Year 11 Mathematics Advanced

Topic: Working with Functions (1 of 12)

Review Questions (Worked Solutions): Algebraic Techniques

Term 3, 2026

Indices, expanding and factorising

1. Index laws (positive, negative, zero and fractional indices)

Q1 Knox 2021 Trial Q1 · 1 mark

Simplify $\dfrac{(2p)^3}{\sqrt{p}}$.

(A) $8\sqrt{p^5}$

(B) $8\sqrt[5]{p^2}$

(C) $6\sqrt{p^5}$

(D) $6\sqrt[5]{p^2}$

Criteria	Marks
Correct option (A)	1

Solution.

Cube the bracket and write the surd as a fractional index:

$$\frac{(2p)^3}{\sqrt{p}} = \frac{8p^3}{p^{1/2}} = 8p^{3-\frac{1}{2}} = 8p^{5/2} = 8\sqrt{p^5}.$$

The answer is (A).

Q2 Killara 2023 Prelim Q11 · **2 marks**

Simplify $(q^2)^3 \times \left(\dfrac{q^5}{q}\right)^2$ using the index laws.

Criteria	Marks
Simplifies each factor: $(q^2)^3 = q^6$ and $\left(q^5/q\right)^2 = q^8$	1
Correct final answer q^{14}	1

Solution.

Apply the power and quotient laws to each factor, then multiply:

$$(q^2)^3 = q^6, \qquad \left(\frac{q^5}{q}\right)^2 = \left(q^4\right)^2 = q^8.$$

$$(q^2)^3 \times \left(\frac{q^5}{q}\right)^2 = q^6 \times q^8 = q^{14}.$$

Q3 Knox 2023 Prelim Q1 · **1 mark**

Which expression is equivalent to $\sqrt[6]{64x^{12}y^3}$?

(A) $2x^{72}y^{36}$

(B) $2x^6\sqrt{y}$

(C) $4x^2y^{\frac{3}{2}}$

(D) $2x^2\sqrt{y}$

Criteria	Marks
Correct option (D)	1

Solution.

Write the sixth root as a power $\frac{1}{6}$ and apply it to each factor:

$$\sqrt[6]{64x^{12}y^3} = \left(64x^{12}y^3\right)^{1/6} = 64^{1/6}\,x^{12/6}\,y^{3/6} = 2x^2y^{1/2} = 2x^2\sqrt{y}.$$

The answer is (D).

Q4 Girraween 2021 Prelim Q1(a)(i) · **2 marks**

Simplify $\left(\dfrac{64x^{-3}}{y^6}\right)^{\frac{1}{3}}$.

Criteria	**Marks**
Applies the index $\frac{1}{3}$ to each factor ($64^{1/3}=4$, x^{-1}, y^{-2})	1
Correct final answer $\dfrac{4}{xy^2}$	1

Solution.

Distribute the power $\frac{1}{3}$ over the numerator and denominator:

$$\left(\frac{64x^{-3}}{y^6}\right)^{\frac{1}{3}} = \frac{64^{1/3}\,x^{-3/3}}{y^{6/3}} = \frac{4x^{-1}}{y^2} = \frac{4}{xy^2}.$$

Q5 North Sydney Boys 2024 Prelim Q7 · **2 marks**

Simplify $\dfrac{12^x \times 18^x}{3^x \times 2^x}$.

Criteria	**Marks**
Combines like bases: numerator 216^x and denominator 6^x (or equivalent)	1
Correct final answer 36^x	1

Solution.

Group each product into a single base using $a^x b^x = (ab)^x$:

$$\frac{12^x \times 18^x}{3^x \times 2^x} = \frac{(12\times 18)^x}{(3\times 2)^x} = \frac{216^x}{6^x} = \left(\frac{216}{6}\right)^x = 36^x.$$

Q6 Girraween 2022 Prelim Q11(a) · **3 marks**

Simplify $\left(\dfrac{12x^{-3}y^2}{xyz^{\frac{1}{2}}}\right)^2 \div \dfrac{6x^2y^3}{z^{-4}}$.

