Year 11 Mathematics Advanced

Topic: Working with Functions (2 of 12)

Review Questions (Worked Solutions): Introduction to Functions and Relations

Term 3, 2026

Functions and relations

1. Evaluate and use function notation

Q1 James Ruse 2022 Trial Q13(a) · **1 mark**

Let $f(x) = -x^2 + x + 4$. Find $f(3)$.

Criteria	Marks
Correct value $f(3) = -2$	1

Solution.

Substitute $x = 3$:

$$f(3) = -(3)^2 + 3 + 4 = -9 + 3 + 4 = -2.$$

Q2 Baulkham Hills 2024 Prelim Q22(a) · **1 mark**

If $f(x) = \sqrt{x-4}$, find $f(5)$.

Criteria	Marks
Correct value $f(5) = 1$	1

Solution.

Substitute $x = 5$:

$$f(5) = \sqrt{5-4} = \sqrt{1} = 1.$$

Q3 Sydney Grammar 2021 Prelim Q12(d)(i) · **1 mark**

Consider the function $g(x) = 1 - x$. Find $g(-2a)$.

Criteria	Marks
Correct expression $g(-2a) = 1 + 2a$	1

Solution.

Substitute $x = -2a$:

$$g(-2a) = 1 - (-2a) = 1 + 2a.$$

Q4 Trinity Grammar 2020 Trial Q12 · **2 marks**

If $m(k) = k^2$, simplify the expression $m(k+1) - m(k)$.

Criteria	Marks
Expands $m(k+1) = (k+1)^2 = k^2 + 2k + 1$	1
Correct simplified answer $2k + 1$	1

Solution.

With $m(k) = k^2$, evaluate each term and subtract:

$$m(k+1) - m(k) = (k+1)^2 - k^2 = k^2 + 2k + 1 - k^2 = 2k + 1.$$

Q5 Killara 2023 Trial Q20 · **2 marks**

If $f(x+1) = x^2 + 3x + 5$, find $f(x)$.

Criteria	Marks
Substitutes $k = x + 1$ so that $x = k - 1$ (or equivalent)	1
Correct answer $f(x) = x^2 + x + 3$	1

Solution.

Let $k = x + 1$, so $x = k - 1$. Then

$$f(k) = (k-1)^2 + 3(k-1) + 5 = k^2 - 2k + 1 + 3k - 3 + 5 = k^2 + k + 3.$$

Replacing k by x: $f(x) = x^2 + x + 3$.

Q6 North Sydney Boys 2024 Prelim Q26 · **2 marks**

A function $f(x)$ with range $(0, \infty)$ is such that $f(a) \times f(b) = f(a+b)$ for any real numbers a and b. Show that $f\left(\frac{a}{2}\right) = \sqrt{f(a)}$ for any real number a.

Criteria	**Marks**
Uses $b = \frac{a}{2}$ (or $b = a$) to obtain $\left[f\left(\frac{a}{2}\right)\right]^2 = f(a)$	1
Concludes $f\left(\frac{a}{2}\right) = \sqrt{f(a)}$, justified by the range $(0, \infty)$	1

Solution.

Apply the given property with $a \to \frac{a}{2}$ and $b \to \frac{a}{2}$:

$$f\left(\frac{a}{2}\right) \times f\left(\frac{a}{2}\right) = f\left(\frac{a}{2} + \frac{a}{2}\right) = f(a),$$

so $\left[f\left(\frac{a}{2}\right)\right]^2 = f(a)$. Since the range of f is $(0, \infty)$, $f\left(\frac{a}{2}\right) > 0$, and taking the positive square root gives

$$f\left(\frac{a}{2}\right) = \sqrt{f(a)}.$$

2. Classify a relation or function (one-to-one, many-to-one, vertical line test)

Q7 Sydney Boys 2023 Trial Q3 · **1 mark**

The following set of ordered pairs is a relation.

$$\{(-2, 3),\ (-4, 5),\ (3, 2),\ (-2, 1),\ (7, 2)\}$$

Which of the following types is this relation?

