$

Q37 Sydney Girls 2025 Trial Q16 · **3 marks**

The points $P(1,-2)$ and $Q(7,1)$ lie on the circumference of a circle. Show that the centre of the circle lies on the line $4x+2y=15$.

Criteria	Marks
Recognise the centre is equidistant from P and Q, i.e. lies on the perpendicular bisector of PQ	1
Midpoint $\left(4,-\frac{1}{2}\right)$ and gradient $m_{PQ}=\frac{1}{2}$, so $m_{\perp}=-2$	1
Derive the perpendicular bisector and show it is $4x+2y=15$	1

Solution.

The centre is equidistant from P and Q, so it lies on the perpendicular bisector of PQ. Midpoint of PQ is

$$\left(\frac{1+7}{2},\frac{-2+1}{2}\right)=\left(4,-\frac{1}{2}\right),\qquad m_{PQ}=\frac{1-(-2)}{7-1}=\frac{3}{6}=\frac{1}{2},\qquad m_{\perp}=-2.$$

The perpendicular bisector passes through $\left(4,-\frac{1}{2}\right)$ with gradient -2:

$$y+\frac{1}{2}=-2(x-4)\implies y+\frac{1}{2}=-2x+8.$$

Multiplying by 2: $2y+1=-4x+16$, so $4x+2y=15$. Hence the centre lies on the line $4x+2y=15$, as required.