Criteria	**Marks**
Simplifies inside the bracket to $12x^{-4}yz^{-1/2}$ (or equivalent)	1
Squares the bracket to $144x^{-8}y^2z^{-1}$ and writes the divisor as $6x^2y^3z^4$	1
Correct final answer $\dfrac{24}{x^{10}yz^5}$	1

Solution.

First simplify inside the bracket with the quotient law:

$$\frac{12x^{-3}y^2}{xyz^{\frac{1}{2}}} = 12x^{-3-1}y^{2-1}z^{-\frac{1}{2}} = 12x^{-4}yz^{-\frac{1}{2}}.$$

Square it:

$$\left(12x^{-4}yz^{-\frac{1}{2}}\right)^2 = 144x^{-8}y^2z^{-1}.$$

Rewrite the divisor with a positive power of z and divide (multiply by its reciprocal):

$$\frac{6x^2y^3}{z^{-4}} = 6x^2y^3z^4, \qquad \frac{144x^{-8}y^2z^{-1}}{6x^2y^3z^4} = 24\,x^{-8-2}\,y^{2-3}\,z^{-1-4} = 24x^{-10}y^{-1}z^{-5} = \frac{24}{x^{10}yz^5}.$$

2. Expand and factorise algebraic expressions

Q7 Killara 2023 Prelim Q1 · **1 mark**

Factorise the expression $x^2 - 2x - 3$.

(A) $(x-3)(x-1)$

(B) $(x-3)(x+1)$

(C) $(x+3)(x+1)$

(D) $(x+3)(x-1)$

Criteria	Marks
Correct option (B)	1

Solution.

Seek two numbers with product -3 and sum -2: these are -3 and $+1$. Hence

$$x^2 - 2x - 3 = (x-3)(x+1),$$

which is option (B).

Q8 Ascham 2022 Trial Q11 · **1 mark**

Factorise $4x^2 - (w+y)^2$.

Criteria	Marks
Correct factorisation	1

Solution.

This is a difference of two squares, $A^2 - B^2 = (A-B)(A+B)$, with $A = 2x$ and $B = w + y$:

$$4x^2 - (w+y)^2 = \big(2x - (w+y)\big)\big(2x + (w+y)\big) = (2x - w - y)(2x + w + y).$$

Q9 Sydney Boys 2020 Trial Q11 · **1 mark**

Factorise $8x^3 + 125$.

Criteria	Marks
Correct factorisation	1

Solution.

This is a sum of two cubes, $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$, with $a = 2x$ and $b = 5$:

$$8x^3 + 125 = (2x)^3 + 5^3 = (2x + 5)\big(4x^2 - 10x + 25\big).$$

Q10 Baulkham Hills 2024 Prelim Q1 · **1 mark**

Which of the following is equivalent to $5x - (x + 3)(2x - 1)$?

(A) $3 - 2x^2$

(B) $-2x^2 + 10x + 3$

(C) $2x^2 - 3$

(D) $2x^2 - 10x - 3$

Criteria	Marks
Correct option (A)	1

Solution.

Expand the product, then subtract:

$$(x + 3)(2x - 1) = 2x^2 + 5x - 3,$$

$$5x - (2x^2 + 5x - 3) = 5x - 2x^2 - 5x + 3 = 3 - 2x^2,$$

which is option (A).

Q11 James Ruse 2022 Trial Q12 · **1 mark**

Factorise $2x^2 + 3x - 2xy - 3y$.

Criteria	Marks
Correct factorisation	1

Solution.

Group the terms in pairs and take out a common factor from each pair:

$$2x^2 + 3x - 2xy - 3y = x(2x + 3) - y(2x + 3) = (2x + 3)(x - y).$$

Q12 Baulkham Hills 2024 Prelim Q12(a) · **2 marks**

Fully factorise $4x^3 - x^5$.

Criteria	Marks
Takes out the common factor x^3: $x^3(4 - x^2)$	1
Completes with the difference of two squares	1

Solution.

Take out the highest common factor x^3, then factorise the remaining difference of two squares:

$$4x^3 - x^5 = x^3(4 - x^2) = x^3(2 - x)(2 + x).$$

3. Simplify algebraic fractions

Q13 Kambala 2024 Prelim Q4 · **1 mark**

Which of the following fractions is equivalent to $\frac{3x-1}{2} - \frac{x-3}{4}$?