(A) one to one

(B) one to many

(C) many to many

(D) many to one

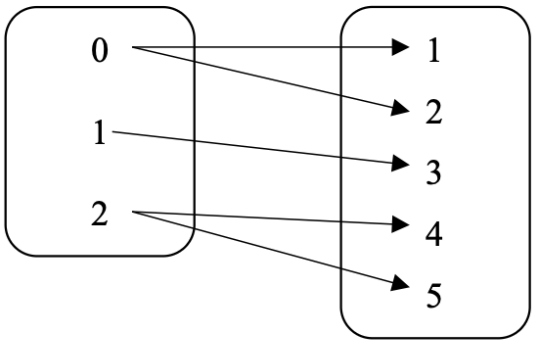
Criteria	**Marks**
Correct option (C)	1

Solution.

Check whether the x-values (inputs) and y-values (outputs) repeat. The input $x = -2$ appears twice, giving two different outputs (3 and 1), so one input maps to *many* outputs. The output $y = 2$ is produced by two different inputs ($x = 3$ and $x = 7$), so *many* inputs map to one output. Because both the x-coordinates and the y-coordinates are shared, the relation is **many to many**, which is option (C).

Q8 Hornsby Girls 2021 Trial Q1 · **1 mark**

What type of relation is shown in the mapping diagram below?



(A) One-to-One

(B) One-to-Many

(C) Many-to-One

(D) Many-to-Many

Criteria	Marks
Correct option (D)	1

Solution.

Read the arrows in the mapping diagram. At least one element of the left-hand set has two arrows leaving it (one input mapped to several outputs), and at least one element of the right-hand set has two arrows arriving at it (several inputs mapped to one output). Since inputs are repeated *and* outputs are repeated, the relation is **Many-to-Many**, which is option (D).

Q9 Sydney Grammar 2023 Trial Q12(a) · **1 mark**

Classify the relation $y = 3x - 4$ as one-to-one, many-to-one, one-to-many, or many-to-many.

Criteria	Marks
Correctly classifies as one-to-one	1

Solution.

The graph of $y = 3x - 4$ is a straight line with non-zero gradient. Every vertical line meets it exactly once (each input x gives one output y), and every horizontal line meets it exactly once (each output y comes from one input x, since $x = \frac{y+4}{3}$). Hence the relation is **one-to-one**.

Q10 Sydney Grammar 2023 Trial Q12(b) · **1 mark**

Classify the relation $x^2 + y^2 = 25$ as one-to-one, many-to-one, one-to-many, or many-to-many.

Criteria	Marks
Correctly classifies as many-to-many	1

Solution.

The graph of $x^2 + y^2 = 25$ is a circle of radius 5. For a value of x with $-5 < x < 5$ there are two outputs, $y = \pm\sqrt{25 - x^2}$, so one input gives many outputs. Likewise, for such a value of y there are two inputs, $x = \pm\sqrt{25 - y^2}$, so one output comes from many inputs. Since inputs and outputs are both repeated, the relation is **many-to-many**.

Q11 Shore 2023 Prelim Q15 · **3 marks**

(a) Sketch $y = 3^{x-1}$, showing asymptote, intercept and one other point. (b) Explain from the graph why $f(x) = 3^{x-1}$ is a function.

Criteria	Marks
(a) Correct exponential shape with horizontal asymptote $y = 0$	1
(a) Shows the y-intercept $\left(0, \frac{1}{3}\right)$ and one further point such as $(1, 1)$	1
(b) Applies the vertical-line test to justify that f is a function	1

Solution.

(a) $f(x) = 3^{x-1}$ is the curve $y = 3^x$ shifted right 1 unit. It is increasing, with horizontal asymptote $y = 0$ (approached as $x \to -\infty$). y-intercept: $f(0) = 3^{-1} = \dfrac{1}{3}$, so $\left(0, \frac{1}{3}\right)$. One other point: $f(1) = 3^0 = 1$, so $(1, 1)$. Sketch the increasing exponential through these points, lying above the x-axis.