(A) $\frac{5x-5}{4}$

(B) $\frac{5x+1}{4}$

(C) $\frac{10x-10}{4}$

(D) $\frac{10x+2}{4}$

Criteria	Marks
Correct option (B)	1

Solution.

Put over the common denominator 4:

$$\frac{3x-1}{2} - \frac{x-3}{4} = \frac{2(3x-1)}{4} - \frac{x-3}{4} = \frac{6x-2-(x-3)}{4} = \frac{5x+1}{4}.$$

The answer is (B).

Q14 Sydney Grammar 2021 Prelim Q9(d) · **2 marks**

Simplify $\frac{y^2+3y}{y^2-9}$.

Criteria	Marks
Factorises numerator and denominator	1
Cancels the common factor for $\frac{y}{y-3}$	1

Solution.

Factorise top and bottom, then cancel the common factor $(y+3)$:

$$\frac{y^2+3y}{y^2-9} = \frac{y(y+3)}{(y-3)(y+3)} = \frac{y}{y-3}.$$

Q15 Baulkham Hills 2024 Trial Q11 · **2 marks**

Simplify $\dfrac{2}{x-1} - \dfrac{6}{(x-1)(2x+1)}$.

Criteria	Marks
Common denominator $(x-1)(2x+1)$ with numerators combined	1
Simplifies to $\dfrac{4}{2x+1}$	1

Solution.

The common denominator is $(x-1)(2x+1)$:

$$\frac{2}{x-1} - \frac{6}{(x-1)(2x+1)} = \frac{2(2x+1)-6}{(x-1)(2x+1)} = \frac{4x-4}{(x-1)(2x+1)} = \frac{4(x-1)}{(x-1)(2x+1)} = \frac{4}{2x+1}.$$

Q16 Killara 2024 Prelim Q11 · **2 marks**

Fully simplify the following expression.

$$\frac{a^2-3a}{2ax} \times \frac{4a^2x}{2ax-6x}$$

Criteria	Marks
Factorises each numerator/denominator	1
Cancels common factors for $\dfrac{a^2}{x}$	1

Solution.

Factorise, then cancel:

$$\frac{a^2-3a}{2ax} \times \frac{4a^2x}{2ax-6x} = \frac{a(a-3)}{2ax} \times \frac{4a^2x}{2x(a-3)} = \frac{a(a-3)\cdot 4a^2x}{2ax\cdot 2x(a-3)} = \frac{4a^3x}{4ax^2} = \frac{a^2}{x}.$$

Q17 Abbotsleigh 2024 Prelim Q13 · **2 marks**

Simplify fully $\dfrac{5}{x^2-3x+2}-\dfrac{3}{x-1}$.

Criteria	**Marks**
Factorises $x^2-3x+2=(x-1)(x-2)$ and forms common denominator	1
Simplifies to $\dfrac{11-3x}{(x-1)(x-2)}$	1

Solution.

Since $x^2-3x+2=(x-1)(x-2)$, the common denominator is $(x-1)(x-2)$:

$$\frac{5}{(x-1)(x-2)}-\frac{3}{x-1}=\frac{5-3(x-2)}{(x-1)(x-2)}=\frac{5-3x+6}{(x-1)(x-2)}=\frac{11-3x}{(x-1)(x-2)}.$$

Q18 Girraween 2023 Prelim Q6(c) · **3 marks**

Factorise the denominators and simplify $\dfrac{3}{x^2-5x+6}-\dfrac{2}{x^2+2x-8}$.

Criteria	**Marks**
Factorises both denominators	1
Correct common denominator and numerator expansion	1
Correct simplified answer	1

Solution.

$x^2-5x+6=(x-2)(x-3)$ and $x^2+2x-8=(x-2)(x+4)$. Common denominator $(x-2)(x-3)(x+4)$:

$$\frac{3(x+4)-2(x-3)}{(x-2)(x-3)(x+4)}=\frac{3x+12-2x+6}{(x-2)(x-3)(x+4)}=\frac{x+18}{(x-2)(x-3)(x+4)}.$$

Surds

4. Expand and simplify surds (exact values)

Q19 Sydney Grammar 2022 Prelim Q9(c) · 1 mark

Simplify $\sqrt{8} - \sqrt{2}$.