(b) By the vertical-line test, every vertical line $x = a$ meets the graph in exactly one point (each input x gives a single value of 3^{x-1}). Since no vertical line cuts the curve more than once, $f(x) = 3^{x-1}$ is a function.

Q12 Gosford 2023 Prelim Q7 · **1 mark**

What domain makes the function $f(x) = -2x^2 - 8x + 10$ one-to-one?

(A) $(-\infty, -2]$

(B) $(-\infty, 18]$

(C) $(-\infty, \infty)$

(D) $(-\infty, 2]$

Criteria	**Marks**
Correct option (A)	1

Solution.

The graph of $f(x) = -2x^2 - 8x + 10$ is a parabola. Its axis of symmetry is at

$$x = -\frac{b}{2a} = -\frac{-8}{2(-2)} = -2.$$

A parabola fails the horizontal line test across the whole real line (two inputs share each output), so it is one-to-one only when the domain is restricted to one side of the vertex $x = -2$. Of the options, only $(-\infty, -2]$ keeps the function strictly monotonic (increasing up to the vertex), so the answer is $(-\infty, -2]$, option (A).

Domain, range and reading graphs

3. Find the natural domain of a function

Q13 Barker 2021 Prelim Q3 · **1 mark**

What is the domain of the function $y = \sqrt{5x - 2}$?

(A) $[0, \infty)$

(B) $\left[\frac{2}{5}, \infty\right)$

(C) $\left(\frac{2}{5}, \infty\right)$

(D) $\left(-\infty, \frac{2}{5}\right)$

Criteria	Marks
Correct option (B)	1

Solution.

The expression under a square root must be non-negative, so

$$5x - 2 \geq 0 \quad \Longrightarrow \quad x \geq \frac{2}{5}.$$

The domain is $\left[\frac{2}{5}, \infty\right)$, which is option (B). (It is closed at $\frac{2}{5}$ because $\sqrt{0} = 0$ is defined.)

Q14 Kambala 2023 Prelim Q6 · **1 mark**

The domain for the function $f(x) = \sqrt{2-x}$ is:

(A) $[0, \infty)$

(B) $(-\infty, 2]$

(C) $[2, \infty)$

(D) $(-\infty, \infty)$

Criteria	Marks
Correct option (B)	1

Solution.

The radicand must be non-negative:

$$2 - x \geq 0 \quad \Longrightarrow \quad x \leq 2.$$

The domain is $(-\infty, 2]$, which is option (B).

Q15 Fort St 2022 Trial Q1 · **1 mark**

What are the values of x for which $f(x) = \sqrt{x^2 - 3x}$ is defined?

(A) $0 \leq x \leq 3$

(B) $x \geq 3$

(C) $x \leq 0$ or $x \geq 3$

(D) all real x

Criteria	Marks
Correct option (C)	1

Solution.

We need $x^2 - 3x \geq 0$, i.e. $x(x-3) \geq 0$. This upward parabola is non-negative outside its roots $x = 0$ and $x = 3$:

$$x \leq 0 \quad \text{or} \quad x \geq 3,$$

which is option (C).

Q16 Knox 2022 Prelim Q4 · **1 mark**

Let $f(x) = \dfrac{1}{\sqrt{x-5}}$. Which of the following is the natural domain of $f(x)$?

(A) $(-\infty, 5)$

(B) $[5, \infty)$

(C) $(5, \infty)$

(D) $[-5, 5]$

Criteria	Marks
Correct option (C)	1

Solution.

The radicand must be positive: it cannot be zero because $\sqrt{x-5}$ sits in the denominator. Hence

$$x - 5 > 0 \quad \Longrightarrow \quad x > 5.$$

The domain is $(5, \infty)$, which is option (C).

Q17 Caringbah 2021 Trial Q34 · **1 mark**

State the natural domain of the function $f(x) = \dfrac{1}{\sqrt{2-x^2}}$.

Criteria	Marks
Correct domain $-\sqrt{2} < x < \sqrt{2}$	1

Solution.

The radicand must be strictly positive, since $\sqrt{2-x^2}$ is in the denominator:

$$2 - x^2 > 0 \quad \Longrightarrow \quad x^2 < 2 \quad \Longrightarrow \quad -\sqrt{2} < x < \sqrt{2}.$$

The natural domain is $\left(-\sqrt{2}, \sqrt{2}\right)$.

Q18 Killara 2024 Trial Q4 · **1 mark**

What is the domain of $f(x) = \log_2(1 - 2x)$?

(A) $x > \frac{1}{2}$

(B) $x > -\frac{1}{2}$

(C) $x < \frac{1}{2}$

(D) $x < -\frac{1}{2}$

Criteria	Marks
Correct option (C)	1

Solution.

A logarithm is defined only for a positive argument:

$$1 - 2x > 0 \implies 2x < 1 \implies x < \tfrac{1}{2},$$

which is option (C).

Q19 Abbotsleigh 2023 Trial Q5 · **1 mark**

What is the domain of the function $f(x) = \ln(3 - x) + \sqrt{x - 2}$?

(A) $(2, 3)$

(B) $[2, 3)$

(C) $(2, 3]$

(D) $[2, 3]$

Criteria	Marks
Correct option (B)	1

Solution.

Both terms must be defined simultaneously. The logarithm needs a positive argument, $3 - x > 0$, i.e. $x < 3$. The square root needs a non-negative radicand, $x - 2 \geq 0$, i.e. $x \geq 2$. Intersecting these:

$$2 \leq x < 3,$$

so the domain is $[2, 3)$, which is option (B).

4. Find the range and state domain and range

Q20 Blacktown Boys 2024 Trial Q1 · **1 mark**

Which of the following is the range of the function $f(x) = x^2 + 3$?

(A) $[3, \infty)$

(B) $(3, \infty)$

(C) $(-\infty, 3)$

(D) $(-\infty, \infty)$

Criteria	**Marks**
Correct option (A)	1

Solution.

For every real x, $x^2 \geq 0$, so $x^2 + 3 \geq 3$, with equality at $x = 0$. The smallest output is 3 and it is attained, while the outputs increase without bound:

$$\text{range} = [3, \infty),$$

which is option (A).

Q21 Hurlstone 2023 Trial Q1 · **1 mark**

The linear function $f(x) = 5 - x$ has range $[-4, 5)$. Which of the following gives the domain of the function?

(A) $(0, 9]$

(B) $(0, 1]$

(C) $[5, -4)$

(D) $(-9, 0)$

Criteria	**Marks**
Correct option (A)	1

Solution.

Rearranging $y = 5 - x$ gives $x = 5 - y$, and x is a decreasing function of y, so an endpoint's open/closed status carries across. The closed end $y = -4$ maps to $x = 5 - (-4) = 9$ (included), and the open end $y = 5$ maps to $x = 5 - 5 = 0$ (excluded). Hence

$$\text{domain} = (0, 9],$$

which is option (A).

Q22 SCEGGS 2022 Prelim Q12 · **2 marks**

State the domain and range for the function $y = \sqrt{2x - 3}$.

Criteria	**Marks**
Domain $\left[\frac{3}{2}, \infty\right)$	1
Range $[0, \infty)$	1

Solution.

The radicand must be non-negative:

$$2x - 3 \geq 0 \quad \Longrightarrow \quad x \geq \tfrac{3}{2},$$

so the domain is $\left[\frac{3}{2}, \infty\right)$. A square root returns values ≥ 0, with $y = 0$ attained at $x = \frac{3}{2}$ and growing without bound, so the range is $[0, \infty)$.

Q23 Knox 2023 Prelim Q32 · **2 marks**

State the domain and range for the function $y = 2\sqrt{x} - 3$.