Criteria	Marks
Correct simplification $\sqrt{2}$	1

Solution.

$$\sqrt{8} - \sqrt{2} = 2\sqrt{2} - \sqrt{2} = \sqrt{2}.$$

Q20 Barker 2021 Prelim Q2 · 1 mark

Which of the following is the expansion of $\left(5 - 2\sqrt{3}\right)^2$?

(A) -11

(B) $25 - 4\sqrt{3}$

(C) 37

(D) $37 - 20\sqrt{3}$

Criteria	Marks
Correct option (D)	1

Solution.

Expand using $(a - b)^2 = a^2 - 2ab + b^2$:

$$\left(5 - 2\sqrt{3}\right)^2 = 25 - 2(5)(2\sqrt{3}) + \left(2\sqrt{3}\right)^2 = 25 - 20\sqrt{3} + 12 = 37 - 20\sqrt{3}.$$

The answer is (D).

Q21 St George Girls 2023 Prelim Q7 · **2 marks**

If $11\sqrt{3} - 2\sqrt{27} = p\sqrt{3}$, find the value of p.

Criteria	Marks
Simplifies $\sqrt{27} = 3\sqrt{3}$	1
Concludes $p = 5$	1

Solution.

Simplify the second surd: $\sqrt{27} = \sqrt{9 \times 3} = 3\sqrt{3}$. Then

$$11\sqrt{3} - 2\sqrt{27} = 11\sqrt{3} - 2(3\sqrt{3}) = 11\sqrt{3} - 6\sqrt{3} = 5\sqrt{3}.$$

Hence $p\sqrt{3} = 5\sqrt{3}$, so $p = 5$.

Q22 Sydney Grammar 2023 Prelim Q10(c) · **2 marks**

Find the values of p and q if $\left(2\sqrt{3} - 1\right)\left(5 - 3\sqrt{3}\right) = p + q\sqrt{3}$.

Criteria	Marks
Correctly expands the product	1
Equates rational and surd parts: $p = -23$, $q = 13$	1

Solution.

Expand the product term by term:

$$\left(2\sqrt{3} - 1\right)\left(5 - 3\sqrt{3}\right) = 10\sqrt{3} - 2(3)(3) - 5 + 3\sqrt{3} = 10\sqrt{3} - 18 - 5 + 3\sqrt{3}.$$

Collecting like terms gives $-23 + 13\sqrt{3}$. Comparing with $p + q\sqrt{3}$,

$$p = -23, \qquad q = 13.$$

Q23 Hurlstone 2024 Prelim Q12 · **2 marks**

A rectangle has an area of 12 cm^2 and length of $\left(2+\sqrt{7}\right)$ cm. Find its width in the form $a+b\sqrt{7}$, where a and b are integers.

Criteria	Marks
Sets up $w=\dfrac{12}{2+\sqrt{7}}$ and rationalises the denominator	1
Simplifies to $w=-8+4\sqrt{7}$	1

Solution.

From area = length × width, $12=\left(2+\sqrt{7}\right)w$, so $w=\dfrac{12}{2+\sqrt{7}}$. Rationalise by multiplying by the conjugate:

$$w=\frac{12}{2+\sqrt{7}}\times\frac{2-\sqrt{7}}{2-\sqrt{7}}=\frac{12\left(2-\sqrt{7}\right)}{4-7}=\frac{12\left(2-\sqrt{7}\right)}{-3}=-4\left(2-\sqrt{7}\right)=-8+4\sqrt{7}.$$

So $a=-8$ and $b=4$, giving width $=-8+4\sqrt{7}$ cm.

Q24 James Ruse 2020 Trial Q12 · **2 marks**

For x and y rational numbers, $\left(\sqrt{x}+\sqrt{y}\right)^2=x+y+2\sqrt{xy}$. Write $\sqrt{16+2\sqrt{55}}$ in the form $\sqrt{x}+\sqrt{y}$.