Criteria	**Marks**
Domain $[0, \infty)$	1
Range $[-3, \infty)$	1

Solution.

The square root requires $x \geq 0$, so the domain is $[0, \infty)$. Since $\sqrt{x} \geq 0$, we have $2\sqrt{x} \geq 0$ and hence $y = 2\sqrt{x} - 3 \geq -3$, with the minimum $y = -3$ attained at $x = 0$ and y increasing without bound. The range is $[-3, \infty)$.

Q24 Hurlstone 2024 Trial Q11 · **2 marks**

State the domain and range of the function $y = 2\sqrt{16 - x^2}$.

Criteria	**Marks**
Domain $[-4, 4]$	1
Range $[0, 8]$	1

Solution.

The radicand must be non-negative:

$$16 - x^2 \geq 0 \quad \Longrightarrow \quad x^2 \leq 16 \quad \Longrightarrow \quad -4 \leq x \leq 4,$$

so the domain is $[-4, 4]$. On this interval $16 - x^2$ ranges from 0 (at $x = \pm 4$) up to 16 (at $x = 0$), so $\sqrt{16 - x^2}$ ranges over $[0, 4]$ and $y = 2\sqrt{16 - x^2}$ ranges over $[0, 8]$. The range is $[0, 8]$.

Q25 St George Girls 2021 Trial Q1 · **1 mark**

Which interval gives the range of the function $y = \dfrac{\sqrt{25 - x^2}}{5}$?

(A) $[0, 5]$

(B) $[0, 1]$

(C) $[-5, 5]$

(D) $[-1, 1]$

Criteria	**Marks**
Correct option (B)	1

Solution.

This is the upper half of the circle $x^2 + y'^2 = 25$ (radius 5), scaled by $\frac{1}{5}$. The numerator $\sqrt{25 - x^2}$ ranges over $[0, 5]$: it is 0 at $x = \pm 5$ and 5 at $x = 0$. Dividing by 5 gives

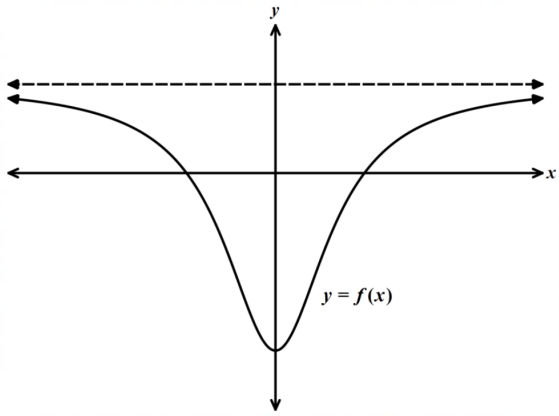
$$\text{range} = [0, 1],$$

which is option (B).

5. Zeros, intercepts and reading values from a graph

Q26 Pymble 2022 Prelim Q4 · 1 mark

The sketch of $y = f(x)$ is shown.



Which of the following is *not* a solution to $f(x) = 0$?

(A) $x = -5$

(B) $x = 1$

(C) $x = 4$

(D) $y = 20$

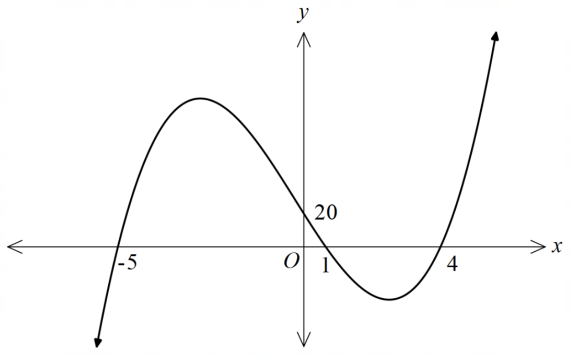
Criteria	Marks
Correct option (D)	1

Solution.

The solutions of $f(x) = 0$ are the x-values where the curve cuts the x-axis. From the graph these are $x = -5$, $x = 1$ and $x = 4$, so options (A), (B) and (C) are all solutions. Option (D), $y = 20$, is a y-value (the height of the curve, not a root), so it is not a solution to $f(x) = 0$. The answer is (D).

Q27 Hurlstone 2023 Trial Q16(a) · **1 mark**

Consider the graph $y = f(x)$ shown below.



Evaluate $f(-3) + 2f(2)$.

Criteria	**Marks**
Reads $f(-3) = -2$ and $f(2) = 2$ from the graph and evaluates to 2	1

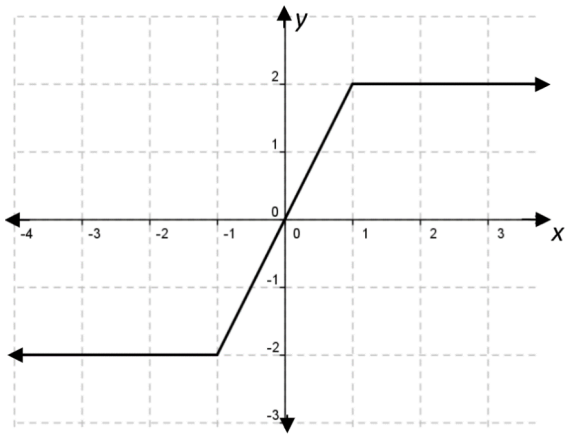
Solution.

Read the two required function values off the graph: at $x = -3$ the curve has height $f(-3) = -2$, and at $x = 2$ it has height $f(2) = 2$. Substituting,

$$f(-3) + 2f(2) = -2 + 2(2) = -2 + 4 = 2.$$

Q28 North Sydney Girls 2020 Trial Q3 · **1 mark**

The following diagram shows the graph of $y = f(x)$.



Which of the following is the solution to $f(x) \geq 0$?

(A) $(-\infty, -2)$

(B) $(-2, 0) \cup (0, \infty)$

(C) $(-2, 0) \cup [0, \infty)$

(D) $(-\infty, -2] \cup [0, \infty)$

Criteria	**Marks**
Correct option (D)	1

Solution.

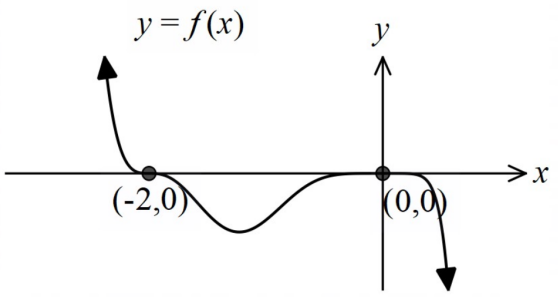
We need the x-values where the curve is on or above the x-axis. The graph cuts the x-axis at $x = -2$ and $x = 0$; it lies above the axis for $x \leq -2$ and again for $x \geq 0$, and below it on $-2 < x < 0$. Because the inequality is $f(x) \geq 0$ (not strict), the endpoints $x = -2$ and $x = 0$ are included, giving

$$(-\infty, -2] \cup [0, \infty).$$

The answer is (D).

Q29 St George Girls 2021 Prelim Q7(a) · **1 mark**

Write an equation for the graph below.



Criteria	Marks
Correct equation $y = \cos x$	1

Solution.

The curve oscillates between $y = 1$ and $y = -1$ with period 2π, starting at its maximum value $y = 1$ when $x = 0$. A curve with maximum at $x = 0$, minima at $x = \pm\pi$, and x-intercepts at $x = \pm\frac{\pi}{2}$ is the cosine curve, so

$$y = \cos x.$$

Q30 North Sydney Boys 2024 Prelim Q10(a) · **2 marks**

In the space below, sketch the graph of the function $y = \sqrt{4 - x}$, showing the intercepts on the axes.