Criteria	Marks
Sets up $x+y=16$ and $xy=55$	1
Solves to give $\sqrt{16+2\sqrt{55}}=\sqrt{11}+\sqrt{5}$	1

Solution.

Using the given identity, $\left(\sqrt{x}+\sqrt{y}\right)^2=x+y+2\sqrt{xy}$, so matching $16+2\sqrt{55}$ requires

$$x+y=16,\qquad xy=55.$$

The two numbers with sum 16 and product 55 are 11 and 5. Therefore

$$\sqrt{16+2\sqrt{55}}=\sqrt{11}+\sqrt{5}.$$

5. Rationalise the denominator (conjugates)

Q25 Killara 2023 Prelim Q12(a) · **1 mark**

Rationalise the denominator of $\dfrac{6}{\sqrt{3}}$, leaving your answer in simplest form.

Criteria	Marks
Correct simplest form $2\sqrt{3}$	1

Solution.

Multiply numerator and denominator by $\sqrt{3}$:

$$\frac{6}{\sqrt{3}} = \frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}.$$

Q26 Sydney Grammar 2021 Prelim Q9(b) · **1 mark**

Rationalise the denominator of $\dfrac{3}{\sqrt{5}-2}$.

Criteria	Marks
Correct rationalised form $3\sqrt{5}+6$ (or equivalent)	1

Solution.

Multiply top and bottom by the conjugate $\sqrt{5}+2$:

$$\frac{3}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2} = \frac{3(\sqrt{5}+2)}{(\sqrt{5})^2-2^2} = \frac{3(\sqrt{5}+2)}{5-4} = 3\sqrt{5}+6.$$

Q27 Moriah 2021 Prelim Q11(b) · **2 marks**

If $x = \dfrac{\sqrt{5}-2}{\sqrt{5}}$, express x with a rational denominator.

Criteria	**Marks**
Multiplies numerator and denominator by $\sqrt{5}$	1
Correct simplified form $\dfrac{5-2\sqrt{5}}{5}$	1

Solution.

Multiply numerator and denominator by $\sqrt{5}$:

$$x = \frac{\sqrt{5}-2}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}\cdot\sqrt{5}-2\sqrt{5}}{5} = \frac{5-2\sqrt{5}}{5}.$$

Equivalently, $x = 1 - \dfrac{2\sqrt{5}}{5}$.

Q28 Fort St 2021 Prelim Q1(a) · **2 marks**

Rationalise the denominator of $\dfrac{4+\sqrt{2}}{4-\sqrt{2}}$, simplifying your answer.

Criteria	**Marks**
Multiplies by the conjugate $\dfrac{4+\sqrt{2}}{4+\sqrt{2}}$	1
Correct simplified answer $\dfrac{9+4\sqrt{2}}{7}$	1

Solution.

Multiply top and bottom by the conjugate $4+\sqrt{2}$:

$$\frac{4+\sqrt{2}}{4-\sqrt{2}} \times \frac{4+\sqrt{2}}{4+\sqrt{2}} = \frac{(4+\sqrt{2})^2}{4^2-(\sqrt{2})^2} = \frac{16+8\sqrt{2}+2}{16-2} = \frac{18+8\sqrt{2}}{14} = \frac{9+4\sqrt{2}}{7}.$$

Q29 Sydney Grammar 2022 Prelim Q10(d) · **2 marks**

By rationalising the denominator or otherwise, show that $\dfrac{2}{5-\sqrt{2}}+\dfrac{2}{5+\sqrt{2}}$ is rational.

Criteria	**Marks**
Combines over the common denominator $(5-\sqrt{2})(5+\sqrt{2})=23$	1
Simplifies to the rational number $\dfrac{20}{23}$	1

Solution.

Combine over the common denominator $(5-\sqrt{2})(5+\sqrt{2})=25-2=23$:

$$\frac{2}{5-\sqrt{2}}+\frac{2}{5+\sqrt{2}}=\frac{2(5+\sqrt{2})+2(5-\sqrt{2})}{(5-\sqrt{2})(5+\sqrt{2})}=\frac{20}{23}.$$

Since $\dfrac{20}{23}$ is a ratio of two integers, the expression is rational.