Criteria	Marks
Finds both intercepts: x-intercept $(4, 0)$ and y-intercept $(0, 2)$	1
Correct shape: half of a sideways parabola, defined only for $x \leq 4$, decreasing from the y-axis to $(4, 0)$	1

Solution.

For the x-intercept put $y = 0$: $\sqrt{4 - x} = 0$, so $4 - x = 0$ and $x = 4$, giving the point $(4, 0)$. For the y-intercept put $x = 0$: $y = \sqrt{4} = 2$, giving the point $(0, 2)$. The expression under the root must be non-negative, $4 - x \geq 0$, so the graph exists only for $x \leq 4$. The curve is the upper half of a sideways parabola: it starts at $(0, 2)$ on the y-axis, rises as x decreases, and falls to meet the x-axis at $(4, 0)$, where it stops.

6. Number of solutions and intersections from graphs

Q31 Pymble 2022 Trial Q32(a) · **1 mark**

The diagram shows the region bounded between the two curves $y = \sqrt{x}$ and $y = x$. The point P is the point of intersection of the curves (other than the origin). Write down the coordinates of P.

Criteria	Marks
Correct coordinates $P = (1, 1)$	1

Solution.

At an intersection $\sqrt{x} = x$. Squaring, $x = x^2$, so $x^2 - x = 0$, giving $x(x - 1) = 0$ and hence $x = 0$ or $x = 1$. The origin $(0, 0)$ is the other intersection, so P has $x = 1$, and then $y = \sqrt{1} = 1$. Therefore

$$P = (1, 1).$$

Q32 Penrith 2020 Trial Q27(a) · **1 mark**

The curves $y = x^2$ and $y = \sqrt{x}$ meet at a point B in the first quadrant. Show that the x-coordinate of B is 1.

Criteria	Marks
Sets the curves equal and solves to show $x = 1$	1

Solution.

At an intersection the y-values are equal, so $x^2 = \sqrt{x}$. Squaring both sides gives $x^4 = x$, i.e. $x^4 - x = 0$, so

$$x\left(x^3 - 1\right) = 0.$$

Hence $x = 0$ or $x^3 = 1$, i.e. $x = 1$. The point B lies in the first quadrant (not the origin), so its x-coordinate is $x = 1$, as required.

Q33 Sydney Grammar 2025 Trial Q4 · **1 mark**

How many solutions does the equation $x^2 = \cos x$ have?

(A) 0

(B) 1

(C) 2

(D) 3

Criteria	**Marks**
Correct option (C)	1

Solution.

Sketch $y = x^2$ (an upward parabola through the origin) and $y = \cos x$ (oscillating between -1 and 1) on the same axes. A solution occurs at each intersection. Since $\cos x \leq 1$, any solution needs $x^2 \leq 1$, i.e. $-1 \leq x \leq 1$. On this interval $y = x^2$ rises from 0 up to 1 while $y = \cos x$ falls from 1 down to $\cos 1 \approx 0.54$; by symmetry the two curves cross once for $x > 0$ and once for $x < 0$ (near $x \approx \pm 0.82$). There are therefore 2 solutions, which is option (C).

Q34 Trinity Grammar 2020 Trial Q7 · **1 mark**

The graph of $y = e^x(1 + x)$ is shown below.

Diagram to be inserted:
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How many solutions are there to the equation $e^x(1 + x) = x^3 + 1$?

(A) 0

(B) 1

(C) 2

(D) 3

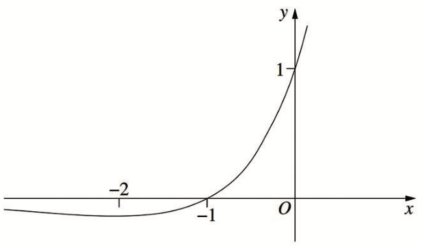
Criteria	Marks
Correct option (C)	1

Solution.

The number of solutions of $e^x(1+x) = x^3 + 1$ is the number of intersections of the given curve $y = e^x(1+x)$ with the cubic $y = x^3 + 1$. Both pass through $(0, 1)$ (at $x = 0$, $e^0(1) = 1$ and $0^3 + 1 = 1$) and through $(-1, 0)$ (at $x = -1$, $e^{-1}(0) = 0$ and $(-1)^3 + 1 = 0$). Checking the gap $g(x) = e^x(1 + x) - (x^3 + 1)$: it is positive for $x < -1$, negative on $-1 < x < 0$, and positive again for $x > 0$, so the curves cross exactly at $x = -1$ and $x = 0$ and nowhere else. There are 2 solutions, which is option (C).

Q35 Knox 2023 Trial Q10 · **1 mark**

The graph shows $y = f(x)$ and $y = g(x)$, where $f(x) = \ln x$ and $g(x) = x^2 - 1$.



NOT TO SCALE

How many solutions does the equation $\left[f(x)\right]^2 - \left[g(x)\right]^2 = 0$ have?

(A) 0

(B) 1

(C) 2

(D) 3

Criteria	Marks
Factorises as $(f-g)(f+g) = 0$ (equivalently $\lvert f\rvert = \lvert g\rvert$)	1

Solution.

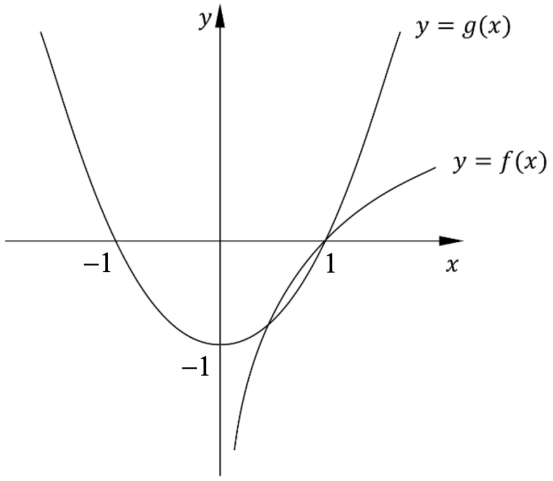
Factor the difference of two squares:

$$\left[f(x)\right]^2 - \left[g(x)\right]^2 = \big(f(x) - g(x)\big)\big(f(x) + g(x)\big) = 0,$$

so a solution occurs where $f(x) = g(x)$ or where $f(x) = -g(x)$ (with $x > 0$, the domain of $\ln x$). From the graph, $y = \ln x$ and $y = x^2 - 1$ meet twice: at $x = 1$ and at one point in $0 < x < 1$ (giving $f = g$). The reflected curve $y = -(x^2 - 1)$ meets $y = \ln x$ only at $x = 1$ (giving $f = -g$). Since $x = 1$ is common to both cases, the distinct solutions are these 2 values, which is option (C).

Q36 James Ruse 2020 Trial Q22 · **2 marks**

The diagram below shows the graph of $y = e^x(x + 1)$.



Using a graphical method, determine the number of solutions to the equation

$$2e^x(1 + x) = 2 - 3x - 2x^2.$$

You may use the diagram above in your solution.

Criteria	Marks
Rearranges to compare the given curve with a parabola, e.g. $e^x(1 + x) = 1 - \frac{3}{2}x - x^2$	1
Reads off the number of intersections: 2 solutions	1

Solution.

Divide the equation by 2 so that the left side matches the plotted curve:

$$e^x(1 + x) = 1 - \tfrac{3}{2}x - x^2.$$

The number of solutions equals the number of intersections of $y = e^x(1 + x)$ (the given curve) with the parabola $y = 1 - \frac{3}{2}x - x^2$. This parabola is concave down, has y-intercept 1, and cuts the x-axis at $x = -2$ and $x = \frac{1}{2}$. Superimposing it on the given diagram, the two curves cross exactly twice: once near $x \approx -2$ and once at $x = 0$ (where $e^0(1) = 1$ and $1 - 0 - 0 = 1$). There are 2 solutions.